Multifactor Indexes Made Simple: *A Review of Static and Dynamic Approaches*

Mehdi Alighanbari and Chin Ping Chia

Mehdi Alighanbari
is an executive director
at MSCI in London, U.K.
mehdi.alighanbari@msci.com

Chin Ping Chia
is a managing director
and head of research for
the Asia Pacific at MSCI
in Hong Kong.
chin-ping.chia@msci.com

Factor investing is gaining rapid acceptance by institutional and retail investors alike. Investors who were attracted by the long-term outperformance of risk premia generally started with single-factor allocations. However, factor index returns have been cyclical, and their active returns are weakly correlated.[1] As a result, investors are increasingly turning their attention towards multifactor index allocations. Not only have their returns been smoother, but historically, they have offered a diversification effect.

We examined nine multifactor index strategies, which include one simple equal-weighted strategy (simple diversification); five rules-based/optimization-based weighting strategies (inverse of variance, inverse of tracking error, trend following, risk parity, and tracking error optimization); and three fundamentals-based weighting strategies (valuation-based, quality-based, and blended factors); except for equal weighting, the other eight strategies involve dynamic adjustment of factor weights. The relative merit of each strategy as measured by the information ratio versus turnover—a key element of cost—can be seen in Exhibit 1.

Our analyses yield two key conclusions:

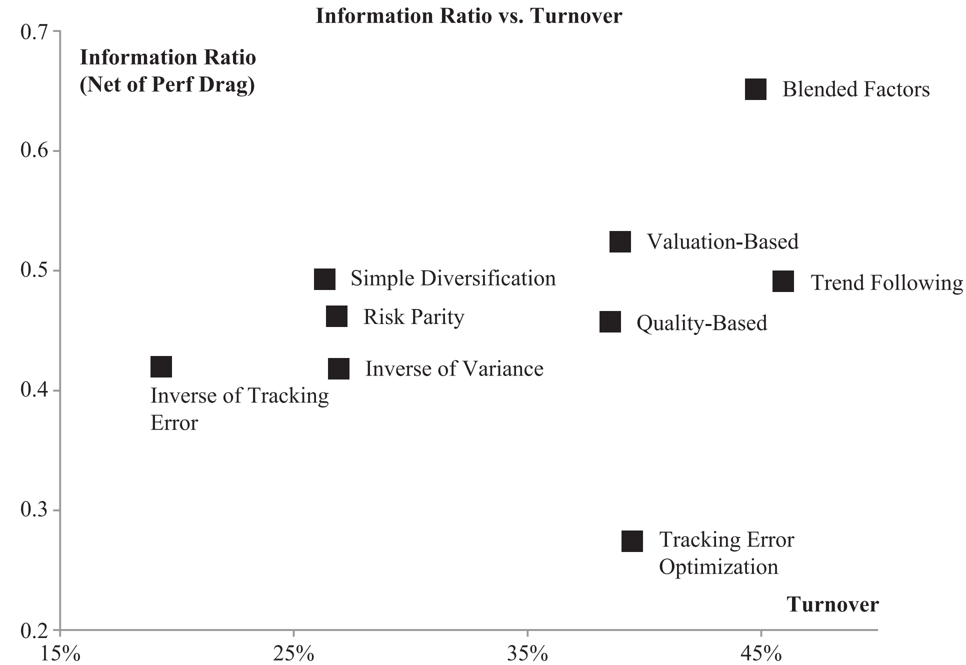
1. The *simple diversification strategy* has been highly effective historically.[2] Many simple, rules-based, dynamic-weighting strategies have failed to match this equal-weighted strategy's performance after accounting for index turnover cost.
2. The *fundamentals-based approaches* have produced attractive results in simulation. The three strategies tested in this study have delivered higher active returns against the simple diversification strategy—pointing to the potential benefits of exploiting fundamental insights in the construction of a multifactor index. Such strategies are active in nature, however, and typically come with the extra "costs" of higher turnover and greater complexity.

Understanding how factors can be combined as well as the merits and disadvantages of various approaches can help investors make more-informed investment decisions. As investors explore the frontier of multifactor investing, it is reassuring to know that a simple equal-weighted approach has provided a compelling risk-and-return profile historically. This strategy index brings simplicity, transparency, and robustness to the investment process and can serve as an attractive starting point for factor allocation—especially in the absence of active investment views and skills.

MSCI has published several articles on factor investing, ranging from the foundation of factor investing and implementing factor

EXHIBIT 1

Information Ratio vs. Turnover of Multifactor Indexes



indexes in equity portfolios, to the behavior of factor indexes in different macroeconomic environments (see Aylor Subramanian et al. [2013a]; Aylur Subramanian et al. [2013b]; Suryanarayanan et al. [2014]; and Melas, Briand, and Urwin [2011]). We have also discussed how combining multifactor indexes has historically offered diversification and provided a smoother return stream than single-factor indexes. In this article, we explore different multifactor investing approaches in greater detail, based on 36 years of MSCI factor index history. We compare the return and risk characteristics of various dynamic factor allocation strategies against the simple diversification strategy and discuss the benefits and trade-offs of different approaches to factor allocation.

The main questions we address are as follows:

1. How does a simple equally weighted combination of factor indexes (simple diversification) compare with individual factor indexes?
2. Can factor indexes be combined in a dynamically managed manner that improves upon the simple diversification approach?
3. What are the trade-offs between the simple diversification approach to factor allocation and more dynamic approaches?

We identify six risk premia—size, value, quality, momentum, low volatility, and yield—as the basic building blocks for this analysis. These risk premia have been proved to earn long-term excess return over the market historically and are solidly grounded in academic literature.

We create the simple diversification multifactor index using six corresponding MSCI World factor indexes. Using a series of rules-based and optimization algorithms to mimic dynamic approaches to index construction, we demonstrate how factor indexes can be combined beyond a simple equal-weighted strategy. We also illustrate how a rules-based interpretation of fundamental or valuation signals can be employed in the construction of multifactor indexes.

To ensure the robustness of analysis, we perform a multiperiod rolling window analysis to eliminate possible biases from picking arbitrary start and end dates. We study the historical probability of factor indexes achieving excess returns or lower risk against the capitalization-weighted benchmark with different time horizons. The analysis, which can be found in Appendix A, also sheds light on the implications of investment horizons for factor investing.

COMBINING FACTORS: POSSIBLE APPROACHES

A Six-Factor Simple Diversification Index

A simple diversification multifactor index arguably provides the simplest combination of factors. It is created by equally weighting factor indexes. We use six MSCI World factor indexes—Equal Weighted, Value Weighted, Quality, Momentum, Minimum Volatility, and High Dividend Yield—to represent six well-researched risk premia. The index is rebalanced semi-annually in May and November, consistent with the rebalancing schedule of the MSCI Factor Indexes. We consider simple diversification a "static" approach to factor allocation, as the weight for each factor is defined as $1/n$.

The historical index performances of the simple diversification strategy and its underlying factor indexes can be seen in Exhibit 2. The multifactor index captured the same long-term risk premia but offered smoother performance than any of the underlying factor indexes. This result is not surprising: The long-term outperformance and low active correlations among the MSCI Factor Indexes help explain this phenomenon, as can be seen in Exhibit 3.

The simple diversification multifactor index also exhibited lower volatility (13.9%) than any single-factor index except for the Minimum Volatility Index, as can be seen in Exhibit 4.[3]

Although a simple diversification multifactor index may look naive in terms of construction, it represents a reasonable starting point for an investor who wants to gain exposure to systematic risk premia but does not have specific views on the expected risk or return of the underlying factor indexes nor the skills to actively manage factor exposures.

EXHIBIT 2

Simple Diversification Has Historically Offered a Smoother Ride

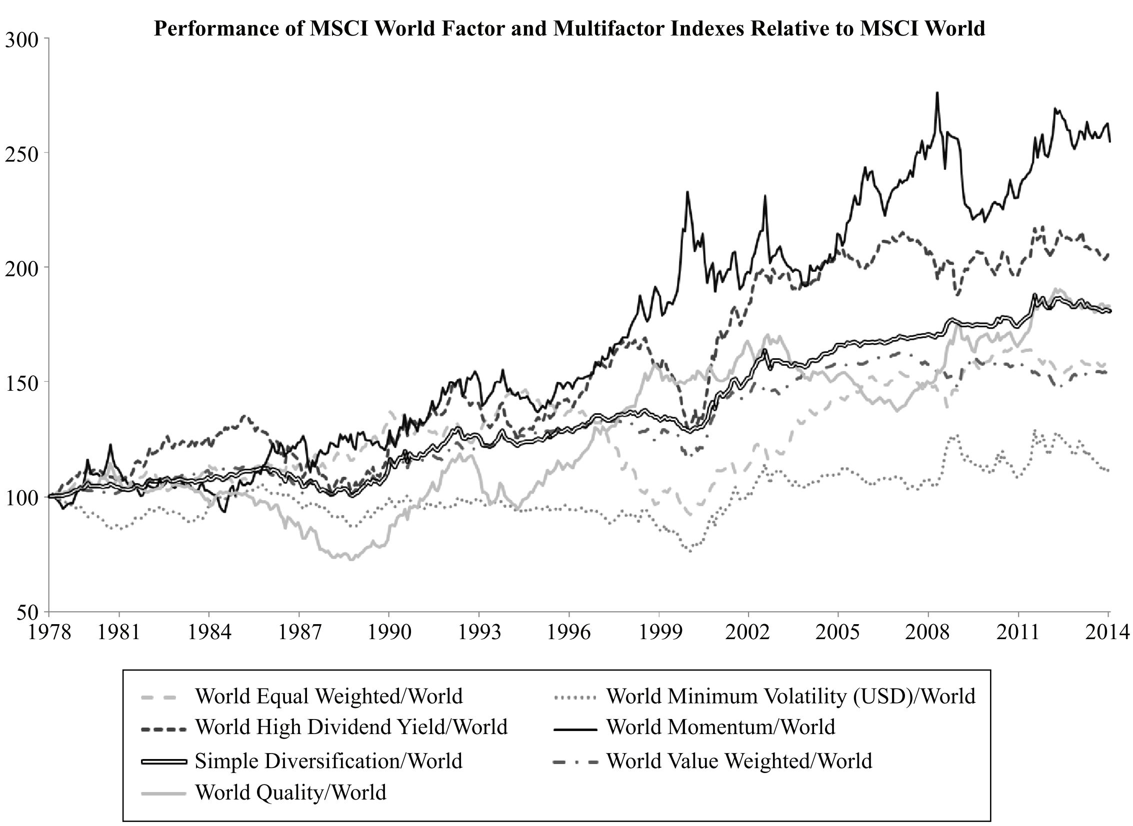


EXHIBIT 3

Active Return Correlations among MSCI World Factor Indexes Are Low

Active Return Correlation	Equal Weighted	Minimum Volatility (USD)	Value Weighted	High Dividend Yield	Momentum	Quality
Equal Weighted						
Minimum Volatility (USD)	0.15					
Value Weighted	0.58	0.16				
High Dividend Yield	0.26	0.45	0.73			
Momentum	–0.15	0.09	–0.33	–0.03		
Quality	–0.15	0.18	0.15	0.44	0.25	

EXHIBIT 4

Return/Risk Profiles of Single-Factor Indexes vs. Six-Factor Simple Diversification Index

	MSCI World	MSCI World Equal Weighted	MSCI World Min Volatility	MSCI World Value Weighted	MSCI World High Div Yield	MSCI World Momentum	MSCI World Quality	Simple Diversification
Total Return*	10.6%	12.0%	10.9%	11.9%	12.9%	13.5%	12.5%	12.4%
Total Risk*	15.1%	15.7%	12.0%	15.2%	14.6%	16.3%	14.5%	13.9%
Return/Risk	0.70	0.77	0.91	0.79	0.88	0.83	0.86	0.90
Maximum Drawdown	–53.7%	–54.8%	–43.1%	–57.3%	–58.8%	–52.6%	–44.5%	–52.0%
Active Return*		1.4%	0.4%	1.4%	2.3%	3.0%	1.9%	1.9%
Performance Drag (bps)**		17.2	26.8	18.1	20.2	91.0	22.7	26.3
Active Return (Net of Performance Drag)		**1.3%**	**0.1%**	**1.2%**	**2.1%**	**2.1%**	**1.7%**	**1.6%**
Tracking Error*		5.0%	6.1%	3.5%	6.4%	8.4%	6.0%	3.3%
Information Ratio***		**0.25**	**0.02**	**0.34**	**0.33**	**0.24**	**0.28**	**0.49**
Maximum Active Drawdown		–37.2%	–27.4%	–13.3%	–24.3%	–23.7%	–36.6%	–10.7%
One-way Index Turnover****	3.0	17.2	26.8	18.1	20.2	91.0	22.7	
Separate Mandates								35.4
Combined Mandate								26.3

*Notes: *Annualized gross return (USD) from 11/30/1978 to 03/31/2014. **Performance drag calculated based on annualized two-way index turnover for combined mandate assuming a transaction cost of 50 bps. ***Information ratio is calculated using active return (net of performance drag). ****Annualized one-way index turnover for the 05/31/1999 to 03/31/2014 period.*

Simple Rules-Based and Optimization-Weighting Approaches

Going beyond simple diversification in a dynamic multifactor index requires active views on factors together with the skills to manage the related exposures. A dynamic factor allocation model adjusts weights regularly; it typically involves overweighting factors that are expected to outperform and underweighting factors that are expected to underperform. The underpinning investment belief is that factors have different return streams and active factor allocation can and will add value.

There are many possible approaches to achieve a dynamic factor allocation. In this article, we will focus only on some examples that can be replicated with a set of mechanical rules and do not seek to represent all possible dynamic factor weighting strategies.

We start with a set of simple rules-based and optimization-based weighting strategies:

- The *inverse of variance* and *risk parity* strategies can be considered as risk-based approaches to factor allocation. The underlying investment beliefs are that overweighting factors with lower volatility or balancing the risk contribution of each factor in the multifactor index could improve risk diversification and help achieve better risk-adjusted returns.

- The *inverse of tracking error* and *tracking error optimization* approaches bring a risk-budgeting dimension into the picture. The former aims to minimize the tracking error of the multifactor index without optimization. The latter seeks to maximize the return outcome using mean–variance optimization subject to a tracking error constraint.
- Finally, the *trend following* strategy takes a conventional momentum strategy and applies it to factor allocation.

All dynamic strategies are rebalanced in May and November. The underlying investment belief, possible approach, and weighting scheme of each strategy are outlined in Exhibit 5.

Return/Risk Profiles of Simple Rules-Based and Optimization-Based Strategies

We examine the return/risk profiles of the rules-based and optimization-based factor allocation approaches along with the simple diversification strategy. As we can see in Exhibit 6, the inverse of variance and risk parity strategies produced risk and return characteristics that were fairly similar to those of the simple diversification strategy during the November 1978 to March 2014 period. This result can be partly explained by the fact that weights of various factor indexes are fairly stable in these two strategies and did not significantly differ from equal weighting.[4]

Inverse weighting each factor index based on its tracking error would not have added much value either. In fact, the inverse of tracking error approach generated a lower return but higher total risk than the simple diversification strategy. The result suggests that a multifactor index that simply overweights low tracking error factor indexes may not necessary lead to better risk or return outcomes.

Optimization techniques are typically employed when investors have a set of objectives and constraints they would like to incorporate into their portfolios.[5] But optimization can be complex, often requiring accurate risk and return inputs as well as careful calibration of optimization parameters.

EXHIBIT 5

Examples of Simple Rules-Based and Optimization-Based Strategies

Multifactor Strategy	Investment Belief	Possible Approach	Weighting Scheme
Inverse of Variance	Factors with lower volatility could produce better risk/return profile or improved risk diversification	Weight factors by inverse of volatility	$w_i = \frac{1}{\sigma_i^2}$
Risk Parity	Balancing the risk contribution of factors would improve risk diversification	Weight factors so that the marginal contributions of all factors to overall risk are equal	$RC_i = RC_j \quad \forall i, j \in \{1, \ldots, 6\}$ where $RC_i = w_i \frac{\partial \sigma_P}{\partial w_i}$
Inverse of Tracking Error	A multifactor index will add value but this may be at the cost of higher tracking error risk	Weight factors by inverse of squared tracking error to parent index	$w_i = \frac{1}{TE_i^2}$
Tracking Error Optimization	Multifactor indexes will add value but will be subject to a tracking error constraint	Optimize the index using mean–variance optimization	$\max_{w_i} \sum_{i=1}^{6} w_i r_i$ $s.t\ TE_{36m}$ Expected alphas: Past 3-year factor index returns Tracking error constraint: Realized 36-month tracking error of an equally weighted multifactor index
Trend Following	Factors with strong past performance will deliver strong future performance	Weight by past performance; overweighting winners and underweighting losers	$w_i = \frac{Mom\, Z^i_{factor}}{\sum_i Mom\, Z_{factor}}$

Note: See Appendix B for detailed descriptions of strategies.

EXHIBIT 6

Performance of Rules-Based and Optimization-Based Multifactor Strategies

	MSCI World	Simple Diversification	Inverse of Variance	Risk Parity	Inverse of Tracking Error	Tracking Error Optimization	Trend Following
Total Return*	10.6%	12.4%	12.3%	12.4%	12.0%	11.8%	12.8%
Total Risk*	15.1%	13.9%	13.6%	13.7%	14.3%	14.6%	14.1%
Return/Risk	0.70	0.90	0.91	0.91	0.84	0.81	0.91
Maximum Drawdown	–53.7%	–52.0%	–50.8%	–51.4%	–54.9%	–53.8%	–49.6%
Active Return*		1.9%	1.8%	1.8%	1.4%	1.2%	2.2%
Performance Drag (bps)**		26.3	26.9	26.8	19.3	39.5	45.9
Active Return (Net of Performance Drag)		**1.6%**	**1.5%**	**1.6%**	**1.2%**	**0.8%**	**1.7%**
Tracking Error*		3.3%	3.6%	3.4%	2.9%	3.0%	3.5%
Information Ratio* **		**0.49**	**0.42**	**0.46**	**0.42**	**0.27**	**0.49**
Maximum Active Drawdown		–10.7%	–11.9%	–11.0%	–13.0%	–12.9%	–10.7%
One-way Index Turnover******	3.0						
Separate Mandates		35.4	38.9	37.1	35.1	74.0	87.0
Combined Mandate		26.3	26.9	26.8	19.3	39.5	45.9

*Notes: *Annualized gross return (USD) from 11/30/1978 to 03/31/2014. **Performance drag calculated based on annualized two-way index turnover for combined mandate assuming a transaction cost of 50 bps. ***Information ratio is calculated using active return (net of performance drag). ****Annualized one-way index turnover for the 05/31/1999 to 03/31/2014 period.*

Optimization can be performed at the individual equity level or at the factor index allocation level. For this study, optimization was employed at the factor allocation level. Based on the selected parameters, the tracking error optimization multifactor index outperformed the cap-weighted benchmark but underperformed other multifactor strategies including simple diversification. The tracking error optimization strategy produced an information ratio of only 0.27, the lowest in this study.

The result can be partially attributed to the fact that we used a rather simplistic covariance estimate based on only six MSCI factor indexes in the study. An alternative and more robust approach would be to optimize the multifactor strategy from the security level with more refined risk estimation; we will explore this approach in a subsequent study.

The only rules-based strategy that outperformed the simple diversification strategy is the trend following approach. The strategy produced slightly higher return/risk and information ratios, suggesting that factor indexes exhibited some forms of momentum behavior that could be exploited.[6] However, the trend following strategy would have experienced greater variations in factor weights and hence higher index turnover.[7] Taking into account the performance drag, the trend following strategy generated only 10 basis points (bps) of outperformance against the simple diversification multifactor index.

The Fundamentals-Based Approach

The fundamentals-based approach to multifactor indexing refers to the systematic implementation of fundamental or valuation-based investment strategies following specified rules or algorithms. The core tenet of the approach is that fundamental data contain important signals that can be used to understand the drivers of volatilities and correlations among assets. For example, valuation plays an important role in any fundamental investment strategy. The same concept can be applied to building multifactor index strategies: Equity factor indexes can be over- or undervalued versus their historical valuation level. Investing in factors that are cheap compared with their historical mean could provide a better chance of achieving excess returns. In addition, valuation could also serve as an indicator of crowding. A factor index that is trading above its "fair price" may be perceived as a crowded trade and would therefore be avoided.

Some investors may prefer to use quality metrics such as profitability. For example, the spread of return on equity (ROE) between high-quality companies and the broad market could be considered a more appropriate indicator for such factors as quality and yield.[8] The hypothesis is that the market will pay a premium to high ROE and high dividend companies when there are huge differences in terms of the average level of profitability and dividend yield of companies.

Although using valuation or a measure of quality to weight each factor index is a rational and sensible approach, we also recognize that each factor premium may be better captured by a different fundamental signal. For instance, the Minimum Volatility Index has historically delivered superior risk-adjusted returns during high-volatility regimes. A volatility indicator such as the CBOE Volatility Index (VIX) may provide a better signal to help manage the volatility factor exposure. Thus, we can anchor different factor exposures to relevant signals. We call this the *blended factors* approach.

The three main fundamentals-based strategies are summarized in Exhibit 7. Similar to the previous strategies, all fundamentals-based strategies are rebalanced in May and November.

Return/Risk Profiles of Fundamentals-Based Strategies

Historically, the use of valuation or other fundamental signals would have improved the performance of multifactor indexes without significant increases in the total risk, as can be seen in Exhibit 8. We make the following observations:

- The Valuation-Based and Quality-Based multifactor indexes produced broadly similar risk and return characteristics over the November 1978 to March 2014 period, but the valuation-based index produced a higher information ratio and a lower maximum active drawdown.
- The Blended Factors multifactor index provided the strongest return, outpacing the simple diversification strategy by 100 bps without a significant increase in risk (or 80 bps after the performance drag). This outperformance was accompanied by an information ratio improvement to 0.65 from 0.49, suggesting that an investor might have been able to add value to a multifactor index by managing factor exposures with the right signals.

CONCLUSION

There are many possible ways to construct multifactor indexes. We use nine weighting strategies to proxy different investment approaches and examine the return/risk characteristics over a 36-year period. The results highlight that a *simple diversification* approach to constructing multifactor indexes has historically proved more effective than many of the more complex approaches—pointing to its potential as a way to combine factors, especially in the absence of active investment views and skills.

EXHIBIT 7
Examples of Fundamentals-Based Strategies

Multifactor Strategy	Investment Belief	Possible Approach	Weighting Scheme
Valuation Based	Factor indexes may become overcrowded and/or expensive which may impair performance	Overweight cheap factor indexes/underweight expensive ones	Normalized current E/P level[a]
Quality Based	Factor indexes with higher ROE will outperform ones with lower ROE	Overweight high ROE indexes/underweight low ROE ones	Normalized current ROE[a]
Blended Factors	Factor indexes perform well when the underlying signal is strong	Weight each factor index based on the strength of its underlying signal	Normalized E/P spread[a] (Value) Normalized effective number of stocks[a] (Size) Normalized ROE spread[a] (Quality) Normalized D/P spread[a] (Yield) Normalized VIX (Low Volatility) Normalized past 6-month momentum[a] (Momentum)

Notes: See Appendix B for more detailed descriptions of strategies. [a]*Compared to its own history.*

EXHIBIT 8
Performance of Fundamental Signal Strategies

	MSCI World	Simple Diversification	Valuation Based	Quality Based	Blended Factors
Total Return*	10.6%	12.4%	13.0%	12.9%	13.4%
Total Risk*	15.1%	13.9%	13.9%	13.8%	14.0%
Return/Risk	0.70	0.90	0.94	0.93	0.96
Maximum Drawdown	–53.7%	–52.0%	–51.9%	–51.5%	–49.7%
Active Return*		1.9%	2.4%	2.3%	2.9%
Performance Drag (bps)**		26.3	39.0	38.5	44.8
Active Return (Net of Performance Drag)		**1.6%**	**2.0%**	**1.9%**	**2.4%**
Tracking Error*		3.3%	3.9%	4.2%	3.7%
Information Ratio* **		**0.49**	**0.52**	**0.46**	**0.65**
Maximum Active Drawdown		–10.7%	–9.7%	–12.2%	–10.9%
One-way Index Turnover******	3.0				
Separate Mandates		35.4	63.8	64.5	76.1
Combined Mandate		26.3	39.0	38.5	44.8

*Notes: *Annualized gross return (USD) from 11/30/1978 to 03/31/2014. **Performance drag calculated based on annualized two-way index turnover for combined mandate assuming a transaction cost of 50 bps. ***Information ratio is calculated using active return (net of performance drag). ****Annualized one-way index turnover for the 05/31/1999 to 03/31/2014 period.*

Dynamic factor allocation strategies, however, have their merits as well. Our study shows that the *fundamentals-based* approach to factor allocation, especially the Blended Factors strategy index, which weights each factor index in the strategy based on the strength of fundamental signals, would have provided the best overall return/risk profile among the dynamic strategies analyzed. Understanding how to use fundamental signals in constructing multi-factor index strategies while controlling turnover could provide fertile ground for future research.

Clearly, there are trade-offs between the various approaches. In considering whether to manage a multifactor index via simple equal-weighting or more dynamic weighting strategies, the decision depends on the investor's investment beliefs and process and, critically, whether the investor is confident of possessing the insight or skills to manage factor exposures dynamically.

APPENDIX A

MULTIFACTOR ALLOCATION AND INVESTMENT HORIZON

Investment horizon plays a critical part in factor investing. Due to the cyclicality of factors, a long horizon is essential in ensuring the success of factor index implementation. To investigate the effect of time, we analyzed the frequency of relative outperformance of multifactor indexes over multiple time horizons. We used rolling windows of different lengths to eliminate possible biases from picking arbitrary start and end dates, running simulations for individual MSCI Factor Indexes and the three best-performing multifactor strategies.

Exhibit A1 reveals several key findings:

- As the time horizon expanded, the frequency of outperformance against the MSCI World Index increased. The observation is consistent across both single- and multifactor indexes, thus highlighting the importance of having a long horizon in factor investing.
- By combining multiple factors into a single index, the frequency of outperformance improved compared with single-factor strategies. This finding validates the argument that a multifactor index would have been more effective in cushioning the effects of market cycles.
- For an investor who has a sufficiently long investment horizon (10 years or more), the historical probability of outperforming the market (regardless of which of the three multifactor strategies was used) was virtually the same.
- The simple diversification strategy produced comparable results in terms of frequency of outperformance versus dynamic approaches such as trend following and blended factors—highlighting that a strategic allocation

Exhibit A1

Historical Frequency of Outperformance of MSCI Factor Indexes vs. MSCI World Index

	Single-Factor Indexes						Multifactor Portfolios		
Rolling Window	Equal Weighted	Minimum Volatility	Value Weighted	High Div Yield	Momentum	Quality	Simple Diversification	Trend Following	Blended Factors
1 Y	60%	46%	68%	60%	67%	56%	71%	74%	72%
3 Y	70%	55%	67%	70%	75%	59%	86%	88%	90%
5 Y	81%	60%	75%	80%	90%	64%	90%	90%	94%
10 Y	73%	69%	100%	98%	99%	84%	100%	100%	100%
15 Y	82%	75%	100%	98%	100%	94%	100%	100%	100%
20 Y	88%	86%	100%	100%	100%	100%	100%	100%	100%
25 Y	100%	100%	100%	100%	100%	100%	100%	100%	100%

Note: On a monthly rolling basis for the period of 11/30/1978 to 3/31/2014.

Exhibit A2

Frequency of Factor Indexes Experiencing Lower Volatility than MSCI World Index

	Single-Factor Indexes						Multifactor Portfolios		
Rolling Window	Equal Weighted	Minimum Volatility	Value Weighted	High Div Yield	Momentum	Quality	Simple Diversification	Trend Following	Blended Factors
1 Y	42%	92%	52%	63%	34%	58%	80%	72%	78%
3 Y	32%	99%	60%	73%	35%	66%	89%	86%	87%
5 Y	28%	100%	66%	72%	28%	67%	100%	95%	96%
10 Y	15%	100%	78%	80%	12%	79%	100%	100%	100%
15 Y	9%	100%	74%	100%	0%	73%	100%	100%	100%
20 Y	1%	100%	69%	100%	0%	81%	100%	100%	100%
25 Y	12%	100%	57%	100%	0%	90%	100%	100%	100%

Note: On a monthly rolling basis for the period of 11/30/1978 to 3/31/2014.

of factors even when the weights are static has historically produced favorable results.

Exhibit A2 repeats the same analysis with respect to index volatility. By construction, single-factor indexes such as the Minimum Volatility Index tended to exhibit lower volatility than the market. Defensive factors, such as high dividend yield and quality, also tended to display lower volatility than the market, especially as the time horizon increased. In contrast, cyclical factors such as momentum and equal weighted showed higher volatility on average than the market.

The three multifactor indexes—whether static or dynamically managed—demonstrated very attractive volatility profiles compared with most single-factor indexes. Significantly, the simple diversification approach has the highest historical probability of achieving a lower risk than the market.

Appendix B

DESCRIPTION OF MULTIFACTOR STRATEGIES

Simple Diversification

Simple diversification is the simplest way of allocating to multiple factor indexes. It does so by giving the same weight to each of the factor indexes. The strategy rebalances every six months back to equal weights, selling the better-performing factors and buying underperformers, potentially capturing mean reversion of factors.

Inverse of Variance

The rationale for this strategy is to allocate to each factor index based on its level of risk, as defined by historical volatility based on trailing 36-month standard deviation. The result will overweight (underweight) factor indexes that have lower (higher) volatility. One expectation for this strategy is to have lower total volatility than the simple diversification strategy. This strategy, as well as all other strategies explained in this article, adheres to a six-month rebalancing frequency.

Risk Parity

The objective of this optimization-based approach is to achieve an equal risk contribution of each factor index. In creating the risk parity factor strategy, individual factor index correlations are taken into account.

Inverse of Tracking Error

This approach is similar to the inverse of variance approach, but instead of using the variance of the factor index returns, the strategy employs the squared tracking error (variance) of the active returns, based on a trailing 36-month window. This strategy overweights (underweights) factors that have lower (higher) tracking error to the parent index. By construction, the resulting index is expected to have low tracking error to the parent index.

Tracking Error Optimization

This approach aims to maximize the expected active return while constraining the tracking error using mean–variance optimization. Expected alphas are estimated for this type of optimization. For this simulation, we base expected alphas on actual factor index returns of the past three years and the tracking error constraint is based on the realized 36-month tracking error of an equally weighted multifactor strategy index.

Trend Following

The trend following approach allocates to different factor indexes based on their recent (six-month) performance. The assumption is that momentum exists in factor performance and the factors that have performed well over the recent past will continue to perform well over the next six months.

Valuation Based

We use valuation to measure how cheap or expensive a factor index is and to adjust the weight of each index in the multifactor strategy based on its valuation. Although the aim here is to avoid expensive indexes or stocks, the strategy may also help to identify crowded strategies. Several fundamental parameters can be used individually or in combination to measure value. In this study, earnings yield (E/P) is used as a measure for value. It is recognized that factor indexes have inherent valuation biases. Factor indexes, such as value weighted, tend to be cheap by construction, while momentum or quality tend to be more expensive in terms of relative valuation. Therefore, there will be a constant overweight (underweight) for inherently cheap (expensive) factor indexes, which is not desirable in this study.

To avoid systematic valuation biases of factor indexes, we first normalize valuation of a factor index of time t against its own history. To capture as much information as possible, the normalization is performed on an expanded history; that is, a shorter history is used at the beginning period of the simulation and a full history is used at the ending period of the simulation.

$$Z_i(t) = \text{Normalized } \frac{E}{P}(t) \text{ within } \left\{ \frac{E}{p}(t-1), \frac{E}{P}(t-2), \ldots, \frac{E}{P}(t-n) \right\} \tag{B-1}$$

We repeat the same process for each factor index and use normalized scores at time t to allocate more weight to factor indexes that are cheap compared with their historical long-term average.

Quality Based

Looking at the cheapness or richness of an index is one way to weight factors. A similar exercise can be made for other fundamental variables such as return on equity (ROE), a measure of quality.

Blended Factors

In the Blended Factors approach, we weight each factor index based on the relevant signal strength. This time, we use the normalized earnings yield spread (the difference between the factor index and the parent index) at time t by comparing it with its own history (the long-term historical average) for the Value Index, the dividend yield spread for the High Dividend Index, and the quality spread for the Quality Index. For the size index, we use the effective number of stocks of the parent index (MSCI World) as the proxy for index concentration at time t and compare it to its own history. For the Minimum Volatility Index, we use the normalized volatility score based on the CBOE VIX at time t and compare it with its own history. For the Momentum Index, we use the recent six-month performance spread against its own history.

Exhibit B1

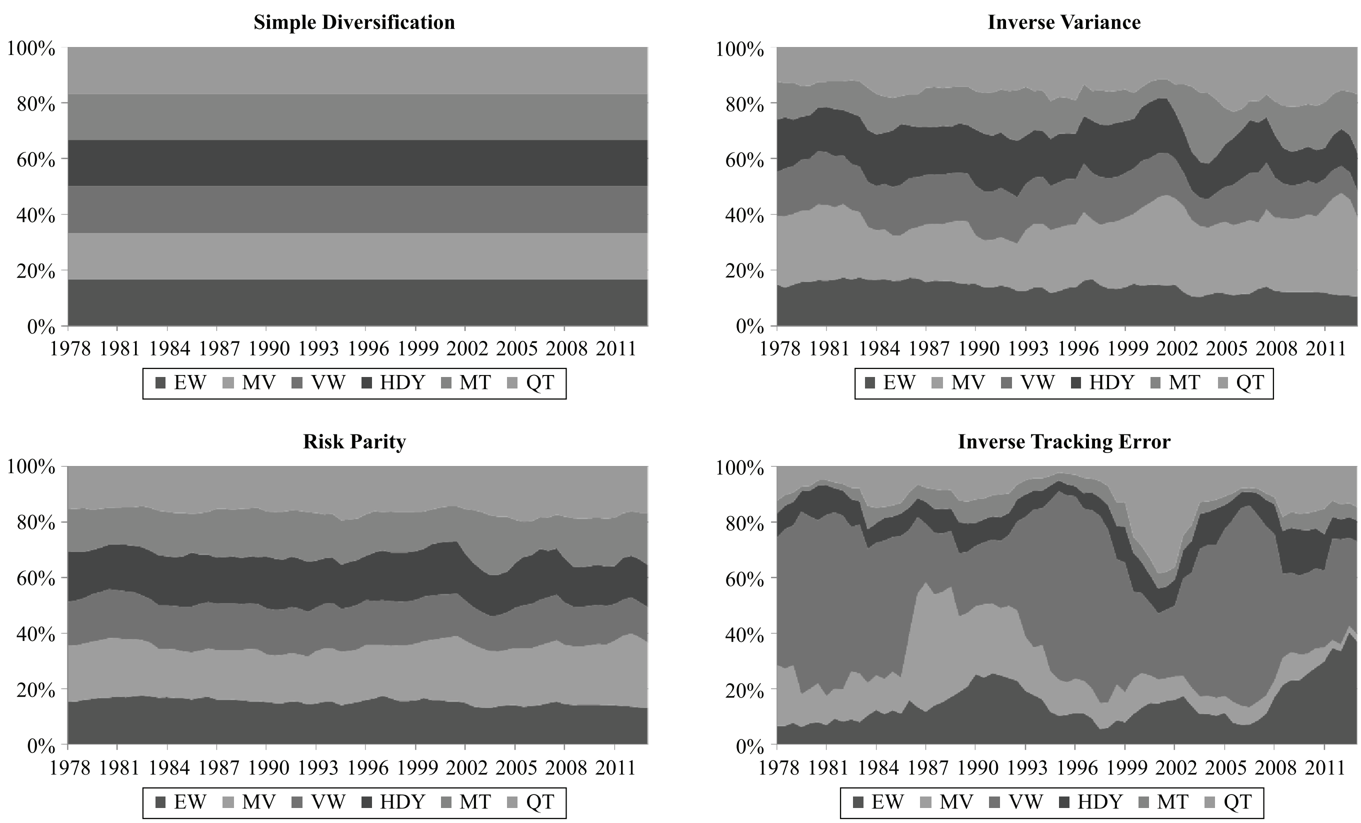
Summary of Signals Used in the Blended Factors Strategy

Factor Index	Factor-Specific Signal	Weighting Scheme
Value Weighted	Earnings yield spread	Overweight (underweight) **Value Weighted Index** when the earnings yield spread against the parent index is high (low) relative to its historical range
Equal Weighted	Effective number of stocks—a measure of index concentration	Overweight (underweight) **Equal Weighted Index** when the effective number of stocks of the parent index (MSCI World) is low (high) relative to its historical range
Momentum	Six-month performance spread	Overweight (underweight) **Momentum Index** when the six-month return differential with the parent index is high (low) relative to its historical range
Minimum Volatility	VIX	Overweight (underweight) **Minimum Volatility Index** when VIX is high (low) relative to the historical range
Quality	ROE spread	Overweight (underweight) **Quality Index** when ROE spread against the parent index is high (low) relative to its historical range
High Dividend Yield	Dividend yield spread	Overweight (underweight) **High Dividend Yield Index** when the dividend yield spread against the parent index is high (low) relative to its historical range

Appendix C

Exhibit C1

Weights of Different Factor Indexes within Multifactor Strategies

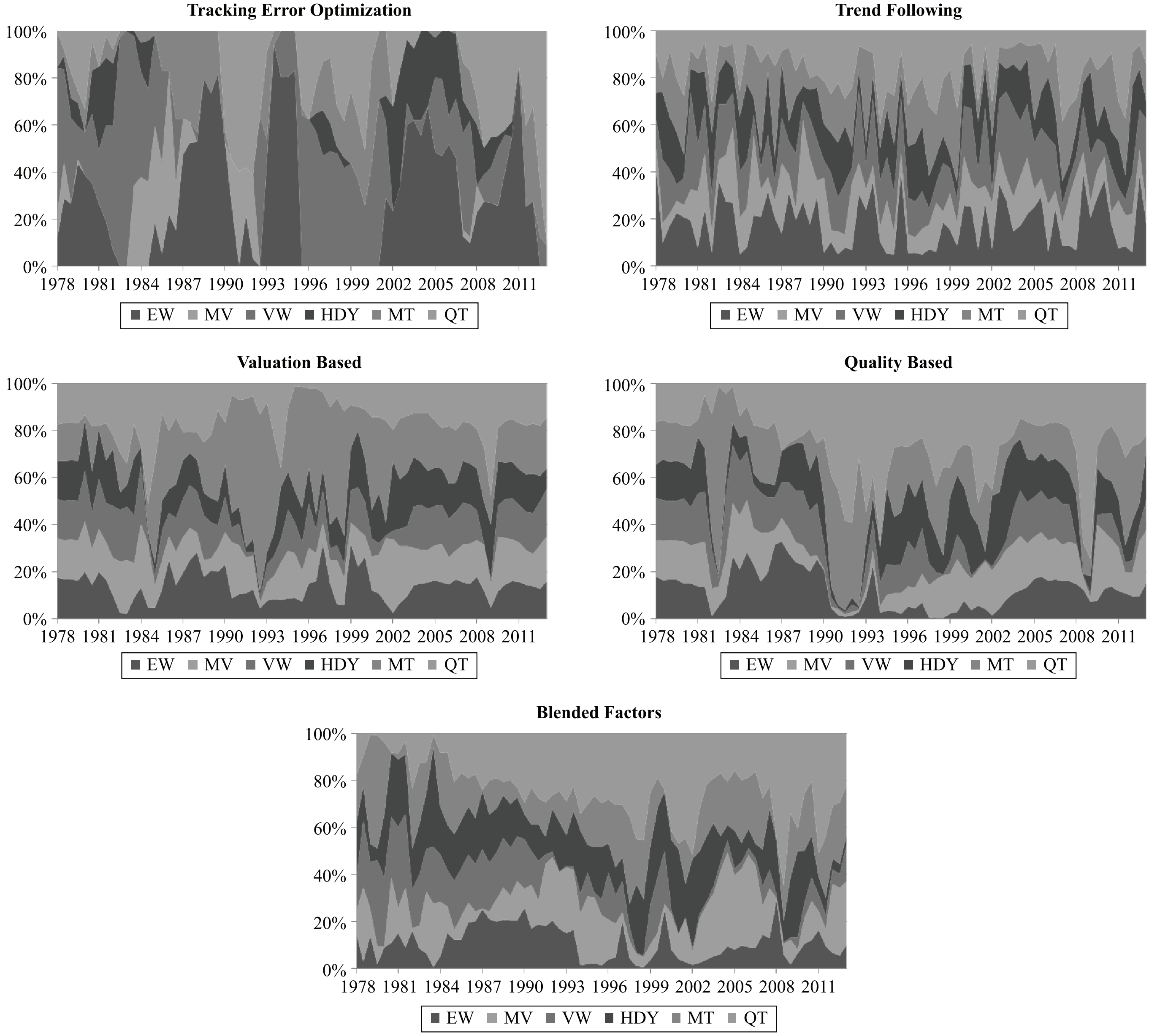


Note: Legend left to right = chart bottom to top.

(continued)

EXHIBIT C1 (*continued*)

Weights of Different Factor Indexes within Multifactor Strategies



Note: Legend left to right = chart bottom to top.

ENDNOTES

[1]MSCI Factor Indexes provide exposure to six factors—size, value, quality, momentum, low volatility, and yield—that have produced excess returns over long time periods while maintaining transparency, investability, and replicability.

[2]Previous research also has demonstrated that it is very difficult for an optimal portfolio to outperform one employing simple diversification. See DeMiguel, Garlappi, and Uppal [2009].

[3]Throughout the article, we use data from November 30, 1978, to March 31, 2014, for our simulations. The exception is Turnover, for which we use data from May 31, 1999, to

March 31, 2014, due to availability and limitation of data and analyses.

[4]See Appendix C for factor weight changes.

[5]We use the Barra open optimizer to construct the risk parity and tracking error optimization strategies. Given that optimization is performed at the factor index allocation level, historical MSCI factor index level returns are used to create the covariance matrix and the expected return estimation. The expected return of a strategy index is estimated using the last 36 months of index returns. The covariance matrix is computed directly using the last 36 months of index returns. Indexes are optimized every six months and then rebalanced.

[6]We used the past six months of return data to be consistent both with the MSCI Momentum Index and the rebalancing frequency of the strategy. We performed the same analysis by varying the length of the momentum signal but did not see material differences in their performance characteristics.

[7]See Appendix C for more detail.

[8]The MSCI High Dividend Yield Index explicitly incorporates a quality screen in selecting high yield index constituents.

REFERENCES

Aylur Subramanian, R., J. Bender, R. Briand, and D. Melas. "Foundations of Factor Investing." MSCI Research Insight, 2013a.

Aylur Subramanian, R., J. Bender, R. Briand, D. Melas, and M. Subramanian. "Deploying Multi-Factor Index Allocations in Institutional Portfolios." MSCI Research Insight, 2013b.

DeMiguel, V., L. Garlappi, and R. Uppal. "Optimal versus Naïve Diversification: How Inefficient Is the 1/N Portfolio Strategy?" *Review of Financial Studies*, Vol. 22, No. 5 (2009), pp. 1915–1953.

Melas, D., R. Briand, and R. Urwin. "Harvesting Risk Premia with Strategy Indices." MSCI Research Insight, 2011.

Suryanarayanan, R., K. Varga, A. Gupta, and A. Kassam. "Index Performance in Changing Economic Environments." MSCI Research Insight, 2014.

To order reprints of this article, please contact Dewey Palmieri at dpalmieri@iijournals.com or 212-224-3675.