Solving Shape-Analysis Problems in Languages with Destructive Updating

#SAV Presentation

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Motivation

Analyse the shape of heap allocated data structures.

Verify that a program preserves shape properties such as:

- *list-ness*
- *circular list-ness*
- *tree-ness*

This analysis algorithm can be used to find aliases, and therefore to optimise code (no alias means it can be more easily parallelised).

List Reversal – Normalisation

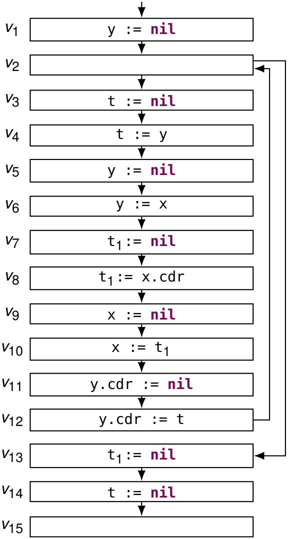
```
// x points to an unshared list
y := nil
while x ≠ nil do

  t := y

  y := x




  x := x.cdr

  y.cdr := t
od

t := nil
```

List Reversal – Normalisation

```
// x points to an unshared list
y := nil
while x ≠ nil do
  t := nil
  t := y
  y := nil
  y := x
  t1 := nil
  t1 := x.cdr
  x := nil
  x := t1
  y.cdr := nil
  y.cdr := t
od
t1 := nil
t := nil
```

List Reversal – Normalisation

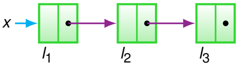
```
// x points to an unshared list
y := nil
while x ≠ nil do
  t := nil
  t := y
  y := nil
  y := x
  t₁ := nil
  t₁ := x.cdr
  x := nil
  x := t₁
  y.cdr := nil
  y.cdr := t
od
t₁ := nil
t := nil
```



Shape-Graph $SG = \langle E_v, E_s \rangle$

What is a **shape-graph**?

- directed graph with nodes and edges
- nodes are called **shape-nodes**
  - runtime locations, i.e. heap memory, or *cons-cells*
  - implicitly defined by edges and $shape\_nodes(SG)$
- edges are divided into 2 categories:
  - E_v: **variable-edges** of the form $[x, n]$
  - E_s: **selector-edges** of the form $\langle s, sel, t \rangle$



DSG: Deterministic Shape-Graph

A shape-graph is deterministic if

1. no variable points to more than one node
2. no node has a selector pointing to more than one node

In other words:

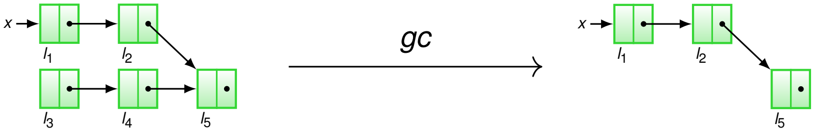
It is deterministic when edges are behaving like pointers.

gc: Garbage Collection

The *gc* function removes runtime location that are not reachable from any program variable:

$gc : SG \rightarrow SG$

$gc\,(\langle E_v, E_s \rangle) \stackrel{\text{def}}{=} \langle E_v, E'_s \rangle$ where $E'_s \subseteq E_s$ and $\langle s, sel, t \rangle \in E'_s$ if and only if there exists $[x, r] \in E_v$ such that there is a path of selector-edges in E_s from r to s.



DSG Transformers

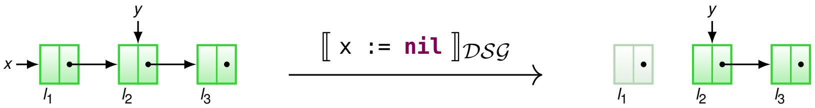
A program statement is represented as a transform function on *DSG*, denoted $[\![\ \texttt{st}\ ]\!]_{\mathcal{DSG}}$.

Thanks to program normalisation, there are only 6 statements and they are simple:

1. $[\![\ \texttt{x := } \textbf{nil}\ ]\!]_{\mathcal{DSG}}$
2. $[\![\ \texttt{x.sel := } \textbf{nil}\ ]\!]_{\mathcal{DSG}}$
3. $[\![\ \texttt{x := } \textbf{new}\ ]\!]_{\mathcal{DSG}}$
4. $[\![\ \texttt{x := y}\ ]\!]_{\mathcal{DSG}}$
5. $[\![\ \texttt{x := y.sel}\ ]\!]_{\mathcal{DSG}}$
6. $[\![\ \texttt{x.sel := y}\ ]\!]_{\mathcal{DSG}}$

DSG Transformers: 1

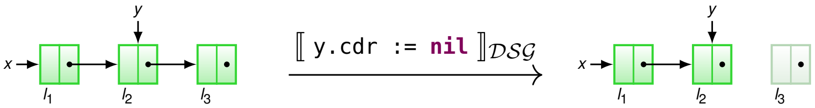
$$[\![\ \texttt{x := nil} \ ]\!]_{\mathcal{DSG}} (\langle E_v, E_s \rangle)$$
$$\stackrel{\text{def}}{=} \langle E_v - \{[\mathrm{x}, *]\}, E_s \rangle$$



NB: *gc* can do some cleaning.

DSG Transformers: 2

$$[\![\ \texttt{x.sel := }\textbf{nil}\ ]\!]_{\mathcal{DSG}}(\langle E_v, E_s\rangle) \stackrel{\text{def}}{=} \langle E_v, E_s - \{\langle s, \texttt{sel}, *\rangle \mid [\mathrm{x}, s] \in E_v\}\rangle$$



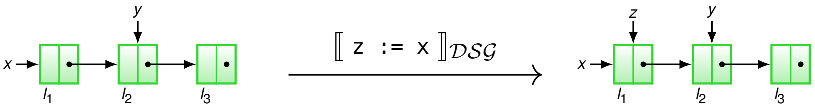
NB: *gc* can do some cleaning.

DSG Transformers: 3

$$
\llbracket\ \texttt{x := new}\ \rrbracket_{\mathcal{DSG}} \left(\langle E_v, E_s \rangle\right) \stackrel{\text{def}}{=} \langle E_v \cup \{[\mathrm{x}, \mathrm{n}_{new}]\}\ ,\ E_s \rangle
$$

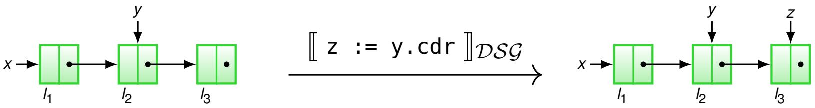
DSG Transformers: 4

$$
\llbracket \texttt{x := y} \rrbracket_{\mathcal{DSG}} (\langle E_v, E_s \rangle) \stackrel{\text{def}}{=} \langle E_v \cup \{[\mathrm{x}, \mathrm{n}] \mid [\mathrm{y}, \mathrm{n}] \in E_v\}, E_s \rangle
$$



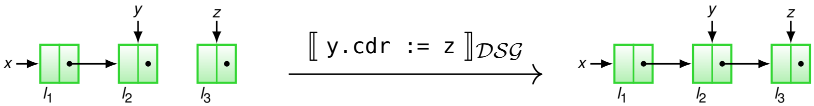
DSG Transformers: 5

$$\llbracket\ \texttt{x := y.sel}\ \rrbracket_{\mathcal{DSG}}\left(\langle E_v, E_s\rangle\right)$$
$$\stackrel{\text{def}}{=} \langle E_v \cup \{[\mathrm{x},\mathrm{t}] \mid [\mathrm{y},\mathrm{s}] \in E_v \wedge \langle \mathrm{s},\mathrm{sel},\mathrm{t}\rangle \in E_s\}\ , E_s\rangle$$



DSG Transformers: 6

$$\llbracket\ \texttt{x.sel := y}\ \rrbracket_{\mathcal{DSG}}\left(\langle E_v, E_s\rangle\right) \stackrel{\text{def}}{=} \langle E_v, E_s \cup \{\langle \mathrm{s}, \mathrm{sel}, \mathrm{t}\rangle \mid [\mathrm{x}, \mathrm{s}] \in E_v \wedge [\mathrm{y}, \mathrm{t}] \in E_v\}\rangle$$



Collecting Semantics

Each DSG models one runtime behaviour (of many).

We need a tool to perform analysis on **all** possible execution paths, not just one.

Hence the collecting function $cv : V \rightarrow 2^{DSG}$

$$cv(v) \stackrel{\text{def}}{=} \{ [\![\texttt{st}(v_k)]\!]_{\mathcal{DSG}} (\cdots ([\![\texttt{st}(v_1)]\!]_{\mathcal{DSG}} (\langle \varnothing, \varnothing \rangle))) \mid [v_1, \ldots, v_k] \in pathsTo(v) \}$$

V is the vertex set of the *regular* control flow graph $G = \langle V, A \rangle$.

Abstraction, Abstractly

DSG

Abstraction, Abstractly

$\subseteq \quad \cup$

$\{DSG\}$

Abstraction, Abstractly

$\subseteq \quad \cup$

$\sqsubseteq \quad \sqcup$

$$\{DSG\} \xrightarrow{\alpha} SSG$$

Abstraction, Abstractly

$\subseteq \quad \cup$

$\sqsubseteq \quad \sqcup$

$$\{DSG\} \xrightarrow{\quad\alpha\quad} SSG$$

Static shape graphs: $SSG = \langle SG, is\_shared \rangle$

- SG is a shape graph
- $is\_shared : shape\_nodes(SG) \rightarrow \{T, F\}$
  is a predicate identifying nodes that were shared in the DSG

Abstraction – Helpers

- Grouping nodes by variable labels

$$\alpha_s[DSG] \colon \mathit{shape\_nodes}(DSG) \to \{n_X \mid X \subseteq \mathit{PVar}\}$$

$$\alpha_s[DSG](r) \stackrel{\text{def}}{=} n_{\{x \in \mathit{PVar} \mid [x,r] \in E_v\}}$$

- Initialisation of the sharing predicate

$$\mathit{induced\_is\_shared}[DSG] \colon \mathit{shape\_nodes}(DSG) \to \{T, F\}$$

$$\mathit{induced\_is\_shared}[DSG](t) \stackrel{\text{def}}{=} |\{\langle *, *, t\rangle \in E_s\}| \leq 2$$

- Projection (a.k.a. quotienting) of SGs with respect to f

$$\langle SG, p\rangle \downarrow f$$

Abstraction

- Projection/quotient of a single DSG

$$\hat{\alpha} : \mathcal{DSG} \to \mathcal{SSG}$$

$$\hat{\alpha}(DSG) \stackrel{\text{def}}{=} \langle DSG', induced\_is\_shared[DSG'] \rangle \downarrow \alpha_s[DSG']$$
$$\text{where } DSG' = gc(DSG)$$

- Abstraction function

$$\alpha : 2^{\mathcal{DSG}} \to \mathcal{SSG}$$

$$\alpha(S) \stackrel{\text{def}}{=} \bigsqcup_{DSG \in S} \hat{\alpha}(DSG)$$

Abstract Interpretation, Abstractly

DSG_1

Abstract Interpretation, Abstractly

$$\begin{array}{c} DSG_1 \\ \Big\downarrow{\scriptstyle [\![s]\!]_{\mathcal{DSG}}} \\ DSG_2 \end{array}$$

Abstract Interpretation, Abstractly

$\subseteq \quad \cup$

$$\begin{array}{c}\{DSG_1\} \\ \Big\downarrow{\scriptstyle \{[\![s]\!]_{\mathcal{DSG}}\}} \\ \{DSG_2\}\end{array}$$

Abstract Interpretation, Abstractly

$\subseteq \quad \cup$ $\qquad\qquad$ $\sqsubseteq \quad \sqcup$

$$
\begin{tikzcd}
\{ DSG_1 \} \arrow[rr, "\alpha"] \arrow[d, "{\{ [\![s]\!]_{\mathcal{DSG}} \}}"'] & & SSG_1 \\
\{ DSG_2 \} & &
\end{tikzcd}
$$

Abstract Interpretation, Abstractly

$\subseteq \quad \cup$ $\qquad\qquad$ $\sqsubseteq \quad \sqcup$

$$
\begin{tikzcd}
\{DSG_1\} \arrow[rr, "\alpha"] \arrow[d, "{\{[\![s]\!]_{\mathcal{DSG}}\}}"'] & & SSG_1 \\
\{DSG_2\} \arrow[r, "\alpha"] & SSG_2 &
\end{tikzcd}
$$

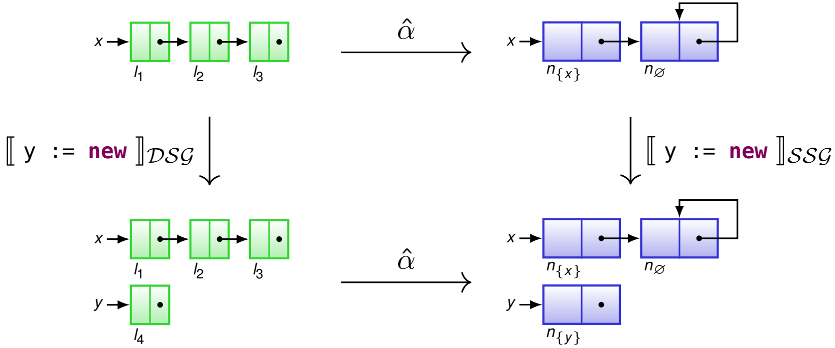
Abstract Interpretation, Abstractly

$$\boxed{\subseteq \quad \cup} \qquad\qquad\qquad \boxed{\sqsubseteq \quad \sqcup}$$

$$\begin{tikzcd}
\{DSG_1\} \arrow[r, "\alpha"] \arrow[d, "{\{[\![s]\!]_{\mathcal{DSG}}\}}"'] & SSG_1 \arrow[d, "{[\![s]\!]_{\mathcal{SSG}}}"] \\
\{DSG_2\} \arrow[r, "\alpha"] & SSG_2 \sqsubseteq SSG_2'
\end{tikzcd}$$

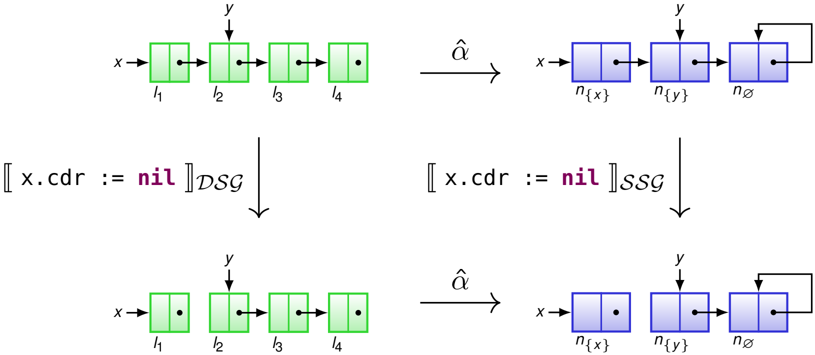
Abstract Interpretation – Examples

Allocating a new node.



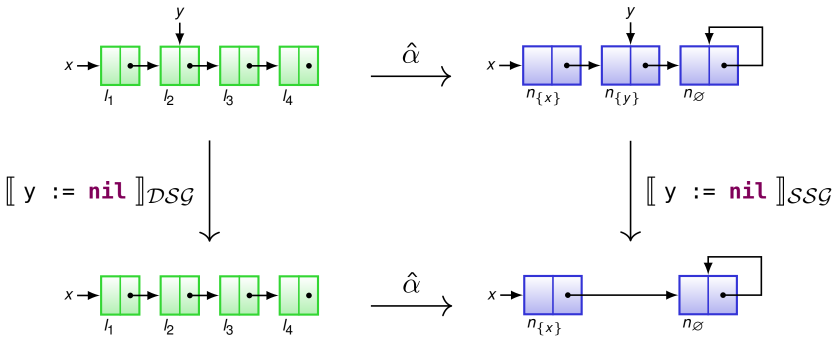
Abstract Interpretation – Examples (cont.)

Assigning **nil** to a field.



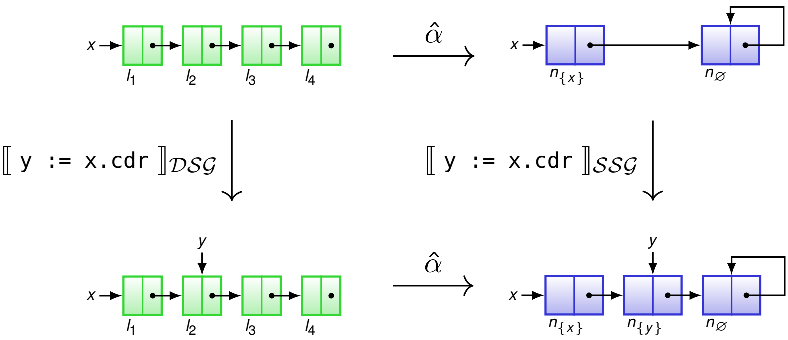
Abstract Interpretation – Examples (cont.)

Assigning **nil** to a variable.



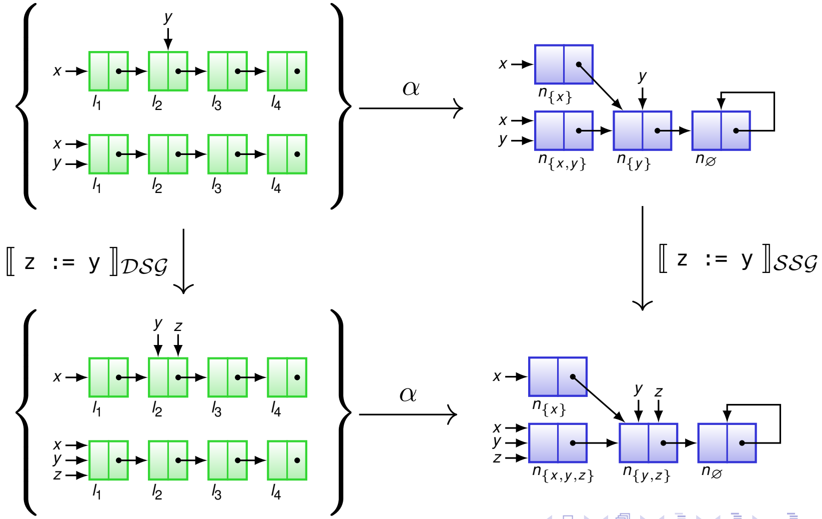
Abstract Interpretation – Examples (cont.)

Materialising a node n_y from the summary node $n_\varnothing$.



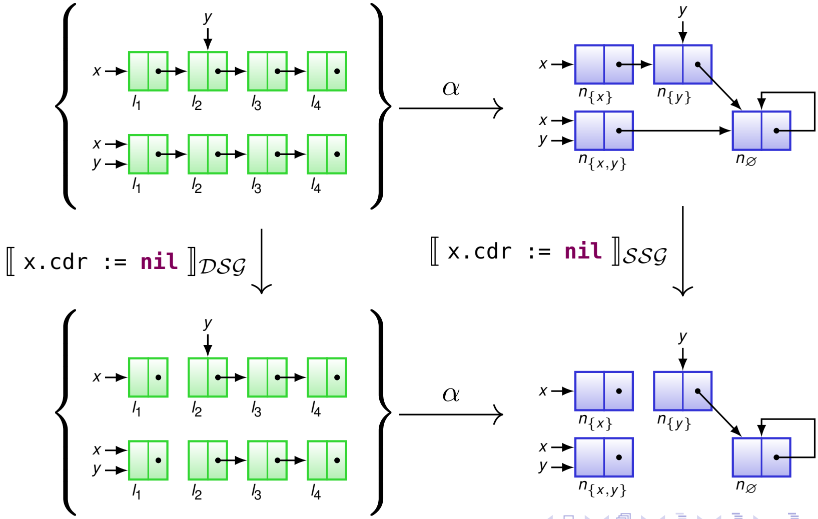
Abstract Interpretation – Examples (cont.)

Variable assignment.



Abstract Interpretation – Examples (cont.)

Strong nullification.



List Insertion – Normalisation

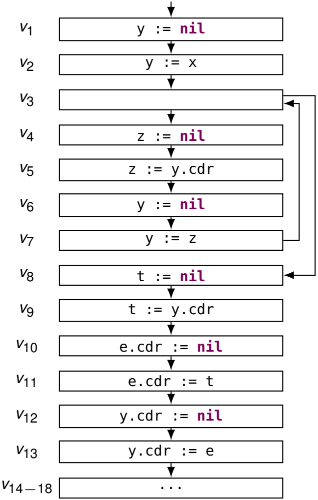
```
// x is an unshared list
// e the element to insert

y := x
while y.cdr ≠ nil ∧... do

  z := y.cdr

  y := z
od

t := y.cdr

e.cdr := t

y.cdr := e
t := nil
z := nil
e := nil
y := nil
```

List Insertion – Normalisation

```
// x is an unshared list
// e the element to insert
y := nil
y := x
while y.cdr ≠ nil ∧... do
  z := nil
  z := y.cdr
  y := nil
  y := z
od
t := nil
t := y.cdr
e.cdr := nil
e.cdr := t
y.cdr := nil
y.cdr := e
t := nil
z := nil
e := nil
y := nil
```

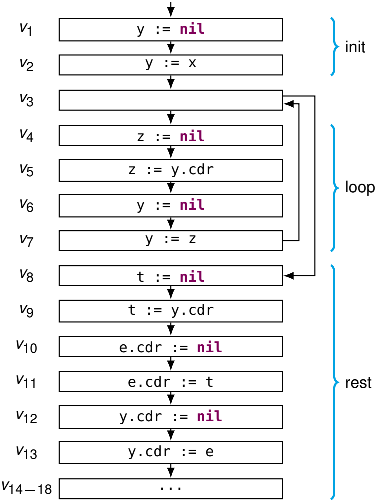
List Insertion – Normalisation

```
// x is an unshared list
// e the element to insert
y := nil
y := x
while y.cdr ≠ nil ∧... do
  z := nil
  z := y.cdr
  y := nil
  y := z
od
t := nil
t := y.cdr
e.cdr := nil
e.cdr := t
y.cdr := nil
y.cdr := e
t := nil
z := nil
e := nil
y := nil
```



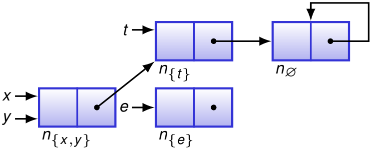
List Insertion – Normalisation

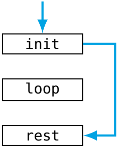
```
// x is an unshared list
// e the element to insert
y := nil
y := x
while y.cdr ≠ nil ∧... do
  z := nil
  z := y.cdr
  y := nil
  y := z
od
t := nil
t := y.cdr
e.cdr := nil
e.cdr := t
y.cdr := nil
y.cdr := e
t := nil
z := nil
e := nil
y := nil
```



Sharing

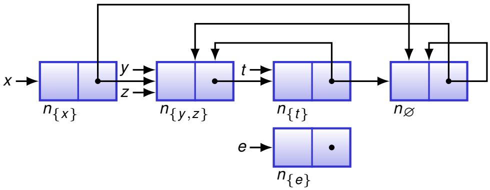
From v_1 to v_{11} *without entering* the loop.
Executing $[\![$ `e.cdr := nil` $]\!]$ – $n_{\{t\}}$ is **not** shared.

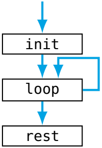




Sharing

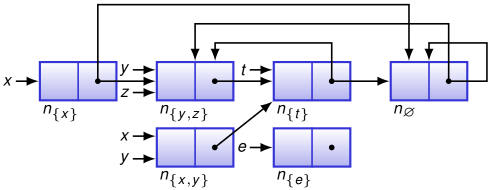
From v_1 to v_{11} *through* the loop.
Executing $[\![$ `e.cdr := nil` $]\!]$ – $n_{\{t\}}$ is **not** shared.

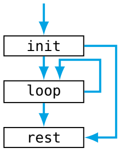




Sharing

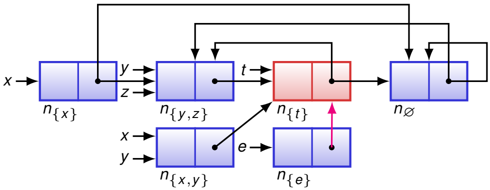
From v_1 to v_{11} by all possible paths.
Executing $[\![$ `e.cdr := nil` $]\!]$ – $n_{\{t\}}$ is still **not** shared.

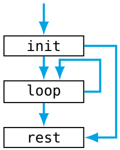




Sharing

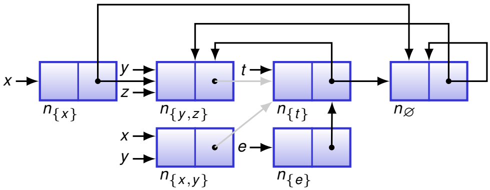
From v_1 to v_{12} by all possible paths.
Executing $[\![$ `e.cdr := t` $]\!]$ – $n_{\{t\}}$ **is** shared.

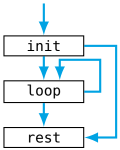




Sharing

From v_1 to v_{13} by all possible paths.
Executing $[\![$ `y.cdr := nil` $]\!]$ – **strong nullification.**





Extensions

Merging Shape-Nodes to avoid a huge number of nodes ($\leq 2^{|PVar|}$), a widening operator can be introduced.

Finding Aliases and Sharing testing whether x and y are aliases at some point of the program can be extended to test whether two paths can alias by introducing two extra variables.

Interprocedural Analysis *shape-graph-transformations* can be introduced to accurately model procedures.

Representing Definitely Circular Structures with extra special nodes ($n_{atom}, n_{nil}, n_{uninit}$), definitely cyclic data structures can be modelled.

Thank you!

Questions?