Sorting Algorithms

The Sorting Problem

- **Input:**
  - A sequence of **n** numbers $a_1, a_2, \ldots, a_n$
- **Output:**
  - A permutation (reordering) $\mathbf{a_1', a_2', \ldots, a_n'}$ of input sequence such that $\mathbf{a_1' \leq a_2' \leq \cdots \leq a_n'}$

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Some Definitions

- **Internal Sort**
  - The data to be sorted is all stored in the computer's **main memory**.
- **External Sort**
  - Some of the data to be sorted might be stored in some **external, slower, device**.
- **In Place Sort**
  - The amount of **extra space** required to sort the data is **constant with the input size**.

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Insertion Sort

- **Idea: sorting a hand of playing cards**
  - Start with an empty left hand and the cards facing down on the table.
  - Remove **one card at a time** from the table, and insert it into the correct position in the left hand
    - **compare** it with each of the cards already in the hand, **from right to left**
  - The cards held in the left hand are sorted

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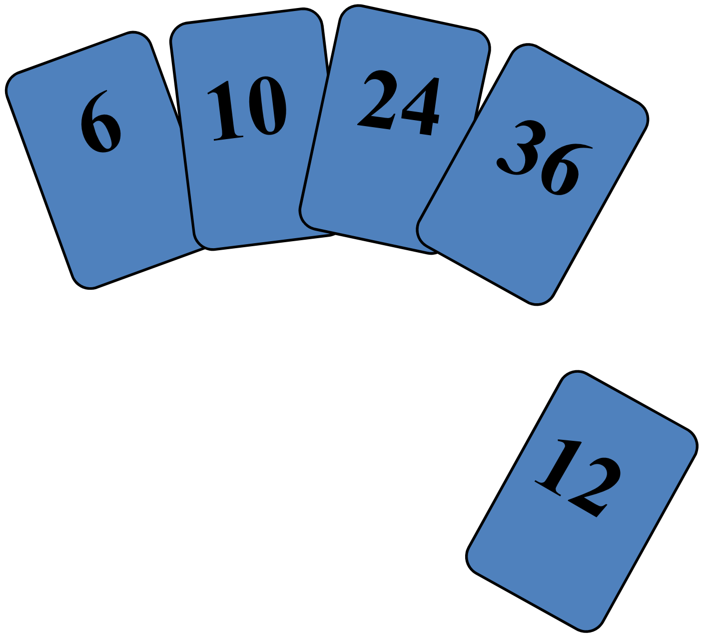
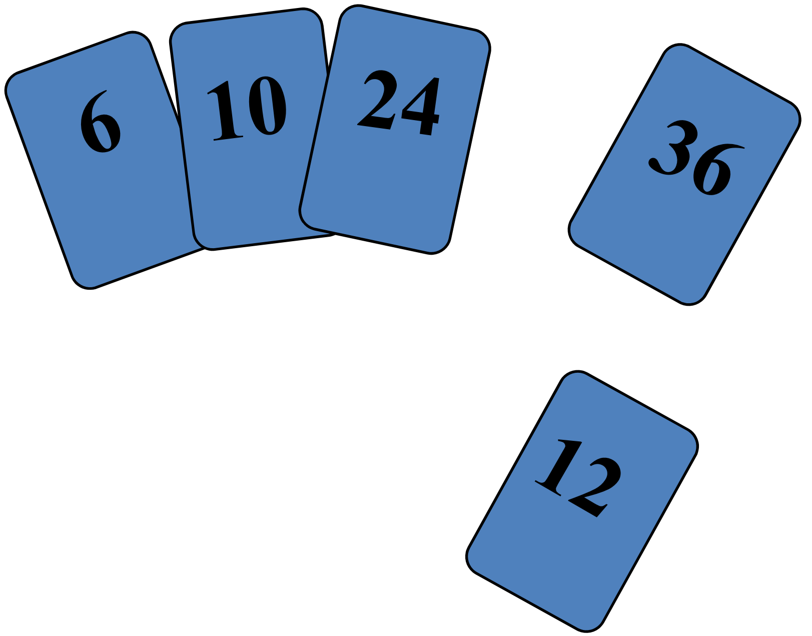
Insertion Sort



To insert 12, we need to make room for it by moving first 36 and then 24.

Insertion Sort



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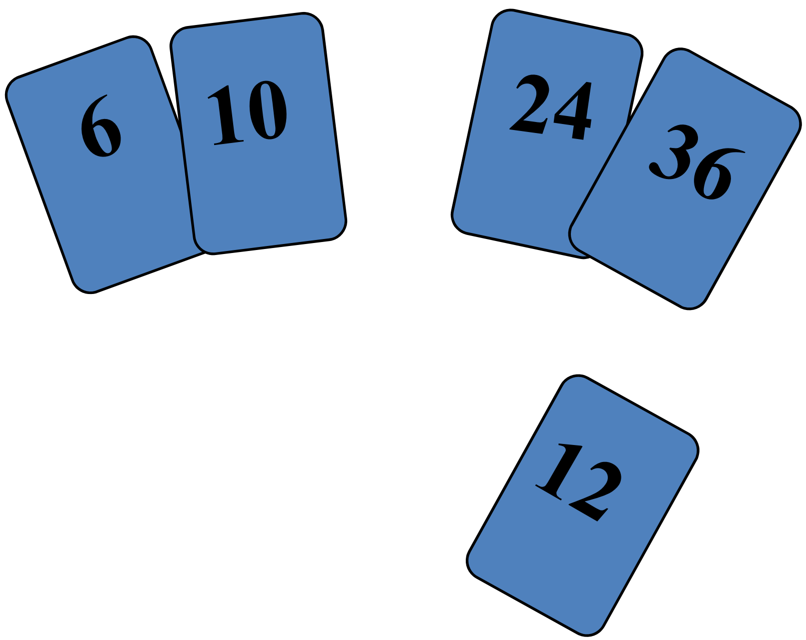
Insertion Sort



Insertion Sort

Sorts the elements in place

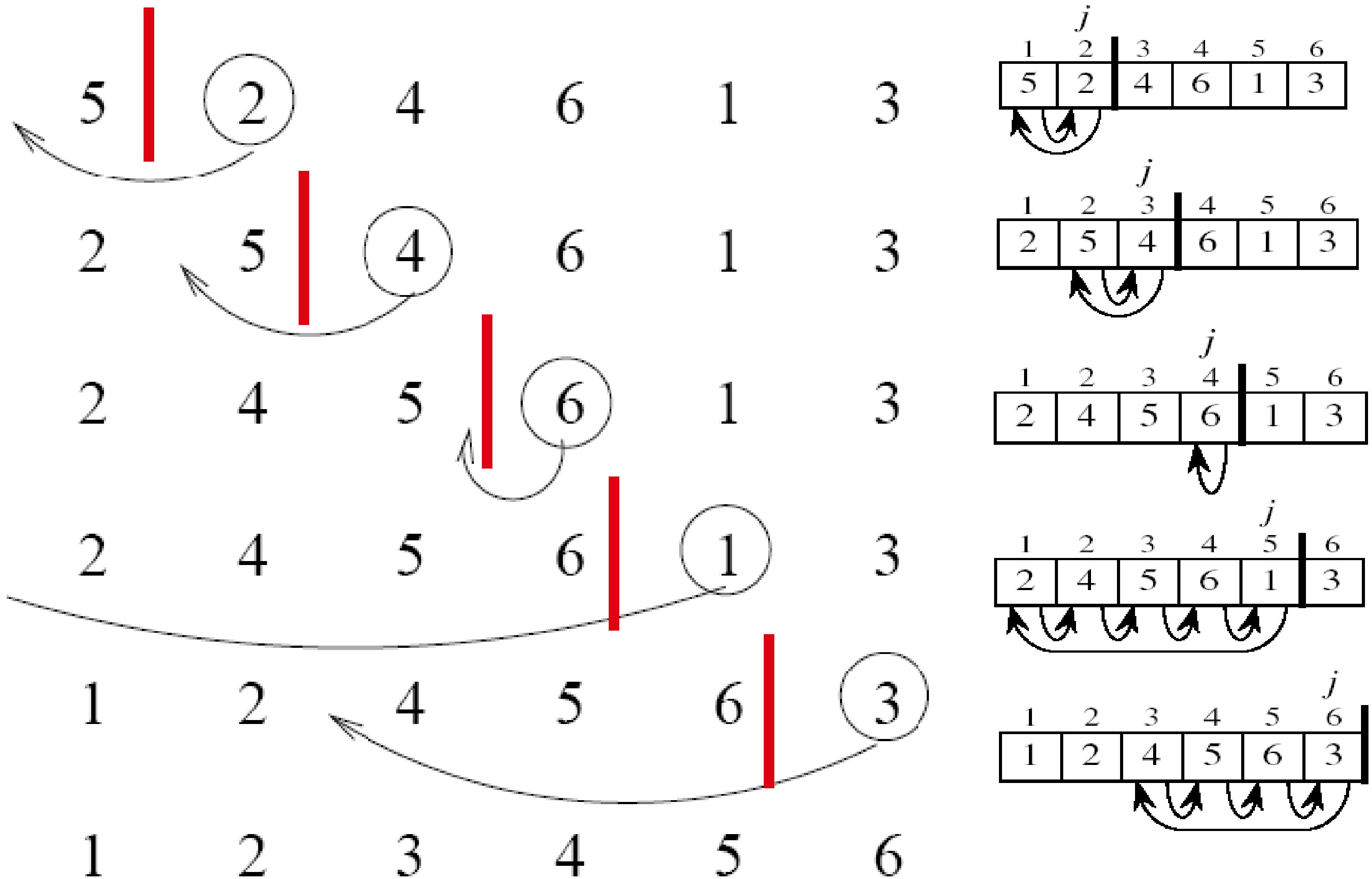
```
void insertionsort( int x[], int n )
{
    int temp, k, i;
    for( j = 2; j < = n; j ++)
    {
        temp = x[j];
        for( i = j-1; i >= 1 && temp < x[i]; i--)

                x[i+1]=x[i];
        x[i+1]=temp;
    }
}
```

temp ***holds the value to be inserted in the sorted array x [1..j-1]***

Shift the elements larger than temp to the right

Insertion Sort



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Insertion Sort

- **Best case**
  - Array is already sorted
  - Outer loop executes n-1 times
  - Complexity $T(n)$ = **$O(n)$**

- **Worst case**
  - Array is in reverse order
  - $T(n) = 1 + 2 + \ldots. + (n-1) =$ **$O(n^2)$**

Insertion Sort

- **Average case**
  - The probability that $\mathbf{k^{th}}$ insertion requiring 1, 2, .., k number of comparisons is same = $\mathbf{1/k}$.
  - The expected number of comparisons for $\mathbf{k^{th}}$ insertion =1/k + 2/k +……….+ k/k = $\mathbf{(k+1)/2}$

$$T(n)=2/2 + 3/2 + 4/2 + \ldots\ldots\ldots + n/2$$

$$= \mathbf{O(n^2)}$$

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Selection Sort

- **Idea:**
  - Find the **smallest** element in the array
  - Swap it with the element in the **first position**
  - Find the **second smallest** element and **swap** it with the element in the **second position**
  - Continue until the array is sorted

Selection Sort

```
void  selectionsort ( int a [], int n )
{
     int i, j, index;
     for ( i = 0; i < n ; i ++)
     {
          index=i;
          for ( j = i +1 ; j < n ; j++)
          {
               if ( a[j] < a[index] )
                    index = j;
          }
          swap ( &a[i] , &a[index] );
     }
}
```

8	4	6	9	2	3	1
1	4	6	9	2	3	8
1	2	6	9	4	3	8
1	2	3	9	4	6	8
1	2	3	4	9	6	8
1	2	3	4	6	9	8
1	2	3	4	6	8	9
1	2	3	4	6	8	9

Selection Sort

- Irrespective of the order of the element, you have to find out the minimum element in every iteration.
- Number of comparisons required to find out the minimum element in i^{th} iteration is n-i.

$$T(n)= (n-1) + (n-2) + \ldots\ldots\ldots\ldots + 3 + 2 + 1$$

$$=O(n^2)$$

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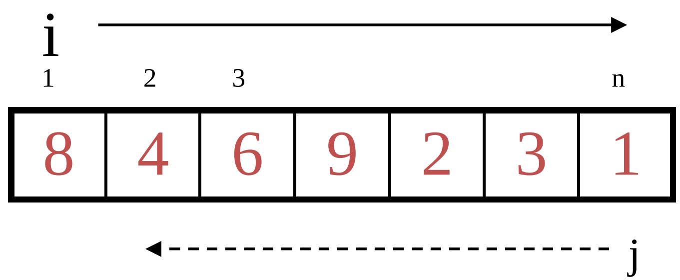
Bubble Sort

- **Idea:**
  - Repeatedly pass through the array
  - Swaps adjacent elements that are out of order



Bubble Sort

8	4	6	9	2	3	1

i = 1 j

8	4	6	9	2	1	3

i = 1 j

8	4	6	9	1	2	3

i = 1 j

8	4	6	1	9	2	3

i = 1 j

8	4	1	6	9	2	3

i = 1 j

8	1	4	6	9	2	3

i = 1 j

1	8	4	6	9	2	3

i = 1 j

1	8	4	6	9	2	3

i = 2 j

1	2	8	4	6	9	3

i = 3 j

1	2	3	8	4	6	9

i = 4 j

1	2	3	4	8	6	9

i = 5 j

1	2	3	4	6	8	9

i = 6 j

1	2	3	4	6	8	9

i = 7

j

Bubble Sort

```
Void bubblesort (int a [], int n)
{
    int i, j;
    for ( i = 1; i <= n; i++)
        for ( j = n; j > i; j --)
            if (a [j] < a [j-1])
                swap (&a [j], &a [j-1]);
}
```

Complexity = $O(n^2)$

Quick Sort

- Example of **Divide and Conquer** algorithm
- **Two phases**
  - Partition phase
    - Divides the work into half
  - Sort phase
    - Conquers the halves!

Quick Sort

- **Partition**
  - Choose a **pivot**
  - Find the position for the pivot so that
    - all elements to the **left are less**
    - all elements to the **right are greater**

< pivot	**pivot**	**> pivot**

Quick Sort

- **Conquer**
  - Apply the same algorithm to each half

< pivot				> pivot		
< p’	p’	> p’	pivot	< p”	p”	> p”

Quick Sort

```
quicksort( void *a, int low, int high )
{
    int pivot;
    if ( high > low )
    {
        pivot = partition( a, low, high );
        quicksort( a, low, pivot-1 );
        quicksort( a, pivot+1, high );
    }
}
```

Divide

Conquer

Quick Sort

```
int partition( int *a, int low, int high )
{
    int left, right, pivot_item;
    pivot_item = a[low];
    left = low + 1;
    right = high;
    while ( left < right )
    {
        while( a[left] < pivot_item ) left++;
        while( a[right] > pivot_item ) right--;
        if ( left < right ) SWAP(a, left, right);
    }
    a[low] = a[right];
    a[right] = pivot_item;
    return right;
}
```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Quick Sort

```
int partition( int *a, int low, int high )
{
    int left, right, pivot_item;
    pivot_item = a[low];
    left = low + 1;
    right = high;
    while ( left < right )
    {
        while( a[left] < pivot_item ) left++;
        while( a[right] > pivot_item ) right--;
        if
    }
    a[low] = a[right];
    a[right] = pivot_item;
    return r
}
```

Any item will do as the pivot, choose the leftmost one!

23	12	15	38	42	18	36	29	27

low high

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Quick Sort

```
int partition( int *a, int low, int high )
{
    int left, right, pivot_item;
    pivot_item = a[low];
    left = low + 1;
    right = high;
    while ( left < right )
    {
        whi      eft] < pivot_item ) left++;
        while( a[right] > pivot_item ) right--;
        if ( lef  < right ) SWAP(a, left, right);
    }
    a[l      = a[right];
    a[ri   t] = pivot_item;
    ret      ht;
}
```

Set left and right markers

left

right

23	12	15	38	42	18	36	29	27

low

pivot: 23

high

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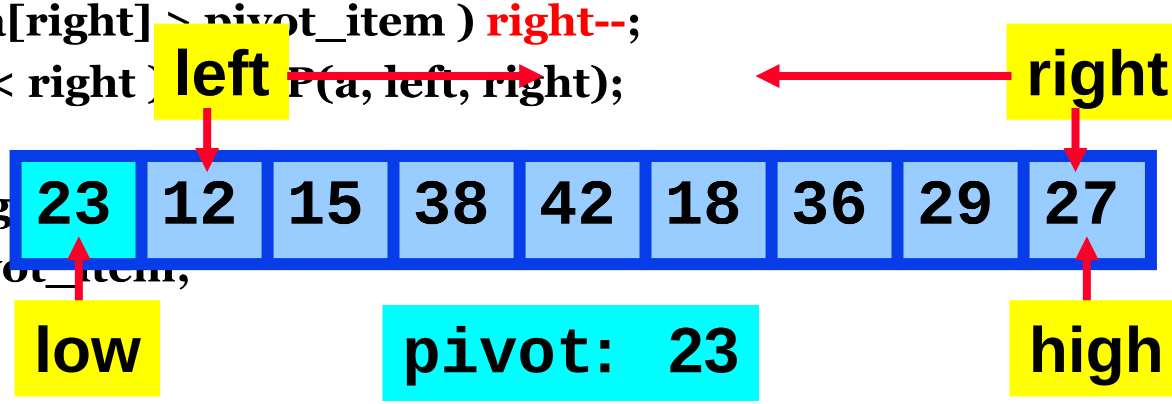
Quick Sort

```
int partition( int *a, int low, int high )
{
    int left, right, pivot_item;
    pivot_item = a[low];
    left = low + 1;
    right = high;
    while ( left < right )
    {
        while( a[left] < pivot_item ) left++;
        while( a[right] > pivot_item ) right--;
        if ( left < right ) P(a, left, right);
    }
    a[low] = a[rig
    a[right] = pivot_item;
    return right;
}
```

Move the markers until they cross over



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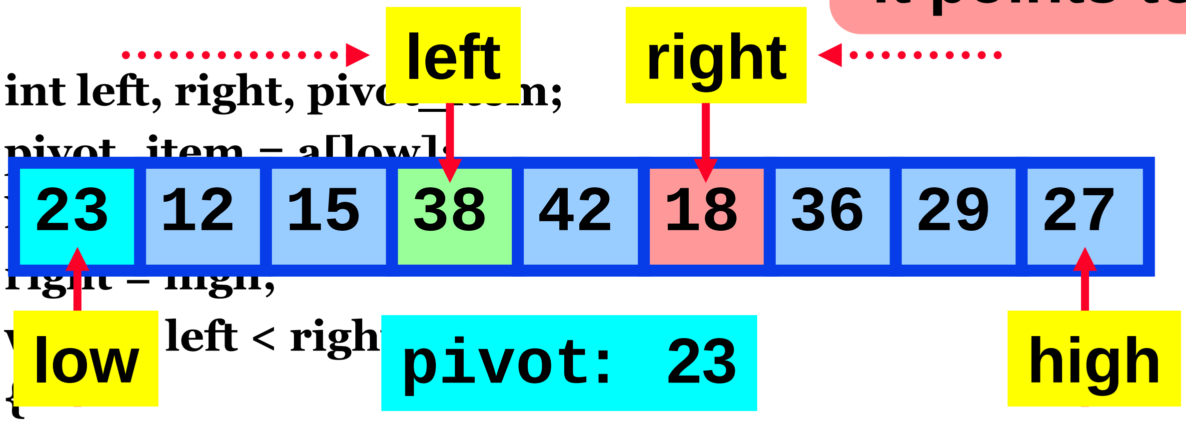
Quick Sort

Move the left pointer while it points to items <= pivot

Move right similarly

```
int partition( int *a, int low, int high )
{
    int left, right, pivot_item;
    pivot_item = a[low];
    right = high;
    while ( left < right )
    {
        while( a[left] < pivot_item ) left++;
        while( a[right] > pivot_item ) right--;
        if ( left < right ) SWAP(a, left, right);
    }
    a[low] = a[right];
    a[right] = pivot_item;
    return right;
}
```



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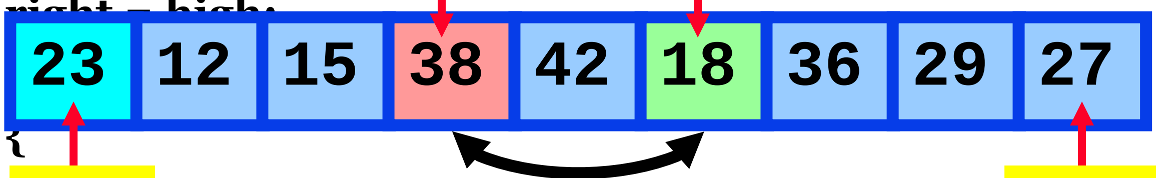
Quick Sort

```
int partition( int *a, int low, int hig
{
    int left, right, pivot_item;
    pivot_item = a[lo
    left = low + 1;
    right = high;
    {
        while( a[left] < pivot_item ) left++;
        while( a[right] > pivot_item ) right--;
        if ( left < right ) SWAP(a, left, right);
    }
    a[low] = a[right];
    a[right] = pivot_item;
    return right;
}
```



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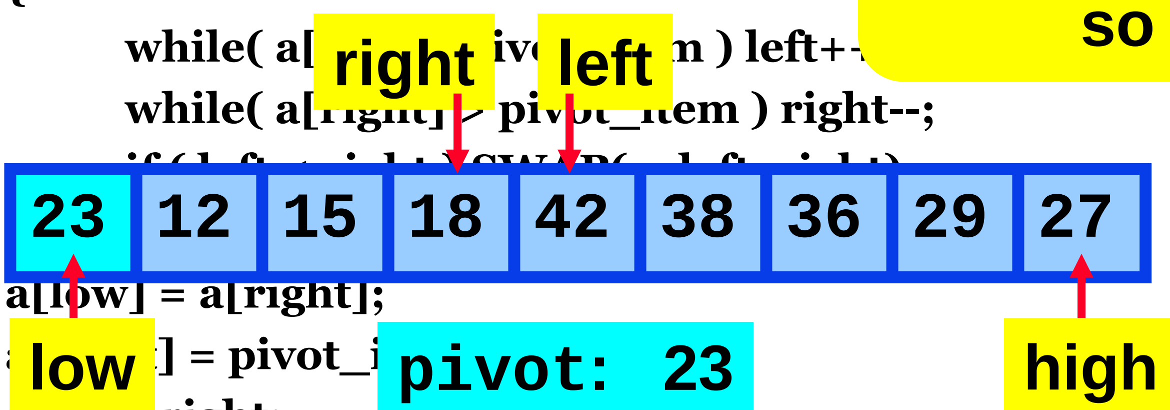
Quick Sort

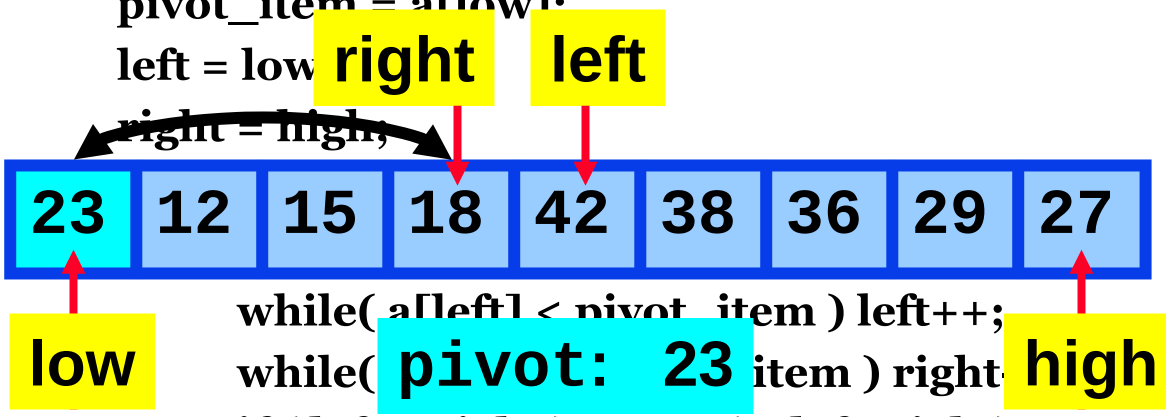
```
int partition( int *a, int low, int high )
{
    int left, right, pivot_item;
    pivot_item = a[low];
    left = low + 1;
    right = high;
    while ( left < right )
    {
        while( a[left] <= pivot_item ) left++;
        while( a[right] > pivot_item ) right--;
        if ( left < right ) SWAP(a,left,right);
    }
    a[low] = a[right];
    a[right] = pivot_item;
    return right;
}
```

`left` and `right` have swapped over, so stop



Quick Sort

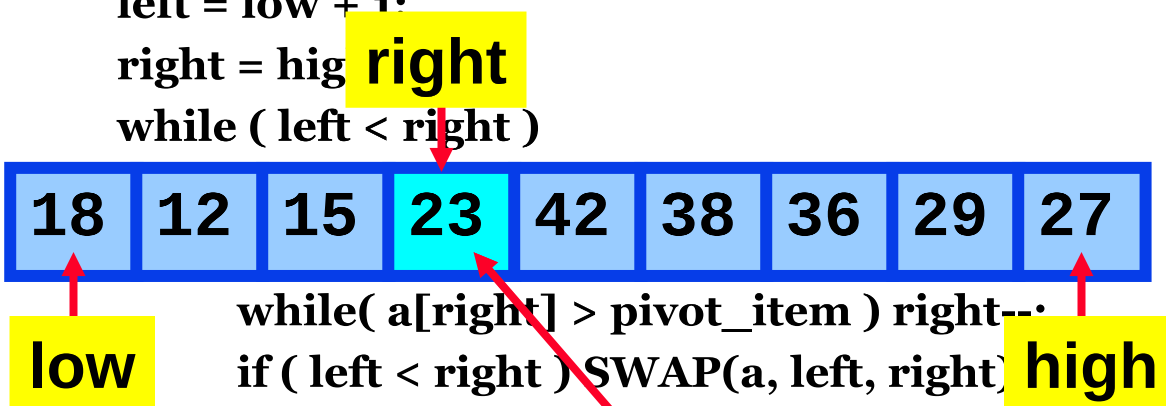
```
int partition( int *a, int low, int high )
{
    int left, right, pivot_item;
    pivot_item = a[low];
    left = low;
    right = high;
        while( a[left] < pivot_item ) left++;
        while( a[right] > pivot_item ) right--;
        if ( left < right ) SWAP(a, left, right);
    }
    a[low] = a[right];
    a[right] = pivot_item;
    return right;
}
```



Finally, swap the pivot and `right`

Quick Sort

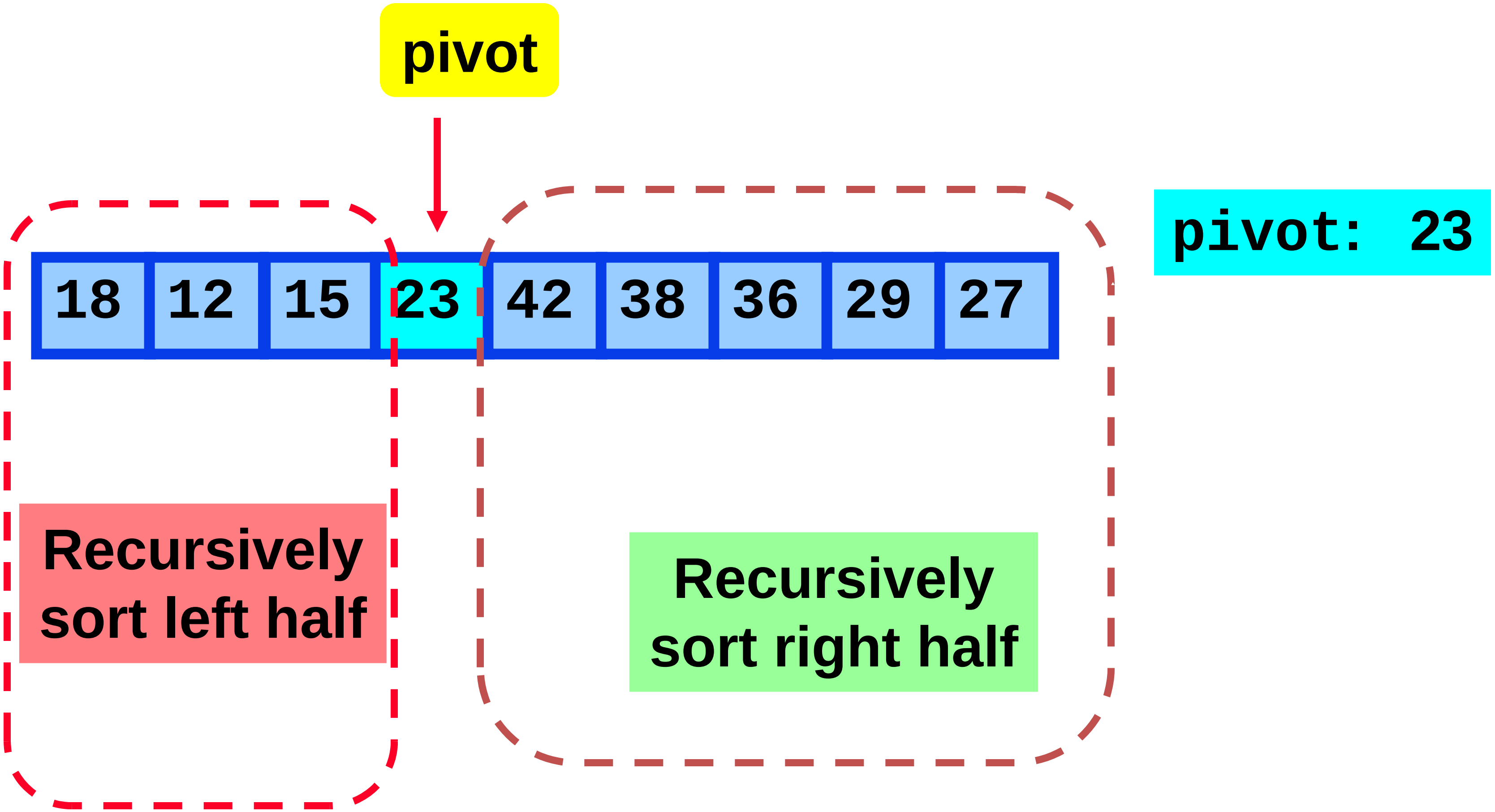
```
int partition( int *a, int low, int high )
{
    int left, right, pivot_item;
    pivot_item = a[low];
    left = low + 1;
    right = hig
    while ( left < right )

        while( a[right] > pivot_item ) right--;
        if ( left < right ) SWAP(a, left, right)
    }
    a[low] = a[right];
    a[right] = pivot_item;
    return right;
}
```



pivot: 23

Return the position of the pivot

Quick Sort



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Quick Sort

- **Partition**
  - Check every item once $O(n)$
- **Conquer**
  - Divide data in half $O(\log_2 n)$
- **Total**
  - Product $O(n \log n)$
- ***But there's a catch ....***

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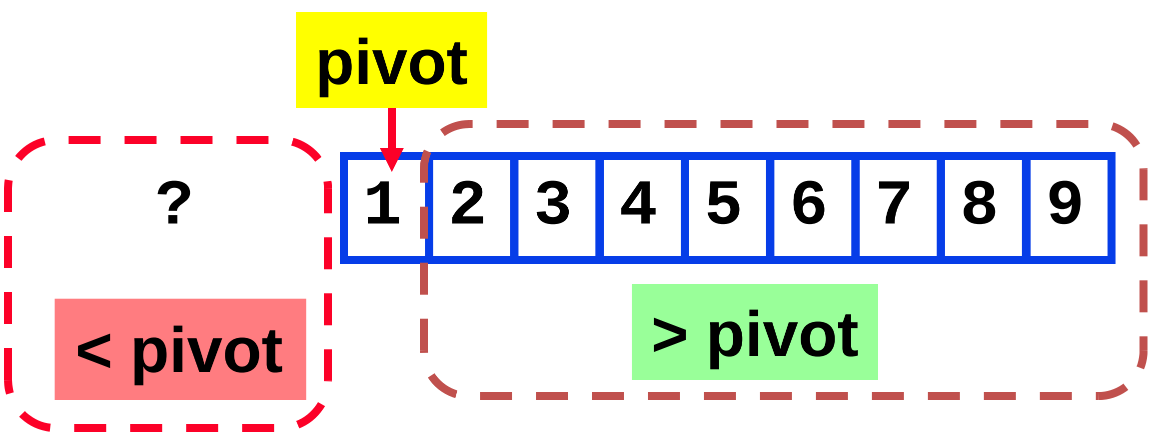
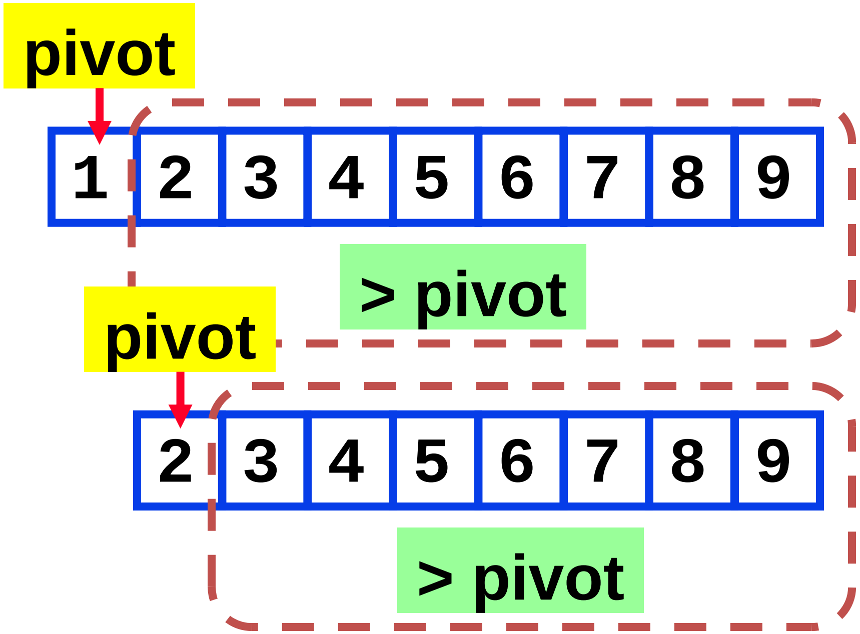
Quick Sort

- What happens if we use quicksort on data that's already sorted
  (or nearly sorted)
- ***We'd certainly expect it to perform well!!!!***



Quick Sort

- **Each partition produces**
  - a problem of size 0
  - and one of size *n*-1!
- **Number of partitions?**
  - *n* each needing time $O(n)$
  - Total $nO(n)$ or $O(n^2)$



? ***Quicksort is as bad as bubble or insertion sort***

Merge Sort

- **To sort an array A[p . . r]:**
  - **Divide**
    - Divide the n-element sequence to be sorted into two subsequences of n/2 elements each
  - **Conquer**
    - Sort the subsequences recursively using merge sort
      - When the size of the sequences is 1 there is **nothing more to do**
  - **Combine**
    - ***Merge*** **the two sorted subsequences**

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Merge Sort

```
void mergesort (int A [], int p, int r)
{
    int q;
    if ( p < r)
        {
            q = ( p + r ) / 2;        Divide
            mergesort ( A, p, q);
            mergesort (A, q+1, r);
            merge (A, p, q, r);       Combine
        }
}
```

Conquer

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Merge Sort

```
void merge( int A [], int p, int q, int r)
{
    int B [100], i, j, k;
    i = p;    j=q+1;     k = 0;
    while ( i <= q &&   j <= r)
    {
        if ( A [i] < A [j])
                B [ k ++] = A [ i ++];
        else
                B [ k ++] = A [ j ++];
    }
    while ( i <= q)
        B [ k ++] = A [ i ++];
    while ( j <= r)
        B [ k ++] = A [ j ++];
}
```


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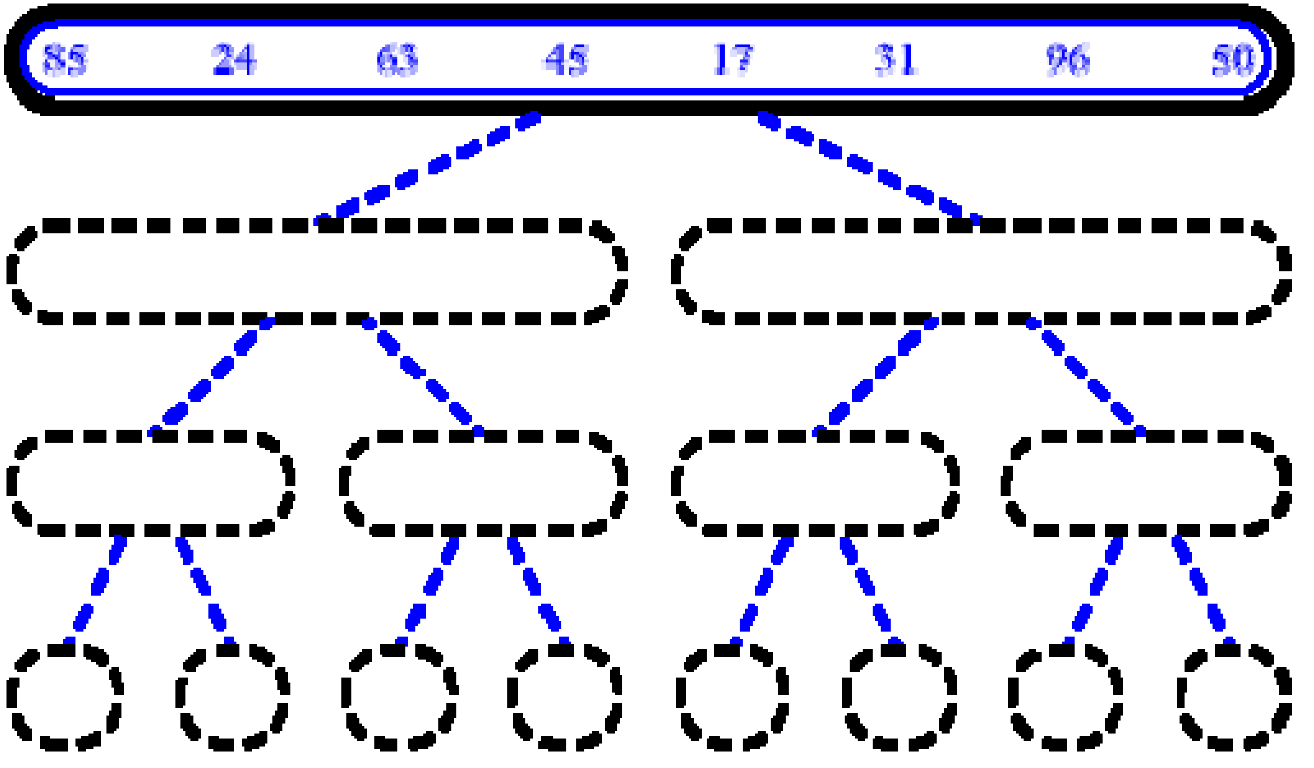
Merge Sort



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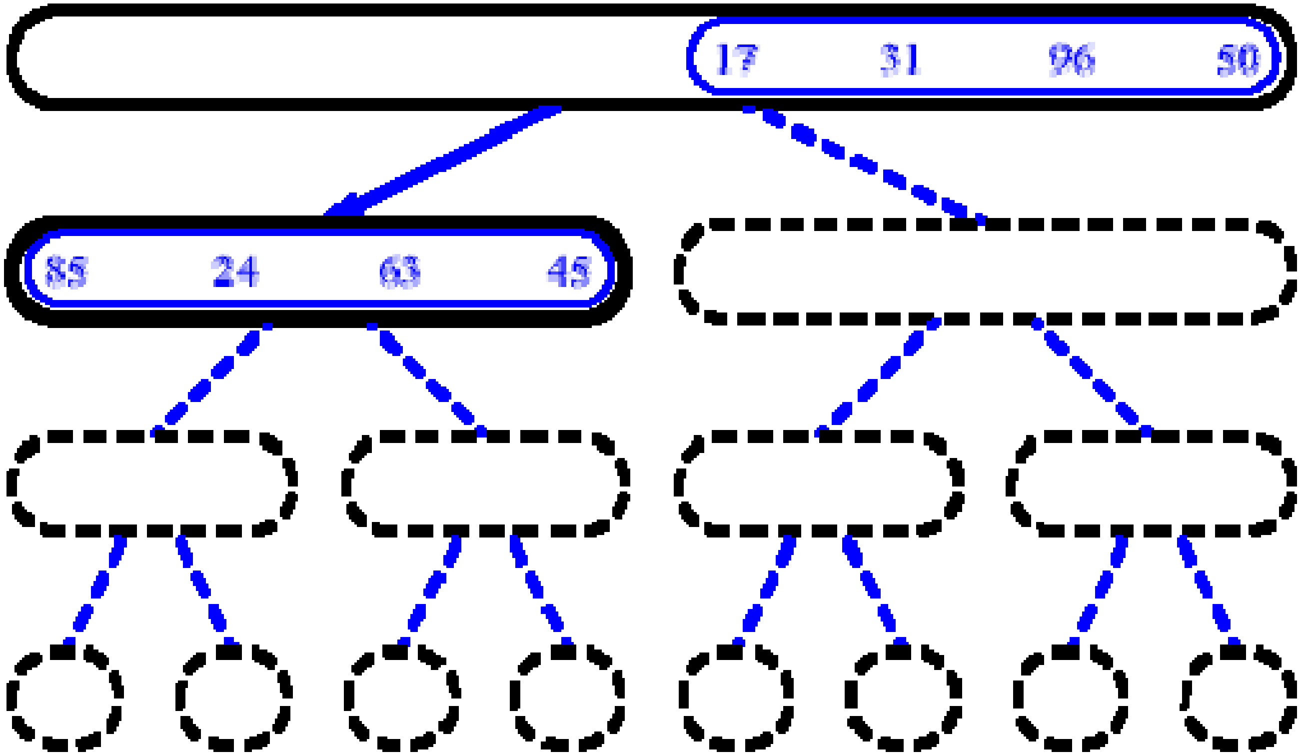
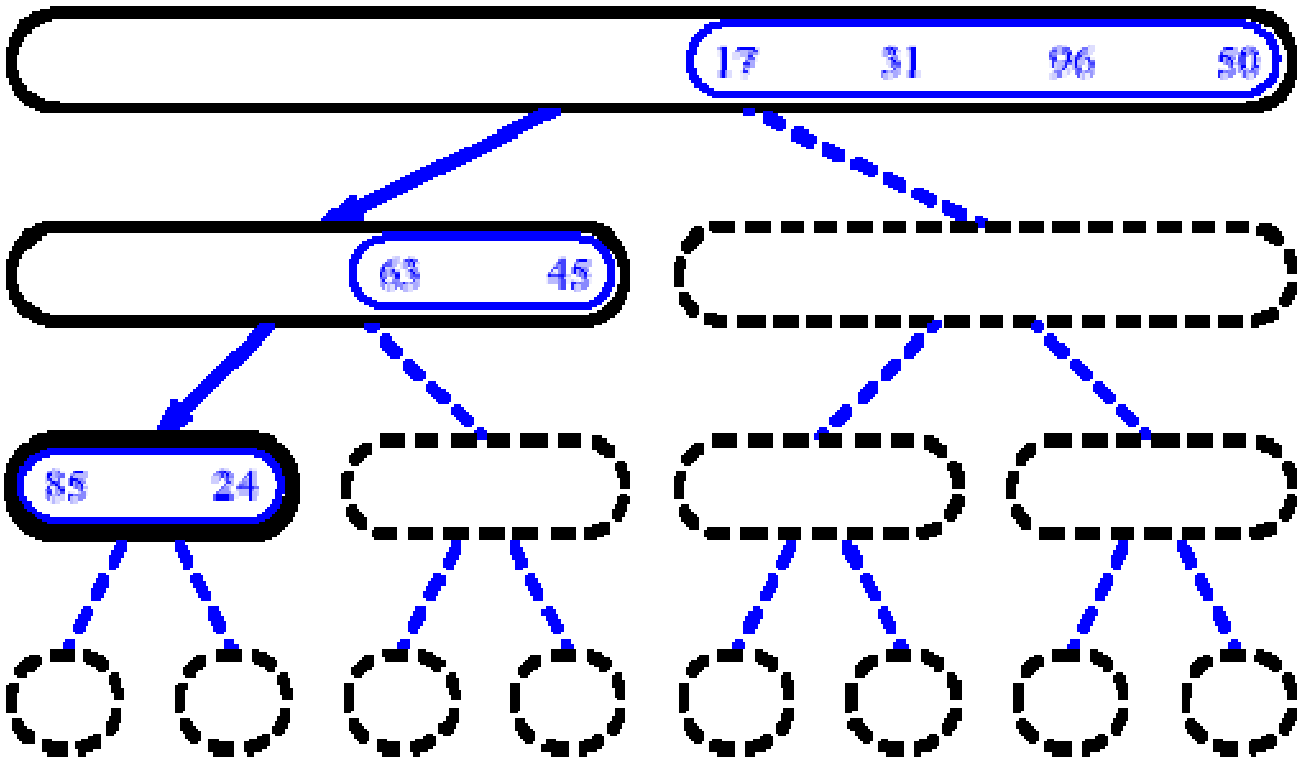
Merge Sort



Merge Sort



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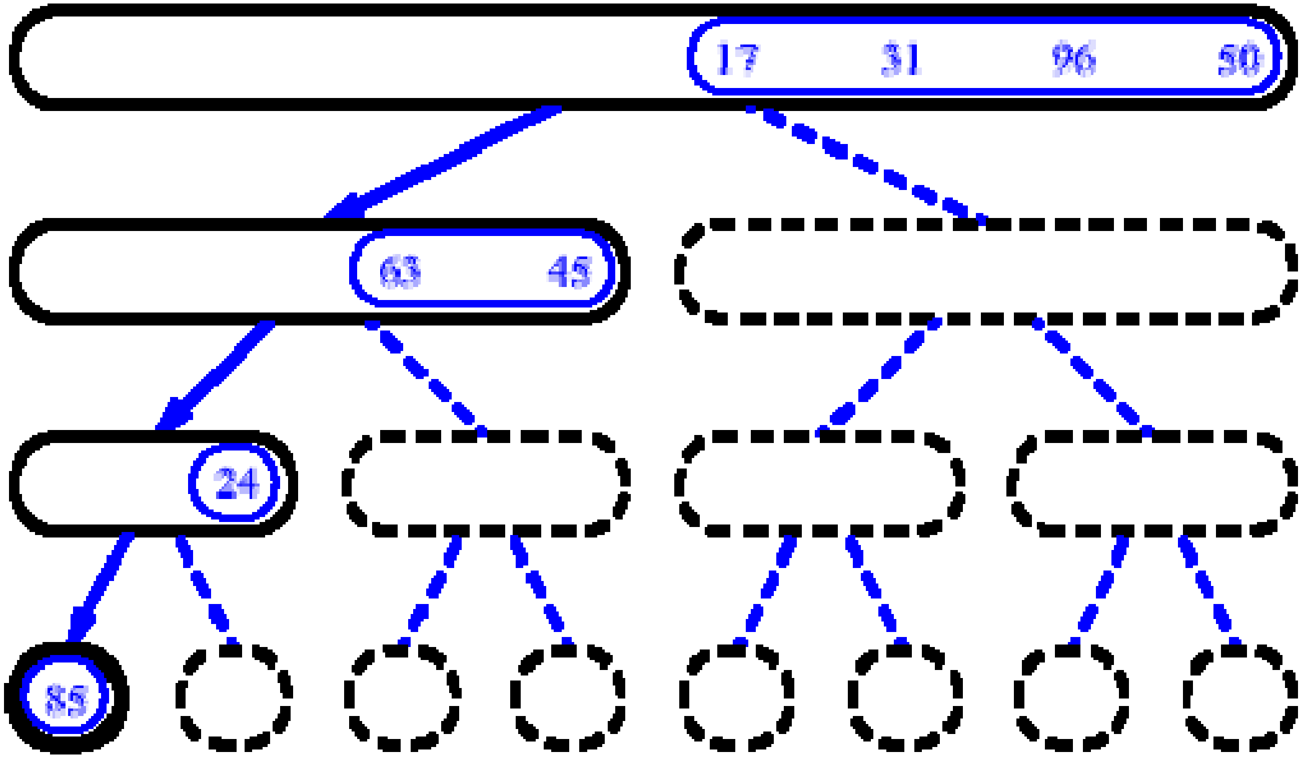
Merge Sort



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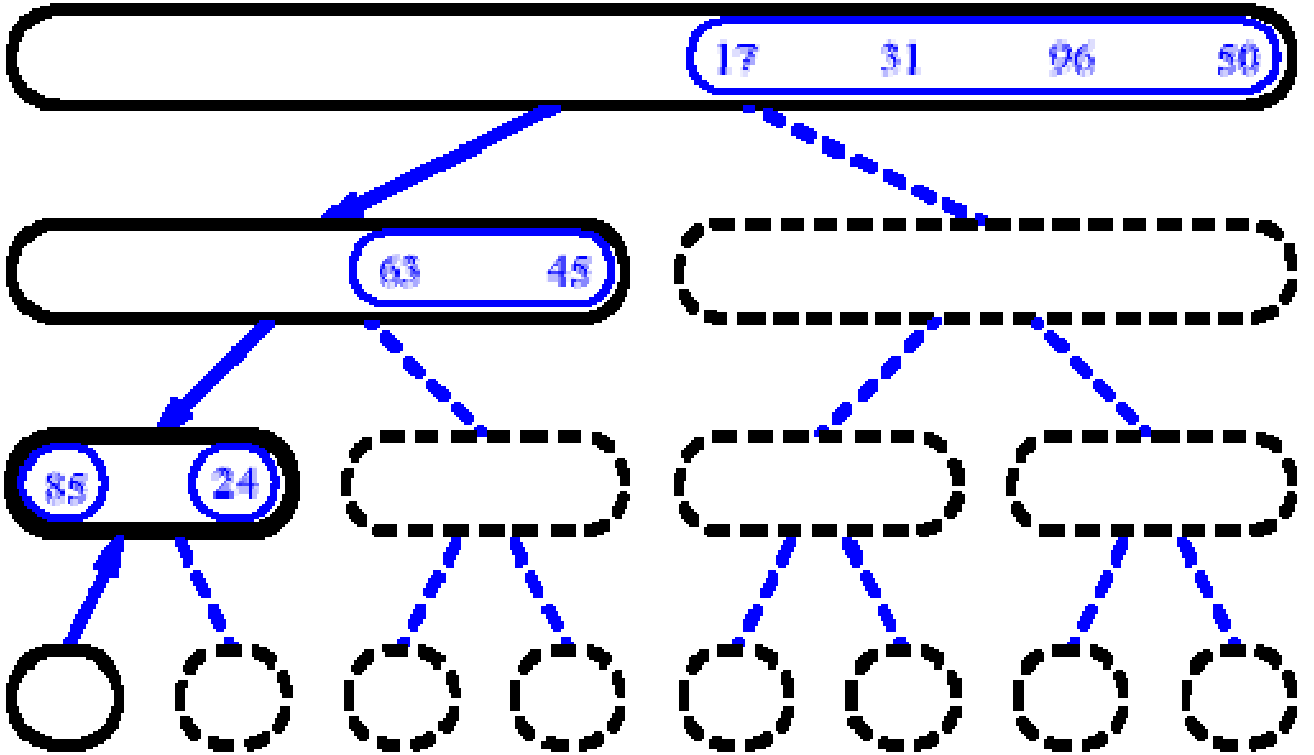
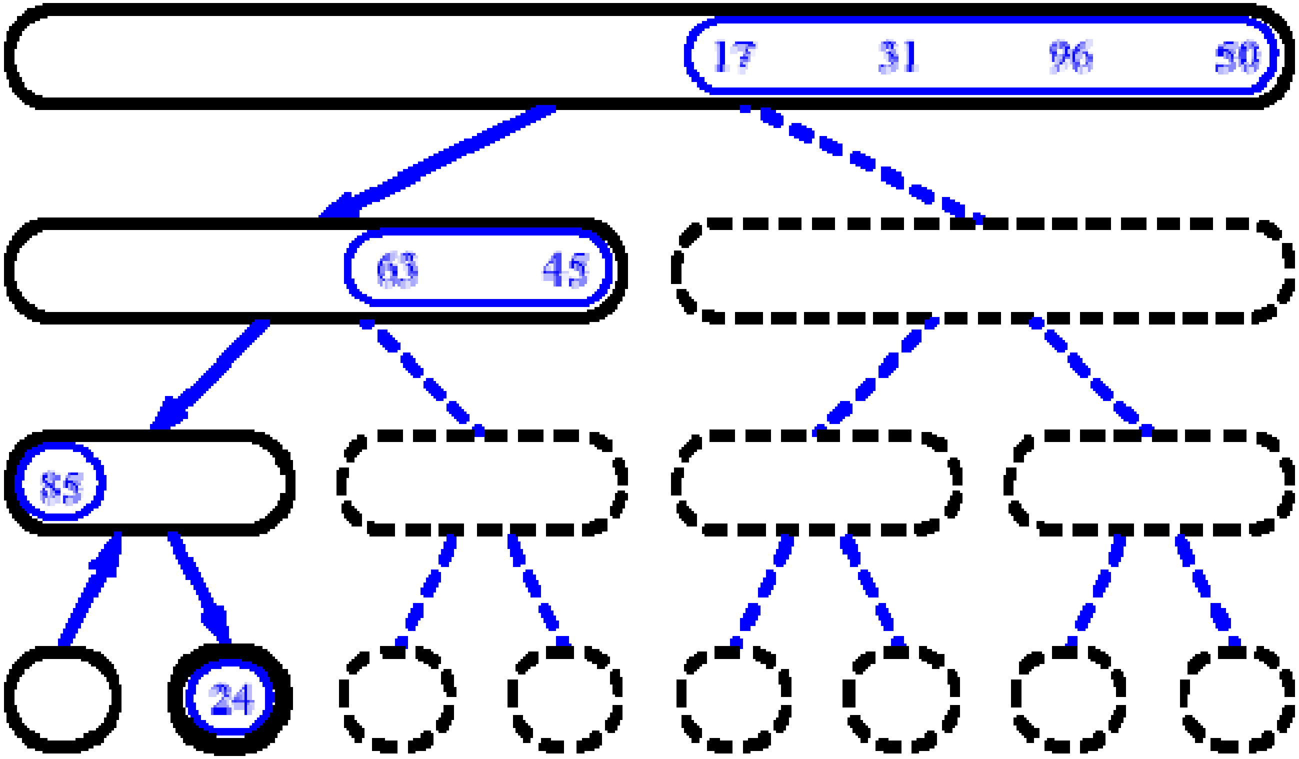
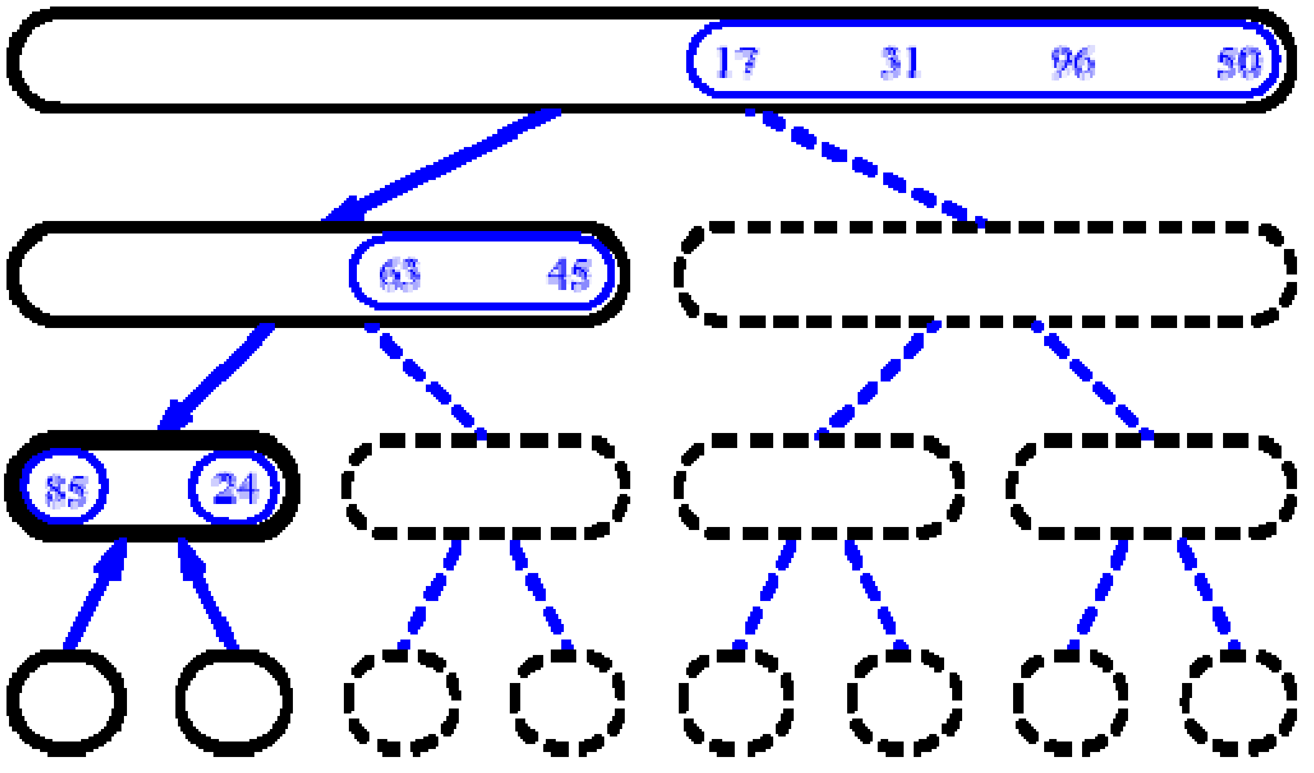
Merge Sort



Merge Sort



Merge Sort



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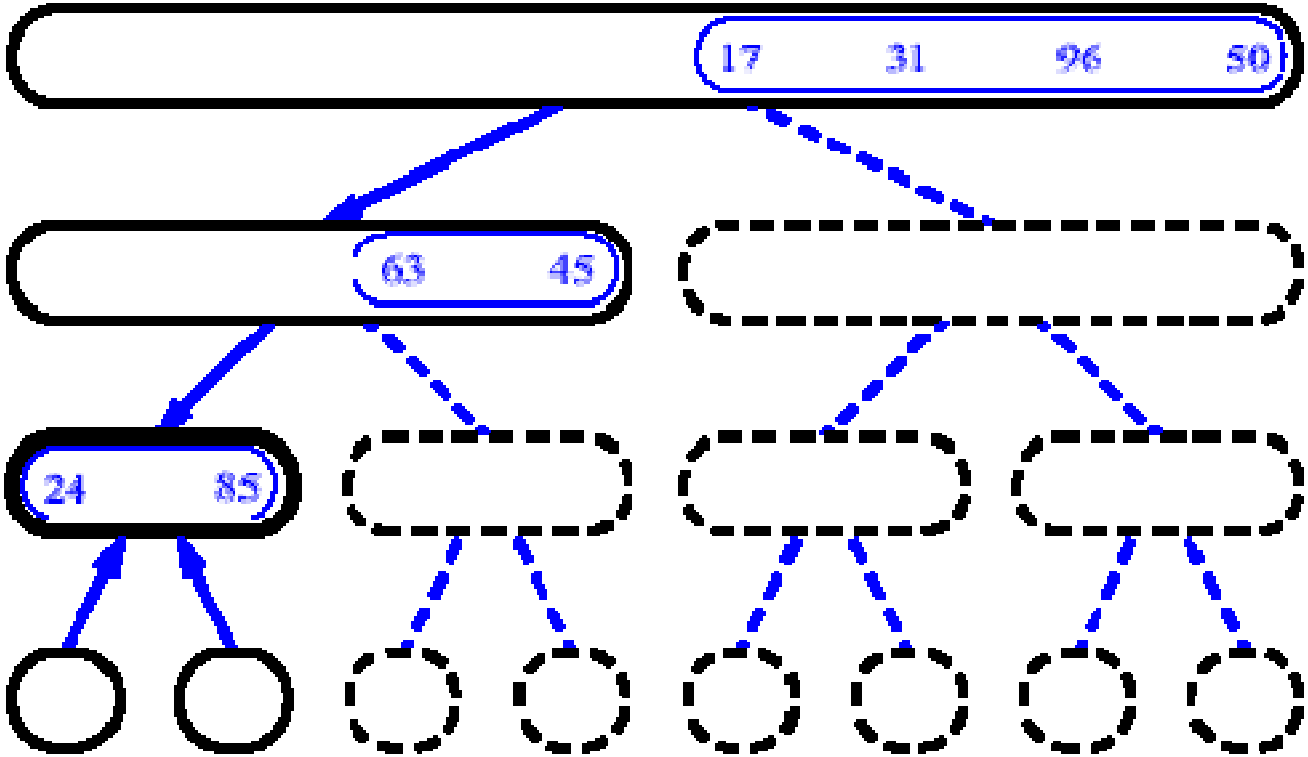
Merge Sort



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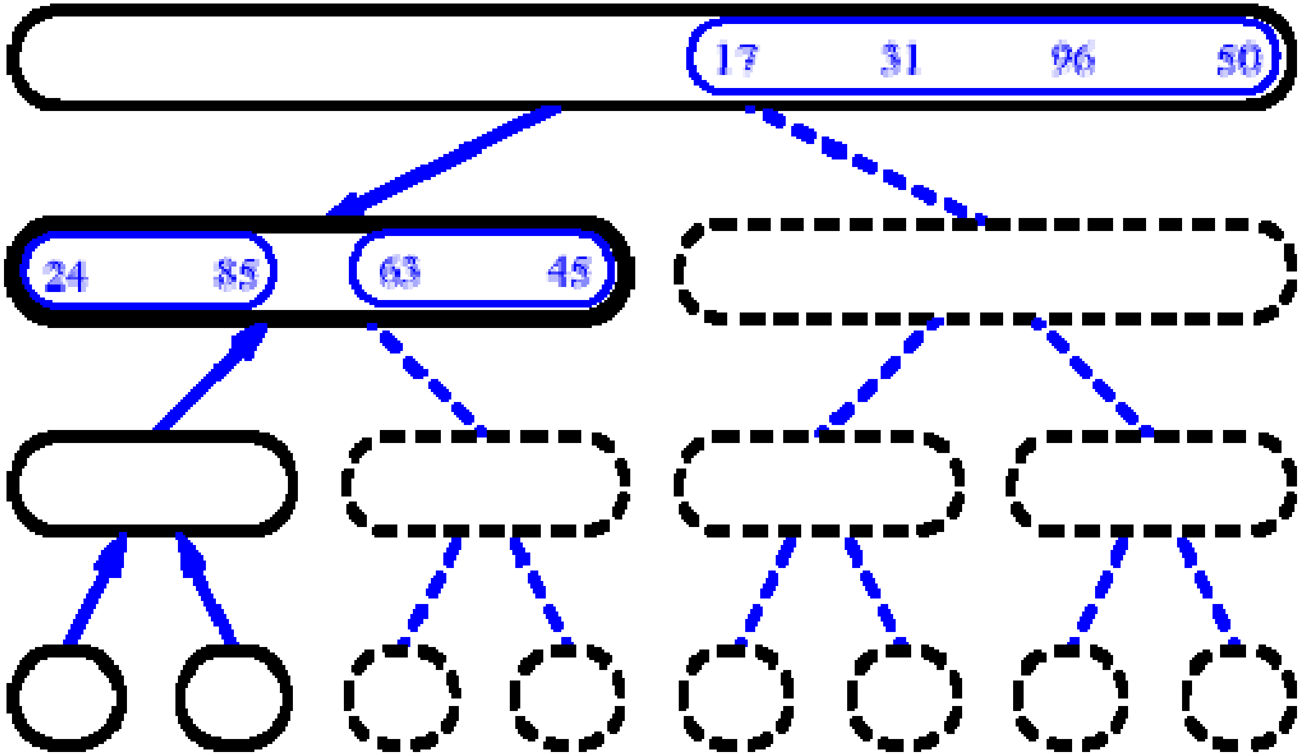
Merge Sort



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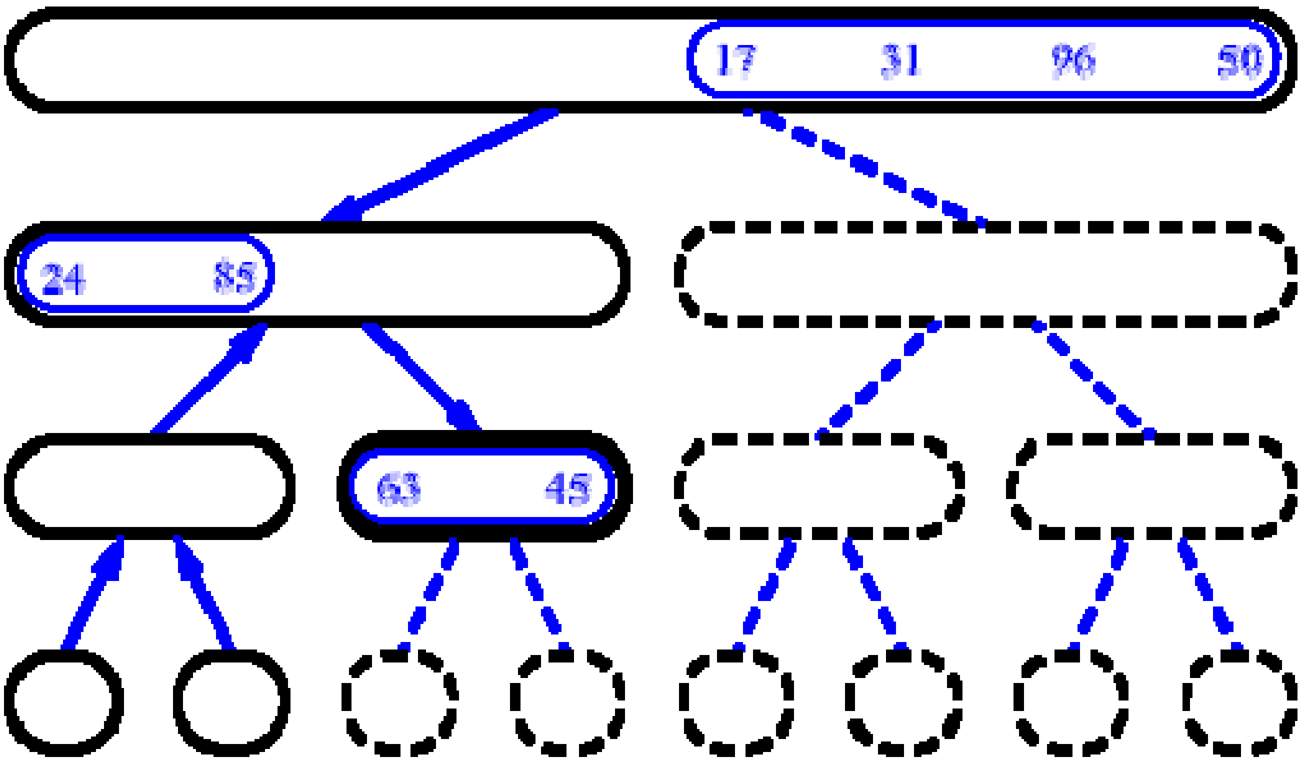
Merge Sort



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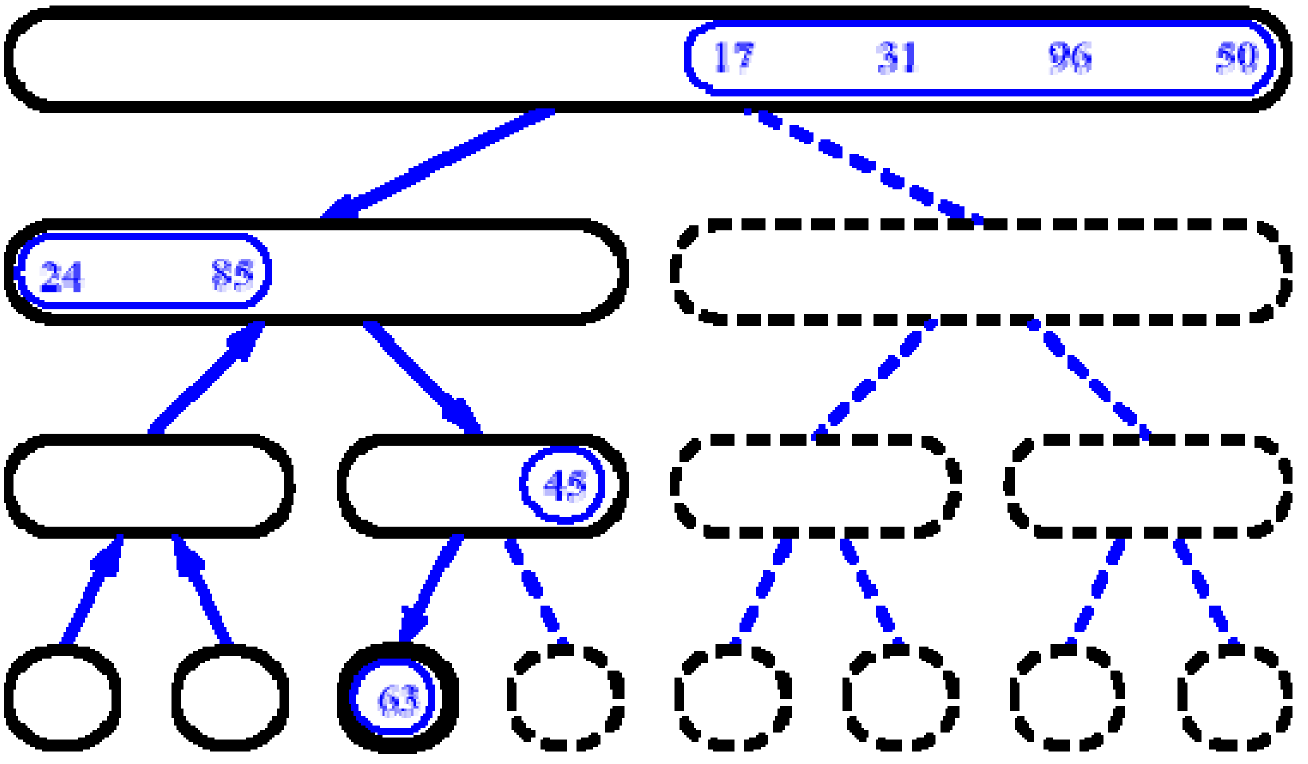
Merge Sort



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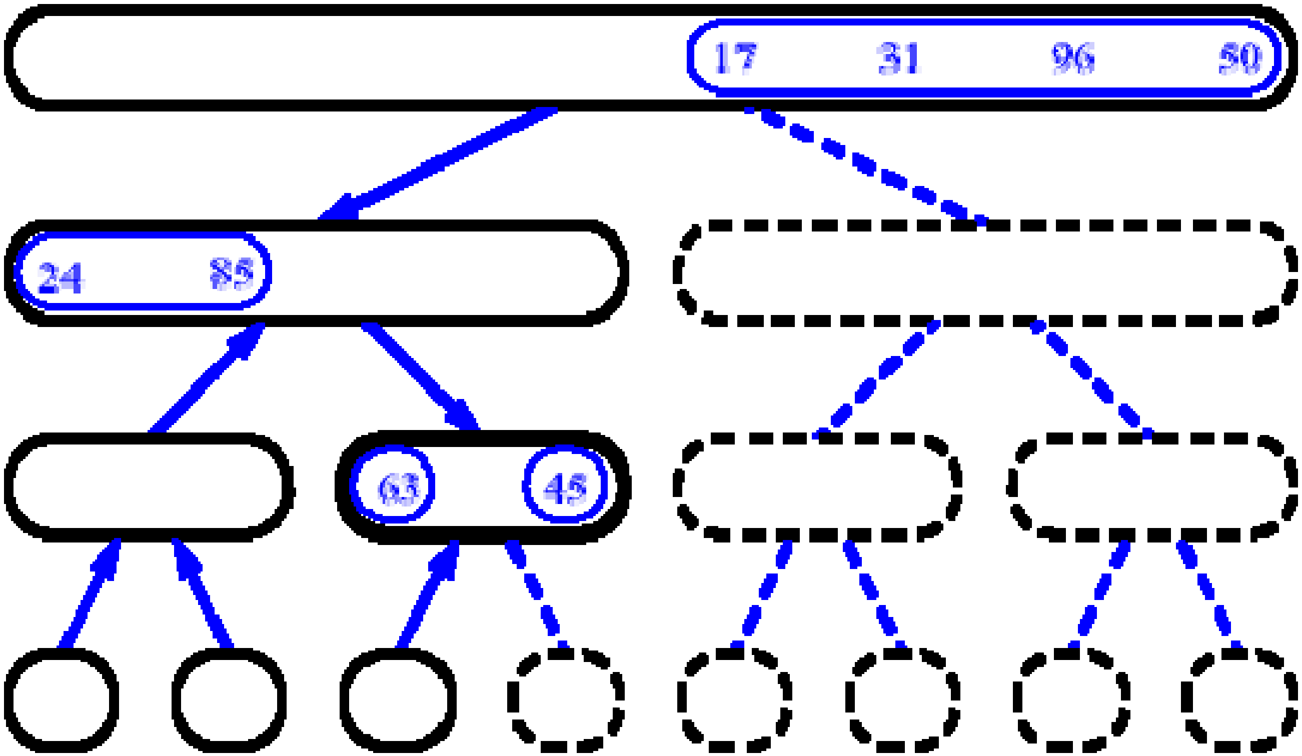
Merge Sort



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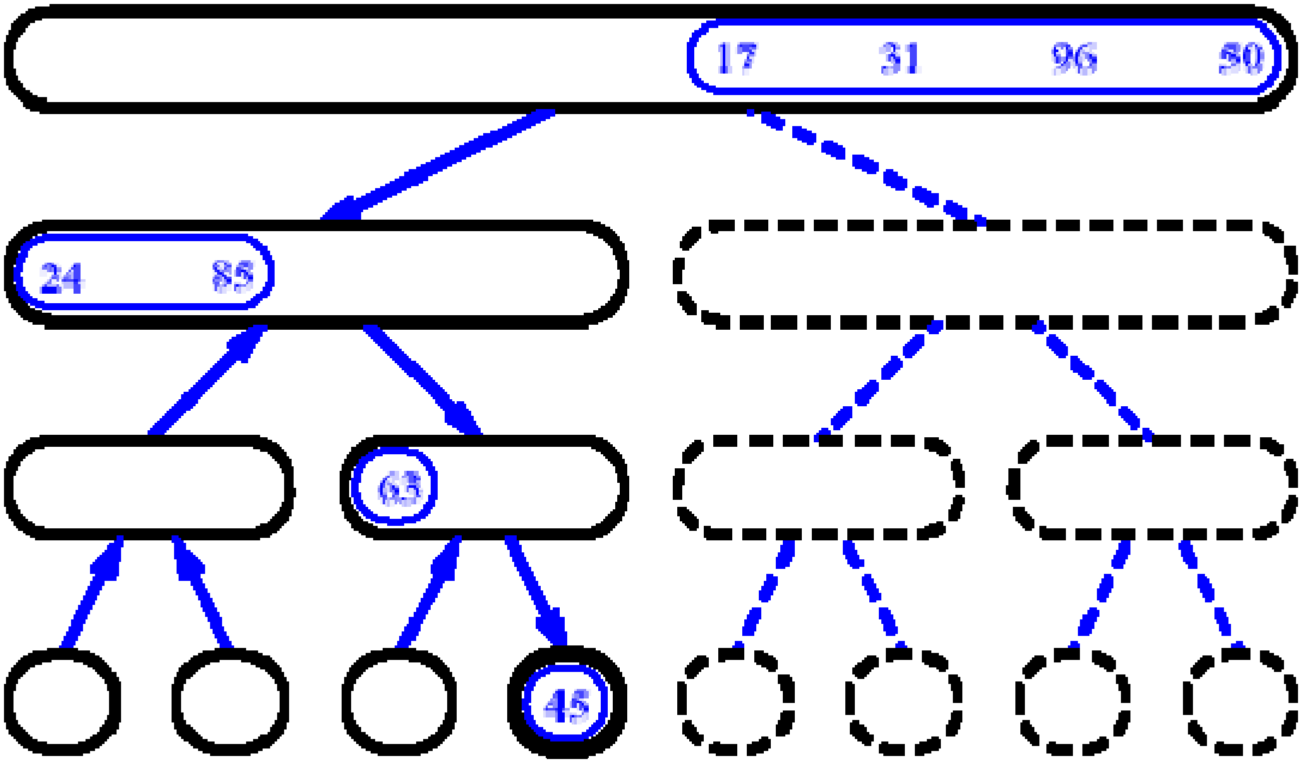
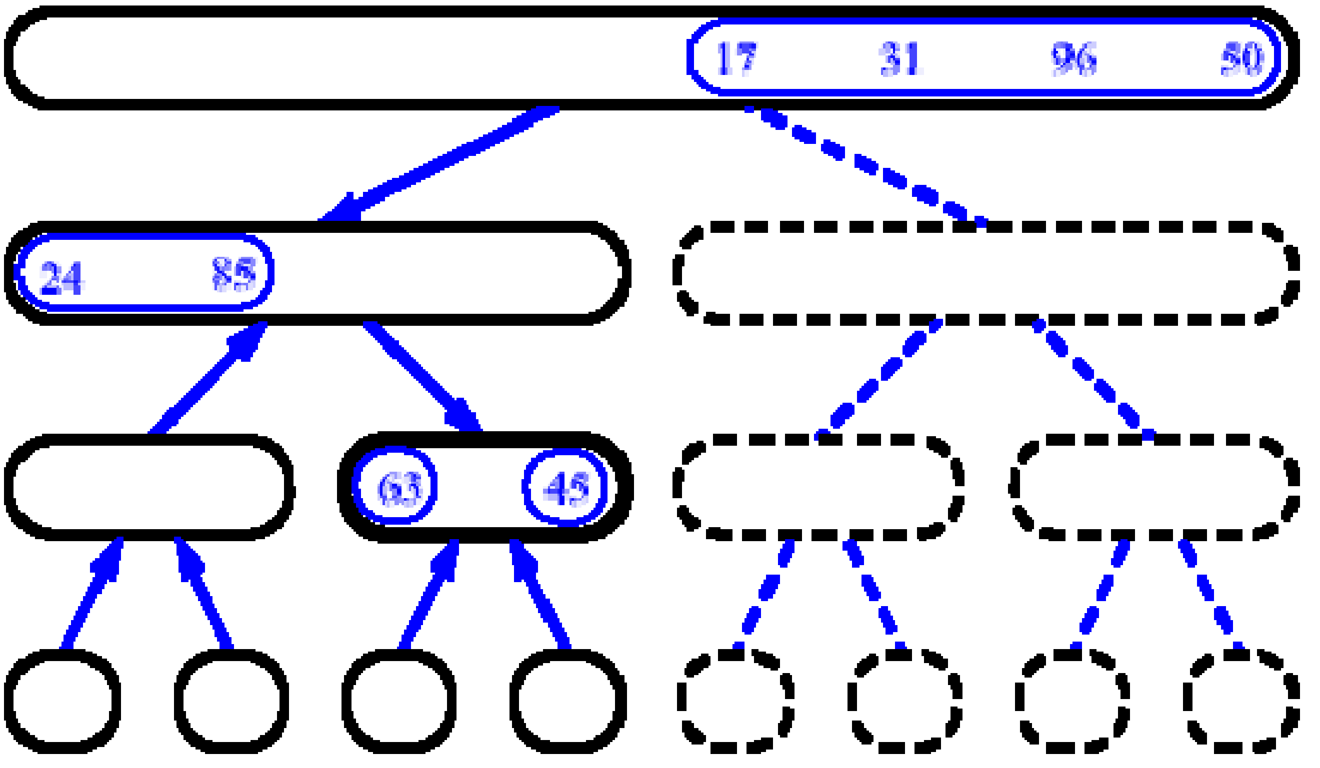
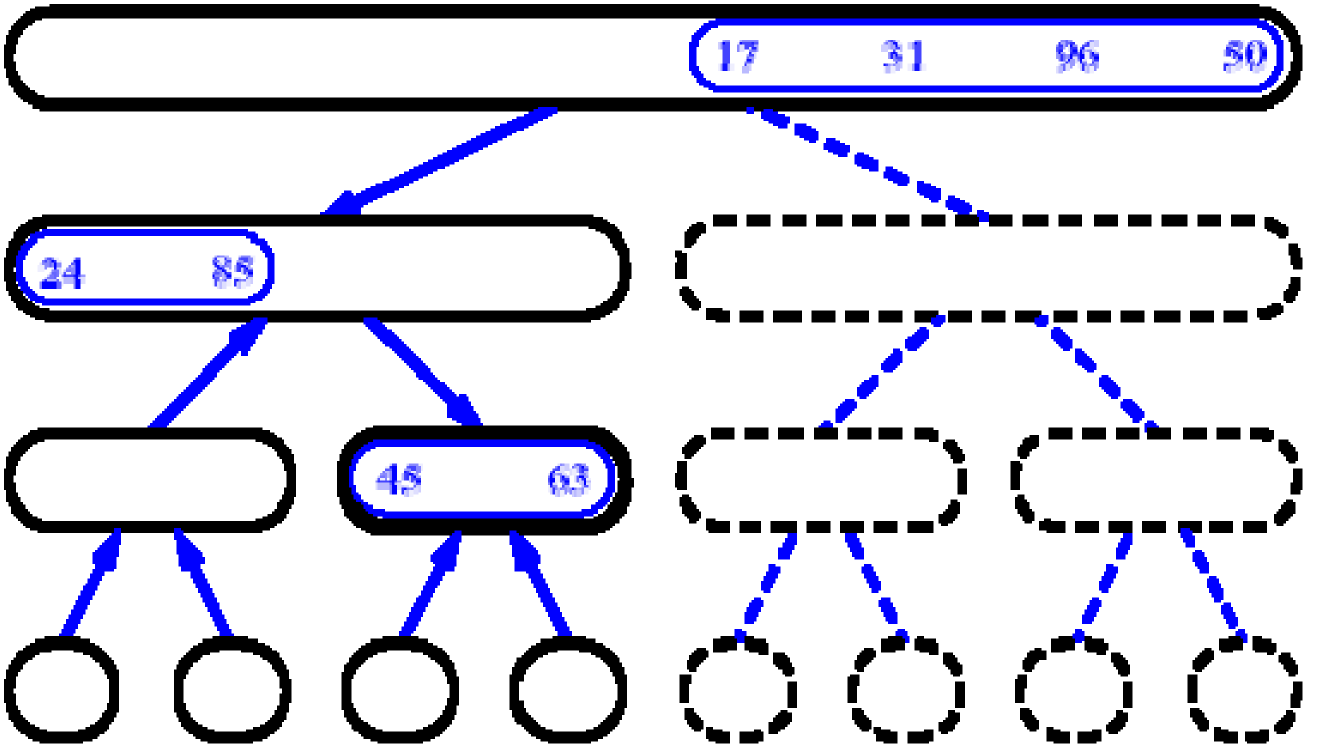
Merge Sort



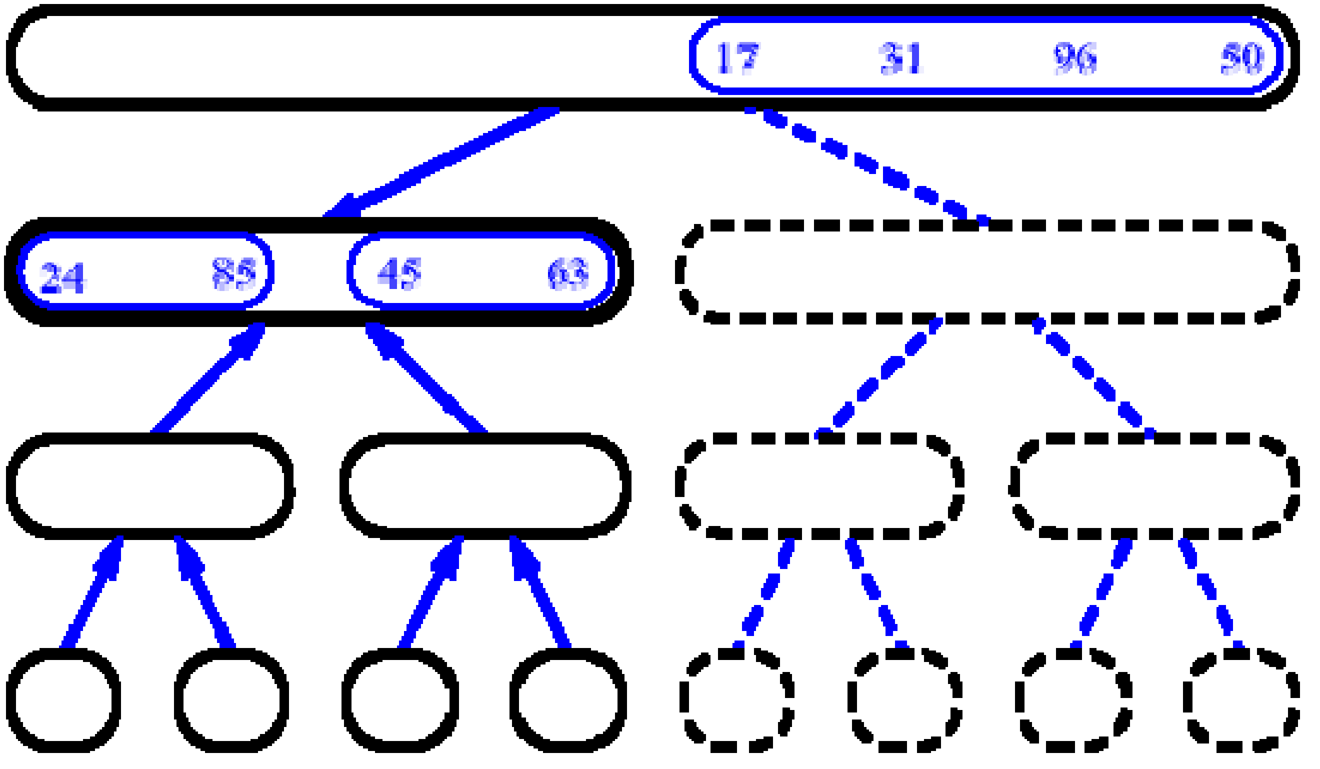
Merge Sort



Merge Sort



Merge Sort



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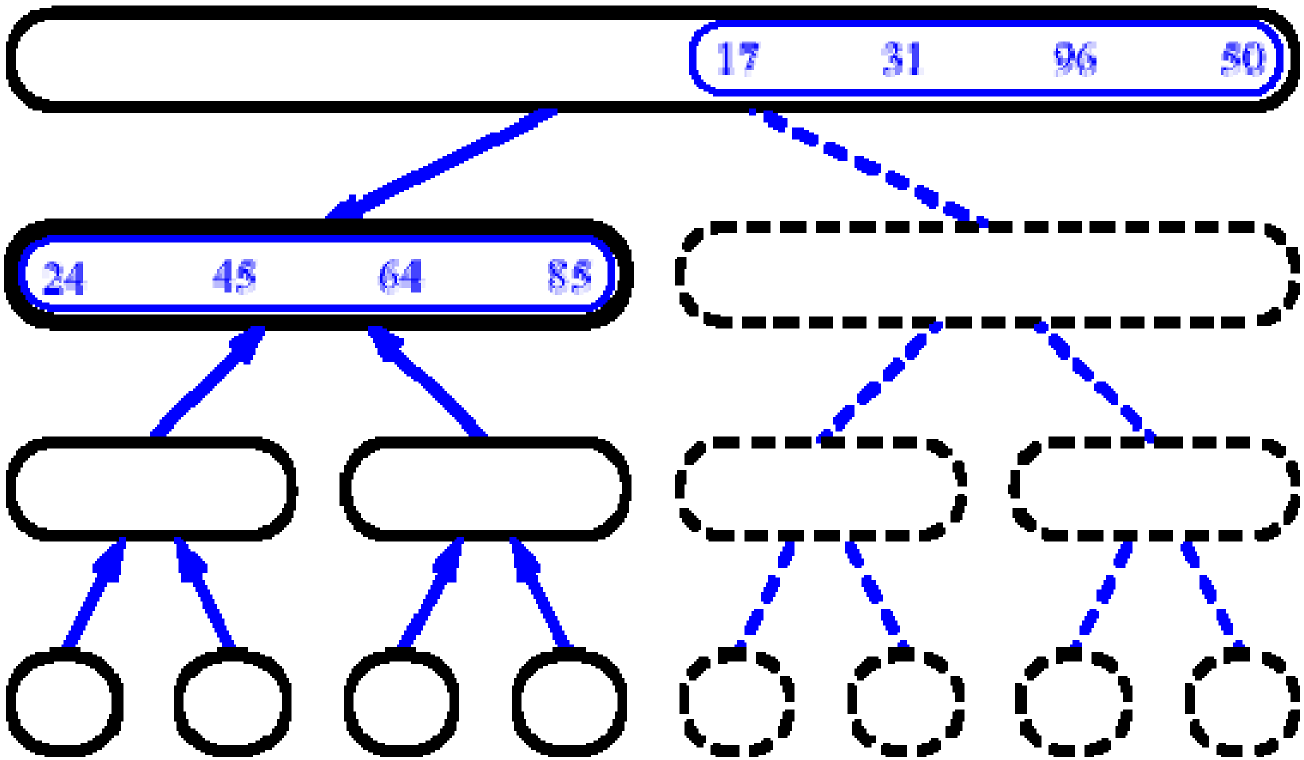
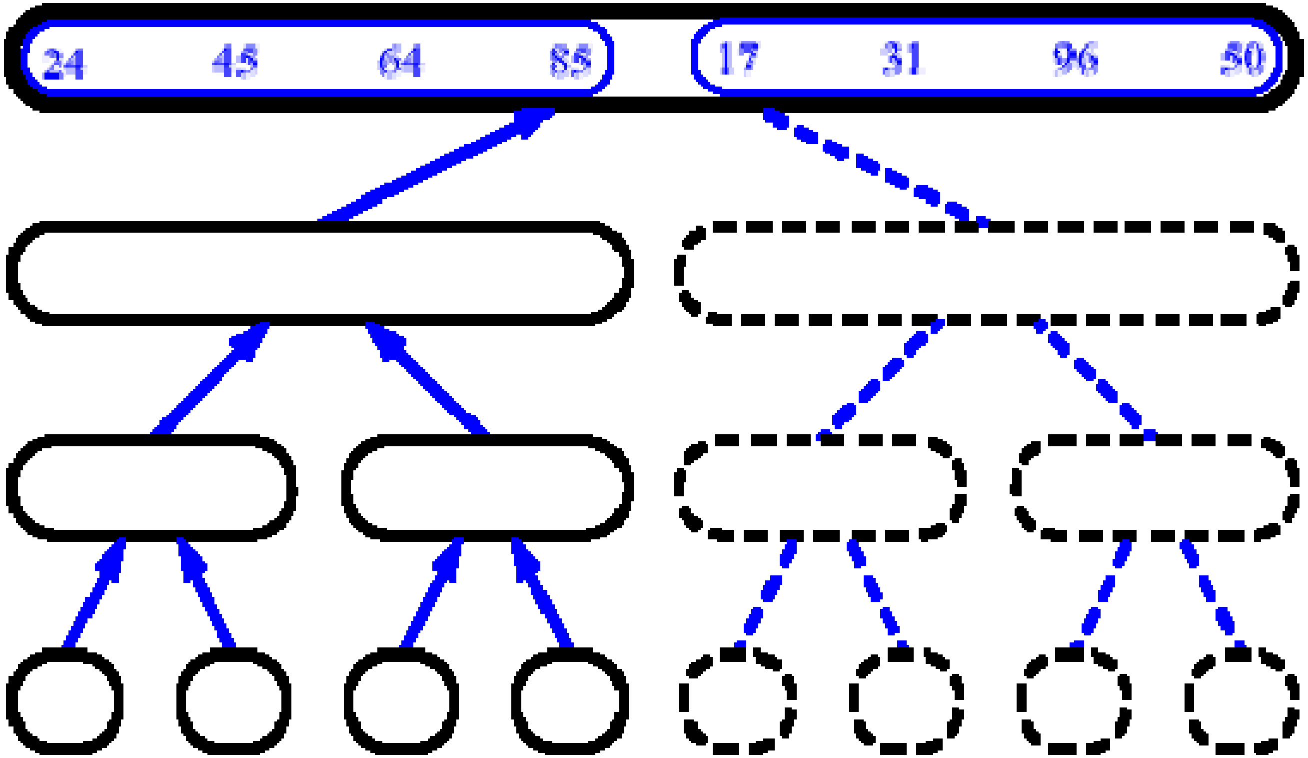
Merge Sort



Merge Sort



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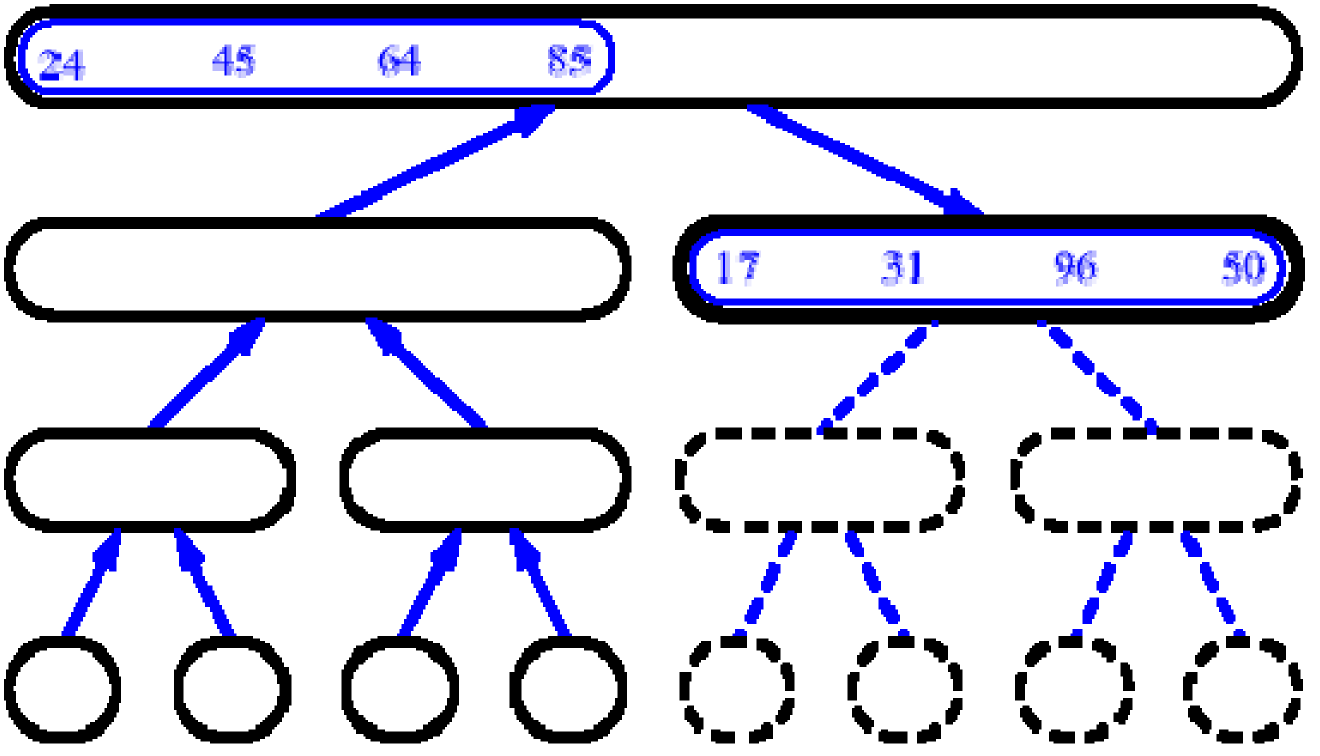
Merge Sort



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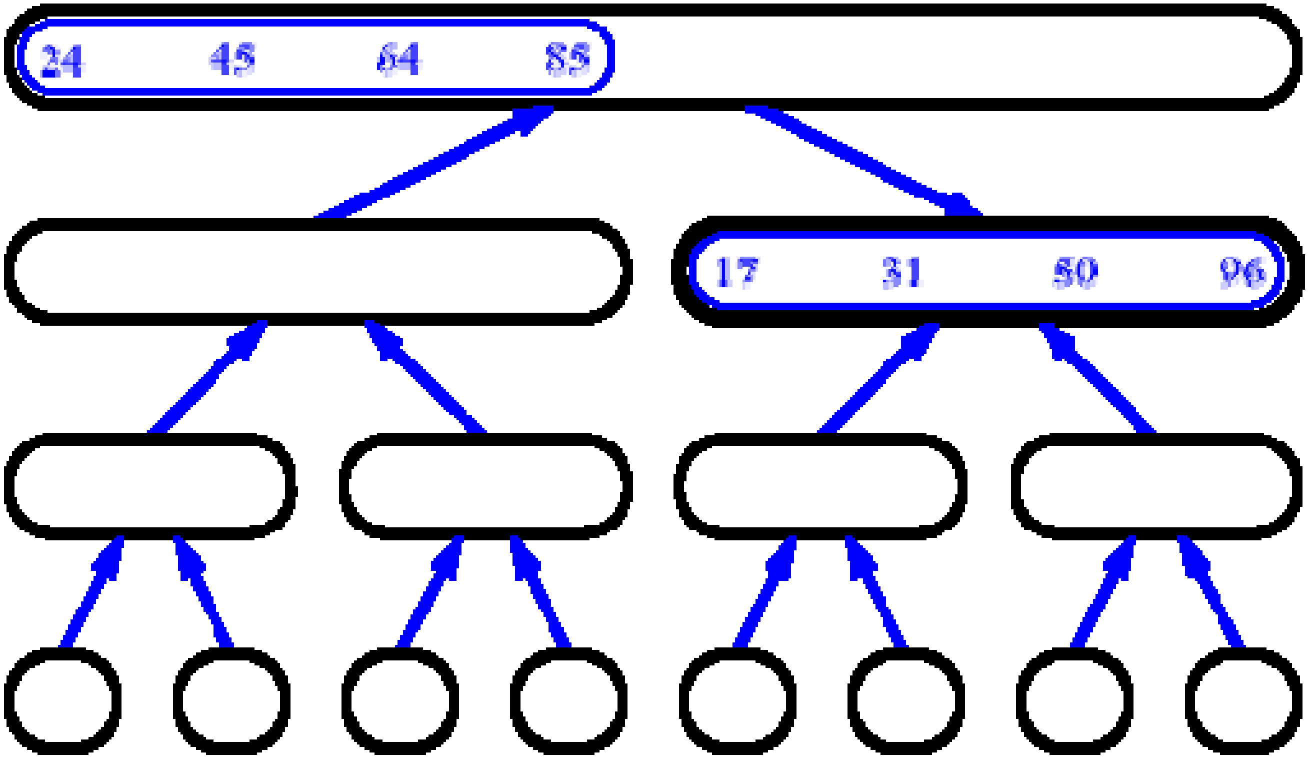
Merge Sort



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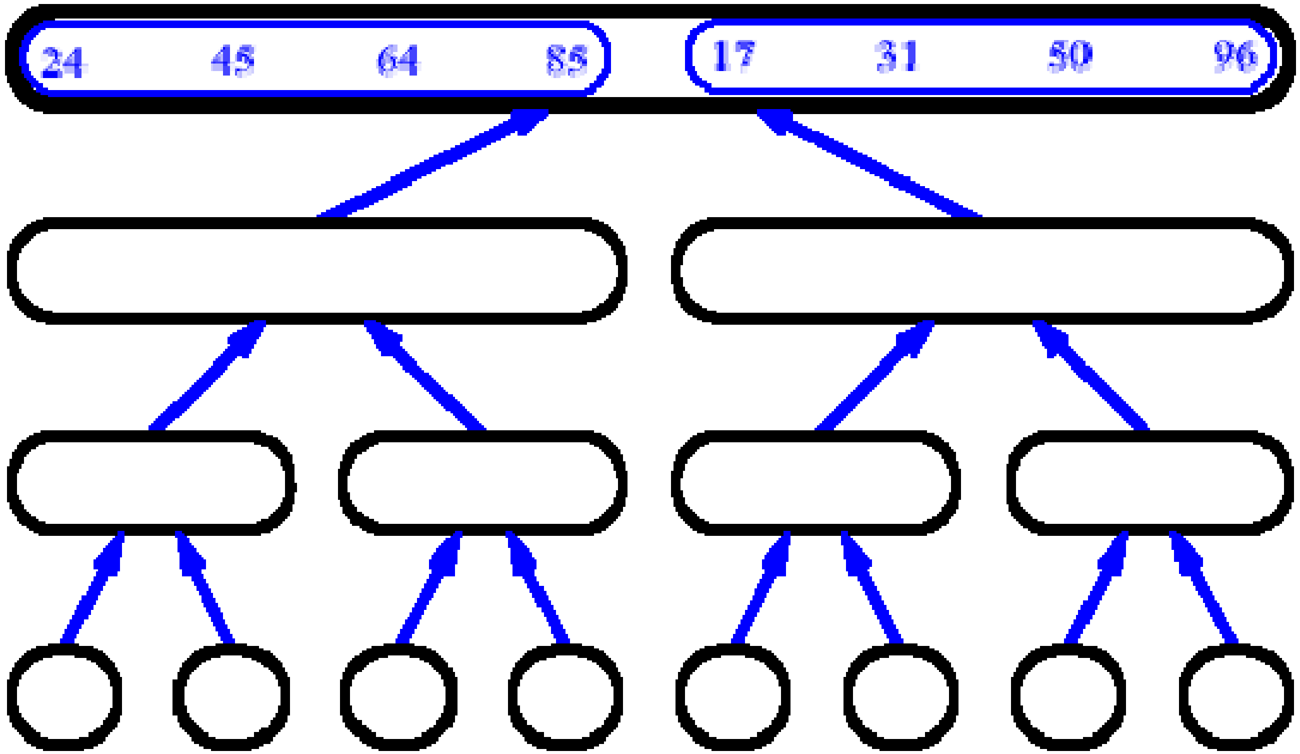
Merge Sort



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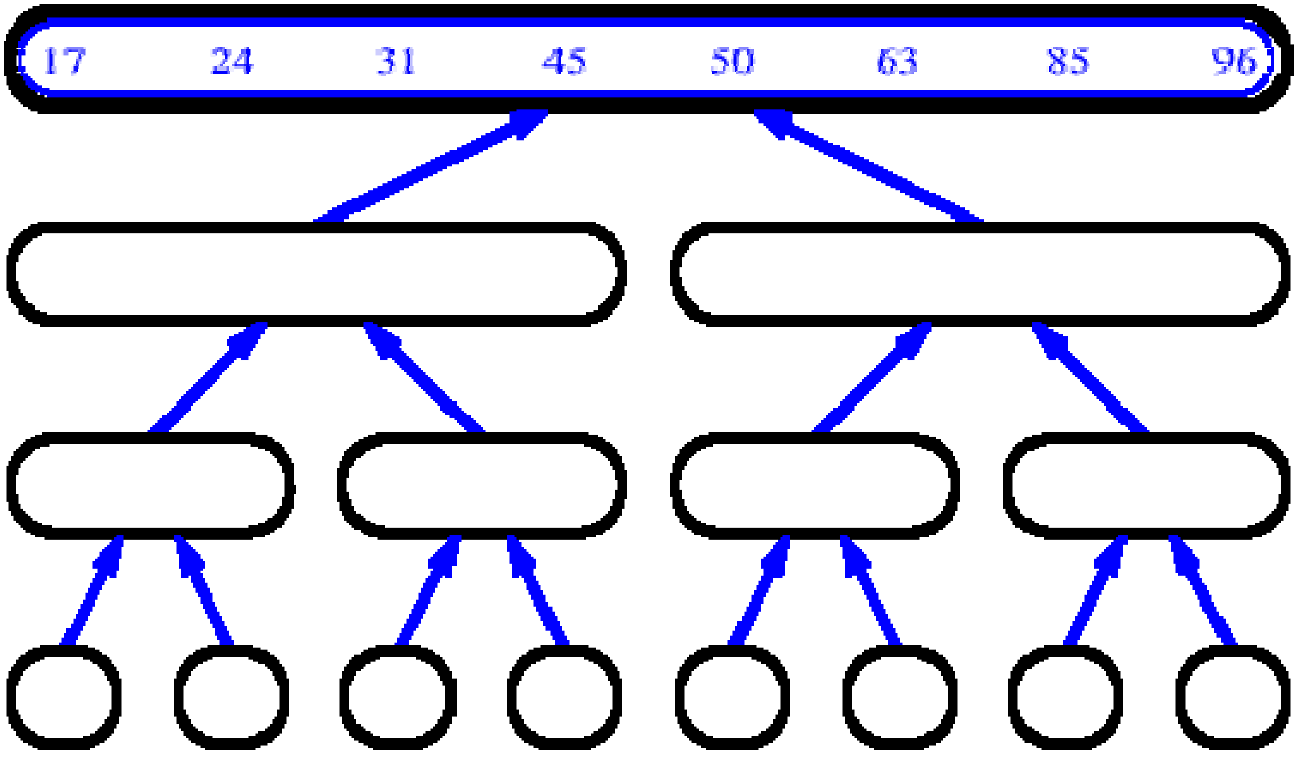
Merge Sort



Merge Sort

- **Running time $T\ (n)$ of Merge Sort:**
  - **Divide:** computing the middle takes **O(1)**
  - **Conquer:** solving 2 sub-problems takes $\mathbf{2T(n/2)}$
  - **Combine:** merging n elements takes $\mathbf{O(n)}$
  - **Total:**

$$T(n) = 2T(n/2) + O(n)$$

$$\Rightarrow T(n) = O(n \log n)$$

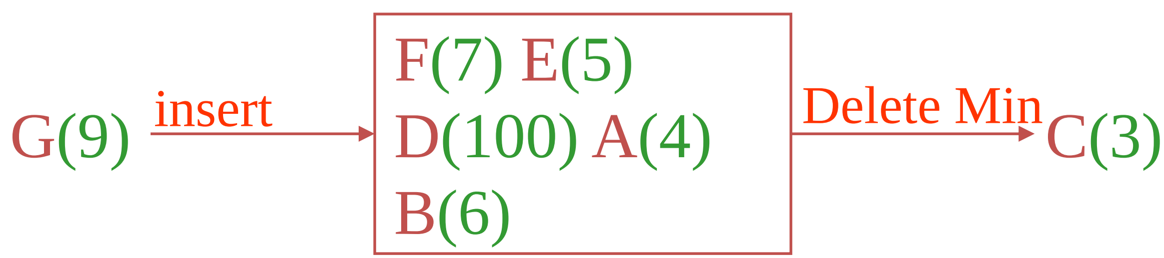
Priority Queues and Heaps

- **Consider applications**
  - **Ordering** CPU jobs
  - Printing **Jobs**
  - **Emergency** room admission processing

- **Problems?**
  - short jobs **should go first**
  - Hold jobs for a printer in **order of length**
  - most urgent cases **should go first**

Priority Queues and Heaps

- **Priority Queue property:**
  - For two elements in the queue, ***x*** and ***y***, if ***x*** has a lower ***priority value*** than ***y***, **x** will be ***deleted before y***



Priority Queues and Heaps

- **Priority Queue operations**
  - **Create**
  - **Destroy**
  - **Insert**
  - **Delete Min / Delete Max**
  - **Is_empty**

Priority Queues and Heaps

- **Unordered linked list**
  - **Insert : O(1)**
  - **Delete Min/Max : O(n)**
- **Ordered linked list**
  - **Insert : O(n)**
  - **Delete Min/Max : O(1)**
- **Ordered array**
  - **Insert : O(n)**
  - **Delete Min/Max : O(1)**
- **Balanced BST**
  - **Insert : O(log n)**
  - **Delete Min/Max : O(log n)**

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Priority Queues and Heaps

- A **heap** is a binary tree with two properties:
  - **Structure property**
    - A complete binary tree
      - Height of a complete binary tree with n elements is log n
  - **Heap-order property**
    - Parent's key is smaller (greater) than children's keys in Min Heap (Max Heap)
    - Result: Minimum (maximum) is always at the root

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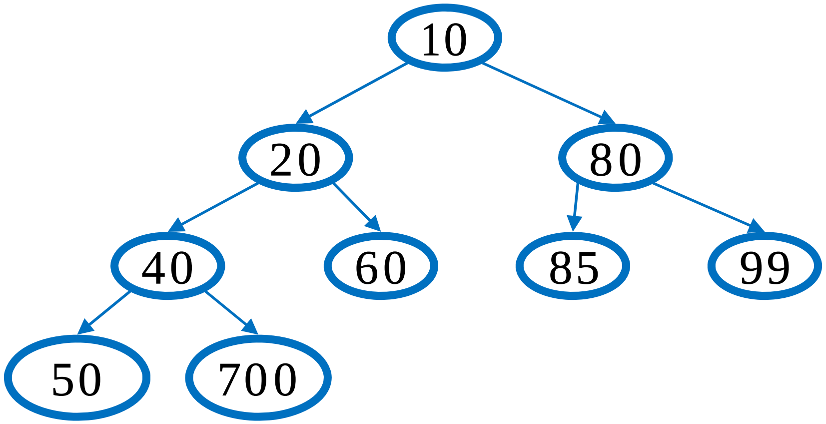
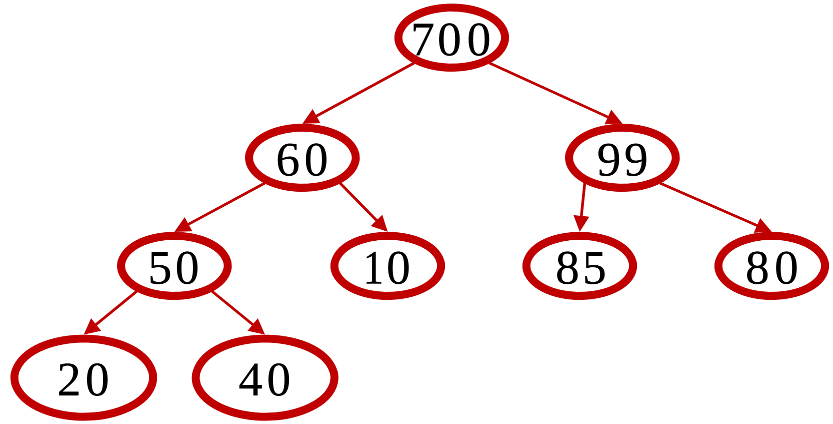
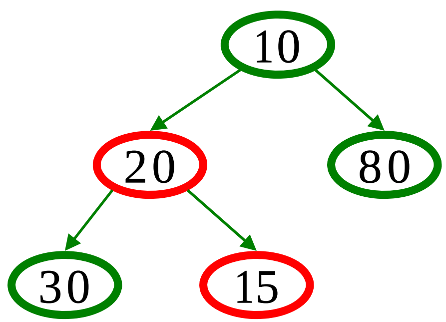
Priority Queues and Heaps



Min Heap



Max Heap



Not a Heap

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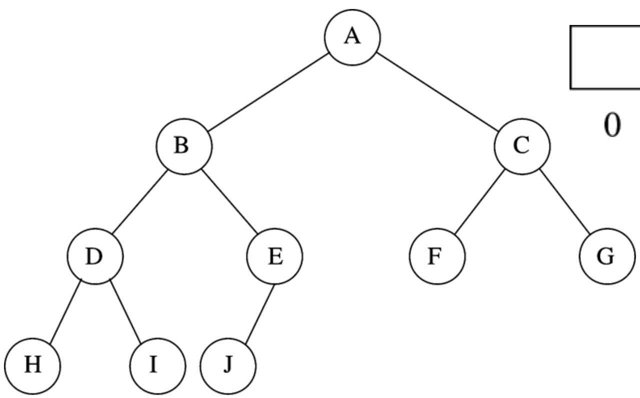
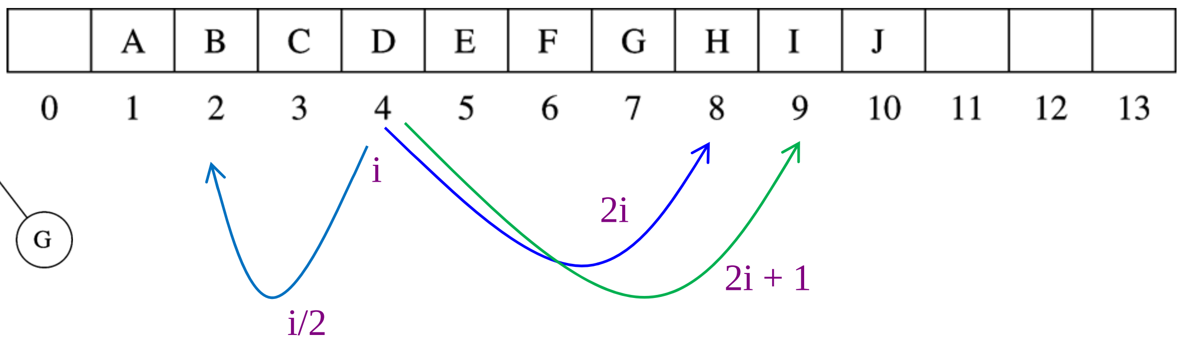
Priority Queues and Heaps

- Given element at position i in the array
  - Left child(i) = at position 2i
  - Right child(i) = at position 2i + 1
  - Parent(i) = at position i/2





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Priority Queues and Heaps

- **Insertion in a Heap**
  - Basic Idea:
    1. Put element at **“next”** leaf position
    2. **Restore heap property** by repeated exchange starting from inserted element until no longer needed

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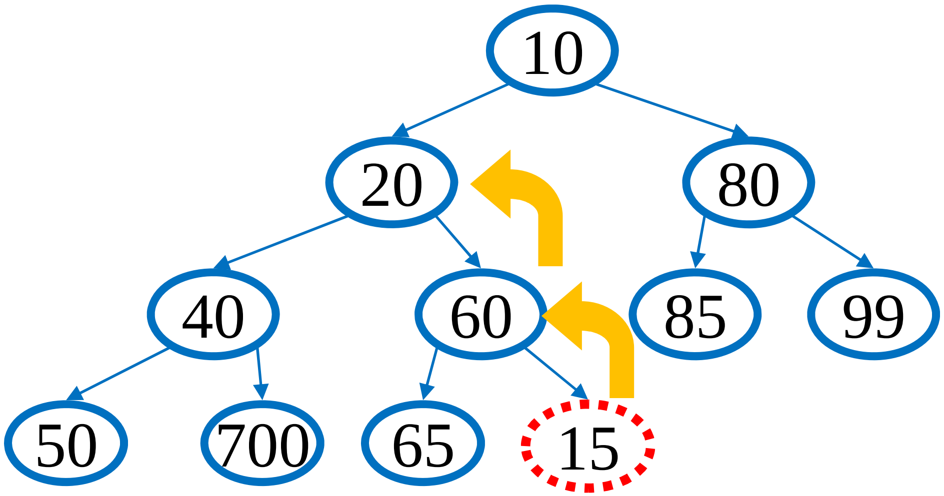
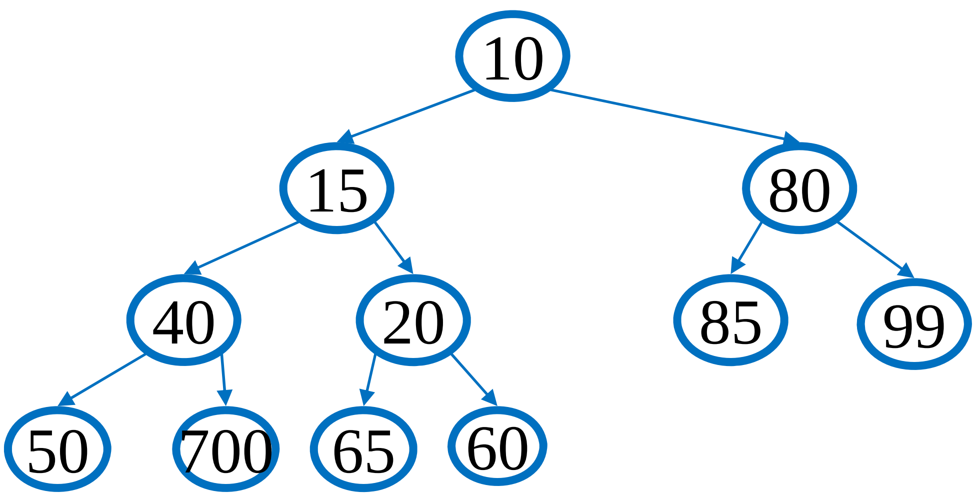
Priority Queues and Heaps




```
void insert (int element)
{
      Pqueue [++n] = element;
      RestoreHeapUp (n);
}

void RestoreHeapUp (int pos)
{
      int element = Pqueue [pos];
      while ( Pqueue [pos/2] >= element)
      {
            Pqueue [pos ] = Pqueue [pos/2];
            pos = pos/2;
      }
      Pqueue [pos] = element;
}
```



Complexity = O (log n)

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Priority Queues and Heaps

- **Deletion from a Heap**
  - Basic Idea:
    1. Remove **root** (that is always the min!)
    2. Put "last" leaf node at **root**
    3. **Restore heap property** by repeated exchange starting from deleted element until no longer needed

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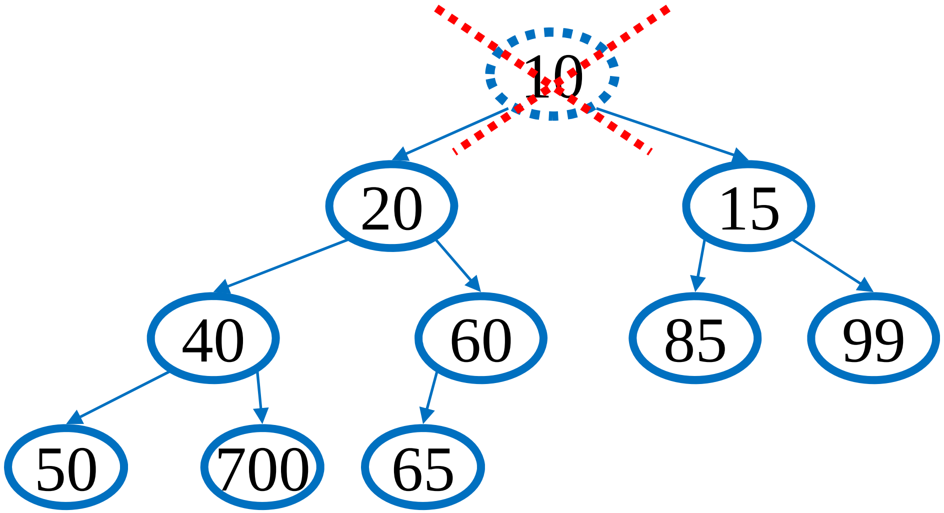
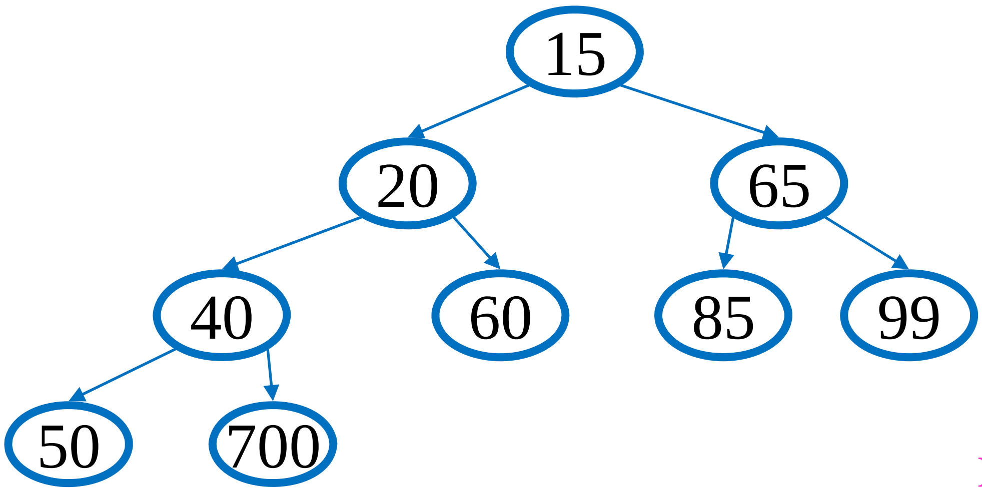
Priority Queues and Heaps






```
int Delete ()
{
        int element = Pqueue [ 1 ];
        Pqueue [ 1 ] = Pqueue [ n --];
        RestoreHeapDown ( 1 );
        return element;
}

void RestoreHeapDown ( int pos)
{
        int i, element = Pqueue [ pos ];
        while ( pos <= n/2)
        {
                i = 2*pos;
                if ( (i<n) && (Pqueue [i] > Pqueue [ i + 1] ) )
                        i ++;
                if ( element <= Pqueue [ i ] )
                        break;
                Pqueue [ pos ] = Pqueue [ i ];
                pos = i;
        }
        Pqueue [ pos ] = element;
}
```

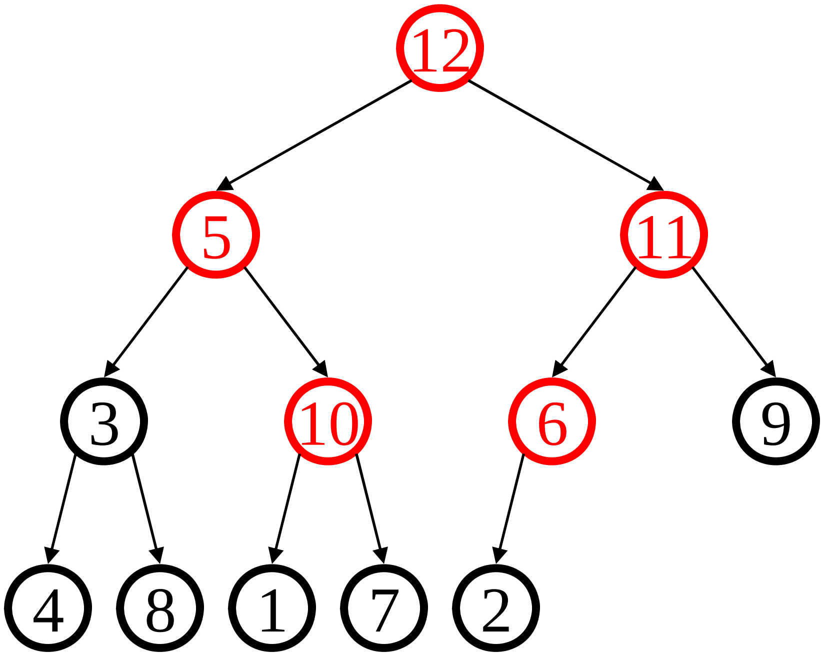
Complexity = O (log n)

Priority Queues and Heaps

- **Heap construction**

12	5	11	3	10	6	9	4	8	1	7	2

Add elements arbitrarily to form a complete tree.
Pretend it's a heap and fix the heap-order property!



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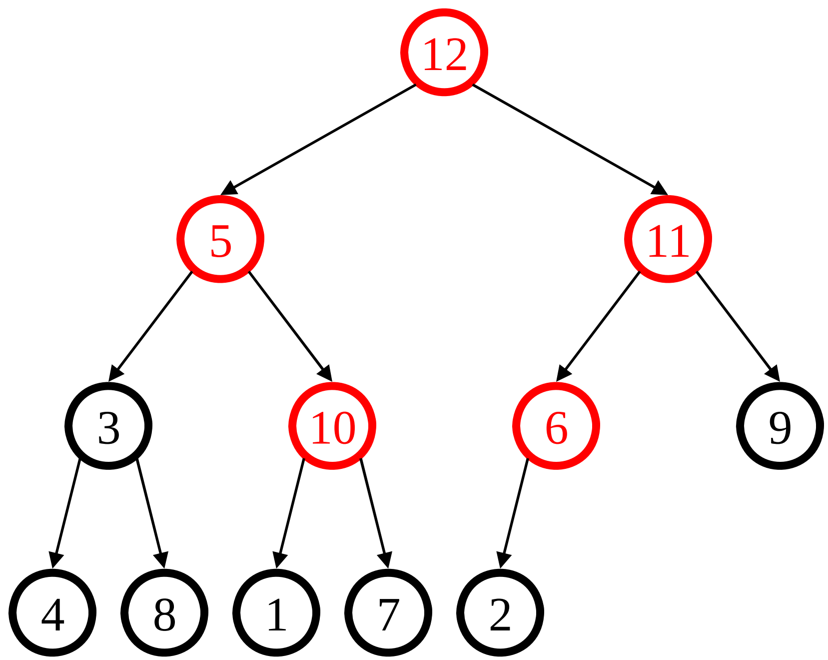
Priority Queues and Heaps

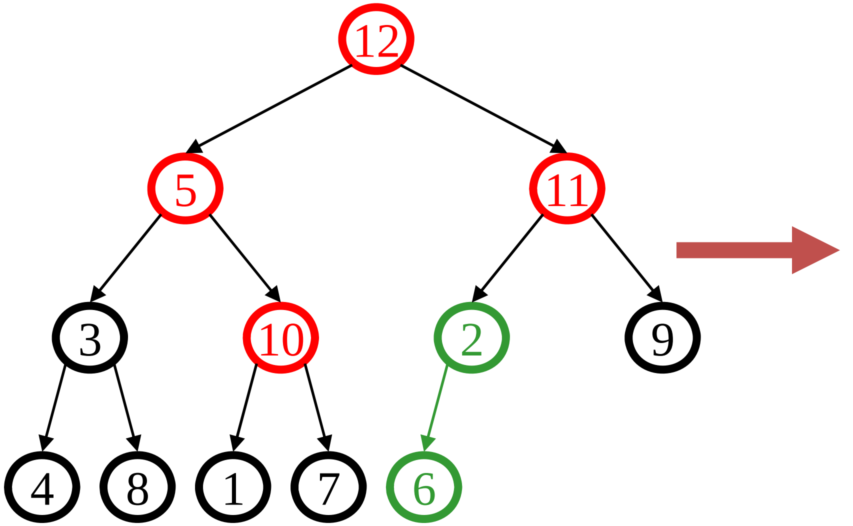
```
void BuildHeap ( )
{
        int i;
        for ( i = n/2; i > 0 ; i --)
        RestoreHeapDown ( i );
}
```

Complexity = O (n)

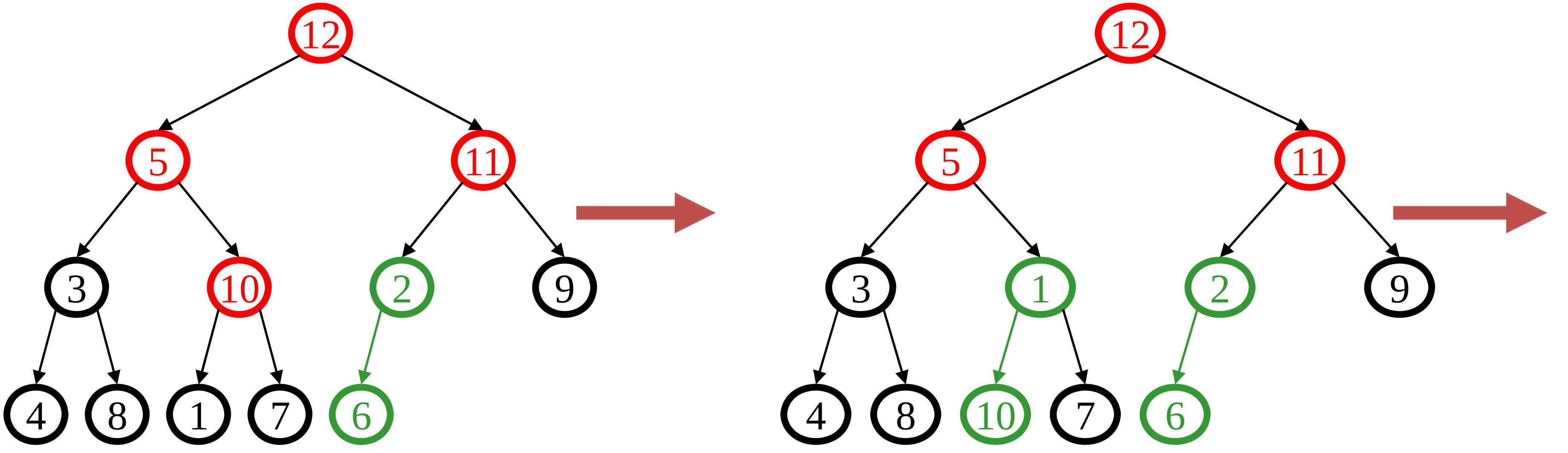
Priority Queues and Heaps



Priority Queues and Heaps



Priority Queues and Heaps



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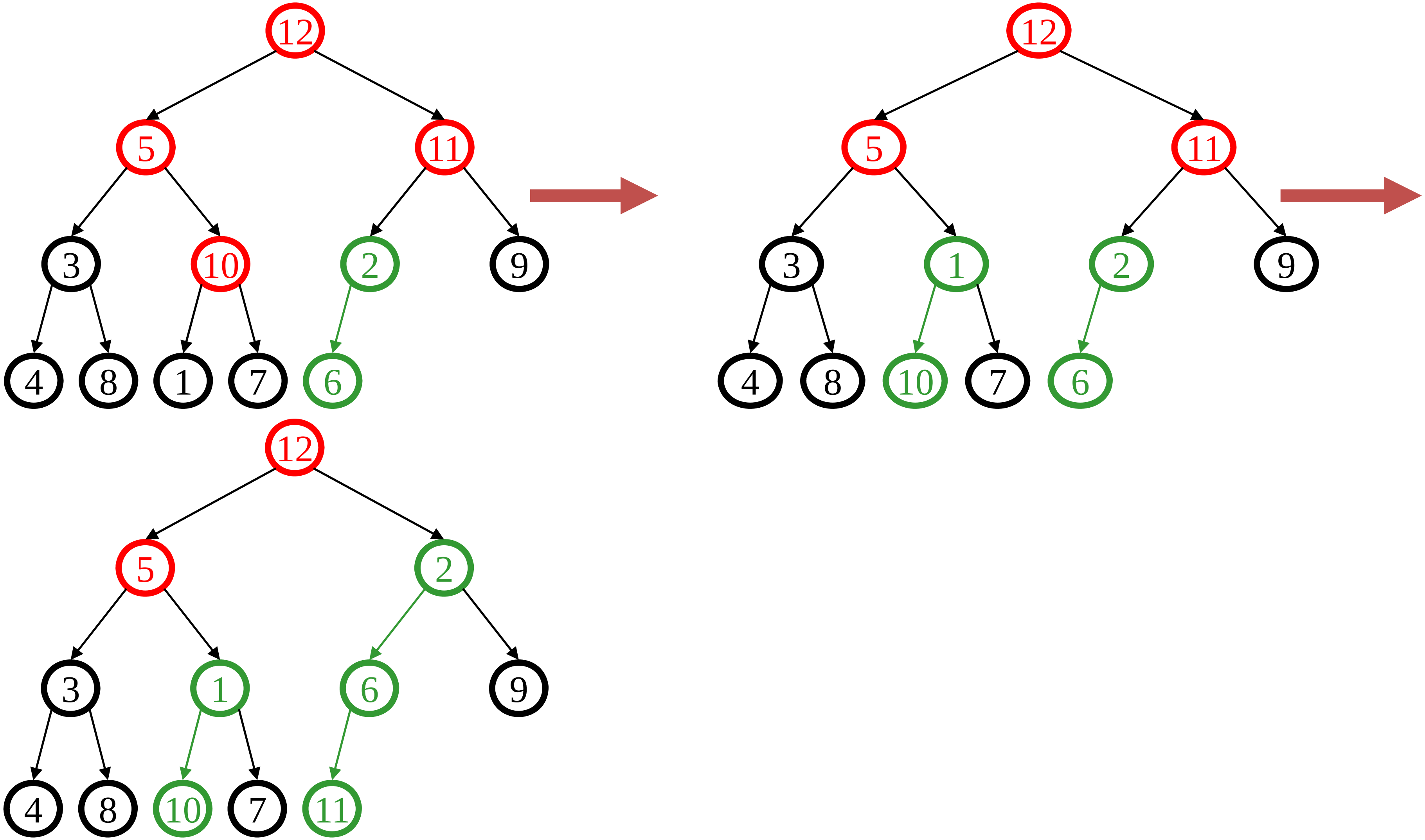
Priority Queues and Heaps



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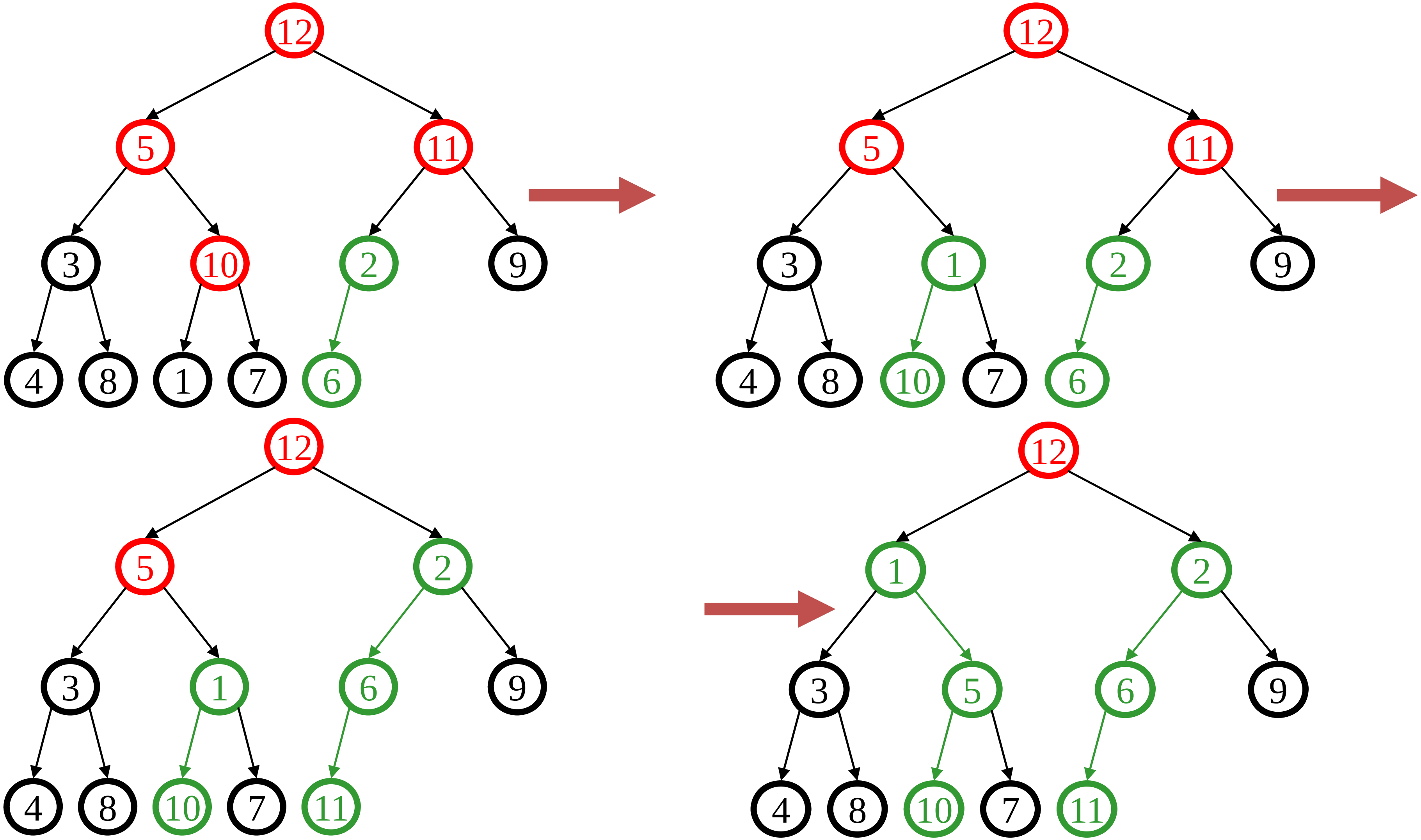
Priority Queues and Heaps



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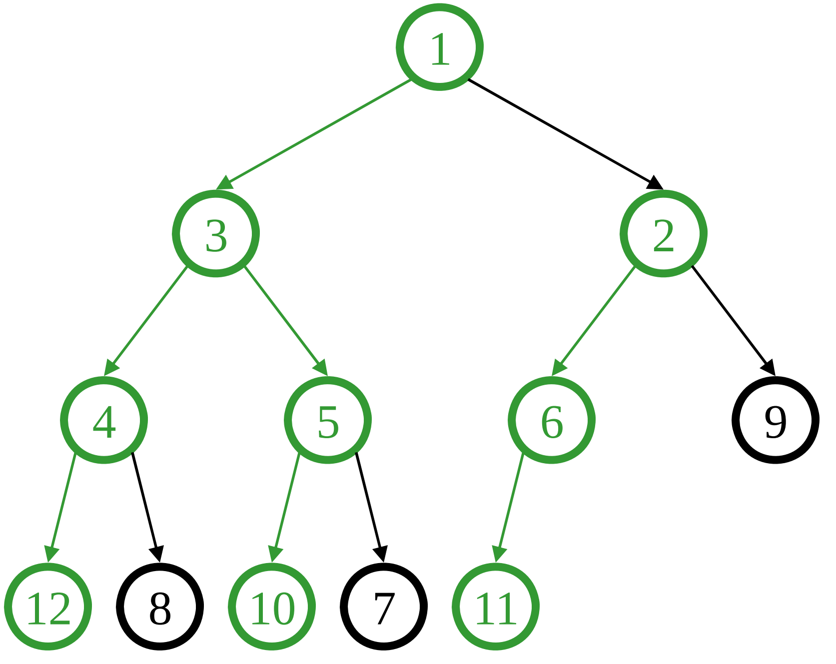
Priority Queues and Heaps



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Heap Sort

- Given **BuildHeap()**, an in-place sorting algorithm is easily constructed:
  - Smallest element is at A[1]
  - Discard by swapping with element at A[n]
    - Decrement **Heap_size**
  - Restore heap property at A[1] by calling **RestoreHeapDown()**
  - Repeat, always swapping A[1] with A[heap_size]

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Heap Sort

```
Heapsort(A)
{
        BuildHeap(A);
        for (i = length(A) downto 2)
        {
                Swap(A[1], A[i]);
                heap_size(A) -= 1;
                RestoreHeapDown(A, 1);
        }
}
```


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Heap Sort

- The call to **BuildHeap()** takes O(n) time
- Each of the **n - 1** calls to **RestoreHeapDown()** takes **O(log n)** time
- Thus the total time taken by **HeapSort()**
  = **O(n) + (n - 1) O(log n)**
  = **O(n) + O(n log n)**
  = **O(n log n)**

Any Doubt ?

- Please feel free to write to me:

bhaskargit@yahoo.co.in