IE - 616
Decision Analysis and Game Theory

Mini Project
Sequential Version of Cournot Duopoly

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Abstract

In today's globalised world, various firms engage in various types of games and reach a Nash equilibrium to gain maximum profit, we analysed one such game in our paper, in which firms engage in quantity competition among themselves with 1 firm being the leader, that is, one firm has a 1st player advantage because of being the market leader, we find out that leader firm is always better off in such a game compared to cournot duopoly and have found out various other results which help in understanding the various market behaviour in the real market.
Also, we found out that the market is economically better off in all three cases of the game we deployed compared to cournot equilibrium.

Introduction

For our game project, we have worked on a different form of cournot Duopoly game in which 2 firms unlike cournot duopoly play games sequentially, that is firm 1 selects the quantity in 1st move to maximise its profit and based on that firm 2 decides its quantity to maximise its profit.

We see many real-life examples of such a game played out between various firms in different sectors of the economy, therefore it's important to find out the advantage if any the firm which moves first has.

1. One such example is the airline industry, where a dominant airline sets the price and schedule for a particular route, and other airlines follow suit by adjusting their prices and schedules to match the dominant airline, American Airlines' hub at Dallas/Fort Worth International Airport serves as a model for other airlines, and they adjust their pricing and scheduling to compete with American Airlines.
2. In the fast food industry, a dominant chain can set prices and menu options, and other chains follow suit by offering similar products at competitive prices. For instance, when McDonald's introduced a new sandwich, other fast-food chains like Burger King and Wendy's followed by developing their own versions of that sandwich with similar prices and ingredients.
3. In the automobile industry, where the dominant manufacturer sets the price and product design, other manufacturers follow by offering similar products at competitive prices. For example, when Tesla introduced its Model S electric vehicle, other carmakers followed suit by developing similar electric vehicles, like the Chevrolet Bolt and the Nissan Leaf, and set their prices at competitive levels.

4. In the oil industry, we can clearly put the OPEC countries as the dominant player and other oil-producing countries make quantity production decisions based on the oil production decision taken by OPEC countries to maximize their profits, thus making the OPEC countries the 1st player firm and non-OPEC countries as the second player firm.

For the game, we will use methods of finding a reaction function for firm 2 based on firm 1 action through the use of backward induction we learnt in the IE-616 course by maximizing the profit function of firm 2 and will use that information to maximize the profit function of firm 1 and find the nash equilibrium for quantities between two firms and compare the solution of our game with that of cournot duopoly game for the same marginal cost and then will compare the total economic output between of sequential duopoly game and the cournot duopoly game.

We will only consider the economic output of the market, that is, we will only look at which game is leading to more quantity being produced in the market, we will for this instance will not look at the outcome based on social equality, as its most probably going to be the case that firm 1 due to its 1st player advantage will get the bigger share of the pie, ceteris paribus.

In our game, we will consider 3 cases of the relation between the marginal cost of 2 firms and then compare the economic outcome for the three cases with the same relation between the marginal cost of two firms for a cournot solution

Theory

Cournot competition is an economic model used to describe an industry structure in which companies compete on the amount of output they will produce, which they decide on independently of each other and at the same time.

The Nash equilibrium is a decision-making theorem within the game theory that states a player can achieve the desired outcome by not deviating from their initial strategy. In the Nash equilibrium, each player's strategy is optimal when considering the decisions of other players.

Backward induction in game theory is an iterative process of reasoning backwards in time, from the end of a problem or situation, to solve finite extensive form and sequential games, and infer a sequence of optimal actions.

In economics, the marginal cost is the change in total production cost that comes from making or producing one additional unit.

Game Formation and its Solution

For our game, we have the following things considered:

1) 2 firms F_1 & F_2, who compete in quantity
2) Let the quantity of goods produced by 2 firms be q_1 & q_2 respectively
3) The marginal cost of production of goods is C_1 for F_1 & C_2 for F_2
4) Profit earned by F_1 is π_1 & by F_2 is π_2
5) q_1^c & q_2^c is the cournot solution for quantity for F_1 & F_2 respectively, π_1^c & π_2^c are cournot solutions for profits of F_1 & F_2 respectively, P^c is price-demand expression value

The total quantity produced in the market is $Q = q_1 + q_2$

Inverse demand curve which both F_1 & F_2 follow is:

$$P = a - b(q_1 + q_2)$$

In our game, unlike cournot both firms do not move simultaneously but sequentially with F_1 moving 1st i.e. F_1 decides the quantity q_1 first that it will produce to maximise its profits & F_2 chooses q_2 as the best response to F_1's selection of q_1 to maximise its profits.
Here, since F_1 knows F_2 will select q_2 based on q_1, F_1 will perform backward induction to find its best strategy for selecting q_1.
Then, applying backward induction of F_2's profit equation π_2:

$$\pi_2 = (P - C_2)\times q_2$$

$$\partial\pi_2/\partial q_2 = 0 \quad => \quad \partial(a- b\times(q_1 + q_2) - C_2)q_2/\partial q_2 = 0$$

$$=> \quad a - b\times q_1 - 2b\times q_2 - C_2 = 0$$

$$=> \quad \mathbf{R_2(q_1): q_2 = a - b\times q_1 - C_2/2b} \qquad \text{-------------------------} \quad \text{(i)}$$

Here, $R_2(q_1)$ is the reaction function of q_2 with change in q_1, now, based on this, we will maximise the profit of F_1 i.e.

$$\pi_1 = (P - C_1)\times q_1$$

$$\pi_1 = (a - b\times(q_1 + q_2) - C_1)\times q_1$$

$$\pi_1 = [a - b\times(q_1 + (a - b\times q_1 - C_2)/2b - C_1]\times q_1$$

$$\pi_1 = [a - b\times q_1 + C_2 -2C_1]\times q_1/2$$

$$\partial\pi_1/\partial q_1 = 0 \quad \Rightarrow \quad a - 2b\times q_1 + C_2 - 2C_1 = 0$$

$$\Rightarrow \quad \mathbf{q_1^* = (a + C_2 - 2C_1)/2b} \qquad \text{------------------------} \quad \text{(ii)}$$

By using (ii) in (i) we get:

$$q_2^* = [a - b\times\{(a + C_2 - 2C_1)/2b\} - C_2]/\ 2b$$

$$q_2^* = (2a - a - C_2 + 2C_1 - 2C_2)/4b$$

$$\mathbf{q_2^* = (a - 3C_2 + 2C_1)/4b} \qquad \text{------------------------} \quad \text{(iii)}$$

Now,

$$P^* = a - b\times[\ (a + C_2 - 2C_1)/2b \ + \ (a - 3C_2 + 2C_1)/4b\ ]$$

$$\mathbf{P^* = (a + C_2 + 2C_1)/4} \qquad \text{------------------------} \quad \text{(iv)}$$

Now,

$$\pi_1 = (P - C_1)\times q_1$$

$$\pi_1 = [\ (a + C_2 + 2C_1)/4 - C_1\ ]\times[\ (a + C_2 - 2C_1)/2b\ ]$$

$$\mathbf{\pi_1^* = (a + C_2 - 2C_1)^2/8b} \qquad \text{------------------------} \quad \text{(v)}$$

Similarly,

$$\pi_2 = (P-C_2)\times q_2$$

$$= [(a + C_2 + 2C_2)/4 - C_2]\times[(a - 3C_2 + 2C_1)/4b]$$

$$\Rightarrow \quad \mathbf{\pi_2^* = (a - 3C_2 + 2C_1)^2/16b} \qquad \text{------------------------} \quad \text{(vi)}$$

Now, we know that for the marginal cost C_1 & C_2, F_1 & F_2 produces in cournot doupoly games following:

$$\mathbf{q_1^c = (a + C_2 - 2C_1)/3b,}$$

$$\mathbf{q_2^c = (a + C_1 - 2C_2)/3b,}$$

$$\mathbf{P^c = (a + C_1 + C_2)/3,}$$

$$\boldsymbol{\pi_1^c = (a + C_2 - 2C_1)^2/9b,}$$

$$\boldsymbol{\pi_2^c = (a + C_1 - 2C_2)^2/9b,}$$

$$\mathbf{Q^c = q_1^c + q_2^c = (2a - C_1 - 2C_2)/3b,}$$

$$\mathbf{Q^* = q_1^* + q_2^* = (a + C_2 - 2C_1)/2b + (a - 3C_2 + 2C_1)/4b,}$$

$$\mathbf{\Rightarrow Q^* = (3a - C_2 - 2C_1)/4b}$$

From the direct comparison, we can say that for any values of C_1 & C_2:

$q_1^* > q_1^c$ i.e. F_1 has a ***1st player advantage*** if the cournot game is played in sequential ways, and F_1 will ***always*** be better off than the standard cournot solution that we get for profits of F_1.

Thus, we can say our game will have 1st player advantage.

Now, making various cases of relation between C_1 & C_2 to compare which game has better ***market efficiency*** in each case. We will view economic market efficiency & view in total Q^c & Q^*, one with higher Q value has better market efficiency.

<u>CASE I:</u>

When marginal costs, $\mathbf{C_1 = C_2 = C}$,

Then,

$$q_1^* = (a - C)/2b, \quad q_2^* = (a - C)/4b, \quad P^* = (a + 3C)/4,$$

$$\pi_1^* = (a - C)^2/8b, \;\; \pi_2^* = (a - C)^2/16b, \;\; Q^* = 3(a - C)/4b,$$

$$q_1^c = (a - C)/3b, \quad q_2^c = (a - C)/3b, \quad P^C = (a + 2C)/3,$$

$$\pi_1^c = (a - C)^2/9b, \;\; \pi_2^c = (a - C)^2/9b, \;\; Q^c = 2(a-C)/3b,$$

Here, we can clearly see, when $C_1 = C_2 = C$,

$$\mathbf{q_1^* > q_1^c, \quad q_2^* < q_2^c,}$$

$$\boldsymbol{\pi_1^* > \pi_1^c, \quad \pi_2^* > \pi_2^c,} \ \&$$

$$\mathbf{Q^* > Q^c}$$

i.e. the market is better off in such a situation in an economic sense, even though F_2 because of the second player disadvantage is making less, overall the market is more quantity for consumers.

$$\mathbf{q_1^* > q_1^c = q_2^c > q_2^*,}$$

$$\boldsymbol{\pi_1^* > \pi_1^c = \pi_2^c > \pi_2^*}$$

<u>CASE II:</u>

We want to find condition b/w C_1 & C_2 such that $\mathbf{q_1^* = q_2^*}$:

i.e., $(a + C_2 - 2C_1)/2b = (a - 3C_2 + 2C_1)/4b$

$C_1 = (a + 5C_2)/6$

Putting C_1 in terms of C_2 in the expressions for q_1^*, q_2^*, P, Q^*, π_1^*, π_2^*:

$q_1^* = (a - C_2)/3b$, $q_2^* = (a - C_2)/3b$,

$P^* = (a + 2C_2)/3$, $Q^* = 2(a - C_2)/3$,

$\pi_1^* = [(a - C_2)/3]^2/2b$, $\pi_2^* = [(a - C_2)/3]^2/b$

Now, if we put,

$(a - C_2)/3 = k$, we get,

$q_1^* = k/b$, $q_2^* = k/b$, $P^* = (a + 2C_2)/3$, $Q^* = 2k/b$,

$\pi_1^* = k^2/2b$, $\pi_2^* = k^2/b$, $q_1^c = 2k/3b$, $q_2^c = 7k/6b$,

$P^C = (7 + 11C_2)/18$, $Q^c = 11k/6b$, $\pi_1^c = (2k/3)^2/b$, $\pi_2^c = (7k/6)^2/b$

We observe that:

$$\mathbf{q_2^c > q_2^* = q_1^* > q_1^c},$$

$$\boldsymbol{\pi_2^c > \pi_2^* > \pi_1^* > \pi_1^c}$$

$$\mathbf{Q^* > Q^c}$$

Here, we see when we made $q_2^* = q_1^*$ & counter the 1st player advantage in quantity produced in our game, firm 2 is having more advantage both in quantity & profit in cournot games, while π_1^c is worst off, which goes in line with our theory of 1st player advantage in our game.

CASE III:

When, $\boldsymbol{\pi_2^* = \pi_1^*}$

$$(a + C_2 - 2C_1)^2/8b = (a - 3C_2 + 2C_1)^2/16b$$

condition is $(a + C_2 - 2C_1) > 0$ & $(a - 3C_2 + 2C_1) > 0$ to make sure $q_1^* > 0$ & $q_2^* > 0$

$$a + C_2 - 2C_1 = (a - 3C_2 + 2C_1)/\sqrt{2}$$

$$\Rightarrow \quad (\sqrt{2} - 1)a + (3 + \sqrt{2})C_2 = (2 + 2\sqrt{2})C_1$$

$$C_1 = [(\sqrt{2} - 1)a + (3 + \sqrt{2})C_2]/(2 + 2\sqrt{2})$$

$$q_1^* = (a + C_2 - 2C_1)/2b$$

$$= \{a + C_2 - 2[(\sqrt{2} - 1)a + (3 + \sqrt{2})C_2]/(2 + 2\sqrt{2})\}/2b$$

$$= [(\sqrt{2} + 1)a + (\sqrt{2} + 1)C_2 - (\sqrt{2} - 1)a - (3 + \sqrt{2})C_2]/2(1 + \sqrt{2})b$$

$$= [2a - 2C_2]/2(1 + \sqrt{2})b$$

=> $\mathbf{q_1^* = (a - C_2)/(1 + \sqrt{2})b}$

$q_2^* = [a - 3C_2 + \{(\sqrt{2} -1)a + (3 + \sqrt{2})C_2\}/(1 + \sqrt{2})]/4b$

$q_2^* = [2\sqrt{2}a - 2\sqrt{2}C_2]/4(1 + \sqrt{2})b$

=> $q_2^* = 2\sqrt{2}(a - C_2)/4(1 + \sqrt{2})b$

=> $\mathbf{q_2^* = (a - C_2)/\sqrt{2}(1 + \sqrt{2})b}$

Now,

$P^* = [a + C_2 + \{(\sqrt{2} -1)a + (3 + \sqrt{2})C_2\}/(1 + \sqrt{2})]/4$

$= [(1 + \sqrt{2})a + (1 + \sqrt{2})C_2 + (\sqrt{2} -1)a + (3 + \sqrt{2})C_2]/4(1 + \sqrt{2})$

=> $\mathbf{P^* = [a + (1 + \sqrt{2})C_2]/\sqrt{2}(1 + \sqrt{2})}$

$\boldsymbol{\pi_1^* = [(a - C_2)(1 + \sqrt{2})]^2/2b}$, $\boldsymbol{\pi_2^* = [(a - C_2)(1 + \sqrt{2})]^2/2b}$

Similarly,

$q_1^c = (a + C_2 - 2C_1)/3b$

$= [a + C_2 - 2[(\sqrt{2} -1)a + (3 + \sqrt{2})C_2]/(2 + 2\sqrt{2})]/3b$

$= (2a - 2C_2)/3(2 + 2\sqrt{2})b$

=> $\mathbf{q_1^c = 2[(a - C_2)/(1 + \sqrt{2})]/3b}$

$q_2^c = (a + C_1 - 2C_2)/3b$

$= [a + \{(\sqrt{2} -1)a + (3 + \sqrt{2})C_2]/(2 + 2\sqrt{2})\} - 2C_2]/3b$

=> $\mathbf{q_2^c = (1 + 3\sqrt{2})[(a - C_2)/(1 + \sqrt{2})]/6b}$

$P^c = (a + C_1 + C_2)/3$

$= [a + \{(\sqrt{2} -1)a + (3 + \sqrt{2})C_2]/(2 + 2\sqrt{2})\} + C_2]/3$

=> $\mathbf{P^c = \{(1 + 3\sqrt{2})a + (5 + 3\sqrt{2})C_2\}/6(1 + 2\sqrt{2})}$

$\pi_1^c = (a + C_2 - 2C_1)^2/9b$

=> $\boldsymbol{\pi_1^c = 4[(a - C_2)/(1 + \sqrt{2})]^2/9b}$

$\pi_2^c = (a + C_1 - 2C_2)^2/9b$

=> $\boldsymbol{\pi_2^c = [(1 + 3\sqrt{2})/6]^2[(a - C_2)/(1 + \sqrt{2})]^2/b}$

If we put, $(a - C_2)/(1 + \sqrt{2}) = \ell$, we get,

$q_1^* = \ell/b$, $q_2^* = \ell/\sqrt{2}b$, $Q^* = \ell(1 + \sqrt{2})/\sqrt{2}b$,

$P^* = [a + C_2(1 + \sqrt{2})]/\sqrt{2}(1 + \sqrt{2})$, $\pi_1^* = \pi_2^* = \ell^2/2b$,

$q_1^c = 2\ell/3b$, $q_2^c = (1 + 3\sqrt{2})\ell/6b$,

$Q^* = (5 + 3\sqrt{2})\ell/6b$, $P^C = [(1 + 3\sqrt{2})a + (5 + 3\sqrt{2})C_2]/6(1 + \sqrt{2})$,

$\pi_1^c = 4\ell^2/9b$, $\pi_2^c = [(1 + 3\sqrt{2})/6]^2\ell^2/b$,

Clearly,

$$\mathbf{q_1^* > q_2^c > q_2^* > q_1^c}$$

$$\boldsymbol{\pi_2^c > \pi_1^* = \pi_2^* > \pi_1^c}$$

& $\mathbf{Q^* > Q^c}$

Results

- Irrespective of the relation between C1 and C2 we find out that every time firm 1 will be better off in this game due to 1st player mover advantage compared to the cournot duopoly for the same set of variables.

- For all the three cases 1) Marginal costs, **C1 = C2 = C**, 2) $\mathbf{q_1^* = q_2^*}$, 3) $\boldsymbol{\pi_2^* = \pi_1^*}$

We find out that **$Q^* > Q^c$,** that is, the market is better off economically in all three cases of the game presented above compared to the cournot duopoly game, irrespective of the pie sharing of the outcome, that is, social justice.

Thus we can say providing 1st player advantage to a firm is leading to better economic outcomes for the market, not so in terms of social equilibria though.

- For case 1) we find $q_1^* > q_1^c = q_2^c > q_2^*$, $\pi_1^* > \pi_1^c = \pi_2^c > \pi_2^*$
- For case 2) we find $q_2^c > q_2^* = q_1^* > q_1^c$, $\pi_2^c > \pi_2^* > \pi_1^* > \pi_1^c$
- For case 3) we find $q_1^* > q_2^c > q_2^* > q_1^c$, $\pi_2^c > \pi_1^* = \pi_2^* > \pi_1^c$

Special Remark

For some of the cases that we have taken here, we have found that $\pi_2^* > \pi_1^*$, that is, the firm moving second is earning more or in one case equal, though in real market scenario it is more than likely and safe assumption that market leader firm, that is, firm 1 will always have better quantity produced as well as more profit because due to its large size and 1st player advantage, it has less marginal cost compared to firm 2 which is true for the examples we have presented at the start of the paper, though it is important to understand the economics and the game theory involved behind such market outcomes of reaching nash equilibrium, therefore we have taken the above cases to understand the economic market and nash equilibrium in a better way.

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