Geometry/Combinatorics Individual Test

Interlake Invitational Math Competition

March 14, 2015

1. [**3**] Interlake Math Club is electing new officers. There are 5 positions: a president, a vice-president, and three secretaries. There are 10 people running for office. In how many ways could the positions be distributed among the ten people?

2. [**3**] I am cutting my standard cubic die into an octahedral die. The vertices of the new octahedron will be the centers of the faces of the original cube. What is the ratio of the volume of the new octahedron to the volume of the original cube?

3. [**3**] Brian is flying from Seattle to Boston. Seattle is located at $(0,0)$ and Boston is located at $(3,7)$ on the coordinate plane. Unfortunately, there are no direct flights, but Brian can choose a connection flight at any point along the line $y = x - 4$. (So Brian will fly from $(0,0)$ to that point, and then from that point to $(3,7)$). What is the location of the point on the line $y = x - 4$ that minimizes Brian's total travel distance?

4. [**3**] A ladybug is crawling along the gridlines of a coordinate plane. Starting from the point $(0,0)$, it moves either one unit right or one unit up every second. It wants to reach the point $(10,10)$ after 20 seconds. However, there is a dangerous puddle of acid at every point where both coordinates are odd. How many paths can it take to reach its destination which do not go through any acid puddles?

5. [**4**] An isosceles triangle has side lengths $5, 5$, and 8. Let I be the incenter of the triangle, and O be the circumcenter. Find the length IO.

6. [**4**] There are 4 different pairs of socks in my drawer. Every day for 4 days I randomly pick two socks from my drawer and put them on, and don't return them to the drawer afterward. Out of these 4 days, what is the expected number of days on which my socks will match?

7. [**4**] The sides of a regular dodecagon are labeled from 1 to 12 in clockwise order. Sides $3, 6, 9$, and 12 are extended to form a square of area 1. Find the area of the dodecagon.

8. [**5**] Find the number of sequences beginning with 1 of length 5 of positive integers such that the absolute value of the difference between every pair of adjacent integers is a different power of 2 in the set $\{2^1, 2^2, 2^3, 2^4\}$.

9. [**5**] A triangle $\triangle ABC$ with side lengths $AC = 4$, $AB = 5$, and $BC = 6$ has incircle I. Let I_a be the circle tangent to BC and the extensions of rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Suppose that point D is on the extension of ray $\overrightarrow{AB}$ and that E is on the extension of $\overrightarrow{AC}$ such that DE is tangent to I_a and parallel to BC. Find the length DE.

10. [**6**] A cyclic quadrilateral $ABCD$ is inscribed in a circle O with radius 1. If $136AB = 255BC = 120CA$, and BD is perpendicular to AC, find the length of BD.