1. **[5]** A cube of side length 4 units is painted on 5 faces. When cut into 64 small cubes of side length 1 unit each, how many cubes are painted on exactly 2 sides?

2. **[5]** Determine the perimeter of a triangle ABC with $AB = 6$, $BC = 12$, and $\angle ABC = 120°$.

3. **[5]** Find the minimum possible value of $2x^2 - 8x + 15$.

4. **[5]** Al, Bob, and Cody are running a race. How many total ways can they be ranked if ties are allowed?

5. **[5]** How many ways are there to arrange the letters in the "word" "PIDAY" if the "P" and "I" need to be consecutive (in either order)?

6. **[5]** The Interlake Invitational Math Competition has 26 volunteers, and wishes to divide them such that there are A runners, B proctors, and C graders. Given that all volunteers must be in one of the three positions and that each role must be taken by at least 5 volunteers, how many ordered triples (A, B, C) are possible?

7. **[5]** How many factors of 2400 are **not** perfect squares?

8. **[5]** Triangle ABC satisfies $AB = AC = 10$ and $BC = 16$. Find the sum of the area and perimeter of triangle ABC.

9. **[6]** John has \$32, Jane has \$28, and Jordan has \$25, all of which are in \$10, \$5, and \$1 dollar bills. If they pool all their money together, what is the difference between the greatest possible total number of bills and the least possible total number of bills?

10. **[6]** Compute the sum of all positive integers that are not divisible by 2 or 3 that are at most 60.

11. **[6]** Two circles with centers O_1 and O_2 have radii 10 and 16, respectively. A point P is given such that the lengths of the tangents from P to both circles are both 2015. Find the value of $|PO_1^2 - PO_2^2|$.

12. **[6]** Quadrilateral $ABCD$ satisfies $AB = 4, BC = 3, CD = 12$ and $\angle ABC = \angle ACD = 90°$. Find the area of quadrilateral $ABCD$.

13. **[6]** Find the sum of the factors of 300.

14. **[6]** Call a positive integer *pure* if it is a perfect power of a prime. How many pure numbers are there under 100? (Include 1.)

15. **[6]** In right triangle ABC with AB as the hypotenuse, $AB = 16$. Semicircles with diameters AB, BC, CA are erected on the outside of the triangle. Find the sum of the areas of these semicircles.

16. **[6]** Positive integers x and y satisfy $x^2 - 12x + 6 = y^2 - 8y - 6$. If $x > y$, find the value of x.

17. **[8]** Ryan is taking a walk from the point $(0,0)$ to the point $(4,4)$. At point (a,b), Ryan walks either to point $(a+1,b)$ or to point $(a,b+1)$. If every possible such walk is equally likely, what is the probability Ryan will pass through the point $(2,2)$?

18. **[8]** Let a and b be positive reals satisfying $a^2+ab+a=12$ and $b^2+ab+b=30$. Find the value of $a+b$.

19. **[8]** A regular 31-gon is inscribed in a circle with center O. How many triangles determined by 3 vertices of the 31-gon contain O?

20. **[8]** Given that $\sqrt{-26+6i\sqrt{3}}$ can be expressed in the form $a+bi\sqrt{c}$ for positive integers a, b, and c, find $a+b+c$.

21. **[8]** In isosceles triangle ABC, $AB=AC=20$ and $BC=24$. If I is the incenter of $\triangle ABC$, find the length of AI.

22. **[8]** Find the remainder when 3^{98} is divided by 41.

23. **[8]** Evaluate

$$\sum_{k=1}^{20}\frac{1}{(2k-1)(2k+1)}$$

24. **[8]** Mike's magic cone company can produce cones of any dimension at a price of \$1 every πm^3. If Sally orders an infinite set of similar cones with her first cone having radius $3m$ and height $4m$, and her i^{th} cone having radius $\frac{3}{2^{i-1}}m$, what is the price, in dollars, of her total purchase? Answer as a common fraction.

25. **[10]** Call a positive integer *fat* if and only if the sum of its even proper divisors is greater than itself. What is the sum of the two smallest fat numbers?

26. **[10]** Seven couples are at a party. Some pairs of them shake hands, with the condition that nobody shakes the hand of both members of a couple. What is the maximum possible number of total handshakes at the party?

27. **[10]** The monic polynomial $P(x)$ has degree 4 and satisfies $P(0)=1, P(1)=2, P(2)=5, P(3)=10$. Find the value of $P(4)$. (Note: A polynomial is *monic* if it has leading coefficient 1.)

28. **[10]** $\triangle ABC$ satisfies $AB=13, BC=14, AC=15$. If M,N,P are the midpoints of BC, CA, AB respectively, find the area of the triangle formed by the centroids of $\triangle APN, \triangle BMP, \triangle CNM$.

29. **[10]** Let $f(n)$ be the value of the greatest integer dividing $n^4 + 16$ that is also a power of two. What is the maximum value of $f(n)$ over all positive integer values of n?

30. **[10]** Caleb needs to edit a ten problem test by changing the order of the problems. Each problem number a can be moved to problem number b if and only if $|a - b| \leq 1$. How many different ways can he order the problems? (Include the original ordering)

31. **[10]** $\triangle ABC$ satisfies $AB = 13, BC = 14$, and $[ABC] = 84$ where $[ABC]$ is the area of $\triangle ABC$. What is the sum of the possible values of AC?

32. **[10]** Evaluate

$$\tan\left(\arctan\frac{1}{2} + \arctan\frac{1}{3} + \arctan\frac{1}{4} + \arctan\frac{1}{5} + \arctan\frac{1}{6}\right)$$

33. **[12]** A small country has 30 cities, some pairs of which are connected by two-way roads. At most how many roads can there be such that there are no three cities in the country A, B, C such that any two of them are connected?

34. **[12]** A very large bathtub is connected to an infinite number of pipes, labeled $P_1, P_2, P_3, \ldots$, each of which can be turned on or off. A "turned off" pipe does nothing. For odd i, the pipe labeled P_i adds water to the bathtub at a rate which, if the bathtub started empty and only pipe P_i was left on, the bathtub would take i^2 hours to fill up completely. For even i, the pipe labeled P_i empties water from the bathtub at a rate which, if the bathtub started full and only pipe P_i was left on, it would take i^2 hours to empty completely. If all pipes are left on and the bathtub is initially empty, how many hours will it take for the bathtub to be filled? Answer ∞ if the bathtub will never be filled.

35. **[12]** There are 32 lights in a row, each with its own switch. Initially, every light is off. Let M be the number of ways to toggle the first 16 lights with 48 total switches such that the first 16 lights are on after the switches. Let N be the total number of ways to toggle the 32 lights with 48 switches such that the first 16 lights are on, and the last 16 lights are off after the switches. Determine $\frac{N}{M}$.

36. **[12]** Find the remainder when $(2 \cdot 4 \cdot 6 \cdot \cdots \cdot 100)^2$ is divided by 101.