(2) (c). Let $A \in \mathbb{R}^{n\times n}$ be positive semidefinite and $B \in \mathbb{R}^{m\times n}$ be arbitrary, where $m, n \in \mathbb{N}$. Is BAB^T PSD? If so prove it. If not give a counter example with explicit A, B.

Soln: — As we know that a matrix $A \in \mathbb{R}^{n\times n}$ is PSD, denoted $A \succeq 0$, for semipositive defn equality holds.

So if $A = A^T$ and $x^T A x \geq 0$ for all $x \in \mathbb{R}^n$.

So, let $A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}_{n\times n}$ is positive semi definite so,

$A = A^T$ symmetric as well as $\underline{x^T A x \geq 0}$. true for all $x \in \mathbb{R}^n$.

and $B \in \mathbb{R}^{m\times n}$

Let $B = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m\times n}$.

BAB^T

$= \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m\times n} \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & \ddots & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}_{n\times n} \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & \vdots & & \\ \vdots & \vdots & & \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{bmatrix}_{n\times m}$

$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & \cdots & & a_{mn} \end{bmatrix}_{m\times n}$ $\begin{bmatrix} \underbrace{(a_{11})^2 + (a_{12})^2 + \cdots (a_{1n})^2}_{11 \text{ position}}, \cdots \\ \underbrace{(a_{11} a_{21}) + \cdots (a_{2n} a_{2n})}_{12} \\ \cdots \\ \underbrace{(a_{11} a_{m1}) + (a_{12} a_{m2}) \cdots}_{1n} (a_{mn} a_{mn}) \end{bmatrix}_{n\times m}$

(
&

(2)

$$= B \cdot \begin{bmatrix} (a_{11})^2 + (a_{12})^2 + \dots (a_{1n})^2, & (a_{11}a_{21}) + \dots (a_{1n})(a_{2n}), & \dots & (a_{11}a_{m1}) + \dots (a_{1n})(a_{mn}) \\ \vdots & \vdots & & \vdots \\ (a_{m1}a_{11}) + (a_{m2})(a_{2n}) + \dots (a_{mn}a_{1n}), & \dots & \dots & (a_{m1}a_{m1}) + \dots (a_{m1}a_{mn}) \end{bmatrix}_{m \times m}$$

$B \sim m \times n$

$$\begin{bmatrix} a_{11} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots \\ a_{m1} & \dots & a_{mn} \end{bmatrix}$$

Result for $m \times m$ matrix i.e. P

P = Bmatrix $(m \times m)$

$P = P^T$ symmetric even

as well as P

leading diagonal is always positive its eigen values are positive. So P is P positive semidefinite

(matrix.

$P = BAB^T$ — Consider PSD $P = P^T$. Upon order for all $x \in \mathbb{R}^n$?

$\because$ if $P = BAB^T$. $\quad x^T P x \geq 0$

$$x^T (BAB^T) x \geq 0$$

$$(x^T B) A (B^T x) \geq 0$$

Both are transpose to each other so, $x^T P x \geq 0$ fulfill the condition

V.V.I

$\because$ $(x^T B)^T$

$= B^T (x^T)^T$

$= B^T x$

$\because (AB)^T = B^T A^T$