

# Introduction of Programming Languages

## Programming Language Concepts

- What is a programming language?
- Why are there so many programming languages?
- What are the types of programming languages?
- Does the world need new languages?

## What is a Programming Languages

- A programming language is a set of rules that provides a way of telling a computer what operations to perform.
- A programming language is a set of rules for communicating an algorithm
- It provides a linguistic framework for describing computations

PS – Introduction

## What is a Programming Language?

*A programming language is a notational system for describing computation in a **machine-readable** and **human-readable** form.*

*A programming language is a tool for developing **executable models** for a class of problem domains.*

## What is a Programming Language

- English is a **natural language**. It has words, symbols and grammatical rules.
- A programming language also has words, symbols and rules of grammar.
- The grammatical rules are called **syntax**.
- Each programming language has a different set of syntax rules.

## Why Are There So Many Programming Languages

- Why does some people speak French?
- Programming languages have evolved over time as better ways have been developed to design them.
  - First programming languages were developed in the 1950s
  - Since then thousands of languages have been developed
- Different programming languages are designed for different types of programs.

## Levels of Programming Languages

High-level program

```
class Triangle {
    ...
    float surface()
        return b*h/2;
}
```

Low-level program

```
LOAD r1,b
LOAD r2,h
MUL r1,r2
DIV r1,#2
RET
```

Executable Machine code

```
0001001001000101
0010010011101100
10101101001...
```

## What Are the Types of Programming Languages

- First Generation Languages
- Second Generation Languages
- Third Generation Languages
- Fourth Generation Languages
- Fifth Generation Languages

## First Generation Languages

- Machine language
  - Operation code – such as addition or subtraction.
  - Operands – that identify the data to be processed.
  - Machine language is machine dependent as it is the only language the computer can understand.
  - Very efficient code but very difficult to write.

## Second Generation Languages

- Assembly languages
  - Symbolic operation codes replaced binary operation codes.
  - Assembly language programs needed to be “assembled” for execution by the computer. Each assembly language instruction is translated into one machine language instruction.
  - Very efficient code and easier to write.

## Third Generation Languages

- Closer to English but included simple mathematical notation.
  - Programs written in **source code** which must be translated into machine language programs called **object code**.
  - The translation of source code to object code is accomplished by a machine language system program called a **compiler**.

## Third Generation Languages (cont'd.)

- Alternative to compilation is interpretation which is accomplished by a system program called an **interpreter**.
- Common third generation languages
  - FORTRAN
  - COBOL
  - C and C++
  - Visual Basic



## Fourth Generation Languages

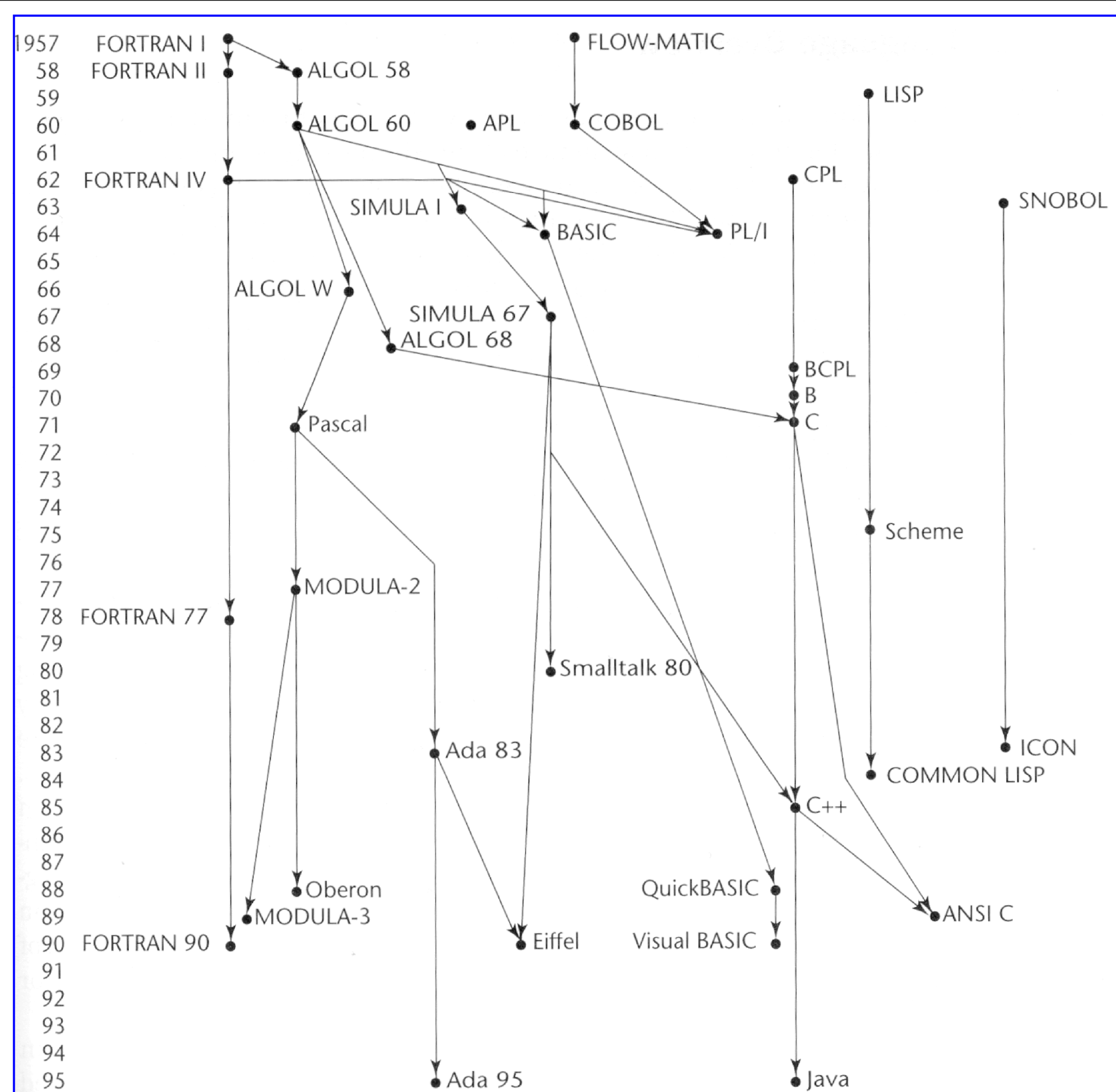
- A high level language (4GL) that requires fewer instructions to accomplish a task than a third generation language.
- Used with databases
  - Query languages
  - Report generators
  - Forms designers
  - Application generators

## Fifth Generation Languages

- Declarative languages
- Functional(?): Lisp, Scheme, SML
  - Also called applicative
  - Everything is a function
- Logic: Prolog
  - Based on mathematical logic
  - Rule- or Constraint-based

# Beyond Fifth Generation Languages

- Though no clear definition at present, natural language programs generally can be interpreted and executed by the computer with no other action by the user than stating their question.
- Limited capabilities at present.





## The principal paradigms

- Imperative Programming (C)
- Object-Oriented Programming (C++)
- Logic/Declarative Programming (Prolog)
- Functional/Applicative Programming (Lisp)

## Programming Languages

- Two broad groups
  - Traditional programming languages
    - Sequences of instructions
    - First, second and some third generation languages
  - Object-oriented languages
    - Objects are created rather than sequences of instructions
    - Some third generation, and fourth and fifth generation languages

## Traditional Programming Languages

- FORTRAN
  - FORmula TRANslation.
  - Developed at IBM in the mid-1950s.
  - Designed for scientific and mathematical applications by scientists and engineers.

## Traditional Programming Languages (cont'd.)

- COBOL
  - COmmon Business Oriented Language.
  - Developed in 1959.
  - Designed to be common to many different computers.
  - Typically used for business applications.

## Traditional Programming Languages (cont'd.)

- BASIC
  - Beginner's All-purpose Symbolic Instruction Code.
  - Developed at Dartmouth College in mid 1960s.
  - Developed as a simple language for students to write programs with which they could interact through terminals.

## Traditional Programming Languages (cont'd.)

- C
  - Developed by Bell Laboratories in the early 1970s.
  - Provides control and efficiency of assembly language while having third generation language features.
  - Often used for system programs.
  - UNIX is written in C.

## Object-Oriented Programming Languages

- Simula
  - First object-oriented language
  - Developed by Ole Johan Dahl in the 1960s.
- Smalltalk
  - First purely object-oriented language.
  - Developed by Xerox in mid-1970s.
  - Still in use on some computers.

## Object-Oriented Programming Languages (cont'd.)

- C++
  - It is C language with additional features.
  - Widely used for developing system and application software.
  - Graphical user interfaces can be developed easily with visual programming tools.

## Object-Oriented Programming Languages (cont'd.)

- JAVA
  - An object-oriented language similar to C++ that eliminates lots of C++'s problematic features
  - Allows a web page developer to create programs for applications, called **applets** that can be used through a browser.
  - Objective of JAVA developers is that it be machine, platform and operating system independent.

## Special Programming Languages

- Scripting Languages
  - JavaScript and VBScript
  - Php and ASP
  - Perl and Python
- Command Languages
  - sh, csh, bash
- Text processing Languages
  - LaTeX, PostScript

## Special Programming Languages (cont'd.)

- HTML
  - HyperText Markup Language.
  - Used on the Internet and the World Wide Web (WWW).
  - Web page developer puts brief codes called **tags** in the page to indicate how the page should be formatted.

## Special Programming Languages (cont'd.)

- XML
  - Extensible Markup Language.
  - A language for defining other languages.



## A language is a language is a language

- Programming languages are languages
- When it comes to mechanics of the task, learning to speak and use a programming language is in many ways like learning to speak a human language
- In both kind of languages you have to learn new vocabulary, syntax and semantics (new words, sentence structure and meaning)
- And both kind of language require considerable practice to make perfect.

## But there is a difference!

- Computer languages lack ambiguity and vagueness
- In English sentences such as *I saw the man with a telescope* (Who had the telescope?) or *Take a pinch of salt* (How much is a pinch?)
- In a programming language a sentence either means one thing or it means nothing

## What determines a “good” language

- Formerly: Run-time performance
  - (Computers were more expensive than programmers)
- Now: Life cycle (human) cost is more important
  - Ease of designing, coding
  - Debugging
  - Maintenance
  - Reusability
- FADS

## Criteria in a good language design

- **Writability:** The quality of a language that enables a programmer to use it to express a computation clearly, correctly, concisely, and quickly.
- **Readability:** The quality of a language that enables a programmer to understand and comprehend the nature of a computation easily and accurately.
- **Orthogonality:** The quality of a language that features provided have as few restrictions as possible and be combinable in any meaningful way.
- **Reliability:** The quality of a language that assures a program will not behave in unexpected or disastrous ways during execution.
- **Maintainability:** The quality of a language that eases errors can be found and corrected and new features added.

## Criteria (Continued)

- ▶ **Generality:** The quality of a language that avoids special cases in the availability or use of constructs and by combining closely related constructs into a single more general one.
- ▶ **Uniformity:** The quality of a language that similar features should look similar and behave similar.
- ▶ **Extensibility:** The quality of a language that provides some general mechanism for the user to add new constructs to a language.
- ▶ **Standardability:** The quality of a language that allows programs written to be transported from one computer to another without significant change in language structure.
- ▶ **Implementability:** The quality of a language that provides a translator or interpreter can be written. This can address to complexity of the language definition.

## Introduction to Logic Programming

## A Little History

- Prolog was invented by Alain Colmerauer, a professor of computer science at the university of Aix-Marseille in France, in 1972
- The first application of Prolog was in natural language processing
- Prolog stands for programming in logic (PROgrammation en LOgique)
- Its theoretical underpinning are due to Donald Loveland of Duke university through Robert Kowalski (formerly) of the university of Edinburgh

## Logic Programming

- Prolog is the only successful example of the family of logic programming languages
- A Prolog program is a theory written in a subset of first-order logic, called Horn clause logic
- Prolog is declarative. A Prolog programmer concentrates on *what* the program needs to do, not on *how* to do it
- The other major language for Artificial Intelligence programming is LISP, which is a functional (or applicative) language

## Knight Moves on a Chessboard

- % This example is from unpublished (to the best of my knowledge) notes by Maarten
- % Van Emden.
- /\* The extensional representation of the (knight) move relation follows. It
- consists of 336 facts; only a few are shown. In particular, all moves from
- position (5,3) on the chess board are shown. \*/
- move(1,1,2,3).
- move(1,1,3,2).
- ....
- move(5,3,6,5).
- move(5,3,7,4).
- move(5,3,7,2).
- move(5,3,6,1).
- move(5,3,4,1).
- move(5,3,3,2).
- move(5,3,3,4).
- move(5,3,4,5).
- ...
- move(8,8,7,6).
- move(8,8,6,7).

## Intensional Representation of Moves

- /\* The intensional representation of the (knight) move relation follows. It
- consists of facts (to define extensionally the relation succ/2) and rules (to
- define the relations move, diff1, and diff2. \*/
- move(X1,Y1,X2,Y2) :- diff1(X1,X2), diff2(Y1,Y2).
- move(X1,Y1,X2,Y2) :- diff2(X1,X2), diff1(Y1,Y2).
- diff1(X,Y) :- succ(X,Y).
- diff1(X,Y) :- succ(Y,X).
- diff2(X,Z) :- succ(X,Y), succ(Y,Z).
- diff2(X,Z) :- succ(Z,Y), succ(Y,X).
- succ(1,2).
- succ(2,3).
- succ(3,4).
- succ(4,5).
- succ(5,6).
- succ(6,7).
- succ(7,8).

## Defining Relations by Facts

- `parent( tom,bob).`
- `parent` is the name of a relation
  - A relation of arity  $n$  is a function from  $n$ -tuples (elements of a Cartesian product) to  $\{\text{true}, \text{false}\}$ . (It can also be considered a subset of the  $n$ -tuples.)
- `parent( pam, bob).` `parent( tom,bob).` `parent( tom,liz).` `parent( bob, ann).` `parent( bob,pat).` `parent( pat,jim).`
- A relation is a collection of *facts*

## Queries

?-parent( bob,pat).

yes

- A query and its answer, which is correct for the relation defined in the previous slide:  
this query *succeeds*

?-parent( liz,pat).

no

- A query and its answer, which is correct for the relation defined in the previous slide:  
this query *fails*



## More Queries

```
?-parent( tom,ben).    /* who is Ben? */
?-parent( X,liz).      /* Wow! */
?-parent( bob,X). /* Bob's children */
?-parent( X,Y). /* The relation, fact by fact */

parent( pam,bob) .
parent( tom,bob) .
parent( tom,liz) .
parent( bob,ann) .
parent( bob,pat) .
parent( pat,jim) .
```

## Composite Queries

- Grandparents:
 

```
parent( pam,bob) .
parent( tom,bob) .
parent( tom,liz) .
parent( bob,ann) .
parent( bob,pat) .
parent( pat,jim) .
```

```
?-parent( Y,jim), parent( X,Y).
    • the comma stands for “and”
?-parent( X,Y), parent(Y,jim).
    • order should not matter, and it does not!
```
- Grandchildren:
 

```
?-parent( tom,X), parent( X,Y).
```
- Common parent, i.e. (half-)sibling:
 

```
?-parent( X,ann), parent( X,pat).
```

## Facts and Queries

- Relations and queries about them
- Facts are a kind of *clause*
  - *Prolog programs consist of a list of clauses*
- The arguments of relations are atoms or variables (a kind of *term*)
- Queries consist of one or more *goals*
- Satisfiable goals succeed; unsatisfiable goals fail

## Defining Relations by Rules

- The offspring relation:  
 For all X and Y,  
 Y is an offspring of X if  
 X is a parent of Y
- This relation is defined by a rule,  
 corresponding to the Prolog clause  
 offspring( Y,X) :- parent( X,Y).
- Alternative reading:  
 For all X and Y,  
 if X is a parent of Y,  
 then Y is an offspring of X

## Rules

- Rules are clauses. Facts are clauses
- A rule has a condition and a conclusion
- The conclusion of a Prolog rule is its *head*
- The condition of a Prolog rule is its *body*
- If the condition of a rule is true, then it follows that its conclusion is true also

## How Prolog Rules are Used

- Prolog rules may be used to define relations
- The offspring relation is defined by the rule  
offspring( Y,X) :- parent( X,Y):
  - if (X,Y) is in the parent relation, then (Y,X) is in the offspring relation
- When a goal of the form offspring( Y,X) is set up, the goal succeeds if parent( X,Y) succeeds
- Procedurally, when a goal matches the head of a rule, Prolog sets up its body as a new goal

## Example

?-offspring(liz,tom).

- No fact matches this query
- The head of the clause *offspring( Y,X) :- parent( X,Y)* does
- Y is replaced with liz, X is replaced with tom
- The instantiated body *parent( tom,liz)* is set up as a new goal
- ?-parent( tom,liz) succeeds
- offspring( liz,tom) therefore succeeds too

```
parent ( pam,bob).
parent ( tom,bob).
parent ( tom,liz).
parent ( bob,ann).
parent ( bob,pat).
parent ( pat,jim).
```

```
offspring( Y,X) :- parent( X,Y).
```

```
% offspring(liz,tom).
```

## More Family Relations

- *female* and *male* are defined *extensionally*, i.e., by facts; *mother* and *grandparent* are defined *intensionally*, i.e., by rules
- female(pam). ... male(jim).
- mother( X,Y) :- parent( X,Y), female( X).
- grandparent( X,Z) :- parent( X,Y), parent( Y,Z).

## Sister

- `sister(X,Y) :- parent(Z,X), parent(Z,Y), female( X).`
- Try:
  - `?-sister(X,pat).`
  - `X = ann;`
  - `X = pat /* Surprise! */`
- (Half-)sisters have a common parent *and* are different people, so the correct rule is:
- `sister(X,Y) :- parent(Z,X), parent(Z,Y), female( X), different(X,Y).`
  - (or: `sister(X,Y) :- parent(Z,X), parent(Z,Y), parent(W,X), parent(W,Y), female(X), different(Z,W), different(X,Y).`)

## Clauses and Instantiation

- Facts are clauses without body
- Rules are clauses with both heads and non-empty bodies
- Queries are clauses that only have a body (!)
- When variables are substituted by constants, we say that they are *instantiated*.

## Universal Quantification

- Variables are universally quantified, but beware of variables that only appear in the body, as in
- *haschild( X) :- parent( X,Y).*
- which is best read as:  
*for all X,*  
 X has a child if  
*there exists* some Y such that X is a parent of Y
  - (I.e.: for all X and Y, if X is a parent of Y, then X has a child)

## Ancestor

- *ancestor( X,Z) :- parent( X,Z).*
  - *ancestor( X,Z) :- parent( X,Y), parent(Y,Z).*
  - *ancestor( X,Z) :- parent( X,Y1),  
                                   parent( Y1,Y2,),  
                                   parent( Y2,Z).*
- etc.
- When do we stop?
  - The length of chain of people between the predecessor and the successor should not arbitrarily bounded.



## A Recursive Rule

- For all X and Z,  
X is a predecessor of Z if  
there is a Y such that  
(1) X is a parent of Y and  
(2) Y is a predecessor of Z.
- predecessor( X,Z) :-  
parent( X,Y),  
predecessor( Y,Z).

## The Family Program

- Comments
  - /\* This is a comment \*/
  - % This comment goes to the end of the line
- SWI Prolog warns us when the clauses defining a relation are not contiguous.

```

% Figure 1.8    The family program.

parent( pam, bob).      % Pam is a parent of Bob
parent( tom, bob).
parent( tom, liz).
parent( bob, ann).
parent( bob, pat).
parent( pat, jim).

female( pam).           % Pam is female
male( tom).             % Tom is male
male( bob).
female( liz).
female( ann).
female( pat).
male( jim).

offspring( Y, X) :-      % Y is an offspring of X if
    parent( X, Y).      % X is a parent of Y

mother( X, Y) :-         % X is the mother of Y if
    parent( X, Y),      % X is a parent of Y and
    female( X).         % X is female

grandparent( X, Z) :-    % X is a grandparent of Z if
    parent( X, Y),      % X is a parent of Y and
    parent( Y, Z).      % Y is a parent of Z

sister( X, Y) :-         % X is a sister of Y if
    parent( Z, X),      % X and Y have the same parent and
    parent( Z, Y),      % X is female and
    female( X),         % X and Y are different
    different( X, Y).

predecessor( X, Z) :-    % Rule pr1: X is a predecessor of Z
    parent( X, Z).

predecessor( X, Z) :-    % Rule pr2: X is a predecessor of Z
    parent( X, Y),
    predecessor( Y, Z).

```

## Declarative Sorting

`sort1(A, B) :- permutation(A,B), sorted(B).`

`permutation([],[]).`

`permutation(B, [A|D]) :- del(A,B,C),  
 permutation(C,D).`

`sorted([]).`

`sorted([X]).`

`sorted([A, B | C]) :- A=<B, sorted([B|C]).`

`del(A, [A|B], B).`

`del(B, [A|C], [A|D]) :- del(B, C, D).`

## Declarative and Procedural Meaning of Prolog Programs

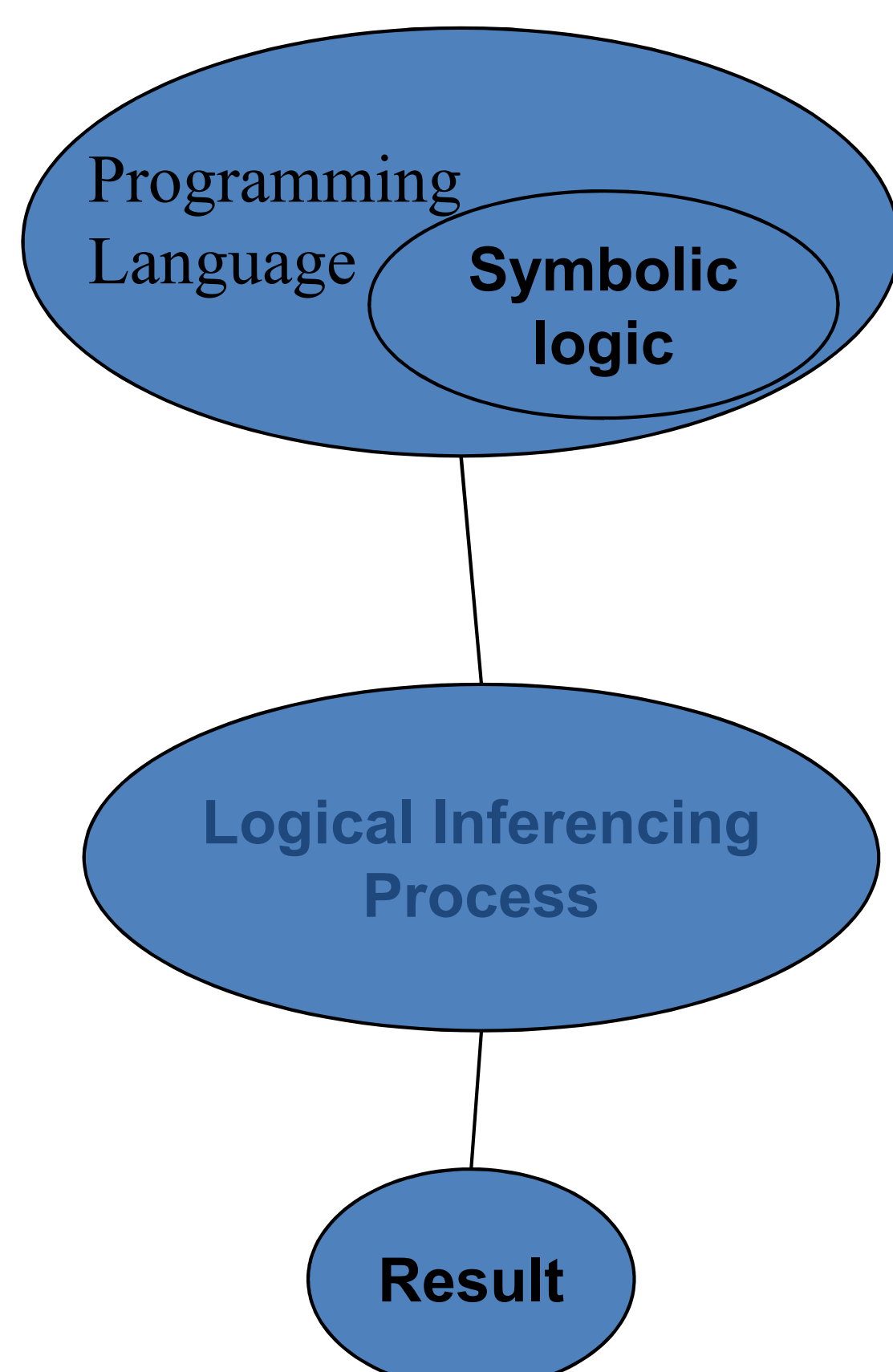
- The declarative meaning is concerned with the relations defined by the program: what the program states and logically entails
- The procedural meaning is concerned with how the output of the program is obtained, i.e., how the relations are actually evaluated by the Prolog system
- It is best to concentrate on the declarative meaning when writing Prolog programs
- Unfortunately, sometimes the programmer must also consider procedural aspect (for reasons of efficiency or even correctness)

## Logic Programming Languages

## Objective

- To introduce the concepts of logic programming and logic programming languages
- To introduce a brief description of a subset of prolog

## Introduction



- Logic programs are declarative programs
  - Specify the desired results
    - true
  - State the fact

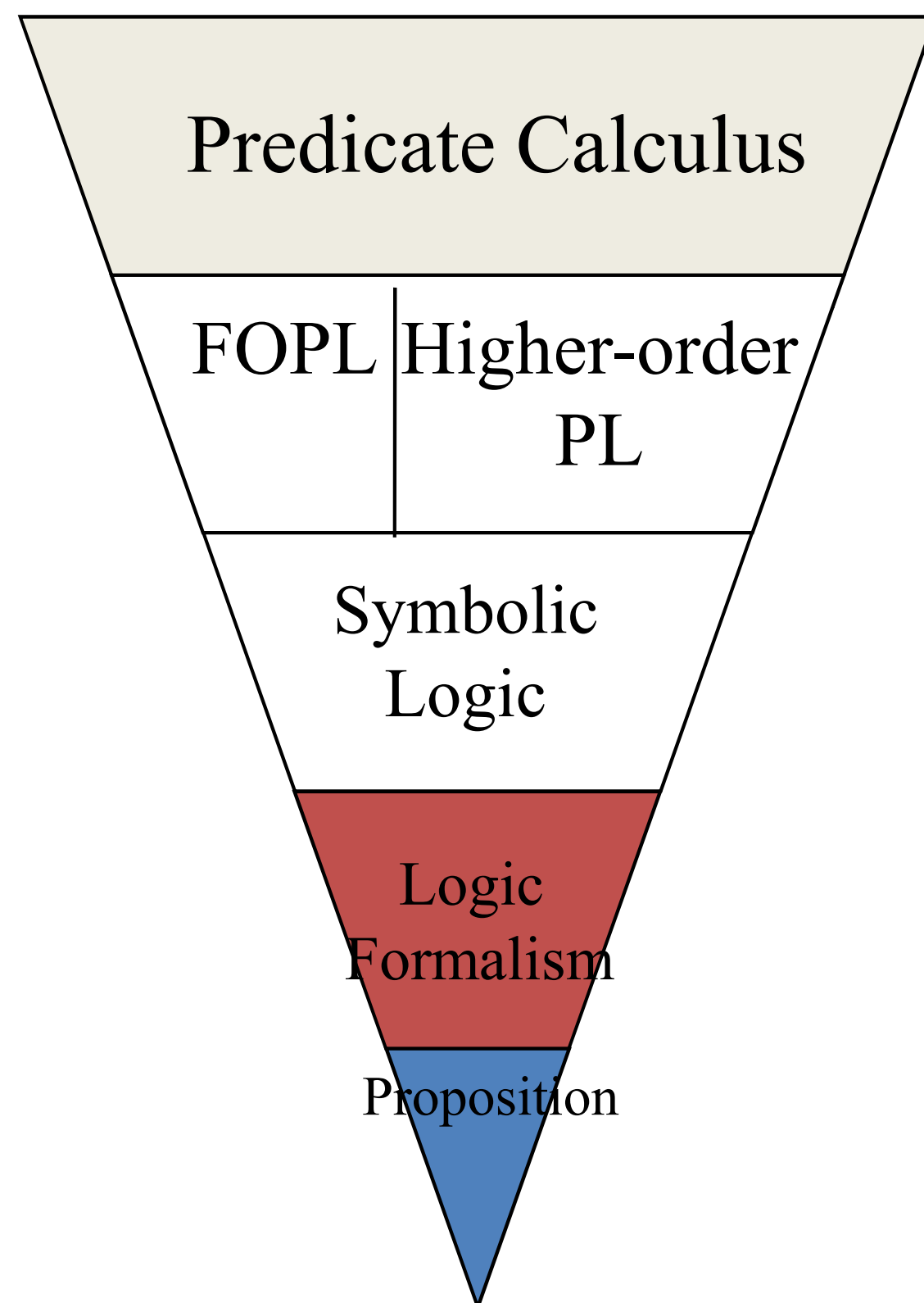
## Introduction

- The major difference between logic programming and other programming languages (imperative and functional)
  - Every data item that exist in logic programming has written in specific representation (symbolic logic)
- Prolog is a logic programming that widely used logic language

## Introduction

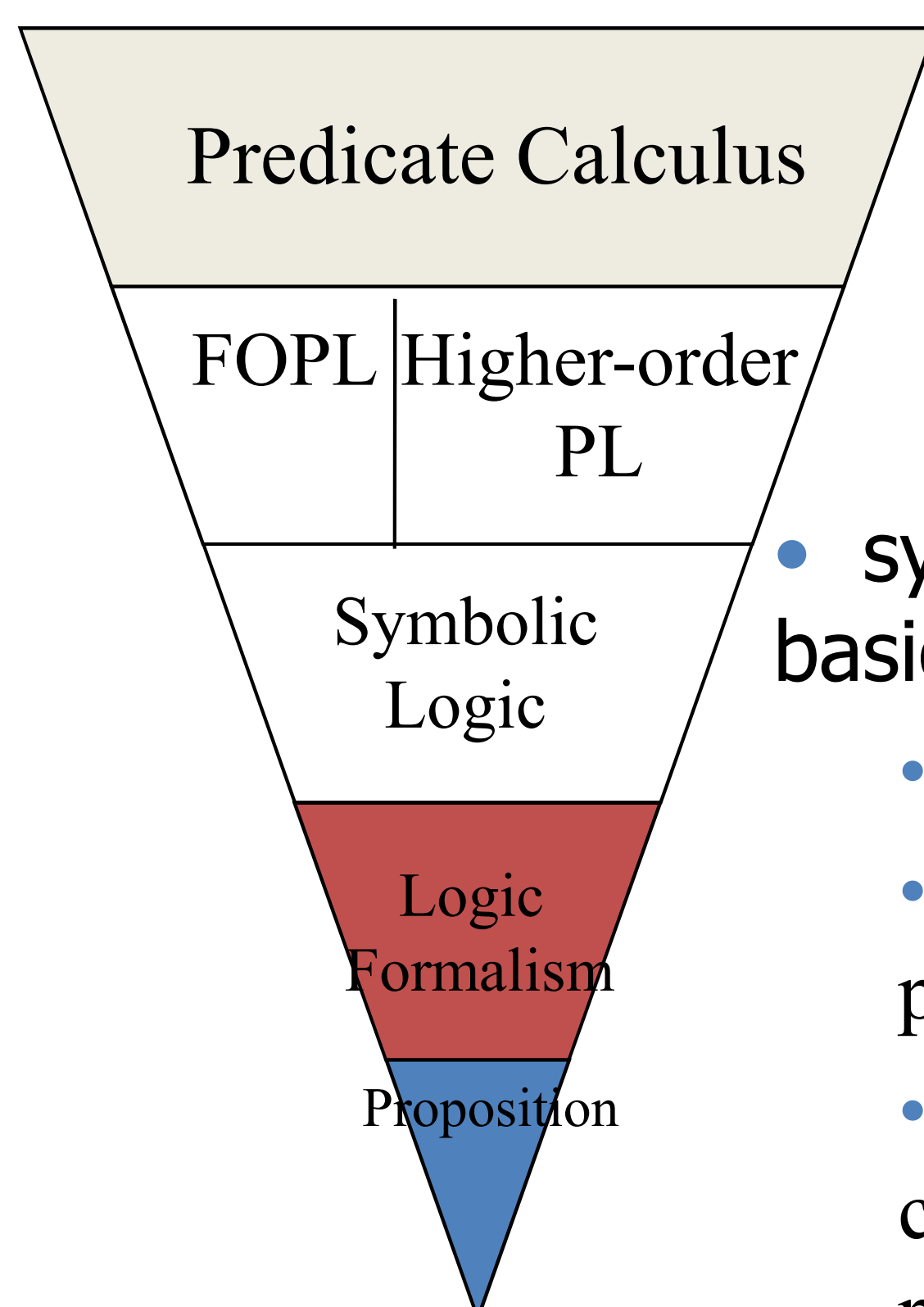
- Prolog specified the way of computer carries out the computation and it is divided to 3 parts:
  - logical declarative semantic of prolog
  - new fact prolog can infer from the given fact
  - explicit control information supplied by the programmer

## Symbolic representation: Predicate Calculus



- mathematical representation of formal logic
  - is a particular form of symbolic logic that is used for logic programming

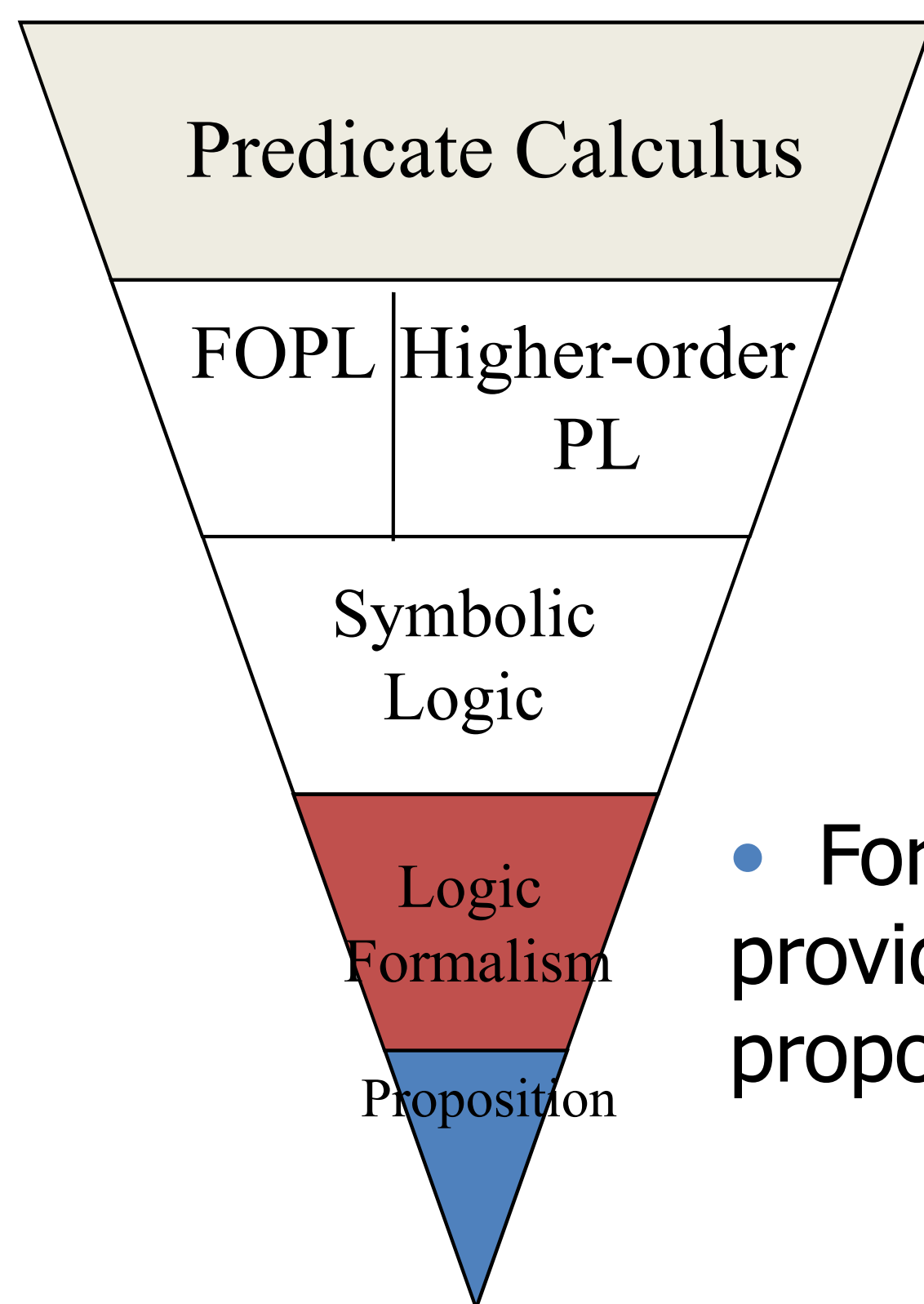
## Symbolic representation: Predicate Calculus



- symbolic logic used for the three basic need of formal logic
  - to express propositions
  - to express the relationships between propositions
  - to describe how new propositions can be inferred from other propositions that are assumed to be true

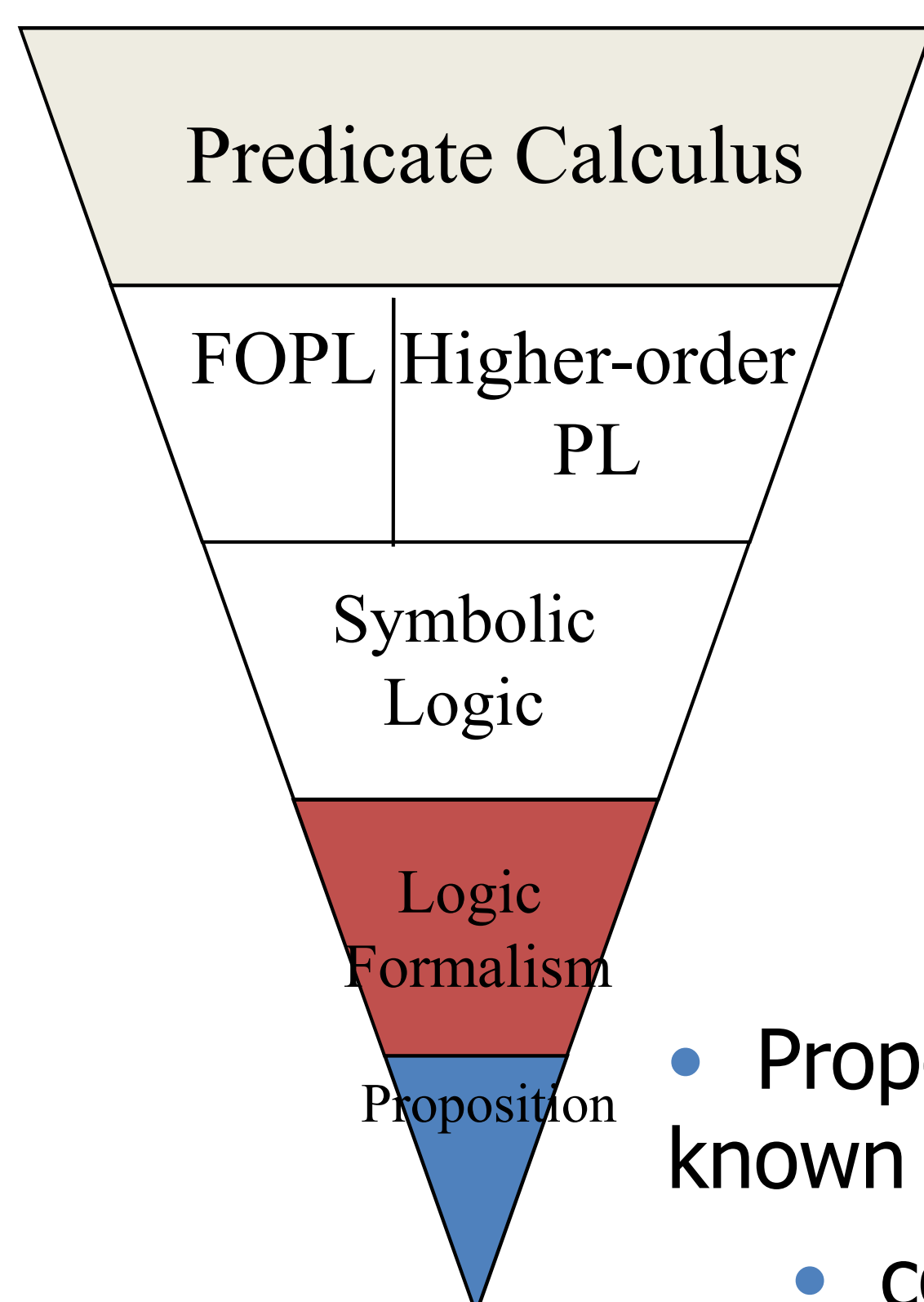


## Symbolic representation: Predicate Calculus



- Formal logic was developed to provide a method for describing proposition.

## Symbolic representation: Predicate Calculus



- Proposition is a logical statement also known as fact
  - consist of object and relationships of object to each other

## Proposition

- Object:
  - Constant represents an object, or
  - Variable represent different objects at different times
- Simple proposition called as atomic propositions, consist of compound terms – one element of mathematic relation which written in a form that has the appearance of mathematical function notation.

Example (constants):

single parameter (1-tuple): `man(jake)`

double parameter (2-tuples): `like(bob, steak)`

## Proposition

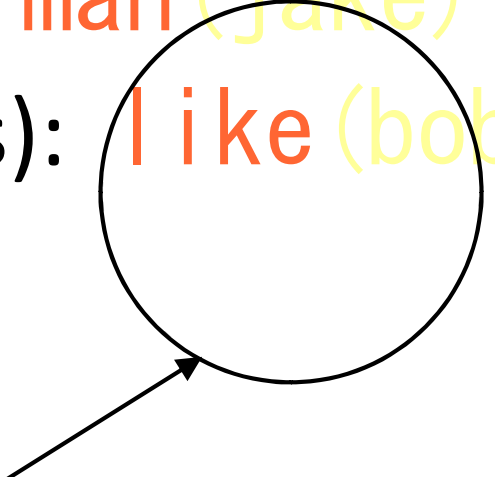
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Example:

single parameter (1-tuple): `man(jake)`

double parameter (2-tuples): `like(bob, steak)`

*functor shows the  
names the relation*



## Proposition

- Object:
  - Constant represents an object, or
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- Simple proposition called as atomic propositions, consist of compound terms – one element of mathematic relation

Example:

single parameter (1-tuple): `man(jake)`

double parameter (2-tuples): `like(bob, steak)`



*list of parameter*

## Proposition

- Two modes for proposition:
  - proposition defined to be true (fact), and
  - the truth of the proposition is something that is to be determined (queries)
- Compound propositions have two or more atomic proposition, which are connected by logical operator (is the same way logic expression in imperative languages)

# Logic operators

<i>Name</i>	<i>Symbol</i>	<i>Example</i>	<i>Meaning</i>
negation	$\neg$	$\neg a$	not a
conjunction	$\cap$	$a \cap b$	a and b
disjunction	$\cup$	$a \cup b$	a or b
equivalence	$\equiv$	$a \equiv b$	a is equivalent to b
implication	$\supset$	$a \supset b$	a implies b
	$\subset$	$a \subset b$	b implies a

# Compound propositions

Example:

$a \cap b \supset c$   
 $a \cap \neg b \supset d$   
 $(a \cap (\neg b)) \supset d$

Precedence:

$\neg$   
 $\cap \cup \equiv$   
 $\supset \subset$

higher  
  
  
lower

## Variables in Proposition

- Variable known as *quantifiers*
- Predicate calculus includes two quantifiers,  $X$  – variable, and  $P$  – proposition

<i>Name</i>	<i>Example</i>	<i>Meaning</i>
universal	$\forall X, P$	For all $X$ , $P$ is true
existential	$\exists X, P$	There exists a value of $X$ such that $P$ is true

## Variables in Proposition

### Example

$\forall X. (\text{woman}(X) \supset \text{human}(X))$

$\exists X. (\text{mother}(\text{mary}, X) \wedge \text{male}(X))$

## Variables in Proposition

### Example

$\forall X. (\text{woman}(X) \supset \text{human}(X))$

$\Rightarrow$  for any value of X, if X is a woman, then X is a human (NL: woman is a human)

$\exists X. (\text{mother}(\text{mary}, X) \cap \text{male}(X))$

## Variables in Proposition

### Example

$\forall X. (\text{woman}(X) \supset \text{human}(X))$

$\Rightarrow$  for any value of X, if X is a woman, then X is a human (NL: woman is a human)

$\exists X. (\text{mother}(\text{mary}, X) \cap \text{male}(X))$

$\Rightarrow$  there exist a value of X such that mary is the mother of X and X is a male (NL: mary has a son)

## Clausal Form

- Simple form of proposition, it is a standard form for proposition without loss of generality
- Why we need to transform PC into CF?
  - too many different ways of stating propositions that have the same meaning

Example:

$\forall X. (\text{woman}(X) \supset \text{human}(X))$

$\forall X. (\text{man}(X) \supset \text{human}(X))$

## Clausal Form

- General syntax for CF
 
$$B_1 \cup B_2 \cup \dots \cup B_n \subset A_1 \cap A_2 \cap \dots \cap A_m$$

$\Rightarrow$  if all the As are true, then at least one B is true

Example:

$\text{human}(X) \subset \text{woman}(X) \cap \text{man}(X)$

$\text{likes}(\text{bob}, \text{trout}) \subset \text{likes}(\text{bob}, \text{fish}) \cap \text{fish}(\text{trout})$



## Clausal Form

Example:

$$\text{likes}(\text{bob}, \text{trout}) \subset \text{likes}(\text{bob}, \text{fish}) \cap \text{fish}(\text{trout})$$

↑  
consequent

↑  
antecedent

- Characteristics of CF:
  - Existential quantifiers are not required
  - Universal quantifiers are implicit in the use of variables in the atomic propositions
  - No operator other than conjunction and disjunction are required

## Clausal Form

Example:

$$\text{likes}(\text{bob}, \text{trout}) \subset \text{likes}(\text{bob}, \text{fish}) \cap \text{fish}(\text{trout})$$

$\Rightarrow$  if bob likes fish and trout is a fish, then bob likes trout

## Clausal Form

Example:

$$\text{father}(\text{louis}, \text{al}) \cup \text{father}(\text{louis}, \text{violet}) \subset \\ \text{father}(\text{al}, \text{bob}) \cap \text{mother}(\text{violet}, \text{bob}) \cap \\ \text{grandfather}(\text{louis}, \text{bob})$$

$\Rightarrow$  if al is bob's father and violet is bob's mother and louis is bob's grandfather, louis is either al's father or violet's father

## Proving Theorems

- Method to infer the collection of proposition
  - use a collection of proposition to determine whether any interesting or useful fact can be inferred from them
- Introduced by Alan Robinson (1965)

## Proving Theorems

- Alan Robinson introduced resolution in automatic theorem proving
  - resolution is an inference rule that allows inferred proposition to be computed from given propositions
  - resolution was devised to be applied to propositions in clausal form

## Proving Theorems

- Idea of resolution:

$P1 \subset P2$  and  $Q1 \subset Q2$

which given

$P1$  is identical to  $Q2$

$\therefore Q1 \subset P2$

## Proving Theorems

Example:

$\text{older}(\text{joanne}, \text{jake}) \subset \text{mother}(\text{joanne}, \text{jake})$

$\text{wiser}(\text{joanne}, \text{jake}) \subset \text{older}(\text{joanne}, \text{jake})$

$\therefore \text{wiser}(\text{joanne}, \text{jake}) \subset \text{mother}(\text{joanne}, \text{jake})$

## Proving Theorems

Example:

$\text{father}(\text{bob}, \text{jake}) \cup \text{mother}(\text{bob}, \text{jake}) \subset \text{parent}(\text{bob}, \text{jake})$

$\text{gfather}(\text{bob}, \text{fred}) \subset \text{father}(\text{bob}, \text{jake}) \cap \text{father}(\text{jake}, \text{fred})$

$\therefore \text{gfather}(\text{bob}, \text{fred}) \cup \text{mother}(\text{bob}, \text{jake})$   
 $\subset \text{parent}(\text{bob}, \text{jake}) \cap \text{father}(\text{jake}, \text{fred})$

## Proving Theorems

- Process of determining useful values for variables during resolution – unification
- Unification
  - Hypotheses : original propositions
  - Goal: presented in negation of the theorem
  - Proposition in unification must be presented in Horn Clauses

## Proving Theorems

- Horn Clauses:
  - Headed Horn Clauses  
 Example:  
 $\text{likes}(\text{bob}, \text{trout}) \subset \text{likes}(\text{bob}, \text{fish}) \cap \text{fish}(\text{trout})$
  - Headless Horn Clauses  
 Example:  
 $\text{father}(\text{bob}, \text{jake})$

## Applications of Symbolic Computation

- Relational databases
- Mathematical logic
- Abstract problem solving
- Understanding natural language
- Design automation
- Symbolic equation solving
- Biochemical structure analysis
- Many areas of artificial intelligent