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Multi-Factor Portfolio Construction for Passively Managed Factor Portfolios

Transparent rule-based index-tracking portfolios that employ alternative weighting schemes have grown rapidly in the last decade, especially within equities. These passively managed factor portfolios can be constructed in many ways, ranging from relatively simple rule-based approaches that specify weights as a function of factor characteristics to more complex optimization-based ways. Both single factor and multiple factor portfolios can be constructed. In the latter case, an often asked question is whether it is better to combine individual factor portfolios or build a multi-factor portfolio from the security level. Here, we show that a bottom-up approach to multi-factor portfolio construction can produce superior results than a combination of individual single factor portfolios, at least for well-known factors such as value, quality, low volatility and momentum. Because the bottom-up approach assigns weights to securities on multiple factor dimensions simultaneously, it accounts for cross-sectional interaction effects in a way that combining single-factor portfolios does not.

19.1. A short history of passively managed factor portfolios

Transparent rule-based index-tracking portfolios that employ alternative weighting schemes have grown rapidly in the last decade, especially within equities. Today, these types of non-market cap-weighted portfolios go by the term "advanced beta", "smart beta", "systematic strategies", "factor-based investing" and more. These strategies are passively implemented in the same way as traditional passive portfolios. Because of this, they retain the benefits of passive management including full transparency and low costs, with the potential to earn higher returns and/or

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deliver lower volatility than market cap-weighted portfolios. Particularly, over long periods, many investors are increasingly viewing them as a more cost-effective way to enhance returns relative to traditional active management. A wealth of papers have been written on these non-market cap-weighted strategies, which we refer to as passively managed factor (PMF) strategies for the remainder of this chapter[1].

The earliest PMF strategies applied an alternative weighting scheme to market capitalization weighting. Examples include Gross Domestic Product (GDP)-weighted portfolios in the 1980s, equal-weighted portfolios in the 1990s and more recently, fundamental-weighted portfolios in the 2000s. GDP weighting applies GDP weights as country weights in a global equity portfolio, while equal weighting and fundamental weighting assign equal weights to securities or weights based on company fundamentals such as book value, respectively. Proponents of these strategies were typically critical of cap weighting and argued that these alternative weighting schemes were superior either because they were more representative of investment value or more diversified (less concentrated).

Since 2008, an alternate way of viewing PMF portfolios has emerged, one that focuses on the underlying factors. This approach focuses on what "pure" factors (value, size, quality, momentum, etc.) the portfolios are exposed to, and derive their returns, from. The pure factors, beginning with the multi-factor models of Ross [ROS 76] are those that have been widely researched in the academic literature, have strong theoretical foundations and have exhibited persistence over multiple decades. Viewing PMF as a way to capture pure factors means it is consistent with the way academics have viewed factors, most widely popularized by Fama and French's seminal three-factor model, and extended over the years by countless others. It also grounds PMF investing in the same broad investing principles that underlie many active management approaches.

These factors represent systematic sources of return and risk, "risk premia" or arise because of mispricing of securities by investors which fail to be arbitraged away or because of market frictions. Increasing familiarity with traditionally academic factor models and newer commercially available factor models has driven greater adoption of this factor-based approach.

19.2. Single-factor portfolio construction

There are a range of techniques that can be used to build single-factor portfolios. Equal weighting, GDP weighting, fundamental indexation and its close companion

1 See [ANG 09, ANG 13, URW 11, BEN 13a] and [BEN 13b].

wealth weighting were compelling because they were intuitive and did not employ a black box algorithm such as optimization, which meant security weights could be directly tied to the securities' observable characteristics. Subsequent factor-based approaches could also be constructed in a similar manner; these can be viewed as "heuristic" or rule-based methods which use a set of rules to specify security weights as a function of the factor characteristics. Fama–French factor portfolios, for instance, while typically not viewed as PMF, are in fact rules-based factor portfolios. However, these were never meant to be investable portfolios, their long-short construct being difficult to scale.

Heuristic methods fall under one of the two categories: benchmark-independent or benchmark-relative. A benchmark-independent approach specifies a function for determining the weights that does not recognize the role of a benchmark (a market cap-weighted portfolio). A benchmark-dependent approach, however, does. Fundamental indexation, equal weighting and risk weighting are all examples of benchmark-independent approaches as shown in equations [19.1]–[19.3].

Equal weights: $w_i = \frac{1}{N}$ [19.1]

Fundamental indexation weights: $w_i = \frac{F_i}{\sum_i^N F_i}$ [19.2]

Risk weights: $w_i = \frac{\frac{1}{\sigma_i^2}}{\sum_i^N \frac{1}{\sigma_i^2}}$ [19.3]

where w_i is the weight of stock i in the portfolio, N is the number of stocks in the universe, F_i is the fundamental value of stock i (e.g. book value, earnings, etc.) and σ_i^2 is the variance of stock i.

Benchmark-relative approaches on the other hand incorporate market cap-weighting explicitly. For instance, one popular way is to apply multipliers to market cap weights:

Tilted factor portfolio weights: $w_i = w_{i,mktcap}\,\gamma_i$ [19.4]

where γ_i is a scalar applied to the market cap weight of each stock. The scalar γ_i can be specified in many ways. It can be the result of a mapping function based on the security's factor characteristics. It can be nonlinear or linear cross-sectionally, and it can be unique for each security or unique for groups of securities.

In addition to the weighting scheme, stock screening decisions also drive the performance and characteristics of the portfolio. These two decisions together determine the main characteristics of the portfolio (risk, return, excess return, relative risk, liquidity, concentration, etc.). In tilted portfolios, for instance, the greater the amount of stocks screened or the more aggressively the weighting scheme departs from cap weighting, the higher the relative risk (or tracking error), the higher the turnover, the lower the liquidity and so forth.

Benchmark-relative approaches have appeared to become the more preferred route in recent years primarily because of several reasons. First, there has been a broad adoption of market cap-weighted indices as performance and policy benchmarks by institutional investors. In this context, factor exposures are viewed as active exposures relative to the market cap-weighted index. Second, benchmark-relative approaches are more consistent with academic models such as Fama–French. In this model, for instance, the market factor is the first factor such that size Small Minus Big (SMB) and value High Minus Low (HML) are meant to capture effects excess or net of the market. When benchmark-relative portfolios are regressed on Fama–French factors, the signs of the exposures are consistent with the targeted factors. In the same vein, "anti-tilted" portfolios which tilt away from a particular factor do in fact underperform the benchmark and exhibit the opposite signs on Fama–French exposures. Further discussion appears in [BEN 15].

Besides heuristic or rules-based approaches, more complex portfolio construction methods can be used. PMF does not preclude per se the use of quadratic optimization or linear or nonlinear algorithms. Optimization has widely been accepted for building minimum volatility portfolios, for instance not least because it is by far the most efficient way to do so. Standard mean-variance quadratic optimization could be used for PMF portfolios. Recall the unconstrained optimal solution from [GRI 00] which maximizes the function:

$$\max \alpha' w - \frac{\lambda}{2} w' \Sigma w \qquad [19.5]$$

where w is the vector of active weights, Σ is the covariance matrix, α is the vector of alphas and λ is the risk aversion parameter. The optimal portfolio is given by:

$$w^* = \frac{1}{\lambda} \Sigma^{-1} \alpha \qquad [19.6]$$

Equation [19.6] is not dissimilar from equations [19.1]–[19.4]. Security weights are a function of alpha (which could just be some normalized factor characteristic or exposure), risk and risk aversion. In practice, however, constraints are typically required to arrive at realistic portfolios, since quadratic optimization tends to select extreme outcomes if no constraints are set and to potentially become error-maximizers, such that estimation noise in the inputs is magnified in the optimal weights; see [MIC 98]. Once constraints are introduced, the closed form solution in equation [19.6] no longer holds and the link between optimization inputs and portfolio weights quickly becomes less clear.

While the use of more complex portfolio construction techniques is not barred in PMF portfolios, because they tend to run contrary to transparency, their usage is likely to be limited. This issue arises because the active decision to own factors is made by the investors, and not by the asset managers. Passive managers hired to track a factor index cannot be held accountable if the factor underperforms since their objective is to track the index. Because the investors own the factor investing decision, they must be comfortable in understanding the methodology behind the indices. More generally, if the goal is broad exposure to one or more factors, which we believe it should be in PMF, we believe that both approaches will achieve the desired result.

19.3. Why combine multiple factors?

So far, we have discussed single-factor portfolio construction in broad terms without referencing the actual factors, arguably the most important point in PMF investing. Factor research (also known as the asset pricing anomaly research) comprises a vast body of academic literature. The most widely discussed factors include the original Fama–French–Carhart factors – value, size and momentum – and a handful of additional factors which have received moderate treatment – (low) volatility, quality, liquidity and yield. Numerous other stock characteristics have also been studied, spanning across income statement and balance sheet measures such as earnings revisions and accruals, technical indicators such as volatility and relative strength (momentum) and even non-financial factors such as media coverage, Internet hits and environmental, social and governance (ESG) themes.

There are several main camps in the debate over what drives factor returns. In the first camp are those who argue that factors earn excess returns because there is systematic risk attached to them. Markets are generally efficient and these factors reflect "systematic" sources of risk. In the second camp, factors are thought to earn excess returns because of investors' systematic errors which lead to persistent mispricing. These systematic behaviors are a result of investors collectively exhibiting behavioral biases and barriers which prevent these from being arbitraged away. A third camp focuses on market frictions giving rise to these anomalies, for instance the fact that many investors cannot use leverage. All three rationales have been proposed for value and size, while momentum, quality and low volatility tend to rest on investor mispricing mistakes or market frictions.

Since factors are unobservable, there is a limit to how certain investors can be around their existence and persistence. Following the old adage that "it is better not to put all of one's eggs in one basket", employing multiple factors has been one manner in which investors have diversified this information uncertainty. This is the first leg of the diversification argument for multi-factor investing.

The second leg of the diversification argument is that empirically, factors also have exhibited variation in performance over time, such that they diversify each other. In Table 19.1, we summarize the performance of factor portfolios over the past two decades. These portfolios are developed market securities formed from the MSCI World Index universe, where securities ranked higher on the relevant factor are overweight relative to the benchmark and securities ranked lower are underweight. (The details behind these portfolios, including the actual metrics used for these factors, are discussed in section 19.6). All five portfolios have historically outperformed the market cap-weighted benchmark, the MSCI World Index, and have exhibited higher return-to-risk ratios and moderate-to-robust information ratios (Table 19.1).

	Value	Volatility	Size	Momentum	Quality	
	Valuation tilt	(Low) Volatility tilt	(Low) Size tilt	Momentum tilt	Quality tilt	MSCI World
Annualized returns	9.15%	8.39%	8.86%	8.69%	9.04%	7.82%
Annualized standard deviation	15.74%	13.01%	15.26%	14.66%	13.85%	14.98%
Return to risk ratio	0.58	0.64	0.58	0.59	0.65	0.52
Annualized excess returns	1.34%	0.57%	1.04%	0.87%	1.22%	–
Tracking error	3.39%	3.44%	2.87%	4.40%	2.36%	–
Information ratio	0.39	0.17	0.36	0.20	0.52	–

Table 19.1. *Tilted factor portfolios, performance (Gross USD Monthly Returns, March 1993–December 2014, Global, Universe = MSCI World Index). The valuation tilted strategy tilts toward stocks with higher than average book-to-price, sales-to-price, earnings-to-price, cash flow to price and dividend yield. The low volatility tilted strategy tilts toward stocks with lower than average historical return volatility, while the low size tilted strategy tilts toward smaller cap stocks in the MSCI World Index (i.e. mid caps). The momentum tilted strategy tilts toward stocks with higher than average trailing 12-month returns, while the quality tilted strategy tilts toward stocks with higher than average return-on-assets and lower than average earnings-per-share variability and long-term debt to equity. Underlying factors are shown in the first row with the relevant tilted strategy underneath. Excess returns are the returns to the tilted strategies minus the benchmark (MSCI World). Tracking error is the standard deviation of excess returns annualized. The information ratio is excess returns divided by tracking error*

Next, Table 19.2 displays the correlations of the factor portfolios over the March 1993–December 2014 period. Correlations are generally low, sometimes negative. The highest correlations are between value and size, and low volatility and quality. The lowest correlations are between value and momentum, value and quality, and size and quality.

	Value	Volatility	Size	Momentum	Quality
	Valuation tilt	(Low) Volatility tilt	(Low) Size tilt	Momentum tilt	Quality tilt
Valuation tilt	1.00				
(Low) Volatility tilt	0.13	1.00			
(Low) Size tilt	0.60	–0.02	1.00		
Momentum tilt	–0.42	0.23	–0.22	1.00	
Quality tilt	–0.38	0.53	–0.36	0.31	1.00

Table 19.2. *Correlation of excess returns (Gross USD Monthly Returns, March 1993–December 2014). Underlying factors are shown in the first row with the relevant tilted strategy underneath. Excess returns are the returns to the tilted strategies minus the benchmark (MSCI World) (source: SSgA, Factset)*

The correlations above are just one view of diversification, measuring month-to-month co-movement between two PMF strategies. Another view of diversification is through their co-movement over longer periods. Rolling excess returns averaged over the preceding 3 years are shown in Figure 19.1. For example, between 2004 and 2007, quality and momentum significantly underperformed the market, but value and size significantly outperformed the market. There have been only a handful of periods, all short-lived, where all the factors performed poorly, for instance in 2003.

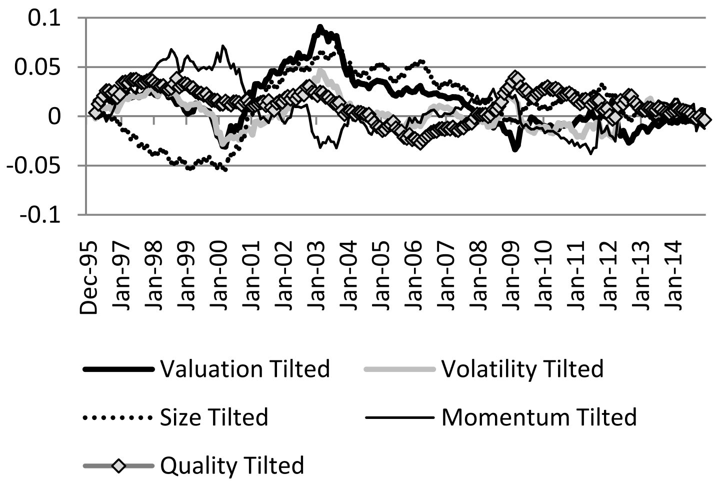

Figure 19.1. *Alleviating timing risk as seen through returns over time (Gross USD Monthly Returns, March 1993–December 2014). The relevant tilted strategies are shown for each underlying factor. Excess returns are the returns to the tilted strategies minus the benchmark (MSCI World) (source: SSgA, Factset)*

Having said that, as evidenced in Figure 19.1, all five factors have historically undergone prolonged periods of underperformance. Combining multiple factors alleviates the problem but does not completely eliminate it. Even if multiple factors are employed, PMF investing requires patience to harvest premiums over the long run.

19.4. Multi-factor portfolio construction

Multi-factor portfolios can be constructed into two main ways. The simplest way is to combine single-factor portfolios, such as the ones shown in Tables 19.1 and 19.2 and Figure 19.1, into one portfolio. Another way is to build the portfolio from the security-level up ("bottom-up"), incorporating all the factor characteristics simultaneously. Intuitively, the latter approach is more compelling since this approach evaluates securities on the multiple dimensions simultaneously. Asness [ASN 97] highlighted, for instance, interaction effects between value and momentum. Mixing portfolios independently constructed may miss these interaction effects. However, analogous to working with "building blocks," combining single-factor portfolios does have benefits for performance attribution and reallocation across factors.

We focus on comparing the "bottom-up" versus the "combination" methods within the context of the tilted portfolio approach where market cap weights are scaled by a multiplier. First, it may be helpful to point out that there is only one condition under which the two approaches will be identical:

– the starting weight (which the multiplier is applied to) is equal weight;

– multipliers do not capture information about the cross-sectional distribution of securities. For instance, ranks are used to identify the relative attractiveness of the 10 securities, not scores, and no two securities have the exact same rank (among any of the factors).

To illustrate this, we show portfolios of 10 securities in Table 19.3 (left panel) that blend three simple "factors" – dividend yield, book-to-price and return-on-assets as of 31 December 2014. For the combination portfolio, we rank securities for each of the three factors. We assign multipliers to equal weights (10%), and cap weights, which are identical to the stocks rank (e.g. a stock ranked 5 has a multiplier of 5). The scaled weights are then rescaled to sum to 100%. Then, we blend the three resulting portfolios into one using equal weights. For the bottom-up Portfolio, after we rank securities, we compute an average (equally weighted) rank across the three factors and multiply this combined rank by security market cap weights (or equal weights). We rescale the weights to sum to 100%.

	Rank-based approach (left panel)				Score-based approach (right panel)			
	Applied to equal weights		Applied to market cap weights		Applied to equal weights		Applied to market cap weights	
	Comb. portfolio	Bottom-up	Comb. portfolio	Bottom-up	Comb. portfolio	Bottom-up	Comb. portfolio	Bottom-up
633987	10.30%	10.30%	3.61%	2.95%	10.42%	12.73%	3.61%	3.15%
B1YW44	6.67%	6.67%	0.28%	0.25%	6.71%	1.82%	0.28%	0.06%
88579Y10	10.91%	10.91%	9.30%	10.04%	10.92%	14.55%	9.30%	11.58%
425304	8.48%	8.48%	10.49%	9.97%	8.54%	7.27%	10.49%	7.39%
425305	7.88%	7.88%	8.91%	8.94%	7.33%	3.64%	8.91%	3.57%
710889	8.48%	8.48%	4.33%	3.59%	8.56%	10.91%	4.33%	3.99%
00282410	14.55%	14.55%	9.29%	8.51%	14.65%	18.18%	9.29%	9.21%
00287Y10	10.30%	10.30%	9.21%	9.27%	10.33%	9.09%	9.21%	7.08%
629210	12.12%	12.12%	43.97%	45.98%	12.15%	16.36%	43.97%	53.73%
000312	10.30%	10.30%	0.61%	0.51%	10.39%	5.45%	0.61%	0.23%
Correlation	1.00		1.00		0.87		0.99	
Absolute sum of weight differences	0.00%		5.70%		32.05%		24.09%	

Table 19.3. *Bottom-up versus combination method, 10-stock example: rank versus score-based (31 December 2014)*

If we apply the multipliers to equal weights, we see in Table 19.3 that the two resulting portfolios are identical; that is the bottom-up multi-factor portfolio is identical to the combination portfolio in the left panel. This is the only scenario in which the two portfolios can be the same; when no cross-sectional distributional information is captured in the multiplier (e.g. ranks are used, each security receives one unique rank and no two ranks are the same), and the multipliers are applied to equal weights. Note that in the right panel, while the correlation between the weights in the two portfolios is 1.0, because the "excess weight" (the difference between the sum of the weights and the 100% target weight) is distributed unevenly across securities so that higher ranked larger stocks receive more weight, the weights for the two portfolios are different.

Similarly, if we use scores instead of ranks, the two outcomes are different, whether or not we apply the multipliers to equal weights or cap weights. For scores, we normalize the raw metrics for each security by subtracting the mean and dividing by the standard deviation across securities, for each factor. Scores preserve the distributional characteristics of each factor in a way that ranks do not. If a security has an extremely high price-to-book relative to the other securities, that "extremeness" will be captured by the score. Since multipliers cannot be negative for long-only portfolios, we use the ranks of the scores, and not the scores themselves, as the multipliers. Note that average rank is not the same as a rank based on average score. The latter is what we use in the bottom-up portfolio and it does indeed capture distributional characteristics. Table 19.3 (right panel) summarizes the difference between the rank-based approach and the score-based approach.

The score-based approach results in portfolios that are meaningfully more different from each other than the rank-based approach. In the case where multipliers are applied to equal weights, the correlation falls to 0.87 and the absolute sum of the weight differences is 32%, considerably higher than both rank-based examples. In the case where multipliers are applied to market cap weights, the correlation remains high at 0.99 but the absolute sum of the weight differences remains quite high at 24%.

We have seen with a highly stylized hypothetical 10-security example that scoring has a far greater impact than ranking when comparing combination versus bottom-up approaches to multi-factor portfolio construction. But, how much of an impact does it have in more realistic portfolios that employ a much larger number of securities?

We conduct the following simulations to further our understanding:

– combination portfolio: for the combination portfolio, we create the following single-factor portfolios: value, low volatility, quality and momentum. The definitions for factors are the same used in the tilted portfolios (section 19.6). Our universe is the MSCI World Index. First, we rank the securities by each metric. Second, we group the securities into 20 fractiles, which we also refer to as subportfolios. We apply a fixed set of multipliers (linearly interpolated between 0.25 and 2.0 in increments of 0.25) to the market cap weight of the security depending on which fractile (subportfolio) it falls in. Finally, we rescale the weights such that they sum to 100%. The combination portfolio is an equally weighted average of the four individual factor portfolios. All portfolios are rebalanced monthly;

– bottom-up portfolio: first, we assign scores to securities for each factor. Second, we average the scores (equally weighting the factors). Third, we group the securities into 20 fractiles/subportfolios based on their average scores. We then apply the same fixed set of multipliers as in the combination portfolio depending on the fractile the security falls in. Finally, the weights are rescaled such that they sum to 100%. The factor definitions, universe, rebalancing frequency are the same as above.

The results are summarized in Table 19.4. The bottom-up returns are higher than any of the underlying component factor returns, and higher than the combinations. The difference is not insignificant, a spread of 86 basis points. Moreover, the volatility of the bottom-up approach is significantly lower and risk-adjusted return increases from 0.73 to 0.84 between the two approaches. This suggests that there are interaction effects important to capture. While these relationships are not likely to be the same across all factor combinations, we suspect that for the most well-known factors – value, size, volatility, momentum, etc. – these interaction effects will exist.

	Value portfolio	Low volatility portfolio	Quality portfolio	Momentum portfolio	Combination portfolio	Bottom-up
Annualized return	11.63%	10.69%	10.40%	10.91%	10.94%	11.80%
Annualized volatility	17.05%	13.77%	15.05%	15.07%	15.06%	14.12%
Risk-adjusted return	0.68	0.78	0.69	0.72	0.73	0.84
Excess return	3.49%	2.55%	2.26%	2.77%	2.80%	3.66%
Tracking error	7.12%	5.19%	4.43%	4.52%	4.78%	5.10%
Information ratio	0.49	0.49	0.51	0.61	0.59	0.72

Table 19.4. *Combination versus bottom-up approach, four-factor portfolios (January 1993–March 2015, Gross USD Returns)*

19.5. Conclusion

PMF portfolios have emerged in recent years as an alternative to investors dissatisfied with market-cap weighting or as an explicit way to achieve exposure to well-known factors that have been shown to drive stock returns. These portfolios employ indices and portfolios are managed to these indices just as in traditional passive investing. Thus, PMF investing has the same benefits as traditional passive investing – transparency, implementation efficiency and low costs. These innovations are changing the investment landscape, which until recently, was composed of traditional passive investing and active management.

Portfolios can be constructed in many ways, ranging from relatively simple rules-based approaches that specify weights as a function of factor characteristics to more quantitatively-oriented ways that utilize more complex functions. Within multi-factor portfolio construction, we show that a bottom-up approach can produce superior results than a combination of individual single-factor portfolios, at least for well-known factors such as value, quality, low volatility and momentum.

19.6. Appendix A: description of tilted factor portfolios

The tilted factor portfolios shown in this chapter employ the following methodology. Each factor uses either a single metric or several metrics (in the latter case, they are equally weighted) as follows:

– value (valuation tilted strategy): price to fundamental (five fundamentals used: earnings, cashflow, sales, dividend and book value);

– low volatility (volatility tilted strategy): trailing 60-month variance of total returns;

– quality (quality tilted strategy): return-on-assets, variability in earnings per share[2] and leverage[3];

– size (size tilted strategy): free float-adjusted market capitalization;

– momentum (momentum tilted strategy): trailing 12-month return.

For each tilted portfolio, we rank all stocks in the benchmark universe by the variable shown. (In the case of value and quality, a normalized score is first calculated and averaged across the individual metrics before ranking.) The stocks are next assigned to 20 ranked subportfolios such that each subportfolio holds 5% of the market capitalization of the universe[4]. The subportfolio with the highest ranking

2 Earnings variability is measured by the standard deviation of earnings per share (EPS) divided by the median earnings for the past 5 years. Dividing the median earnings normalizes the volatility and makes it more comparable across different companies.

3 Leverage is measured by total liabilities divided by shareholders equity. It indicates what percentage of equity and debt companies use to finance their assets. The lower the indicator, the more sound a company's financial strength is, and the higher quality it is, holding all other factors constant.

4 In the simulated strategies shown in this chapter, shares of a stock can straddle two subportfolios.

is subportfolio 20, while the subportfolio with the lowest ranking is subportfolio 1. Stocks within each subportfolio are cap-weighted. Next, a multiplier[5] is assigned to each subportfolio with subportfolios in which the lower ranked subportfolios receive a multiplier less than 1 and the higher ranked subportfolios receive a multiplier greater than 1. This multiplier is then applied to each stock in the subportfolio's market cap weight. All weights are then rescaled to sum to 100%. All simulated strategies are rebalanced annually in March, while the momentum strategy is rebalanced quarterly.

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5 The multiplier used for the valuation tilted strategy is based on the ratio of the universe's weighted valuation ratio relative to each subportfolio's weighted valuation ratio. The same logic is applied to the size tilted strategy and volatility tilted strategies. In these last two, a maximum multiplier of 3 is allowed. For the quality tilted and momentum tilted strategies, the multiplier is 1.95 for subportfolio 20 and 0.05 for subportfolio 1, with a linear interpolation for the subportfolios in between.