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# A Study of Low-Volatility Portfolio Construction Methods

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Low-volatility investing has attracted considerable interest and substantial assets since the global financial crisis, but the concept is not new. Minimum variance, one of the most popular low-volatility strategies, has been known since Markowitz's 1952 paper on mean–variance analysis. The performance advantage of low-volatility or low-beta stocks has been documented by Black [1972]; Haugen and Heins [1975], and others since the early 1970s, preceding even the discovery of the value and size premia.

Over extended periods, the Sharpe ratios of long-only low-volatility portfolios are roughly 0.5 in the United States and slightly higher in international markets. These values are significantly more attractive than those of corresponding cap-weighted indices, which register Sharpe ratios of 0.3 and 0.25, respectively, for the same periods. The higher Sharpe ratios are not solely the result of less variability in the time series of returns; although they have performed poorly relative to cap-weighted market portfolios in strongly up-trending markets, low-volatility portfolios generally provide superior long-term returns.

Unless an investor seeks to use low-volatility portfolios for the purpose of market timing, short-term relative performance is generally uninformative. For long-term investors, the pertinent question is whether a low-volatility portfolio has a place in the strategic asset allocation. Some investment consultants have suggested allocating to low-volatility developed market equities to free up a portion of the risk budget for more aggressive allocations to emerging market equities.[1] Others have recommended replacing traditional equity exposure with low-volatility strategies to reduce the volatility of the standard 60/40 stock/bond portfolio without sacrificing returns. The more aggressive pension plans have considered increasing the total equity allocation through low-volatility equity strategies, which would maintain plan level volatility while increasing expected return.[2]

## THE LOW-VOLATILITY PUZZLE

Empirical support for the outperformance of low-volatility portfolios is robust across time periods and countries. An equity portfolio's volatility is driven by its market beta. Therefore, the outperformance of low-volatility portfolios relates directly to the anomaly that low-beta stocks deliver higher returns than high-beta stocks; this is the famous empirical dagger in CAPM's heart.[3] Haugen hinted at the "valueness" of low-beta stocks in his earlier commentaries on the low-volatility puzzle. Subsequently, Fama and French's three-factor model formalized the more modern understanding that low-beta stocks outperform high-beta stocks because they also tend to be value and small stocks.[4]

However, the recent literature on the low-beta puzzle has uncovered a new and important empirical fact. Frazzini and Pedersen [2011] created a zero-beta factor portfolio, BAB (betting against beta),[5] which goes long low-beta stocks and short high-beta stocks. For obvious reasons, this zero-beta portfolio is, nonetheless, net long in equity exposure. Unlike the theoretical Black zero-beta securities, it has a very high Sharpe ratio. Frazzini and Pedersen found that the low-beta outperformance can be explained more completely by taking the BAB factor into account. They argued that low-beta investing exposes investors to another valuable source of equity return premium in addition to the traditional value and size premia. Baker et al. [2011] hypothesize that the BAB premium could be driven by the leverage constraint/aversion discussed by Black [1972] or by the preference for gambling hypothesized by Barberis and Huang [2008]. Hsu et al. [2013] suggest that sell-side analysts' systematic forecast optimism on volatile stocks contributes to the BAB premium. Finally, regardless of the reasons for this premium, Brennan et al.'s [2012] delegated agency model implies that the high tracking error induced by low-beta stocks discourages portfolio managers from taking advantage of this persistent phenomenon. Soe [2012] compares the risk–return profiles of large-cap, mid-cap, and small-cap low-volatility portfolios whose constituents are weighted on the basis of optimization and the inverse of standard deviation. She finds that both approaches are equally effective in reducing volatility over the long term. Soe breaks out the contributions of systematic, factor, and idiosyncratic risk to total risk but does not examine specific factor loadings.

## THIS ARTICLE'S CONTRIBUTIONS

We seek to contribute to the literature by comparing and contrasting selected low-volatility strategies. We examine different optimized minimum-variance portfolios as well as simple heuristic portfolio constructions based on weighting by the inverse of volatility and beta. Our empirical backtests allow us to study the strategies' long-term performance; sector, country, and stock concentrations; and turnover, liquidity, and capacity characteristics. The Fama–French–Carhart factors, the Fazzini–Pedersen BAB factor, and duration are used to identify the sources of return premia for the different low-volatility strategies. Low-volatility portfolios earn a duration premium because the greater stability in their cash flows tends to lend them a bond-like characteristic. This is a new finding in the literature and helps reconcile the sizeable outperformance of low-volatility strategies.

We find that all the methodologies examined—whether optimized or heuristics based—appear to have similar risk profiles.[6] This observation is unsurprising because portfolio volatility is largely dominated by the equity market beta; in principle, any reasonable methodology that shifts allocations from high-beta stocks to low-beta stocks could be calibrated to have volatility comparable to that of the minimum-variance portfolio. Similarly, we find no evidence that one low volatility portfolio construction methodology stands out from a return perspective. This result is intuitively reasonable because there are no ex ante reasons to believe that one approach to selecting low-beta stocks benefits more from the low-volatility effect than any other.

Factor attribution analyses of the U.S. and global developed low-volatility portfolios reveal that returns in excess of cap-weighted index returns are substantially driven by the value, BAB, and duration premia. The value bias in low-volatility portfolios is not unexpected and is consistent with the interpretations of Haugen, Fama, and French. Growth industries, such as technology, are characterized by high price-to-fundamental ratios and tend to have high betas and volatilities; they are natural underweights in low-volatility portfolios. The BAB bias is also intuitive. The low beta in our test portfolios is created through underweights in high-beta stocks and corresponding overweights in low-beta stocks. (It is important to note that we did not blend in cash to reduce portfolio beta.)[7] A significant duration premium is surprising for an equity portfolio; however, it is unsurprising for low-volatility portfolios, which tend to provide stable income with long-bond-like volatility, to exhibit some fixed-income characteristics. We do not find that other sources of return (small size and momentum) contribute meaningfully to the low-volatility portfolios' outperformance.

Compared with broad, cap-weighted indices, all low-volatility portfolios have a lower exposure to the market risk factor, and added exposures to the value, BAB, and duration factors. The low-volatility portfolio, therefore, has a more diversified allocation to the different sources of returns (value, low volatility, duration, and market) than would traditional core equity strategies, which tend to have market betas near unity and are consequently dominated by the equity market risk factor.[8] The differences in portfolio risk characteristics suggest that a low-volatility portfolio can be a risk-diversifying strategy when blended with standard equity portfolios.

The six-factor framework (Carhart-4 + BAB + duration) proposed in this study is powerful for estimating the forward low-volatility premium. We know that historical excess return can be a poor predictor of future outperformance because factor loadings and the sizes of factor premia can be time-varying. As more low-volatility stocks are bid up in price due to increased flows into the strategy, low-volatility portfolios may lose their value characteristics, which would reduce their forward-looking returns. Also, the likelihood of rising interest rates may suggest that the return to duration risk may be significantly lower than history would suggest. This would further detract from forward low-volatility performance.

Finally, while the various low-volatility methodologies result in comparable risk and return over time, they can differ significantly in industry and country biases as well as in turnover and liquidity characteristics. For instance, some unconstrained low-volatility portfolios might have extremely high concentrations in Japanese stocks and utility companies, whereas others have very illiquid holdings or high rates of turnover. These portfolio profiles and investability measures should be important considerations in the investor's selection criteria. We submit in the conclusion that incorporating simple investment governors into the low-volatility construction methodology would be beneficial and can be accomplished without significantly affecting portfolio volatility.

## METHODOLOGICAL SURVEY OF POPULAR STRATEGIES

In this section, we examine the various methodologies for constructing low-volatility portfolios. We illustrate the risk and return characteristics of each implementation using the most extensive historical data available from CRSP for U.S. backtests and Datastream for global and emerging markets backtests. We build low-volatility portfolios for U.S. large companies, global developed large companies, and emerging market large companies to illustrate robustness and identify interesting cross-regional variations.

### Minimum-Variance Strategies

The most popular version of the low-volatility strategies is the minimum-variance (MV) portfolio. The MV portfolio takes as input the covariance matrix for stocks in the selected universe. Then a numerical optimizer is used to select a set of non-negative stock weights such that the resulting predicted portfolio volatility is minimized. In practice, single stock concentration constraints, such as a 5% position limit, are often imposed.

Because we do not observe the *true* covariance matrix for stocks, we must estimate it. (See Chow et al. [2012] for a detailed discussion of the issues surrounding minimum-variance portfolio optimization.) The more popular methods for estimating covariance matrices are high-frequency sample estimates with shrinkage and factor-based estimates using either statistical factors from a principal components analysis (PCA) or risk factors from mainstream finance models. We will consider two shrinkage-based methods and two factor-based methods.

The shrinkage methods we selected are based on Ledoit and Wolf [2004] and Clarke et al. [2006]. Shrinkage methods assume outliers exist in the sample covariance matrix. To prevent these extreme estimates from unduly influencing the portfolio optimization, the sample covariances are *shrunk* toward a priori targets, which are often related to cross-sectional sample averages.[9] The factor-based approach seeks to exclude idiosyncratic noise in stock returns from the covariance estimation. The factor models we selected are the Carhart four-factor model and a statistical factor model extracted by PCA. The former assumes that the market, size, value, and momentum factors adequately describe stock returns; the latter identifies common factors in stock returns by statistically analyzing the sample data.

Using these methods, we estimated covariance matrices for the 1,000 largest companies, using trailing five years of monthly returns. On the basis of these matrices, we then used a quadratic programming solver to compute minimum-variance long-only portfolios with 5% position limits. The hypothetical performance records of these minimum-variance portfolios are shown in Exhibit 1.

**E X H I B I T 1**
**Minimum-Variance Portfolios Based on Covariance Estimation Techniques**

| | Return | Volatility | Sharpe Ratio | Tracking Error vs. Cap-Weighted | Tracking Error vs. Others |
|---|---|---|---|---|---|
| ***U.S.* (1967 to 2012)** | | | | | |
| Cap-Weighted Index | 9.81% | 15.43% | 0.29 | | |
| 4-Factor Model | 11.15% | 11.44% | 0.51 | 10.13% | 3.91% |
| PCA Factor Model | 11.63% | 11.57% | 0.55 | 9.17% | 3.91% |
| Shrinkage (Ledoit & Wolf) | 11.22% | 11.27% | 0.53 | 11.57% | 5.65% |
| Shrinkage (Clarke et al.) | 11.54% | 11.82% | 0.53 | 8.15% | 4.15% |
| ***Global* (1987 to 2012)** | | | | | |
| Cap-Weighted Index | 7.58% | 15.77% | 0.24 | | |
| 4-Factor Model | 8.15% | 10.45% | 0.42 | 14.29% | 4.85% |
| PCA Factor Model | 7.50% | 10.50% | 0.36 | 12.41% | 4.41% |
| Shrinkage (Ledoit & Wolf) | 9.01% | 9.51% | 0.56 | 14.06% | 5.29% |
| Shrinkage (Clarke et al.) | 8.25% | 11.22% | 0.40 | 12.25% | 5.42% |
| ***EM* (2002 to 2012)** | | | | | |
| Cap-Weighted Index | 14.59% | 23.83% | 0.54 | | |
| 4-Factor Model | 16.03% | 11.81% | 1.22 | 16.63% | 4.27% |
| PCA Factor Model | 15.56% | 11.61% | 1.20 | 16.83% | 4.23% |
| Shrinkage (Ledoit & Wolf) | 16.21% | 10.85% | 1.34 | 17.51% | 5.46% |
| Shrinkage (Clarke et al.) | 18.68% | 14.14% | 1.20 | 12.39% | 6.45% |

*Source: Research Affiliates, based on data from CRSP for U.S. and Datastream for Global and EM.*

Compared with the cap-weighted index, the U.S. MV portfolios have about 25% less volatility and a return advantage of 134 to 182 basis points (averaging 156 basis points). The global developed minimum-variance portfolios reflect a volatility reduction over 30%. This superior lessening in volatility is unsurprising; the average correlation of global stocks is lower than that of U.S. stocks. The global minimum-variance portfolios' return advantage ranges from −8 to 143 basis points (averaging 65 basis points). The emerging markets minimum-variance portfolios have about 50% less volatility than the cap-weighted index; again, this is predictable given the substantially lower cross-correlation for the heterogeneous basket of countries. The outperformance ranges from 97 to 409 basis points (averaging 203 basis points). In all cases, the risk reduction is economically and statistically significant. In all but one case, the portfolio outperformed, and on average, the outperformance is economically large. The value-added return is not statistically meaningful due to the large tracking error, but the resulting improvement in the Sharpe ratio is statistically strong.

It is not obvious, ex ante, which minimum-variance methodology would be the best choice. The differences among them in risk and return are not statistically significant. The portfolio constructed in accordance with the Clarke et al. [2006] shrinkage method appears to have the least volatility reduction, but it does not have a consistently poorer Sharpe ratio. We do not have decisive quantitative evidence in favor of any one of the four covariance-based approaches. Given the 4% tracking error between any pair, investors should avoid concluding that any given minimum variance *product* is superior to another due to short-term relative performance. Additionally, it is important not to treat the insignificant differences in long-term volatility or realized returns as informative statistics for selecting one methodology over the others.

### Heuristic Low-Volatility Methodologies

Assuming a well-specified forward-looking covariance is used, the MV portfolio strategy should produce the portfolio with the lowest ex ante volatility. In this section, we examine alternate weighting heuristics that come close to the MV volatility. Clarke et al. [2011] argue that when stock return fluctuations are well described by a one-factor model (that is, the $R^2$ from a single-factor time-series return regression is high), then the MV solution can be approximated by a simpler methodology: calculating weights as a function of CAPM beta.

We examine two distinct weighting heuristics—weighting driven by the constituent stocks' CAPM betas, on one hand, and total volatilities, on the other. We refer to these two portfolio strategies as inverse beta and inverse volatility. Procedurally, we compute the beta and volatility[10] of each stock in our universe using trailing five-year daily data.[11] We select the 200 lowest beta (volatility) stocks from the 1,000 largest companies[12] and weight them in the portfolio by their inverse beta (volatility). Intuitively, the resulting portfolio will contain the 200 lowest-beta (volatility) stocks, with higher weights allocated to the lower-beta (volatility) stocks. Because stocks with low betas generally have low volatilities, the two heuristics are expected to produce fairly comparable portfolios. We display the portfolio backtests in Exhibit 2. For robustness, we weight the inverse-beta (inverse-volatility) portfolios using both equal weighting and 1/beta (1/volatility) weighting schemes.

It is evident from Exhibit 2 that the heuristic approaches can also be effective in reducing volatility. In developed markets, the volatilities are tolerably close to that of the minimum-variance portfolio. In emerging markets, however, the naïve heuristic approaches only reduce volatility by 35%, appreciably less than the 50% reduction achieved by the MV methodology. Clarke et al. [2011] stated that naïve heuristic approaches based on a single risk parameter would be less successful at volatility reduction when applied to a more heterogeneous basket of countries, such as emerging market countries. The Sharpe ratios calculated for the heuristically constructed portfolios are close to or higher than those of the MV portfolio, but there is no theoretical reason to believe that either approach would invariably produce better risk-adjusted returns. However, we note that the naïve EM MV portfolios are generally too concentrated and illiquid to be practical (see Exhibits 3, 4, and 6). We will return to this point in a later section.

The various heuristic strategies do not appear to provide convincing statistical or economic advantages over one another. This is largely related to the empirical fact that, in the cross-section, low-beta stocks tend to also be low-volatility stocks and vice versa. There is

## EXHIBIT 2
**Low-Volatility Portfolios Based on Heuristic Weighting Schemes**

| | | | | Tracking Error | |
|---|---|---|---|---|---|
| | **Return** | **Volatility** | **Sharpe Ratio** | **vs. Cap-Weighted** | **vs. Others** |
| ***U.S.* (1967 to 2012)** | | | | | |
| Cap-Weighted Index | 9.81% | 15.43% | 0.29 | | |
| Minimum Variance* | 11.38% | 11.52% | 0.53 | 9.75% | |
| Low Volatility (1/Vol) | 11.65% | 12.55% | 0.51 | 8.58% | 2.55% |
| Low Volatility (EW) | 11.77% | 12.68% | 0.51 | 8.28% | 2.53% |
| Low Beta (1/β) | 11.83% | 12.84% | 0.51 | 9.21% | 2.95% |
| Low Beta (EW) | 12.30% | 13.13% | 0.53 | 8.93% | 2.76% |
| ***Global* (1987 to 2012)** | | | | | |
| Cap-Weighted Index | 7.58% | 15.77% | 0.24 | | |
| Minimum Variance* | 8.23% | 10.42% | 0.44 | 13.25% | |
| Low Volatility (1/Vol) | 10.58% | 11.56% | 0.59 | 12.16% | 4.38% |
| Low Volatility (EW) | 10.60% | 11.82% | 0.58 | 11.99% | 4.42% |
| Low Beta (1/β) | 10.40% | 12.44% | 0.54 | 13.12% | 7.04% |
| Low Beta (EW) | 10.36% | 11.70% | 0.57 | 10.70% | 5.12% |
| ***EM* (2002 to 2012)** | | | | | |
| Cap-Weighted Index | 14.59% | 23.83% | 0.54 | | |
| Minimum Variance* | 16.62% | 12.10% | 1.24 | 15.84% | |
| Low Volatility (1/Vol) | 21.14% | 16.21% | 1.20 | 10.63% | 3.90% |
| Low Volatility (EW) | 21.19% | 16.63% | 1.18 | 10.22% | 3.96% |
| Low Beta (1/β) | 23.46% | 16.20% | 1.35 | 12.46% | 5.20% |
| Low Beta (EW) | 22.72% | 17.31% | 1.22 | 11.08% | 4.44% |

**Minimum variance results reflect the average of the portfolios reported in Exhibit 1.*

*Source: Research Affiliates, based on data from CRSP for U.S. and Datastream for Global and EM.*

modestly greater dispersion between the inverse-beta and inverse-volatility strategies in the emerging markets case; this presumably reflects noise in the relationship between CAPM beta and volatility for emerging market stocks.

Exhibit 3 shows the effective *N*, the inverse Herfindahl score, for the various strategies. The effective *N* is a simple, straightforward measure of concentration; the larger the value, the less concentrated the portfolio along the measured dimension.[13] This statistic may be valuable for investors who have explicit or implicit concerns regarding portfolio concentration in industry sectors or countries.[14]

Heuristically constructed low-volatility portfolios typically have lower turnover than MV portfolios. Exhibit 4 shows that annually re-optimized minimum-variance methodologies generate one-way turnover of 45% in the United States, roughly twice as much as the inverse-beta and inverse-volatility implementations. The heuristic approaches' turnover advantage declines in more heterogeneous markets. Note also that the inverse-beta low-volatility portfolio has higher turnover than the inverse-volatility portfolio. This characteristic is related to noise (and instability) inherent in a simplistic beta estimate. Using robust beta estimation will reduce turnover in an inverse-beta low-volatility portfolio meaningfully.[15]

## EXHIBIT 3
**Simple Measure of Stock, Sector, and Country Concentration**

| | Cap-Weighted Index | Minimum Variance (average) | Select & Weight by 1/Vol | Select & Weight by 1/β |
|---|---|---|---|---|
| ***U.S.* (1967 to 2012)** | | | | |
| Effective *N* (sectors) | 9.59 | 4.80 | 4.47 | 4.78 |
| Effective *N* (stocks) | 136.14 | 43.13 | 196.02 | 153.85 |
| Weight in Top-10 Holdings | 20.76% | 40.04% | 6.84% | 12.76% |
| ***Global* (1987 to 2012)** | | | | |
| Effective *N* (countries) | 3.47 | 2.91 | 3.39 | 4.83 |
| Effective *N* (sectors) | 4.47 | 5.52 | 6.02 | 6.61 |
| Effective *N* (stocks) | 319.55 | 36.75 | 194.34 | 122.48 |
| Weight in Top-10 Holdings | 10.92% | 36.94% | 7.38% | 19.29% |
| ***EM* (2002 to 2012)** | | | | |
| Effective *N* (countries) | 11.27 | 8.10 | 9.06 | 9.29 |
| Effective *N* (sectors) | 8.66 | 6.83 | 7.19 | 7.68 |
| Effective *N* (stocks) | 222.15 | 63.00 | 193.82 | 110.79 |
| Weight in Top-10 Holdings | 14.10% | 37.67% | 7.56% | 21.71% |

*Source: Research Affiliates, based on data from CRSP for U.S. and Datastream for Global and EM.*

## EXHIBIT 4
**Low-Volatility Portfolio Strategy: Turnover Characteristics**

| | Cap-Weighted Index | Minimum Variance (average) | Select & Weight by 1/Vol | Select & Weight by 1/β |
|---|---|---|---|---|
| ***U.S.* (1967 to 2012)** | | | | |
| One Way Turnover | 4.42% | 44.90% | 18.92% | 26.91% |
| ***Global* (1987 to 2012)** | | | | |
| One Way Turnover | 5.97% | 47.35% | 24.07% | 35.89% |
| ***EM* (2002 to 2012)** | | | | |
| One Way Turnover | 9.08% | 46.75% | 28.32% | 39.24% |

*Source: Research Affiliates, based on data from CRSP for U.S. and Datastream for Global and EM.*

## UNDERSTANDING LOW-VOLATILITY PERFORMANCE

Why do low-volatility portfolios generally outperform cap-weighted indices? To understand the return profile as well as the added value, we turn to a standard return decomposition tool, the Fama–French–Carhart four-factor (FF-4) model. We also use an augmented model, which additionally incorporates Frazzini and Pedersen's BAB factor as well as a duration factor. Exhibit 5 displays a return decomposition analysis using the two risk models. We note that the augmented model has meaningfully improved $R^2$ and statistically and economically large loadings on the BAB and duration factors. The unexplained alphas are also reduced meaningfully.

We obtained the FF-4 factors for the U.S. and global markets from Kenneth French's data library.[16] For emerging markets, which French does not currently cover, we calculated factors using methods similar to those described in Fama and French [2012]. We also calculated BAB factors for all regions using the methodology set forth in Frazzini and Pedersen [2011].[17] We use the concatenated excess return time series from the Barclays Capital Long U.S. Treasury Index and the Ibbotson SBBI U.S. Government Long Treasury for the U.S. duration factor. For the global duration factor, we use the Citi Developed Bond Index. For the emerging market duration factor, we use the JP Morgan GBI Emerging Market Global Diversified Index (local currency bonds).

First, we observe that the market betas of all the low-volatility portfolios are significantly below one (about 0.7 for the U.S. strategies and lower for the global and emerging markets strategies). An equity portfolio's volatility is generally dominated by its co-movement with the market; accordingly, regardless of the construction methodology chosen, a low-volatility portfolio will generally contain stocks with low market beta.[18] We can interpret the lower-than-one market beta as meaning that the low-volatility portfolios are short market factor risk versus the cap-weighted index. It stands to reason that shifting from a portfolio that is concentrated in the market factor to a more diversified portfolio by adding factors with attractive premia, such as value and BAB, would improve the portfolio's risk–return profile. Replacing a beta one equity portfolio with a low-volatility portfolio will be risk diversifying, generally reducing an investment program's co-movement with the equity market without reducing its overall equity allocation.

Second, we note that the value factor generally contributes meaningfully to the observed outperformance in the U.S. and global low-volatility portfolios. The relationship between low beta and value is intuitively clear. Haugen argued that low volatility stocks tend to be boring stocks without the large price movements that garner media mentions and investor attention. Generally, high-beta stocks tend to belong to growth industries; eliminating them would naturally result in a more value-oriented portfolio. The positive correlation between the value and BAB factors also supports this observation. (Shown in Exhibit A1, the correlations range from 0.37 to 0.57.)[19]

Much of the difference between the returns of the low-volatility portfolios and the cap-weighted indices is due to the fact that low-volatility portfolios have market betas of 0.7 or less. As such, low-beta portfolios will underperform in a bull market and outperform in a bear market. Moreover, the low beta overwhelms the value effect over any short to intermediate horizon—that is, in a bull market, a low-volatility strategy would still lag significantly even if value stocks performed well. Consequently, despite low-volatility portfolios' substantial value loading, their excess returns tend to show weak correlation with value stocks and strong negative correlation with the cap-weighted index.

However, in emerging markets, low-volatility portfolios do not naturally favor value industries and companies. Notice that (in Exhibit 5) the loadings on the value factor (HML) are insignificant even when BAB is excluded as a factor; with the BAB factor included, the loadings on HML become economically negative. Given the attractiveness of the value premium in emerging markets (the EM value Sharpe ratio is about three times that of the developed markets), this moderately anti-value bias seems undesirable. The positive exposure to interest rate risk is intuitive. Very low-volatility (and high-yielding) stocks can often be used as fixed-income replacement by investors. These high-yielding low-volatility stocks could be bid up by investors seeking yield and safety when interest rates are low. This then injects duration exposure into low-volatility portfolios. The 0.2 correlation between the BAB and the duration factor supports this observation.

## Exhibit 5
**Low-Volatility Portfolios' Factor Exposures**

| | Factor Model | Alpha | Alpha (T-stat) | Mkt-RF | SMB | HML | WML | BAB | DUR | $R^2$ |
|---|---|---|---|---|---|---|---|---|---|---|
| ***U.S. Low Volatility Strategies (1967 to 2012)*** | | | | | | | | | | |
| Minimum Variance (PCA) | Carhart 4 | 0.82% | 0.87 | 0.65* | 0.16* | 0.31* | 0.04* | | | 72% |
| | Carhart 4 + BAB + DUR | 0.42% | 0.50 | 0.61* | 0.22* | 0.12* | 0.01 | 0.27* | 0.08* | 78% |
| Low Volatility (1/Vol) | Carhart 4 | 0.66% | 0.75 | 0.74* | 0.05* | 0.40* | 0.00 | | | 79% |
| | Carhart 4 + BAB + DUR | 0.06% | 0.09 | 0.69* | 0.13* | 0.16* | –0.04* | 0.35* | 0.15* | 88% |
| Low Beta (1/β) | Carhart 4 | 0.37% | 0.37 | 0.72* | 0.22* | 0.41* | 0.01 | | | 76% |
| | Carhart 4 + BAB + DUR | –0.22% | –0.29 | 0.66* | 0.31* | 0.14* | –0.03* | 0.39* | 0.12* | 85% |
| Factor Premium | | | | 5.4% | 2.9% | 4.6% | 8.2% | 4.8% | 3.1% | |
| Factor Sharpe Ratio | | | | 0.35 | 0.26 | 0.44 | 0.54 | 0.41 | 0.30 | |
| ***Global Low Volatility Strategies (Nov 1990 to 2012)*** | | | | | | | | | | |
| Minimum Variance (PCA) | Carhart 4 | 0.66% | 0.46 | 0.48* | –0.02 | 0.19* | 0.04 | | | 56% |
| | Carhart 4 + BAB + DUR | –0.88% | –0.65 | 0.47* | –0.02 | 0.07 | 0.00 | 0.15* | 0.19* | 62% |
| Low Volatility (1/Vol) | Carhart 4 | 2.56%* | 2.13 | 0.62* | –0.11* | 0.41* | 0.03 | | | 76% |
| | Carhart 4 + BAB + DUR | 0.46% | 0.48 | 0.61* | –0.11* | 0.22* | –0.03 | 0.26* | 0.13* | 86% |
| Low Beta (1/β) | Carhart 4 | 2.42% | 1.20 | 0.53* | 0.13 | 0.29* | 0.08 | | | 42% |
| | Carhart 4 + BAB + DUR | –0.19% | –0.10 | 0.51* | 0.12 | 0.08 | 0.01 | 0.29* | 0.26* | 54% |
| Factor Premium | | | | 5.1% | 1.0% | 4.7% | 7.2% | 11.4% | 3.7% | |
| Factor Sharpe Ratio | | | | 0.34 | 0.13 | 0.57 | 0.51 | 0.87 | 0.55 | |
| ***EM Low Volatility Strategies (2003 to 2012)*** | | | | | | | | | | |
| Minimum Variance (PCA) | Carhart 4 | 8.51%* | 3.26 | 0.39* | –0.02 | –0.06 | 0.11* | | | 61% |
| | Carhart 4 + BAB + DUR | 6.25%* | 2.58 | 0.21* | 0.03 | –0.14 | 0.16* | 0.09 | 0.44* | 68% |
| Low Volatility (1/Vol) | Carhart 4 | 8.69%* | 4.37 | 0.66* | 0.18* | 0.05 | 0.04 | | | 89% |
| | Carhart 4 + BAB + DUR | 5.97%* | 3.89 | 0.47* | 0.21* | –0.11 | 0.06 | 0.26* | 0.40* | 94% |
| Low Beta (1/β) | Carhart 4 | 8.80%* | 3.52 | 0.65* | 0.26* | 0.13 | 0.12* | | | 82% |
| | Carhart 4 + BAB + DUR | 6.37%* | 2.83 | 0.49* | 0.28* | –0.03 | 0.11* | 0.29* | 0.32* | 86% |
| EM Factor Premium | | | | 17.0% | –1.9% | 10.5% | 7.4% | 12.4% | 10.7% | |
| EM Factor Sharpe Ratio | | | | 0.71 | –0.27 | 1.46 | 0.51 | 1.14 | 0.90 | |

*Note: The global and EM sample periods are shortened by three years due to the availability of Citi and JPM bond indices.*

*Sources: Research Affiliates, based on data from CRSP, Kenneth French's Data Library, and Morningstar Encorr for U.S.; Datastream, Worldscope, Morningstar EnCorr, and Bloomberg for Global and EM.*

The relationship between low-beta stocks and small-capitalization stocks is inconsistent across regions. It is true that many small stocks have higher betas and are significantly more volatile. However, there are some small-cap stocks that have very low betas. Often these are illiquid stocks that trade infrequently with large bid–ask spreads; these stocks tend to exhibit large volatilities and price declines in times of crisis due to illiquidity. This suggests that low-volatility portfolios that contain substantial *small* low-beta stocks would exhibit significantly larger downside betas. We will introduce more liquidity measures and revisit this issue in a later section.

Overall, the factor exposures for the different minimum variance and heuristic strategies are statistically similar within each region. It follows that the different low-volatility methodologies would not appreciably diversify each other from a factor risk perspective. However, each low-volatility portfolio is unique with regard to stock, sector, and country weights. Consequently, it is possible to target different stock, sector, and country weights to some degree without changing the risk and return characteristics of a low-volatility portfolio. This observation will be useful when we consider the desirability of *core-like* portfolio characteristics for low-volatility strategies.

Finally, there is no reason to believe that the low-volatility strategies under consideration would generate skill-related alpha. Aside from the outlier experience in emerging markets, where unexplained alphas are nearly 6% per annum, Exhibit 5 finds the model-adjusted alpha to be statistically and economically similar to zero for all other low-volatility strategies. Unexplained alpha should not be included in estimates of future low-volatility performance; the observed alpha is likely a nonrecurring artifact of the sample data. In Exhibit A2, we compute the expected portfolio returns without the unexplained and probably unrepeatable alphas using our six-factor model and the estimated

## EXHIBIT 6
**Liquidity, Capacity, and Turnover Characteristics**

| | Cap-Weighted Index | Minimum Variance (average*) | Select & Weight by 1/Vol | Select & Weight by 1/β |
|---|---|---|---|---|
| ***U.S.*** | | | | |
| **Average Since 1967:** | | | | |
| One Way Turnover | 4.42% | 44.90% | 18.92% | 26.91% |
| WAMC (Ratio to Cap-Weighted Index) | 100.00% | 23.34% | 23.50% | 9.03% |
| **2012:** | | | | |
| Ave. Market Cap (US$B) | 85.21 | 25.42 | 27.73 | 21.35 |
| Ave. Bid-Ask Spread | 0.09% | 0.13% | 0.12% | 0.14% |
| Ave. Daily Volume (US$M) | 473.76 | 96.85 | 110.21 | 88.33 |
| ***Global*** | | | | |
| **Average Since 1987:** | | | | |
| One Way Turnover | 5.97% | 47.35% | 24.07% | 35.89% |
| WAMC (Ratio to Cap-Weighted Index) | 100.00% | 30.77% | 39.93% | 24.69% |
| **2012:** | | | | |
| Ave. Market Cap (US$B) | 70.93 | 22.35 | 33.66 | 17.88 |
| Ave. Bid-Ask Spread | 0.13% | 0.19% | 0.15% | 0.31% |
| Ave. Daily Volume (US$M) | 305.75 | 61.70 | 97.79 | 37.87 |
| ***EM*** | | | | |
| **Average Since 2002:** | | | | |
| One Way Turnover | 9.08% | 46.75% | 28.32% | 39.24% |
| WAMC (Ratio to Cap-Weighted Index) | 100.00% | 21.49% | 27.72% | 19.22% |
| **2012:** | | | | |
| Ave. Market Cap (US$B) | 26.57 | 6.83 | 7.22 | 5.33 |
| Ave. Bid-Ask Spread | 0.71% | 2.56% | 0.60% | 0.81% |
| Ave. Daily Volume (US$M) | 47.45 | 3.42 | 6.22 | 3.71 |

*Source: Research Affiliates, based on data from CRSP for U.S. and Datastream for Global and EM.*

premia for the factors; we find the resulting Sharpe ratios for emerging market low-volatility portfolios to be more economically believable. This calculation can be very valuable to investors seeking to attach a conservative estimate to the prospective outperformance of low-volatility strategies. In addition to adjusting out the potentially nonrepeatable alpha, this calculation framework allows investors to apply a haircut, for example, to the premium associated with the duration factor, if they suspect a secular rate increase might take place over the next decade.

As more sophisticated low-volatility strategies become available, the six-factor framework can help investors see through some of the more superficial features and look "under the hood" to differentiate among the different products.[20]

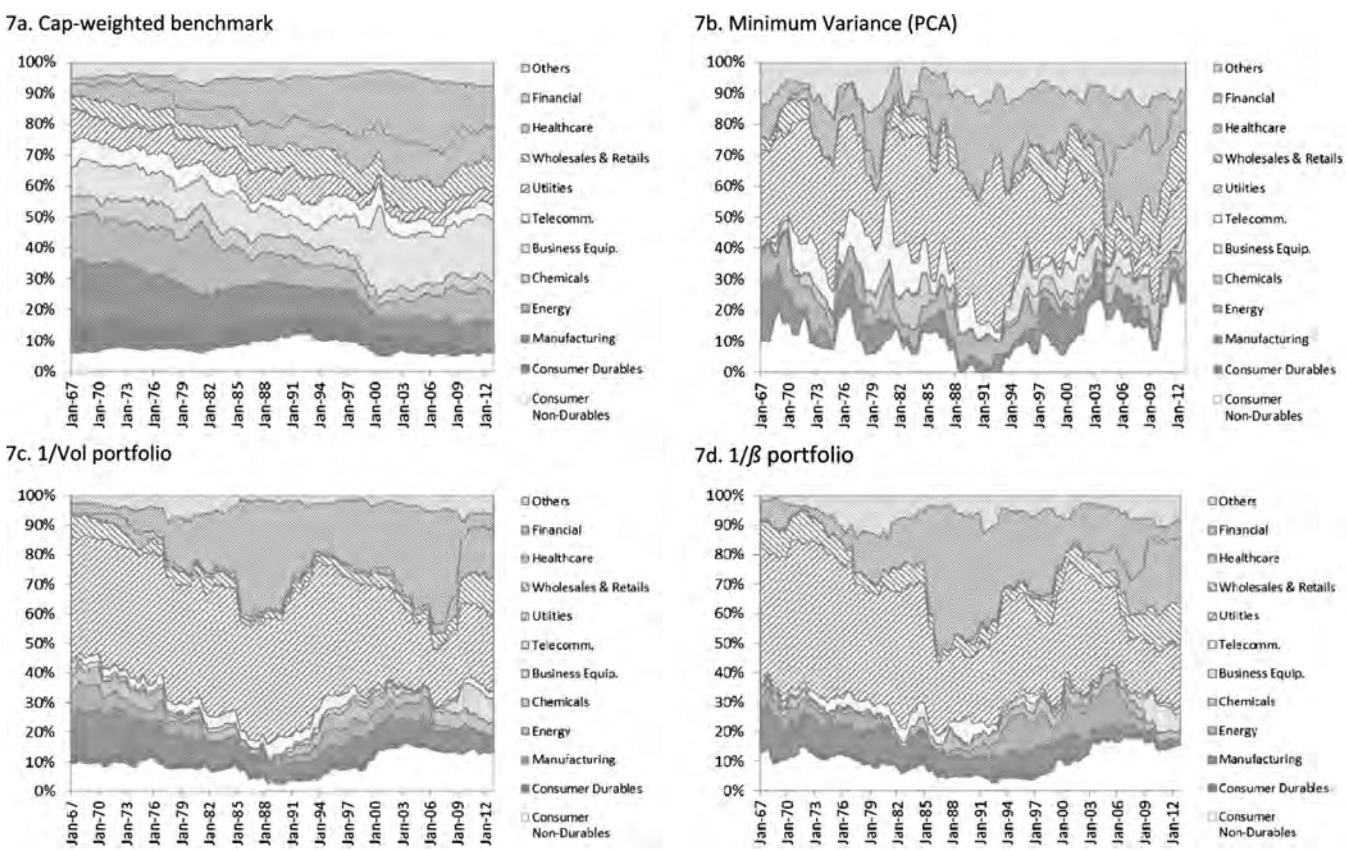
## PORTFOLIO CHARACTERISTICS

Whether the low-volatility strategy can be significantly deployed by pension funds and other institutional investors will depend, in part, on the capacity, liquidity, and implementation costs associated with various implementations. We report the turnover, weighted average market capitalization, weighted average bid–ask spread, and weighted average daily volume for the different low-volatility portfolios in Exhibit 6.

On the basis of the liquidity indicators shown in Exhibit 6, the transaction costs associated with annual reconstitutions of low-volatility portfolios appear to be significantly higher than the cost of trading the cap-weighted market portfolio. In the extreme case, emerging markets MV portfolios hypothetically met an

**E X H I B I T 7**

**Industry Allocations of U.S. Low-Volatility Strategies**




*Note: Legend items and graph categories correspond top to bottom.*

*Source: Research Affiliates, based on data from CRSP with industry definition from Kenneth French's Data Library.*

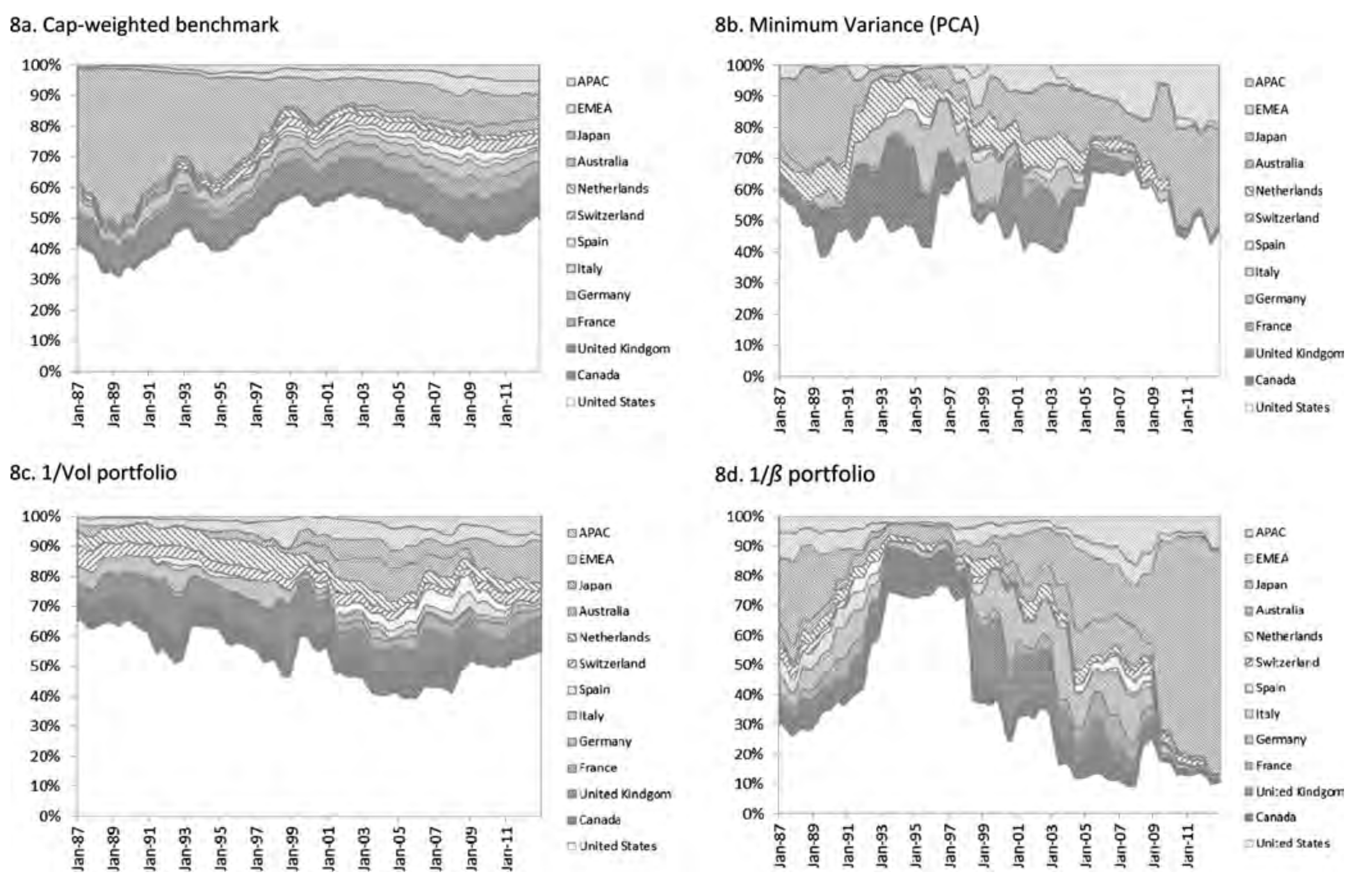
average bid–ask spread of 256 basis points as compared to 71 basis points for the cap-weighted benchmark. The larger bid–ask spread and low capacity and liquidity for low-volatility portfolios are, in part, related to the inclusion of small illiquid stocks, which may appear to have low beta due to stale pricing or asynchronous trading. Although the heuristic approach generally has a lower turnover rate than the MV method, the true takeaway is that naïve low-volatility strategies can generally be significantly more costly to trade and more difficult to implement at scale. Again, we argue that careful portfolio engineering is necessary to improve these portfolios' liquidity and investment capacity.

## CONCERNS, OTHER ISSUES, AND FUTURE RESEARCH

A potential concern regarding low-volatility portfolios is their tendency toward extreme concentrations in a particular industry or country. Highly concentrated low-volatility portfolios can be exposed to tail risk at the idiosyncratic country or sector level. These risk exposures are not captured by volatility measures. Exhibits 7–9 depict various low-volatility portfolios' sector and country allocations over time.

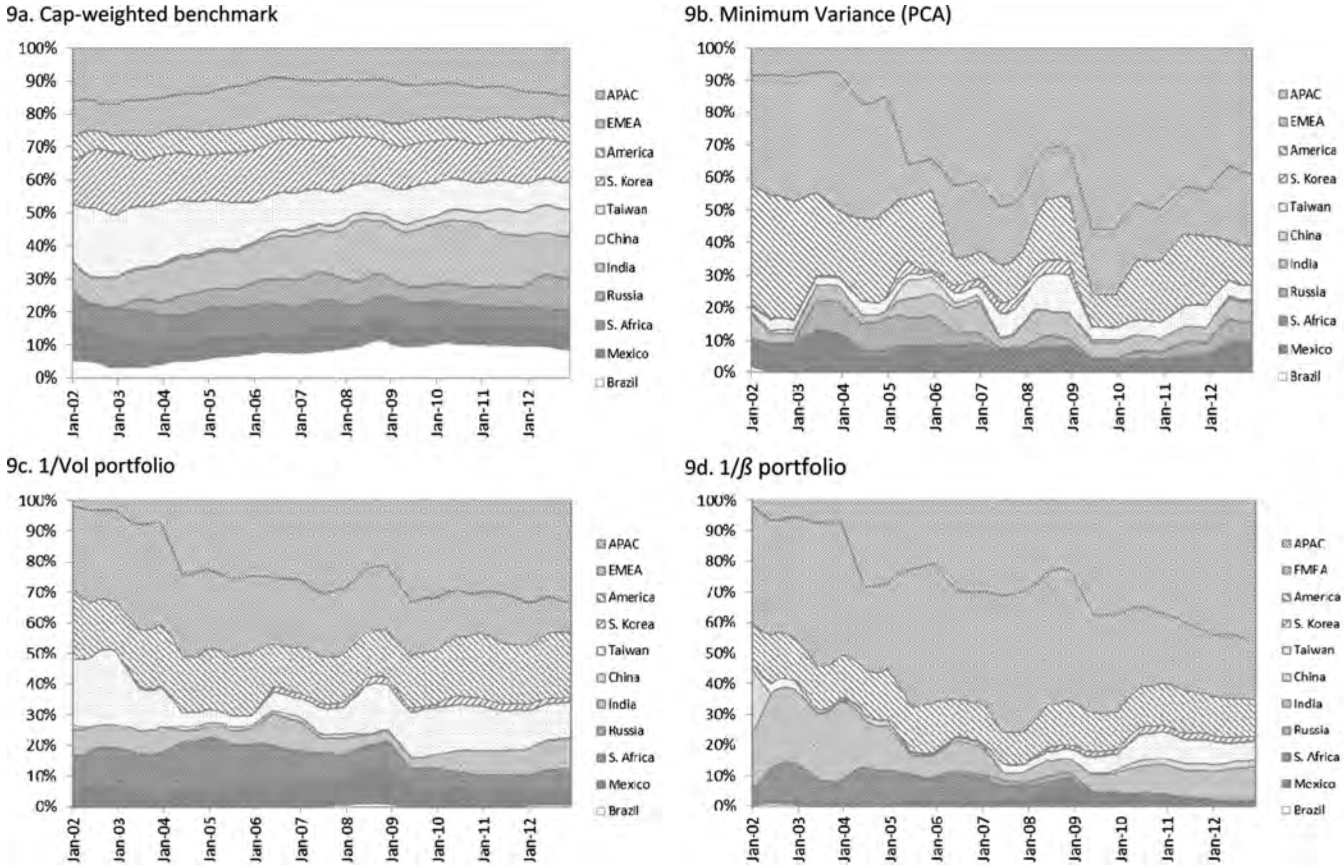
Because there is no natural relationship between volatility for stocks and their economic relevance, naïve low-volatility methodologies might construct a global

### EXHIBIT 8
**Country Allocations of Global Low-Volatility Strategies**




*Note: Legend items and graph categories correspond top to bottom.*
*Source: Research Affiliates, based on data from Datastream and Worldscope.*

## EXHIBIT 9
**Country Allocations of Emerging Markets Low-Volatility Strategies**




*Note: Legend items and graph categories correspond top to bottom.*
*Source: Research Affiliates, based on data from Datastream and Worldscope.*

all-countries portfolio with no allocation to emerging economies or an emerging market portfolio with little BRIC exposure. This can be undesirable from an asset allocation perspective.

Major providers of low-volatility investment products have adopted three distinct positions on country and sector concentrations. The dominant minimum variance index provider, MSCI, constrains its index to reflect the country and sector weights of its cap-weighted benchmark index. The S&P Low Volatility Index does not constrain country and sector allocations. The RAFI Low Volatility Index methodology scales the portfolios by the fundamental weights of constituent companies, resulting in country and sector weights that typically fall between those of the MSCI and S&P low-volatility strategies.

Setting strong constraints is not without cost; they can often increase the portfolio's volatility and reduce returns. Constrained minimum variance indices offered by popular index calculators are shown to have returns that are lower by 1.5% and volatility higher by 1% than the naïve MV portfolio. In addition, recent research finds that the outperformance of low-beta stocks over high-beta stocks is significantly stronger intra-country/sector (see Asness et al. [2013]). More research is required to evaluate different methods for targeting country and sector exposures for low-volatility strategies.

More recently, institutional consultants and investors have expressed doubts regarding the valuation level for low-volatility stocks.[21] This question exposes the one great flaw in low-volatility investing. It cannot be the case that, as investors, we are interested only in minimizing our portfolio's volatility. Would we accept a low-volatility portfolio if it produced 3% volatility but merely a 3% return? A low-volatility strategy can only make sense in the context of superior trade-offs between return and risk. It is fortuitous that low-volatility stocks, which capture an anomalous premium associated with investors' *leverage aversion* or *speculative demand for gambling,* also benefited from the value premium in developed markets. Arguably, if low-volatility stocks were to become more expensive (displaying high price-to-book ratios like growth stocks), then the low-volatility premium could be significantly reduced. Additional research is needed to better understand the interaction between BAB and value as well as to develop methodologies that would favor the cheaper low-beta stocks.[22]

Ultimately, the objective of low-volatility strategies is not so much to lessen volatility as to reallocate exposure from the market factor to additional sources of return, such as BAB, value, and duration, so as to configure a more balanced portfolio; lower volatility is the natural consequence of this risk diversification. Under this interpretation, naïve low-volatility portfolios, such as those constructed with the minimum variance, inverse-volatility, and inverse-beta methodologies, can be blunt instruments. In this, they are comparable to equal-weighted portfolios, which capture the value and small premium but limit investment capacity and generate excessive turnover. Developing more thoughtful low-volatility designs that *explicitly* diversify equity risk premium sources—rather than hoping that low-volatility stocks will also be low-price stocks—might provide more sensible solutions.

## CONCLUSION

Low-volatility investing provides higher returns at lower risk than traditional cap-weighted indexing. The anomaly is persistent over time and across countries. The lower volatility comes from a reduced exposure to the market factor, while the higher return comes from accessing high Sharpe ratio factors such as BAB, value, and duration. There are multiple ways to construct a low-volatility portfolio, including optimization-based (minimum variance) and heuristic-based (inverse beta and inverse volatility). Whereas MV portfolios generally have the lower volatility, heuristic approaches tend to have the higher long-term returns. (We caution against comparing low-volatility strategies on the basis of short-term performance.) The resulting Sharpe ratios are statistically similar.

The primary costs of low-volatility investing are underperformance in upward-trending market environments and the substantial tracking error against traditional cap-weighted benchmarks that investors may find unpalatable. Tracking error bands from 10% in the United States to 17% in emerging markets means investors who are evaluated on the basis of returns relative to the broad market are likely to experience sharp regret after a major bull market.

In addition to high tracking errors, naïve low-volatility strategies also tend to have limited investment capacity, less liquidity, and higher turnover rates. These characteristics result in high implementation costs and may even make the strategy inappropriate at size for larger institutional investors. Refined portfolio engineering techniques would likely be required to construct efficient implementations. Additionally, naïve low-volatility strategies that do not take country and sector information into account can have highly concentrated positions such as 60% Japan or 40% utilities. Conversely, low-volatility portfolios in emerging markets may have small allocations to vital areas such as the BRIC countries. These potential outcomes create asset allocation challenges calling for more thoughtful portfolio construction methodologies. In particular, it is desirable to build in mechanisms that ensure adequate country and sector diversification without significantly affecting portfolio volatility.

Finally, low-volatility investing is not primarily a risk management strategy; it is interesting because it has historically produced high returns. It is useful to recall that the low-volatility anomaly arises from behavioral patterns due to which low-volatility stocks are often undervalued relative to high-volatility stocks. If some low-volatility stocks are trading expensive (that is, at high price-to-book ratios), one would be ill advised to include them in a low-volatility portfolio. Therefore, low-volatility strategies that are unaware of the valuation level for low-beta stocks can deliver forward returns that are very different from those found in backtests.

# Appendix

## Exhibit A1

**Risk Factors' Performance and Correlations**

| | Mkt-RF | SMB | HML | WML | BAB | DUR |
|---|---|---|---|---|---|---|
| ***U.S.* (1967 to 2012)** | | | | | | |
| Factor Premium | 5.42% | 2.90% | 4.57% | 8.21% | 4.82% | 3.13% |
| Factor Sharpe Ratio | 0.35 | 0.26 | 0.44 | 0.54 | 0.41 | 0.30 |
| Correlation Against | | | | | | |
| Mkt-RF | 1.00 | 0.19 | –0.28 | –0.16 | –0.07 | 0.13 |
| SMB | | 1.00 | –0.23 | –0.03 | –0.28 | –0.11 |
| HML | | | 1.00 | –0.15 | 0.57 | 0.00 |
| WML | | | | 1.00 | 0.02 | 0.06 |
| BAB | | | | | 1.00 | 0.19 |
| DUR | | | | | | 1.00 |
| ***Global* (Nov 1990 to 2012)** | | | | | | |
| Factor Premium | 5.07% | 0.98% | 4.72% | 7.24% | 11.36% | 3.69% |
| Factor Sharpe Ratio | 0.34 | 0.13 | 0.57 | 0.51 | 0.87 | 0.55 |
| Correlation Against | | | | | | |
| Mkt-RF | 1.00 | –0.10 | –0.12 | –0.26 | –0.12 | 0.20 |
| SMB | | 1.00 | –0.18 | 0.17 | –0.02 | –0.02 |
| HML | | | 1.00 | –0.25 | 0.37 | 0.05 |
| WML | | | | 1.00 | 0.14 | –0.03 |
| BAB | | | | | 1.00 | 0.19 |
| DUR | | | | | | 1.00 |
| ***EM* (2003 to 2012)** | | | | | | |
| Factor Premium | 17.04% | –1.88% | 10.48% | 7.39% | 12.44% | 10.74% |
| Factor Sharpe Ratio | 0.71 | –0.27 | 1.46 | 0.51 | 1.14 | 0.90 |
| Correlation Against | | | | | | |
| Mkt-RF | 1.00 | –0.22 | 0.05 | –0.20 | 0.31 | 0.80 |
| SMB | | 1.00 | –0.35 | –0.26 | –0.22 | –0.20 |
| HML | | | 1.00 | 0.39 | 0.42 | 0.04 |
| WML | | | | 1.00 | 0.28 | –0.28 |
| BAB | | | | | 1.00 | 0.21 |
| DUR | | | | | | 1.00 |

*Sources: Research Affiliates, based on data from CRSP, Kenneth French's Data Library, and Morningstar Encorr for U.S.; Datastream, Worldscope, Morningstar EnCorr, and Bloomberg for Global and EM.*

## EXHIBIT A2

**Low-Volatility Returns Adjusted for Unexplained "Alphas"**

| | Adjusted | | Unadjusted | | Unexplained |
|---|---|---|---|---|---|
| | Return | Sharpe Ratio | Return | Sharpe Ratio | Alpha (luck) |
| ***U.S.* (1967 to 2012)** | | | | | |
| Cap-Weighted Index | 9.81% | 0.29 | 9.81% | 0.29 | 0.00% |
| Minimum Variance* | 11.37% | 0.53 | 11.38% | 0.53 | 0.01% |
| Low Volatility (1/Vol) | 11.60% | 0.50 | 11.65% | 0.51 | 0.06% |
| Low Volatility (EW) | 11.58% | 0.50 | 11.77% | 0.51 | 0.19% |
| Low Beta (1/β) | 12.05% | 0.53 | 11.83% | 0.51 | –0.22% |
| Low Beta (EW) | 12.21% | 0.53 | 12.30% | 0.53 | 0.09% |
| ***Global* (Nov 1990 to 2012)** | | | | | |
| Cap-Weighted Index | 7.27% | 0.27 | 7.27% | 0.27 | 0.00% |
| Minimum Variance* | 8.38% | 0.55 | 7.89% | 0.50 | –0.49% |
| Low Volatility (1/Vol) | 10.28% | 0.65 | 10.75% | 0.69 | 0.46% |
| Low Volatility (EW) | 10.35% | 0.64 | 10.75% | 0.67 | 0.40% |
| Low Beta (1/β) | 10.20% | 0.59 | 10.01% | 0.58 | –0.19% |
| Low Beta (EW) | 10.62% | 0.68 | 9.93% | 0.62 | –0.69% |
| ***EM* (2003 to 2012)** | | | | | |
| Cap-Weighted Index | 16.88% | 0.63 | 16.88% | 0.63 | 0.00% |
| Minimum Variance* | 12.15% | 0.85 | 18.47% | 1.38 | 6.32% |
| Low Volatility (1/Vol) | 16.74% | 0.91 | 22.71% | 1.27 | 5.97% |
| Low Volatility (EW) | 16.87% | 0.90 | 22.54% | 1.23 | 5.67% |
| Low Beta (1/β) | 17.79% | 0.97 | 24.16% | 1.35 | 6.37% |
| Low Beta (EW) | 18.47% | 0.95 | 23.85% | 1.25 | 5.38% |

**Minimum variance results reflect the average of the portfolios reported in Exhibit 1.*

*Sources: Research Affiliates, based on data from CRSP, Kenneth French's Data Library, and Morningstar Encorr for U.S.; Datastream, Worldscope, Morningstar EnCorr, and Bloomberg for Global and EM*

## ENDNOTES

[1]See Mercer [2010] and Mercer [2011].

[2]Ultimately, the poor information ratio for low-volatility strategies means investors need to consider a more creative way for benchmarking these portfolios. Recognizing that low-volatility investing represents a very distinct equity exposure and establishing an industrywide low-volatility benchmark would obviate a tacit objection to the investment opportunity.

[3]See Black [1972] and Haugen and Heins [1975].

[4]According to a recent interview with Eugene Fama by Bob Litterman for CFA Institute, the low-volatility anomaly does not require the invention of any new finance, behavioral or otherwise. Fama was skeptical of authors who have called for brand-new behavioral stories to explain the outperformance of low-beta stocks. See www.cfapubs.org/doi/pdf/10.2469/faj.v68.n6.1.

[5]We note that some have referred to the BAB premium as also the low-volatility premium; this practice ultimately is confusing. The low-volatility premium is really more appropriately understood as some combination of the value, small, and BAB premia.

[6]For developed markets, the minimum-variance construction methodology has an economically insignificant advantage in volatility reduction. For emerging markets, the volatility differential is, however, economically meaningful.

[7]A low-volatility portfolio with 70% in the S&P 500 Index and 30% in cash would have low beta, but would not load on the BAB factor, nor earn a BAB premium.

[8]We note, however, that the emerging markets low-volatility portfolios characteristically differ in a meaningful way from their counterparts in developed markets. Emerging market low-volatility portfolios do not exhibit a value bias. This result gives us pause. Emerging market low-volatility portfolios tap only two of the three sources of equity premium that developed market low-volatility portfolios harvest, and considering that the value premium is particularly attractive in the emerging markets, the lack of value exposure, in this case, seems particularly unfortunate.

[9]Ledoit and Wolf [2004] define the target covariance of any pair of stocks as the product of sample standard deviations of the two stocks, and the cross-sectional average correlation. Clarke et al. [2006] define more restrictive "shrinkage" targets; the target variance is the average sample variance and the target covariance is the average sample covariance.

[10]There are variations in how volatility and betas can be computed (see Martin and Simin [2003]). In our simulation, we use only the most straightforward calculations (that is, without applying any statistical methods to adjust the measurements).

[11]We require at least three years of daily observations for a stock to be included.

[12]The number 200 is arbitrary, but by selecting a subset of the 1,000 we attempt to construct portfolios that are reasonably comparable to those produced by MV methods. (Optimizers typically assign zero weights to a large number of stocks.) The weighting methods we employ are expected to assign nontrivial weights to all 200 stocks in the portfolios.

[13]The Herfindahl Index is defined as the sum of squared weights. Its inverse ranges from 1 for a portfolio that holds only one stock to $N$ for a portfolio of $N$ equally weighted stocks. Higher effective $N$s indicate less concentration.

[14]See Penfold [2013].

[15]We do not separately show this result or discuss robust beta estimation in the article for simplicity. A research white paper on this topic can be requested from the authors.

[16]http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[17]To improve the risk models' explanatory power, we excluded the bottom 2% of stocks by market capitalization, and stocks in the top and bottom 1% of estimated beta. These changes were motivated by the fact that the long and short portfolios could be heavily influenced by a few micro-cap stocks.

[18]The *market beta* is computed as covariance with the region-specific market portfolio. For example, for the emerging markets strategy, the beta would be relative to the cap-weighted emerging markets index. One can think of the regional market portfolio as a proxy for the first principal component explaining stock returns in the specific region.

[19]Indeed, we note that adding BAB to the FF-4 factor model (not shown in the article) reduces the low-volatility portfolios' loading on HML in favor of BAB while only marginally improving the $R^2$, with very little statistical impact on the unexplained alpha.

[20]Miles [2012] argues that the greatest challenge to a low-volatility investor is the overwhelming choice available owing to an explosion in products. He cautions that the complexity will increase both due diligence cost as well as management fee, while often increasing product turnover.

[21]See Miles [2012], where he concludes, "…the relative valuation appears to be stretched relative to historical levels."

[22]Research on avoiding expensive stocks in the low-volatility portfolio construction process is available from the authors upon request.

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