Research statement

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My research interests broadly focus on theoretical soft matter physics. I use a combination of analytical and numerical techniques to study the physics of soft materials, drawing ideas and tools from statistical mechanics, continuum mechanics, and differential geometry. I am particularly fascinated by the subtle interplay between the mechanical properties of soft materials and their geometry. In this research statement, I will briefly summarize the problems I have worked on during my academic career. I will also briefly comment on ongoing research and plans for future research.

1. Microphase separation in elastomers

It is an everyday culinary observation that certain liquids, water and oil, for example, do not mix and undergo spontaneous demixing when combined. Such a separation of an initially uniform mixture of two or more components into distinct, coexisting phases is termed as phase separation. Typically, the phase-separated domains are not static and usually coarsen over time (macrophase separation). In contrast, in microphase separation, the domains remain stable with a characteristic mesoscopic length scale that is independent of the initial composition of the mixture.

The interplay between elasticity and phase separation has been widely explored in various contexts since Cahn's classic work from the 1960s on spinodal decomposition [2]. For example, a mismatch in the constituents' elastic moduli in metallic alloys can either hinder or speed up phase separation [3]. Besides, mounting evidence now indicates that phase separation and elasticity are both crucial to the development of many membrane-less organelles within biological cells, rekindling interest in the topic [4–8]. To sidestep the complexities of the biological world, several experiments have been conducted with synthetic, *in vitro* model systems in the past few years [9–13]. In one such experiment, a temperature quench was used to trigger microphase separation in elastomers [1]. Elasticity was observed to arrest phase separation, resulting in a stable bicontinuous microstructure or droplets with a characteristic size of a few microns (Fig. 1.1). This microphase separation plausibly arises because of a pronounced difference in the length scales at which thermodynamics and elasticity operate.

In my recent work [14], I considered a minimal phase-field model that captures the key features of microphase separation in swollen elastomers in the limit of weak segregation. We showed that it is possible to address the length-scale discrepancy between elasticity and thermodynamics by employing a continuous order parameter and coarse-graining it further. The resulting long-range interactions lead to the emergence of stable, finite-sized domains whose length scale is governed by the stiffness of the elastomer. The scaling results for the domain size and microphase separation temperature agree with experimental observations (Fig. 1.1).

Ongoing work. At the moment I am working on extending our approach to include effects of anisotropy, e.g., when the initial swelling of the elastomer is anisotropic or when the elastomer has a variable stiffness. Both these situations have been experimentally explored [1] and our initial theoretical predictions broadly agree with the experimental observations.

Future research plans. Nonlinear extensions to the theory using hyperelastic models for gels [15] would make our theory valid for large deformations. Some related questions can also be considered, e.g., cavitation in gels due to drying [16]. Inclusion of viscoelastic effects can capture the forces participating in phase separation inside cellular nuclei. In principle, this would also address other interesting phase separation phenomena, such as metal soap formation in oil paintings [17, 18].

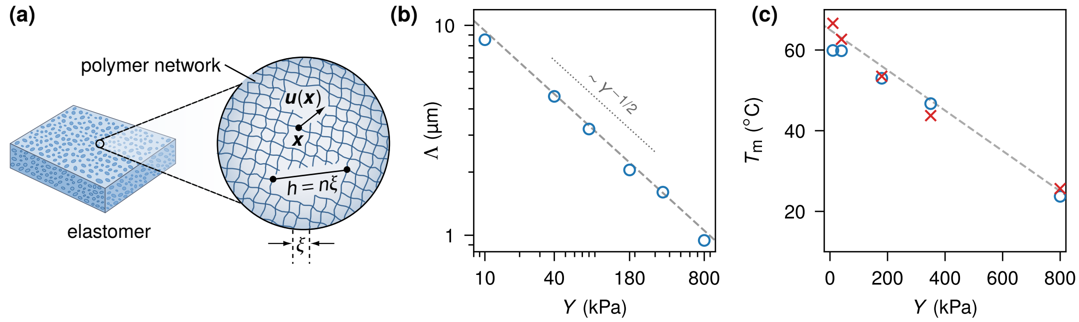

Figure 1.1. (a) An elastomer consists of a polymer network swollen with a solvent. To address the length-scale imbalance between thermodynamics and elasticity, we coarse-grain the displacement field $\boldsymbol{u}(\boldsymbol{x})$ at a length scale $h = n\xi$, where ξ is the mesh size of the polymer network. (b) Domain size Λ and (c) microphase separation temperature T_{m} as a function of the Young's modulus Y of the elastomer. The dashed lines and crosses represent theoretical predictions and the circles represent experimental results from Ref. [1].

2. Free-energy landscapes of singular frameworks

Simply put, a bar-joint framework is a deformable assembly of bars that connect freely rotating joints [19]. Such frameworks have served as highly idealized representations of mechanical structures that underlie a plethora of soft few-body systems, e.g., colloidal clusters, proteins, viruses, etc. More recently, DNA origami has made it possible to make these frameworks at the nanoscale, where they undergo low-temperature thermal fluctuations due to the surrounding medium [20].

Although the bars in a framework are only stiff and not rigid, it is useful to analyze a framework in the limit that the bars become fully rigid, i.e., when the bar lengths are fixed. Indeed, such a situation is an instance of a holonomically constrained classical system, often encountered in elementary mechanics. However, the imposed holonomic constraints need not always be well-behaved. To illustrate this point more generally, consider a particle constrained to move on two intersecting cylinders of equal radius with mutually perpendicular axes [Fig. 2.1(a)]. The configuration space of this particle is not a smooth manifold, precisely because the cylinders have a nontransversal intersection at two singular points, where they share a common tangent plane. Such singularities, which arise when the constraints imposed on a system cease to be linearly independent, are not mere pathological irregularities, and they have been extensively studied in many fields, e.g., robotics and locomotion.

In a recent paper [21], I considered the problem of finding the free energy $\mathcal{A}$ of a framework with singular constraints and equilibrated with a thermal bath, in terms of one of its internal coordinates, e.g., $\mathcal{A}(\rho_1)$ of a triangulated origami as a function of its fold angle ρ_1 [Fig. 2.1(b)]. Free energy is often found using the harmonic approximation by integrating out the fast vibrational modes, which for a framework are along the length of the bars. This, however, fails in the neighborhood of a singularity, where the framework acquires additional softness, and the free energy diverges in a harmonic approximation of the elastic energy [Fig. 2.1(c)]. I was able to show that [21] that these divergences are effectively suppressed by carefully accounting for anharmonic corrections to the energy near the singularities.

Ongoing work. Configuration space singularities of a framework can suddenly disappear if the equilibrium bar lengths are altered. [Going back to the cylinder-cylinder intersection in Fig. 2.1(a), the two singularities completely disappear if one of the cylinders is slightly larger or smaller than the other.] This often leads to a partitioning of the configuration space into disconnected manifolds, and in the presence of thermal fluctuations, the system transitions between these manifolds. I am currently working on an independent project [14] where I study this transition.

Future research plans. Although we only considered few-body frameworks in our work, I

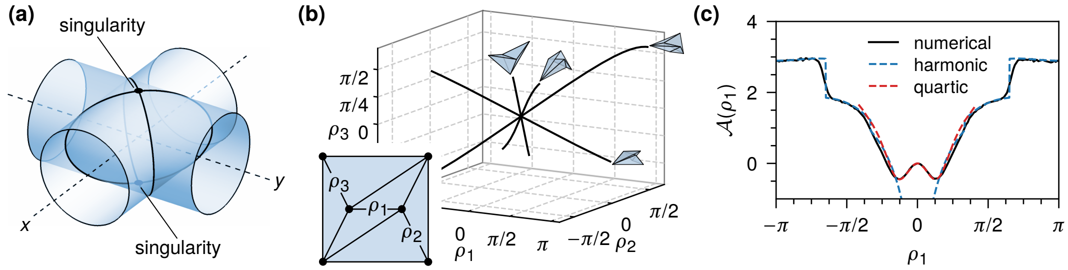

Figure 2.1. (a) The configuration space of a particle constrained to simultaneously move on two cylinders of equal radius is the cylinders' intersection curve, which has a self intersection and ceases to be a smooth manifold. (b) A triangulated origami (lower-left corner) modeled as a framework and its configuration space visualized using its fold angles (the supplement of the dihedral angle at a fold). The configuration space has a singularity when the origami is flat. (c) Free energy of the origami as a function of fold angle ρ_1. Harmonic approximation fails as $\rho_1 \to 0$, but the divergences are regulated by including quartic-order corrections.

also want to look at the thermodynamic limit of singular frameworks. In particular, I want to explore the connection between recent mean-field theories [22] written to describe the anomalous elastic response of cellular tissues and my work. The onset of rigidity in these tissues can be mapped to a critical configuration in a single cell's configuration space, rather similar to the singularity in Fig. 2.1(a). Furthermore, there is evidence that thermodynamic phase transitions of certain classical systems are associated with changes in the systems' configuration space topology. This is interesting because the free-energy landscape of the origami indicates an affinity for it to remain in a flat state, which is reminiscent of the flat phase of polymerized membranes, with the cytoskeleton of a cell membrane being a physical example [23].

3. Wave localization on thin elastic structures

Localized waves are finite-energy solutions to a wave equation that remain spatially confined to a certain region of space. In a recent a paper [24], I tried to understand this localization phenomena using a WKB/semiclassical approach, using a thin rod and a shell with spatially varying curvature as an illustrative example [Fig. 3.1(a)]. A particularly illustrative example of wave localization can be seen in a musical saw made by bending an ordinary wood saw into the shape of an S. The sonorous quality of the saw has been attributed to waves that get localized around its inflection point [25, 26].

An uncurved rod or shell can sustain both purely longitudinal extensional waves driven by stretching, and transverse flexural waves driven by bending. However, if the structure is curved, the flexural and extensional deformations remain coupled, and we can only speak of waves that are *predominantly* flexural or extensional, i.e., waves that respectively become purely flexural or extensional in the limit that the curvature tends to zero [27]. Because the curvature couples the different displacement components, to fully characterize wave propagation on curved structures, we have to consider multicomponent (i.e., vector) waves.

If the structure's curvature does not change significantly along its length, one can resort to a WKB/semiclassical approach to solve the associated eigenvalue equation that describes time-harmonic vibrations of the filament. Because the waves have multiple components, we do not, in general, expect this to be straightforward, partly due to the presence of extra phases (one of which is geometric in nature) that significantly alter the Bohr–Sommerfeld quantization rule used to extract the bound state frequencies [28, 29]. But surprisingly, for the thin elastic structures, these extra phases vanish. This simplifies our analysis considerably and results in remarkable agreement between the numerical experiments and quantization results. For both the rod and the shell, independent of the boundary conditions, waves exhibit robust localization

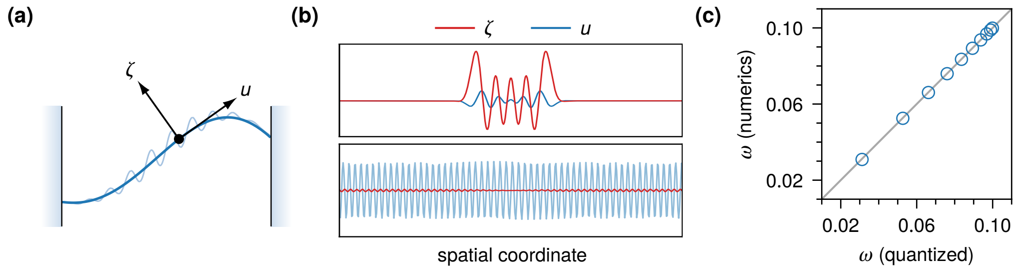

Figure 3.1. (a) Waves propagating on a curved filament have both transverse (ζ) and longitudinal (u) components. (b) Numerical mode profiles illustrating a localized mode centered around the inflection point of a filament (upper panel) and a delocalized mode with (lower panel). (c) Comparison between the eigenfrequencies ω of the localized modes obtained numerically and WKB results. (These results are for a specific set of physical parameters, i.e., length, boundary conditions, etc.)

around points where the absolute curvature has a minimum (Fig. 3.1). Wave localization induced by the presence of an inflection point in an S-shaped musical saw [30] is a special case of this more general observation.

Future research plans. There is a revival of interest in using semiclassical methods to study problems in wave and fluid mechanics. In a recent work [31] they have been successfully used to explain the topological protection of equatorial Kelvin waves. Given this, I want to use these techniques to explore similar phenomena in odd elastic waves [32] and active matter [26]. Apart from this, semiclassical ray tracing methods have been used to predict the phenomenon of branched flows—the spatial branching of rays as a result of random, but weak inhomogeneities in the medium, the end result being the formation of tree-like structures and fluctuations of extreme intensity [33]. In elastodynamics, branched flows have been shown to exist in thin elastic plates and cylinders with varying thickness profiles [34, 35]. It would be interesting to extend these results to consider shells with random curvature profiles, akin to crumpled paper [36].

4. General applied mathematics, nonlinear dynamics, etc.

I maintain a general interest in physical applied mathematics, and in the past, as part of my master's project, I have worked on the problem of synchronizing chaotic oscillators. Coupled chaotic oscillators typically synchronize only when the coupling strength falls in a specific range, i.e., too small or too large a coupling strength results in spontaneous desynchronization. Together with my collaborators, I was able to show that this range could be arbitrarily extended by *uncoupling* the oscillators in certain regions of their phase space [37, 38]. Even in cases where typical methods of synchronization would fail, our method could induce synchronization.

I have also worked on nonlinear time series analysis, a branch of nonlinear dynamics that emerged from attempts to detect the presence of chaos in real-world data sets. We used these analysis techniques to identify the nature of turbulent reversals in Rayleigh–Bénard convection [39]. I also revisited several earlier results that reported evidence of low-dimensional chaos in the observational X-ray light curves of GRS 1915+105, a widely-studied microquasar. Surprisingly, our results were different from what had been reported, and a thorough investigation revealed that these light curves are not, in fact, chaotic [40]. I developed a software package called NoLiTSA while writing Refs. [39] and [40], and it has since been used by other researchers in over 20 publications and has over 150 stars on GitHub [41].

References

[1] C. Fernández-Rico, S. Schreiber, H. Oudich, C. Lorenz, A. Sicher, T. Sai, V. Bauernfeind, S. Heyden, P. Carrara, L. D. Lorenzis, R. W. Style, and E. R. Dufresne, Elastic microphase separation produces robust bicontinuous materials, Nat. Mater. **23**, 124 (2024).

[2] J. W. Cahn, On spinodal decomposition, Acta Metall. **9**, 795 (1961).

[3] P. Fratzl, O. Penrose, and J. L. Lebowitz, Modeling of phase separation in alloys with coherent elastic misfit, J. Stat. Phys. **95**, 1429 (1999).

[4] J.-M. Choi, A. S. Holehouse, and R. V. Pappu, Physical principles underlying the complex biology of intracellular phase transitions, Annu. Rev. Biophys. **49**, 107 (2020).

[5] H. Tanaka, Viscoelastic phase separation in biological cells, Commun. Phys. **5**, 167 (2022).

[6] D. S. W. Lee, A. R. Strom, and C. P. Brangwynne, The mechanobiology of nuclear phase separation, APL Bioeng. **6**, 021503 (2022).

[7] A. Kumar and S. A. Safran, Fluctuations and shape dependence of microphase separation in systems with long-range interactions, Phys. Rev. Lett. **131**, 258401 (2023).

[8] D. Deviri and S. A. Safran, Mechanosensitivity of phase separation in an elastic gel, Eur. Phys. J. E **47**, 16 (2024).

[9] R. W. Style, T. Sai, N. Fanelli, M. Ijavi, K. Smith-Mannschott, Q. Xu, L. A. Wilen, and E. R. Dufresne, Liquid-liquid phase separation in an elastic network, Phys. Rev. X **8**, 011028 (2018).

[10] K. A. Rosowski, T. Sai, E. Vidal-Henriquez, D. Zwicker, R. W. Style, and E. R. Dufresne, Elastic ripening and inhibition of liquid-liquid phase separation, Nat. Phys. **16**, 422 (2020).

[11] K. A. Rosowski, E. Vidal-Henriquez, D. Zwicker, R. W. Style, and E. R. Dufresne, Elastic stresses reverse ostwald ripening, Soft Matter **16**, 5892 (2020).

[12] S. Wilken, A. Chaderjian, and O. A. Saleh, Spatial organization of phase-separated DNA droplets, Phys. Rev. X **13**, 031014 (2023).

[13] J. X. Liu, M. P. Haataja, A. Košmrlj, S. S. Datta, C. B. Arnold, and R. D. Priestley, Liquid-liquid phase separation within fibrillar networks, Nat. Commun. **14**, 6085 (2023).

[14] M. Mannattil, Energy and free-energy landscapes of soft few-body systems with singular constraints, in preparation (2024).

[15] E. Meca, A. Münch, and B. Wagner, Localized instabilities and spinodal decomposition in driven systems in the presence of elasticity, Phys. Rev. E **97**, 012801 (2018).

[16] H. Wang and S. Cai, Drying-induced cavitation in a constrained hydrogel, Soft Matter **11**, 1058 (2015).

[17] K. Keune, P. Novle, A. Van Loon, E. Hendriks, S. Centeno, and G. Osmond, *Metal soaps in art: conservation and research* (Springer, Cham, 2019).

[18] T. Curk and E. Luijten, Phase separation and ripening in a viscoelastic gel, Proc. Natl. Acad. Sci. U.S.A. **120**, e2304655120 (2023).

[19] T. C. Lubensky, C. L. Kane, X. Mao, A. Souslov, and K. Sun, Phonons and elasticity in critically coordinated lattices, Rep. Prog. Phys. **78**, 073901 (2015).

[20] D. Z. Rocklin, V. Vitelli, and X. Mao, Folding mechanisms at finite temperature (2018), arXiv:1802.02704 [cond-mat.soft] .

[21] M. Mannattil, J. M. Schwarz, and C. D. Santangelo, Thermal fluctuations of singular bar-joint mechanisms, Phys. Rev. Lett. **128**, 208005 (2022).

[22] A. Hernandez, M. F. Staddon, M. J. Bowick, M. C. Marchetti, and M. Moshe, Anomalous elasticity of a cellular tissue vertex model, Phys. Rev. E **105**, 064611 (2022).

[23] D. Nelson, T. Piran, and S. Weinberg, *Statistical Mechanics of Membranes and Surfaces*, 2nd ed. (World Scientific, Singapore, 2004).

[24] M. Mannattil and C. D. Santangelo, Geometric localization of waves on thin elastic structures, Phys. Rev. E **109**, 035001 (2024).

[25] J. F. M. Scott and J. H. Woodhouse, Vibration of an elastic strip with varying curvature, Philos. Trans. R. Soc. A **339**, 587 (1992).

[26] S. Shankar, M. J. Bowick, and M. C. Marchetti, Topological sound and flocking on curved surfaces, Phys. Rev. X **7**, 031039 (2017).

[27] J. F. Doyle, *Wave Propagation in Structures*, 3rd ed. (Springer, New York, NY, 2021).

[28] K. Yabana and H. Horiuchi, Adiabatic viewpoint for the WKB treatment of coupled channel system, Prog. Theor. Phys. **75**, 592 (1986).

[29] R. G. Littlejohn and W. G. Flynn, Geometric phases and the Bohr–Sommerfeld quantization of multicomponent wave fields, Phys. Rev. Lett. **66**, 2839 (1991).
[30] S. Shankar, P. Bryde, and L. Mahadevan, Geometric control of topological dynamics in a singing saw, Proc. Natl. Acad. Sci. U.S.A. **119**, e2117241119 (2022).
[31] A. Venaille, Y. Onuki, N. Perez, and A. Leclerc, From ray tracing to waves of topological origin in continuous media, SciPost Phys. **14**, 062 (2023).
[32] A. Souslov, K. Dasbiswas, M. Fruchart, S. Vaikuntanathan, and V. Vitelli, Topological waves in fluids with odd viscosity, Phys. Rev. Lett. **122**, 128001 (2019).
[33] E. J. Heller, R. Fleischmann, and T. Kramer, Branched flow, Phys. Today **74**, 44 (2021).
[34] K. Jose, N. Ferguson, and A. Bhaskar, Branched flows of flexural waves in non-uniform elastic plates, Commun. Phys. **5**, 152 (2022).
[35] K. Jose, N. Ferguson, and A. Bhaskar, Branched flows of flexural elastic waves in non-uniform cylindrical shells, PLoS One **18**, e0286420 (2023).
[36] A. Gopinathan, T. A. Witten, and S. C. Venkataramani, Trapping of vibrational energy in crumpled sheets, Phys. Rev. E **65**, 036613 (2002).
[37] M. Schröder, M. Mannattil, D. Dutta, S. Chakraborty, and M. Timme, Transient uncoupling induces synchronization, Phys. Rev. Lett. **115**, 054101 (2015), arXiv:1508.06545 [nlin.CD] .
[38] A. Tandon, M. Schröder, M. Mannattil, M. Timme, and S. Chakraborty, Synchronizing noisy nonidentical oscillators by transient uncoupling, Chaos **26**, 094817 (2016), arXiv:1611.02298 [nlin.CD] .
[39] M. Mannattil, A. Pandey, M. K. Verma, and S. Chakraborty, On the applicability of low-dimensional models for convective flow reversals at extreme prandtl numbers, Eur. Phys. J. B **90**, 259 (2017), arXiv:1711.01510 [physics.flu-dyn] .
[40] M. Mannattil, H. Gupta, and S. Chakraborty, Revisiting evidence of chaos in X-ray light curves: The case of GRS 1915+105, Astrophys. J. **833**, 208 (2016), arXiv:1611.02264 [astro-ph.HE] .
[41] https://github.com/manu-mannattil/nolitsa.