CS & IT ENGINEERING

DISCRETE MATHS
COMBINATORICS

Lecture No. 04

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TOPICS

01 Binomial coefficient

02 Extended Binomial coefficient

3 Exercise

Binomial coefficient:

no. of ways to take a^2 out

$$(a+b)^2 = 1 \cdot a^2 + 2ab + 1 \cdot b^2$$

$$(a+b) \times (a+b)$$

$$\boxed{a+b} \times \boxed{a+b}$$

$a \cdot a$

ab

$$\boxed{a+b} \times \boxed{a+b}$$

ab

$\rightarrow 3a^2b^1$

$$(a+b)^3 = 1\cdot a^3 + 3a^2b + 3ab^2 + b^3$$

$\{\boxed{a}, \boxed{b}, \boxed{c}\}$

$^3C_1$

a^2b

$$\boxed{a+b} \times \boxed{a+b} \times \boxed{a+b}$$

a^2b.

a^2b

$3a^2b^1$

3 ways to take a^2 out

OR.

3 ways to take 1 b out.

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

1 way.

$$= {}^3C_0\, a^3 b^0 + {}^3C_1\, a^2 b^1 + {}^3C_2\, a\, b^2 + {}^3C_3\, a^0 b^3.$$

3 boxes

we have choose 1 b (which is same as in other boxes we have choose a^2)

${}^3C_1$ $\quad$ b^1

${}^3C_2$ $\quad$ b^2

In a class of 3 students, how many ways we can select 2 students.?

Ans: $^3C_2$.

S1 S2 S3

selected not selected

$(b + a)$

1 1 0

0 1 1

S1 S2 b^2 S3

$(a+b) \times (a+b) \times (a+b)$

$^3C_2\ a\ b^2$

b^2 b^2

$a \rightarrow$ not selected.

$b \rightarrow$ select.

Selecting 2 students is same as finding how many ways we can write b^2 OR coefficient of b^2

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3.$$

$$= {}^3C_0\, a^3 b^0 + {}^3C_1\, a^2 b^1 + {}^3C_2\, a^1 b^2 + {}^3C_3\, a^0 b^3.$$

* power of a ↓
* power of b ↑
* sum of power a & b is always 3.

$$(a+b)^n = {}^nC_0\, a^n b^0 + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 + {}^nC_3\, a^{n-3} b^3$$

$$\ldots\ldots\ {}^nC_n\, a^0 b^n$$

$$(a+b)^n = \sum_{i=0}^{n} {}^nC_i \times a^{n-i} \times b^i$$

binomial coefficient.

→ In a class of 10 students
if we have to select 6.

→ $^{10}C_6$.

Select : b
not select : a.

$(a+b) \times (a+b) \times (a+b) \ldots\ldots$

Selecting 6 students is same as finding coefficient of b^6

$^{10}C_6 \, a^4 b^6$

$$(a+b)^n = \sum_{i=0}^{n} {}^nC_i \, a^{n-i} \cdot b^i$$

$a=1 \quad b=1.$

$$(1+1)^n = \sum_{i=0}^{n} {}^nC_i \, (1)^{n-i} \cdot (1)^i$$

$$2^n = \sum_{i=0}^{n} {}^nC_i = {}^nC_0 + {}^nC_1 + {}^nC_2 \cdots {}^nC_n$$

$$\therefore$$

$${}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + {}^nC_4 \cdots = 2^n.$$

$$(a+b)^n = \sum_{i=0}^{n} {}^nC_i \cdot a^{n-i} \, b^i$$

$a=1 \quad b=-1.$

$$(1+(-1))^n = \sum_{i=0}^{n} {}^nC_i \, (1)^{n-i} \, (-1)^i$$

$$0 = \sum_{i=0}^{n} {}^nC_i \cdot 1 \cdot (-1)^i$$

$$0 = {}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 \cdots$$

$${}^nC_1 + {}^nC_3 + {}^nC_5 \cdots = {}^nC_0 + {}^nC_2 + {}^nC_4$$

$$(a+b)^n = \sum_{i=0}^{n} {}^nC_i \cdot a^{n-i} \cdot b^i$$

$a=1 \quad b=2$

$$3^n = \sum_{i=0}^{n} {}^nC_i \cdot (1)^{n-i} \cdot 2^i$$

$$\boxed{3^n = \sum_{i=0}^{n} {}^nC_i \cdot 2^i}$$

$$(x+y)^{20}$$

what will be coefficient of y^{13}

$$^{20}C_{13}\, x^7 y^{13}$$

$\uparrow$

$$(2x+3y)^{20} \quad \text{coefficient of } y^{13}$$

$$^{20}C_{13}\,(2x)^7 \cdot (3y)^{13}$$

$$\boxed{^{20}C_{13} \cdot 2^7 \cdot 3^{13}} \cdot x^7 \cdot y^{13}$$

$$^nC_k = \frac{n!}{k!\cdot(n-k)!} = \frac{\underbrace{n\cdot(n-1)(n-2)\ldots\ldots(n-k+1)\cancel{(n-k)!}}}{k!\cdot\cancel{(n-k)!}}$$

$$\boxed{^nC_k = \frac{\underbrace{n(n-1)(n-2)\ldots(n-k+1)}}{k!}}$$

$$^{11}C_2 = \frac{11!}{2!\cdot 9!}$$

$$= \frac{11\cdot 10\cdot \cancel{9!}}{2!\cdot\cancel{9!}} = \frac{11\cdot 10}{2!}$$

$$^{11}C_2 = \frac{11\cdot 10}{2!} \qquad ^{11}C_3 = \frac{11\cdot 10\cdot 9}{3!}$$

Extended binomial coefficient :

$$^{n}C_{k} = \frac{n\cdot(n-1)\cdots(n-k+1)}{k!}$$

$$^{-n}C_{k} = \frac{\overset{(-1)}{(-n)}\cdot(-n-1)\cdot(-n-2)\cdots\cdots(-n-k+1)}{k!}$$

-1 common

$$= (-1)^{k}\cdot\frac{(n)(n+1)(n+2)\cdots(n+k-1)}{k!} = (-1)^{k}\,\frac{(n+k-1)\cdots\cdots(n+2)(n+1)(n)}{k!}\,\frac{(n-1)!}{(n-1)!}$$

$$= (-1)^{k}\,\frac{(n+k-1)!}{k!\cdot(n-1)!}$$

$$^{-n}C_k = (-1)^k \frac{(n+k-1)!}{k! \cdot (n-1)!}$$

$$^{n+k-1}C_k$$

$$\frac{(n+k-1)!}{k! \times (n+\cancel{k}-1-\cancel{k})!}$$

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$$\boxed{^{-n}C_k = (-1)^k \; ^{n+k-1}C_k}$$

$$\frac{(n+k-1)!}{k! \, (n-1)!}$$

$-^{11}C_2 = {}^{-n}C_k$

$\boxed{n = 11 \quad k = 2}$

$(-1)^k \; {}^{n+k-1}C_k$

$(-1)^2 \cdot {}^{11+2-1}C_2$

$1 \cdot {}^{12}C_2$

$-^{11}C_2 = {}^{12}C_2$

$K \to$ even
+ve.
$K \to$ odd
-ve.

$-^{11}C_3 = {}^{-n}C_k$

$n = 11 \quad k = 3$

$(-1)^3 \; {}^{n+k-1}C_k$

$(-1)^3 \; {}^{11+3-1}C_3 = -1 \times {}^{13}C_3 = -{}^{13}C_3$

Express each of the sums in closed form

$$\sum_{k=0}^{n}\binom{n}{k}5^k$$

$$\sum_{i=0}^{n}\binom{n}{i}x^i$$

$$\sum_{j=0}^{2n}(-1)^j\binom{2n}{j}x^j$$

$$\sum_{i=0}^{m}\binom{m}{i}p^{m-i}q^{2i}$$

$$\sum_{i=0}^{m}(-1)^i\binom{m}{i}\frac{1}{2^i}$$

$$\sum_{i=0}^{n}(-1)^i\binom{n}{i}5^{n-i}2^i$$

$$\sum_{k=0}^{n}\binom{n}{k}5^k=\sum_{k=0}^{n}\binom{n}{k}1^{n-k}5^k=(1+5)^n=6^n$$

$$\sum_{i=0}^{n}\binom{n}{i}x^i=\sum_{i=0}^{n}\binom{n}{i}1^{n-i}x^i=(1+x)^n$$

$$\sum_{j=0}^{2n}(-1)^j\binom{2n}{j}x^j=\sum_{j=0}^{2n}\binom{2n}{j}1^{2n-j}(-x)^j=(1-x)^{2n}$$

$$\sum_{i=0}^{m}(-1)^i\binom{m}{i}\frac{1}{2^i}=\sum_{i=0}^{m}\binom{m}{i}1^{m-i}\left(-\frac{1}{2}\right)^i$$

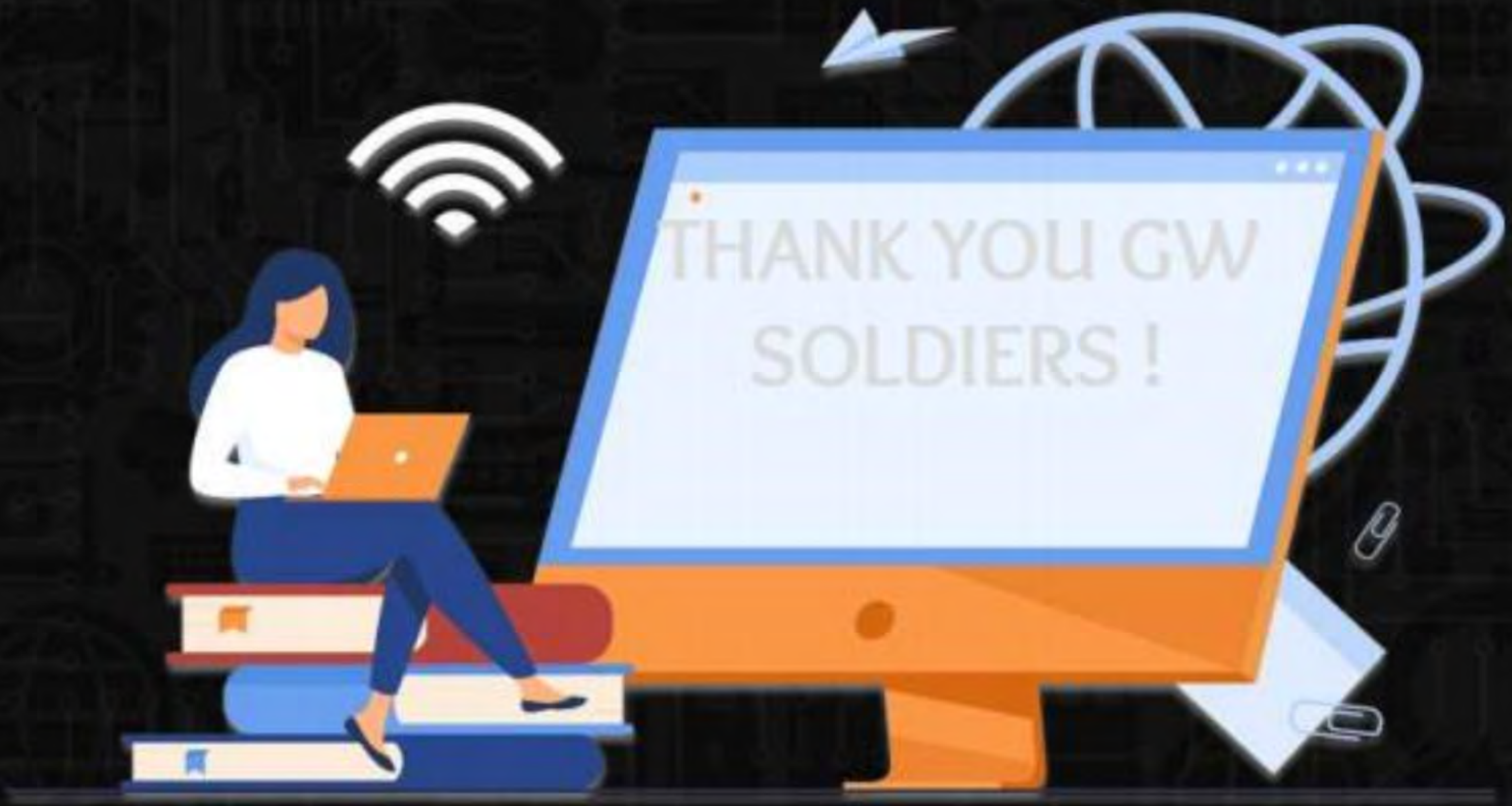
$$=\left(1-\frac{1}{2}\right)^m=\frac{1}{2^m}$$

$$\sum_{i=0}^{n}(-1)^i\binom{n}{i}5^{n-i}2^i=\sum_{i=0}^{n}\binom{n}{i}5^{n-i}(-2)^i=(5-2)^n=3^n$$

Find the coefficient of x^{16} in the expansion of $\left(2x^2 - \frac{x}{2}\right)^{12}$.

$$\binom{12}{k}(2x^2)^{12-k}\left(-\frac{x}{2}\right)^k = \binom{12}{k}2^{12-k}\left(-\frac{1}{2}\right)^k x^{24-k}.$$

We want $24 - k = 16$; thus, $k = 8$. The coefficient is $\binom{12}{8}2^4(-\frac{1}{2})^8 = \frac{1}{16}\binom{12}{8}$
$\frac{495}{16}$.


THANK YOU GW
SOLDIERS !