PROBABILITY AND QUEUEING THEORY

QUESTION BANK

UNIT-I: PROBABILITY AND RANDOM VARIABLES

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- A1. Let the random variable X denote the sum obtained in rolling a pair dice. Determine the probability mass function of Y.

 [AU, A/M 2011]
- A2. A coin is tossed 2 times, if 'x' denotes the number of heads, find the probability distribution of X.

[AU, N/D 2013]

A3. If a fair coin is tossed twice, find $P[x \le 1]$, where X denotes the number of heads in each experiment.

[AU, N/D 2016]

- A4. Let X be the random variable which denotes the number of heads in three tosses of a fair coin. Determine the Probability mass function of X.

 [AU, N/D 2015]
- A5. A random variable X takes the values 1, 2, 3, 4 such that 2P[X=1]=3P[X=2]=P[X=3]=5P[X=4]. Find the probability distribution of X. [AU, N/D 2016]
- A6. If X and Y are two independent random variables with variances 2 and 3, find the variance of 3X-4Y.

 [AU, M/J 2013]
- A7. Let X be a discrete R.V. with probability mass function $P(X = x) = \begin{cases} \frac{x}{10}; & x = 1.2.3.4, \\ 0; & otherwise \end{cases}$. Compute
 - P(X < 3) and $E\left(\frac{1}{2}X\right)$.

[AU, M/J 2016]

- A8. Check whether the function given by $f(x) = \frac{x+2}{25}$ for x = 1, 2, 3, 4, 5 can serve as the probability distribution of a discrete random variable. [N/D 18.]
- A9. A continues random variable X that can assume any value between x = 2 and x = 5 has a density function given by f(x) = k(1+x); Find P[x < 4]. [AU, A/M 2011, N/D 2012 A/M 17
- A10. If a random variable X has the distribution function $F(x) = \begin{cases} 1 e^{-ax} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$, where a is the
- parameter, then find $P(1 \le x \le 2)$. [AU, N/D 2010] A11. A test engineer discovered that the cumulative distribution function of the lifetime of an equipment(in

years) is given by $F_X(x) = 1 - e^{-\frac{x}{5}}, x \ge 0$. what is the expected lifetime of the equipment? [N/D 17.

A12. A continuous random variable X has the probability density function given by

$$f(x) = \begin{cases} \alpha(1+x^2), & 2 \le x \le 5 \\ 0 & elsewhere \end{cases}$$
. Find α and $P[x < 4]$

[AU, M/J 2014]

A13. If the density function of a continuous random variable X is given by $f(x) = \begin{cases} ax; & 0 \le x \le 1 \\ a; & 1 \le x \le 2 \\ 3a - ax; & 2 \le x \le 3 \end{cases}$ then 0; otherwise

find the value of 'a'.

- A14. Test whether $f(x) = \begin{cases} |x|; & -1 \le x \le 1 \\ 0, & otherwise \end{cases}$ can be the probability density function of a continuous random [AU, N/D 2014 A/M 2015] variable.
- A15. A continuous random variable X has a probability density function given by $f(x) = \begin{cases} \frac{3}{4}(2x-x^2): & 0 < x < 2 \\ 0 & Otherwis \end{cases}$. Find P[x > 1].
- A16. A continuous random variable X has the probability density function given by

$$f(x) = \begin{cases} \lambda(1+x^2), & 2 \le x \le 5 \\ 0 & elsewhere \end{cases}$$
. Find λ and $P[x < 4]$ [AU, N/D 2015]
A17. Suppose that a continuous random variable X has the probability density function

$$f(x) = \begin{cases} K(1-x^2); & 0 < x < 1 \\ 0; & elsewhere \end{cases}$$
 Find the value of K. [AU, M/J 2016]

A18. If cumulative function of the random variable X is given by $F_x(x) = \begin{cases} 0, & x < 0 \\ x + \frac{1}{2}, & 0 \le x \le \frac{1}{2}. \end{cases}$ Compute

$$P[x > \frac{1}{4}]$$
 [AU, A/M 2011]

- A19. Check whether the following is a probability density function or not: $f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \ge 0, \lambda > 0 \\ 0 & elsewhere \end{cases}$
- A20. If the probability density function of a random variable X is f(x) = 1/4 in -2 < x < 2 find P(|X| > 1).
- A21. A random variable X has the probability function $f(x) = \frac{1}{2^x}$ for x = 1, 2, 3, Find the moments generating function.
- A22. If a random variable has the moment generating function given by $M_x(t) = \frac{2}{2-t}$, determine the variance [AU, M/J 2012]of X.
- A23. If a R.V X has the moment generating function $M_x(t) = \frac{3}{3-t}$; compute $E[X^2]$. [AU, M/J 2016] [AU, N/D 2014]
- A24. What do you mean by MGF? Why it is called so?
- A25. For a binomial distribution with mean 6 and standard deviation $\sqrt{2}$, find the first two terms of the [AU, M/J 2014]
- A26. The mean of binomial distribution is 20 and standard deviation is 4. Find the parameters of this
- A27. If the probability that a target is destroyed on any one shot is 0.5, find the probability that it would be destroyed on the 6th attempt.
- A28. If X is a binomial distributed random variable with E(x)=2 and $var(x)=\frac{4}{3}$, then find the probability mass function of x.

A29. If $M_X(t) = \frac{pe'}{1 - qe'}$ is the Moment Generating function of X then find the mean and variance of X.

[A/M 18.]

- A30. Balls are tossed at random into 50 boxes. Find the expected number of tossed required to get the first ball
- A31. Every week the average number of wrong number phone calls received by a certain mail order house is seven. What is the probability that they will receive two wrong calls tomorrow?
- [AU, N/D 2011] A32. Given the probability law of poisson distribution and also its mean and variance. [AU, A/M 2015]
- A33. What are the limitations of Poisson distribution?
- A34. Suppose that, on an average, in every three pages of a book there is one typographical error. If the number of typographical errors on a single page of the book is Poisson random variable. What is the probability of [AU, A/M 2015] at least one error on a specific of the book? [AU, M/J 2013]
- A35. State memory less property of exponential distribution.
- A36. What is meant by memoryless property? Which discrete distribution follows this property? [N/D 2015] [AU, A/M 2010]
- A37. Obtain the mean for a Geometric random variables.
- A38. If X is a geometric variate, taking values 1,2,3,...., find P(X is odd).

- [A/M 17.]
- A39. Identity the random variable and name the distribution it follows, from the following statement: "A realtor claims that only 30% of the houses in a certain neighbourhood are appraised at less than Rs. 20 lakhs. A random sample of 10 houses from that neighbour-hood is selected and appraised to check the realtor's [N/D 2012 A/M 17] claim is acceptable are not".
- A40. If X is a normal random variable with mean 3 and variance 9, find the probability that X his between 2 [N/D 17.] and 5.
- A41. What is mean by memory less property? Which continuous distribution follows this property?

[AU, A/M 2010]



B1.1 A Random Variable X has the following probability distribution

(A/M 2016)(N/D 2011)

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X	-2	-1	0	1	2	3	
P(x)	0.1	K	0.2	2k	0.3	3k	

- (i)Find k (ii)Evaluate Evaluate P(X < 1), P(X < 2), $P(-1 < X \le 2)$ and P(-2 < X < 2) (iii)Find the CDF of X and
- (iv) Evaluate the mean of X B1.2 A Random Variable X has the following probability distribution function

(A/M 2015) (M/J 2012)

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X	0	1	2	3	4	5	6		7
P(x)	0	K	2K	2k	3K	l.	ζ ²	$2k^2$	$7k^2 + k$

- (i) Find the value of k (ii) Evaluate P(X < 6), $P(X \ge 6)$ (iii) find P(1.5 < X < 4.5/X > 2) (iv) If $P(X \le c) > \frac{1}{2}$ find the minimum value of c.
- **B1.3** The probability function of an infinite discrete distribution is given by $P(X=j) = \frac{1}{2^j}$, $j=1, 2, 3, ..., \infty$. Verify that the total probability is 1 and find the mean and variance of the distribution. Find also P(X is even), $P(X \ge 5)$ and P(X is even). divisible by 3).
- B1.4 If the RV X takes the values 1, 2, 3 and 4 such that 2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4). Then find the
- B1.5 A Random Variable X takes the values -2,-1,0 and 1 with probabilities $\frac{1}{8}$, $\frac{1}{4}$ and $\frac{1}{2}$ respectively. Find and draw the probability distribution function.
- **B1.6** Let $P(X = x) = \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{x-1}$, x = 1, 2, 3, ..., be the probability mass function of a R.V X compute (i) P(X > 4) (ii) (A/M 2016)P(X > 4/X > 2) (iii) E(X) (iv) Var(X)

TOPIC: Continuous Random Variables

- B1.7 If the density function of X is $f(x) =\begin{cases} ce^{-2x} & 0 < x < \infty \\ 0 & x < 0 \end{cases}$, find c, what is $P(X \ge 2)$ (A/M 2010)

 1.8 The distribution function of a RV X is given by $F(x) = 1 (1 + x)e^{-x}$, $x \ge 0$. Find the density function, Mean and (A/M 2015 N/D 2010)
- 1.9 If the distribution function of a Continuous Random Variable X is given by

$$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ x^2 & \text{for } 0 \le x \le 1/2 \\ 1 - \frac{3}{25}(3 - x^2) & \text{for } 1/2 \le x \le 3 \\ 1 & \text{for } x \ge 3 \end{cases}$$

Find the pdf of X and evaluate $P(|X| \le 1)$ and $P(\frac{1}{3} \le X \le 4)$ using both the pdf and cdf.

1.10 A Continuous Random Variable X has the pdf $f(x) = kx^4$, $-1 \le x \le 0$, find the value of k and also (N/D 2011)

- $P\left\{X > \left(-\frac{1}{2}\right)/X < \left(-\frac{1}{4}\right)\right\}$ (M/J 2013)
- 1.11 If $f(x) = \begin{cases} xe^{-\frac{x^2}{2}}, & x \ge 0 \\ 0, & x < 0 \end{cases}$ then show that f(x) is a pdf and hence find F(x). (N/D 2014)
- 1.12 The cumulative distribution function of a RV X is given by $F_x(x) = \begin{cases} 0 & ; x < 0 \\ x + \frac{1}{2} & ; 0 \le x \le \frac{1}{2} \\ 1 & ; x > \frac{1}{2} \end{cases}$

Draw the graph of the CDF. Compute $P(X > \frac{1}{4})$, $P(\frac{1}{2} < X \le \frac{1}{2})$ (A/M 2015)

- 1.13 If a random variable X has a cumulative distribution function $F(x) = \begin{cases} 0 & \text{for } x \le 0 \\ c(1 e^{x}) & \text{for } x > 0 \end{cases}$ Find the probability density function, the value of c and $P(1 \le x \le 2)$. (N/D 2016)
- 1.14 Let X be a continuous random variable with the probability density function $f(x) = \frac{1}{4}$, $2 \le X \le 6$. Find the expected value and variance of X. (N/D 17.)
- 1.15 If the density function of a continuous R.V. X is given by

$$f(x) = \begin{cases} ax &, 0 \le x \le 1 \\ a &, 1 \le x \le 2 \\ 3a - ax, 2 \le x \le 3 \\ 0 &, otherwise \end{cases}$$

i) find the value of a (ii) find the cdf of X. (iii) If x_1, x_2 and x_3 are 3 independent observation of X. what is the probability that exactly one of these 3 is greater than 1.5? [A/M 04, N/D 08]

- 1.16 Find the MGF and r^{th} moment for the distribution whose p.d.f is $f(x) = ke^{-x}$, $0 \le x \le \infty$. Hence find the mean and
- 1.17 If $f(x) =\begin{cases} \frac{x}{4}e^{-\frac{x}{2}}, & x \ge 0 \\ 0, & x < 0 \end{cases}$ find first four moments about the origin. (A/M 17.)

 1.18 A Continuous Random Variable X has the p.d.f $f(x) = kx^3e^{-x}$, $x \ge 0$. Find the r^{th} order moment about origin.
- MGF and mean and variance of X.

- 1.19 Find the MGF of the RV X having the pdf $f(x) = \begin{cases} x, & 0 < x \le 1 \\ 2 x, & 1 \le x < 2 \\ 0, & otherwise \end{cases}$ (N/D 2013)
- 1.20 Let X be a continuous R.V with probability density function $f(x) = \begin{cases} xe^{-x} & ; x \ge 0 \\ 0 & ; otherwise \end{cases}$ Find (i) the cumulative (A/M 2016)

distribution function of X(ii) Moment generating function $M_x(t)$ of X (iii) P(X < 2) (iv) E(X). 1.21 Find the moment generating function and rth moment for the distribution whose probability density function is (N/D 2016) $f(x) = e^{-x}$, $0 \le x \le \infty$. Also find the first three moments about mean.

1.22 The probability distribution function of a random variable X is given by $f(x) = \frac{4x(9-x^2)}{81}$, $0 \le x \le 3$. Find the mean, variance and third moment about origin.

TOPIC: Binomial Distribution

- 1.23 Find the M.G.F of the binomial random variable with parameters m and p and hence find its mean and variance. (A/M 2011) (N/D 2012) (A/M 2015 17)
- 1.24 6 dies are thrown 729 times. How many times do you except at least three die to show a five (or) a six? (N/D 2013)
- 1.25 In a large consignment of electric bulbs, 10% are defective. A Random Sample of 20 is taken for inspection. Find the probability that

i. All are good bulbs.

ii. At most there are 3 defective bulbs

(M/J 2013)iii. Exactly there are 3 defective bulbs

1.26 Assume that 50% of all engineering students are good in mathematics. Determine the probabilities that among 18 (A/M 2016)engineering students (1) exactly 10 (2) atleast 10 are good in maths.

1.27 It is known that the probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20, find the number of packets containing at least, exactly and at most 2 (N/D 2016) defective items in a consignment of 1000 packets.

1.28 The probability of a man hitting a target is 1/4. If he fires 7 times, what is the probability of his hitting the target atleast twice? And how many times must be fires so that the probability of his hitting the target atleast once is (A/M 17.) greater than 2/3?

1.29 A radar system has a probability of 0.1 of detecting a certain target during a single scan. Use binomial distribution to find the probability that the target will be detected atleast 2 times in four consecutive scans. Also compute the probability that the target will be detected atleast onces in twenty scans. (N/D 18.)

TOPIC: Poisson Distribution

- 1.30 By calculating the MGF of Poisson distribution with parameter λ , prove that the mean and variance of the Poisson (M/J 2014) (N/D 2014) (A/M 2010) distribution are equal.
- 1.31 The number of monthly breakdowns of a computer is a RV having a Poisson distribution with Mean equal to 1.8 .Find the probability that this computer will function for a month (i) Without a breakdown (ii) with only one (N/D 2012 A/M 17) breakdown
- 1.32 If X is a Poisson variate such that P(X=2) = 9P(X=4) + 90P(X=6). Find (i) Mean and $E(X^2)$ (ii) $P(X \ge 2)$ (M/J 12)
- 1.33 A Manufacturer of pins knows that 2% of his products are defective. If he sells pins in boxes of 100 And guarantees that not more than 4 pins will be defective, what is the probability that a box fails to meet the guaranteed (N/D 2013)
- 1.34 The number of typing mistakes that a typist makes on a given page has a Poisson distribution with Mean of 3 (N/D 2015) mistakes. What is the probability that she makes
 - (i) Exactly 7 mistakes
 - (ii) Fever than 4 mistakes
 - (iii) No mistakes on a given page
- 1.35 Message arrive at a switch board in a poisson manner at an average rate of six per hour. Find the probability for each of the following events:
 - (i) exactly two messages arrive within one hour
 - (ii) no message arrives within one hour
 - (iii) at least three messages arrive within one hour

(A/M 15 N/D 2016 17.)

- 1.36 Suppose that telephone calls arriving at a particular switch board follow a Poisson process with an average of 5 calls coming per minute. What is the probability that up to a minute will elapse unit 2 calls have come into the switch (A/M 2011)
- 1.37 Ten percent of the tools produced in a certain manufacturing company turn out to be defective. Find the probability that in a sample of 10 tools chosen at random, exactly 2 will be defective by using (i) Binomial distribution (ii) The poisson approximation to the binomial distribution
- 1.38 The probability of an individual suffering a bad reaction from an injection of a certain antibiotic is 0.001, out of 2000 individual, use poisson distribution to find the probability that exactly three suffer. Also, find the probability of more than two suffer from bad reaction. (N/D 18.)

TOPIC: Geometric Distribution

1.39 Describe the situations in which geometric distributions could be used. Obtain its moment generating function.

(A/M 10)

- (A/M 2015) 1.40 Find the MGF of geometrically distributed random variable and hence find its mean and variance.
- 1.41 Derive mean and variance of a geometric distribution. Also establish the forgetfulness property of the geometric (A/M 2011) (A/M 2016) distribution.
- 1.42 A coin is tossed until the first head occurs. Assuming that the tosses are independent and the probability of a head occurring is 'p'. Find the value of 'p', so that the probability that an odd number of tosses required is equal to 0.6. can you find a value of 'p', so that the probability is 0.5 that an odd number of tosses are required. (N/D 2010)
- 1.43 If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8. what is the (N/D 2012 A/M 17) probability that he will finally pass the test (i) on the 4th trial (2) in fewer than 4 trials?
- 1.44 Suppose that a trainee soldier shoots a target in an independent fashion. If the probability that the target is shot on any one shot is 0.7.
 - (1) What is the probability that the target would be hit on tenth attempt?
 - (2) What is the probability that it takes him less than 4 shots?
 - (3) What is the probability that it takes him an even number of shots?

(M/J 2014)

TOPIC: Exponential Distribution

1.45 Find the moment generating function of an exponential random variable and hence find its mean and variance.

- 1.46 The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = 1/2$. What is the probability that the repair time exceeds 2h? What is the conditional probability that a repair takes at least 10h given (N/D 2013) that its duration exceeds 9h?
- 1.47 The milage which car owners get with a certain kind of radical tire is a random variable having an exponential distribution with mean 40,000 km. Find the probabilities that one of these tires will last at least 20,000 km and at (A/M 2015)most 30,000km.
- 1.48 The life time X of particular brand of batteries is exponentially distributed with mean of 4 weeks. Determine

(i)The mean and variance of X

- (ii) What is the probability that the battery life exceeds 2 weeks?
- (iii) Given that the battery has lasted 6 weeks, What is the probability that it will last another 5 weeks? (N/D 2015)
- 1.49 A component has an exponential time to failure distribution with mean of 10,000 hours. (i)The component has already been in operation for its mean life. What is the probability that it will fail by 15,000 hours?(ii)At 15,000 hours the component is still in operation. What is the probability that it will operate for another 5000 hours? (N/D 2015)
- 1.50 The life (in years) of a certain electrical switch has an exponential distribution with an average life of $\frac{1}{2} = 2$. If 100 of these switches are installed in different systems, find the probability that at most 30 fail during the first year. (A/M 2016)
- 1.51 State and prove forgetfulness property of exponential distribution. Using this property. Solve the following problem: The length of the shower on a tropical island during rainy season has an exponential distribution with parameter 2, time being measured in minutes. If a shower has already lasted for two minutes. What is the (N/D 2016) probability that it will last for at least one more minute?

TOPIC: Uniform Distribution(Rectangular Distribution)

- (M/J 2013)1.52 Find the MGF of Uniform Distribution. Hence find its mean and variance.
- 1.53 Trains arrive at a station at 15 min intervals starting at 4 a.m. If a passenger arrive at a station at a time that is uniformly distributed between 9.00 and 9.30, find the probability that he has to wait for the train for (1) less than 6 mins (2) more than 10 mins.

- 1.54 If a continuous Random Variable X follows uniform distribution in the interval (0,2) and a continuous RV. Y follows exponential distribution with parameter λ . Find λ such that P(X < 1) = P(Y < 1) (N/D 2013)
- 1.55 If X is a Random Variable with a continuous distribution function F(X). Prove that Y = F(X) has a uniform distribution in (0,1), further if $f(x) = \begin{cases} \frac{1}{2}(x-1), & 1 \le x \le 3 \\ 0, & otherwise \end{cases}$ Find the Range of Y. Corresponding to the Range 1.1 $\le x \le 2.9$
- 1.56 Let X be uniformly distributed random variable in the interval (a, 9) and P[3 < x < 5] = 2/7. Find the constant 'a' and compute P[|X-5| < 2]. (A/M 18.)
- 1.57 Electric trains in a particular route run every half an hour between 12. Midnight and 6a.m. using uniform distribution, find the probability that a passenger entering the station at any time between 1.00a.m will 1.30a.m. will have to wait atleast twenty minutes.

 (N/D 18.)
- 1.58 Let X be a uniformly distributed R.V over [-5,5]. Determine (i) $P(X \le 2)$ (ii) P(|X| > 2) (iii) Cumulative distribution function of X (iv) Var(X) (A/M 2016)

TOPIC: Normal Distribution

- 1.59 Find the MGF of $N(\mu, \sigma^2)$ Normal Distribution and hence find its mean and variance (N/D 15 17. A/M 18.)
- 1.60 In a Normal Distribution 31% of the items are under 45 and 8% are over 64. Find the mean and variance of the distribution.

 (N/D 2014)
- 1.61 The peak temperature T, as measured in degrees Fahrenheit, on a particular day is the Gaussian (85, 10) random variable. What is P(T > 100), P(T < 60) and $P(70 \le T \le 100)$? (A/M 2015)
- 1.62 The average percentage of marks of candidates in an examination is 42 with a standard deviation of 10. If the minimum mark for pass is 50% and 1000 candidates appear for the examination, how many candidates can be expected to get the pass mark if the marks follow Normal Distribution? If it is required, that double the number of the candidates should pass, what should be the minimum mark for pass?

 (N/D 2015)
- 1.63 The annual rainfall in inches in a certain region has a normal distribution with a mean of 40 and variance of 16. What is the probability that the rainfall in a given year is between 30 and 48 inches? (N/D 2016)
- 1.64 In a normal population with mean 15 and standard deviation 3.5, it is found that 647 observations exceed 16.25, what is the total number of observations in the population?

 (A/M 17.)
- 1.65 An electrical firm manufactures light bulbs that have a length of life which is normally distributed with mean $\mu = 800$ hours and standard deviation $\sigma = 40$ hours. Find the probability that a bulbs burns between 778 and 834 hours.

 (N/D 18.)
- 1.66 The scores on an achievement test given to 1,00,000 students are normally distributed with mean 500 and standard deviation 100. What should the score of a student be to place him among the top 10% of all students? (A/M 18.)

BAYE'S THEOREM

- 1.67 A bag contains 5 ball and it is not known how many of them are white. Two balls are drawn at random from the bag and they are noted to be white. What is the chance that all the balls in the bag are white?
- 1.68 There are 3 true coins and 1 false coin with 'head' on both sides. A coin is chosen at random and tossed 4 times. If heads occurs all the 4 times, what is the probability that the false coin has been chosen and used?
- 1.69 For a certain binary communication channel, the probability that a transmitted '0' is received as a'0' is 0.95 and the probability that a transmitted '1' is received as '1' is 0.90. If the probability that a '0' is transmitted is 0.4. find the probability that (i) a '1' is received and (ii) a '1' was transmitted given that a '1' was received.
- 1.70 In a coin tossing experiment, if the coin shows head, 1 die is thrown and the result is recorded. But if the coin shows tail, 2 dice are thrown and their sum is recorded. What is the probability that the recorded number will be 2?
- 1.71 An urn contains 10white and 3 black balls. Another urn contains 3white and 5black balls. Two balls drawn at random from the first urn and placed in the second urn and then 1 ball is taken at random from the latter. What is the probability that it is a white ball?
- 1.72 A bolt is manufactured by 3 machines A,B and C.A turns out twice as many items as B, and machines B and C produce equal number of items.2% of bolts produced by A and B are defective and 4% of bolts produced by C are defective. All bolts are put into 1 stock pile and 1 is chosen from this pile. What is the probability that it is defective?