(1) (a) Let $f(x) = g(a^T x)$, where $g: \mathbb{R} \to \mathbb{R}$ is continuously differentiable and $a \in \mathbb{R}^n$ is a vector. $\nabla f(x) = ?$ $\nabla^2 f(x) = ??$

Sol^n:- $a \in \mathbb{R}^n$ is a vector

$$a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \quad \text{so, } a^T = [a_1, a_2, a_3 \ldots a_n]_{1 \times n}$$

and
$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n \times 1}$$

$$\text{so } a^T x = [a_1, a_2 \ldots \ldots a_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$a^T x = [a_1 x_1 + a_2 x_2 + a_3 x_3 + \ldots \; a_n x_n).$$

$$\text{so, } f(x) = g(a_1 x_1 + a_2 x_2 + \ldots \; a_n x_n).$$

$$\nabla f(x) = \begin{bmatrix} \frac{\partial f(x)}{\partial x_1} \\ \frac{\partial f(x)}{\partial x_2} \\ \vdots \\ \frac{\partial f(x)}{\partial x_n} \end{bmatrix} \quad \text{so, } \nabla f(x) = \begin{bmatrix} \frac{\partial(a_1 x_1 + a_2 x_2 + \ldots \; a_n x_n)}{\partial x_1} \\ \frac{\partial(a_1 x_1 + a_2 x_2 + \ldots \; a_n x_n)}{\partial x_2} \\ \vdots \\ \frac{\partial(a_1 x_1 + a_2 x_2 + \ldots \; a_n x_n)}{\partial x_n} \end{bmatrix}$$

$$\Rightarrow \nabla f(x) = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{bmatrix} = a.$$

and $\nabla^2 f(x) = 0$ — Ans