Source Stirring Analysis in a Reverberation Chamber Based on Modal Expansion of the Electric Field

E. Amador[1], P. Besnier[2]

[1]EDF Lab, LME. Moret sur Loing, France
emmanuel.amador@edf.fr
[2]IETR-UMR CNRS 6164, INSA Rennes, France
philippe.besnier@insa-rennes.fr

Joint IEEE International Symposium on Electromagnetic Compatibility and EMC Europe, Dresden 2015

Content

Motivations

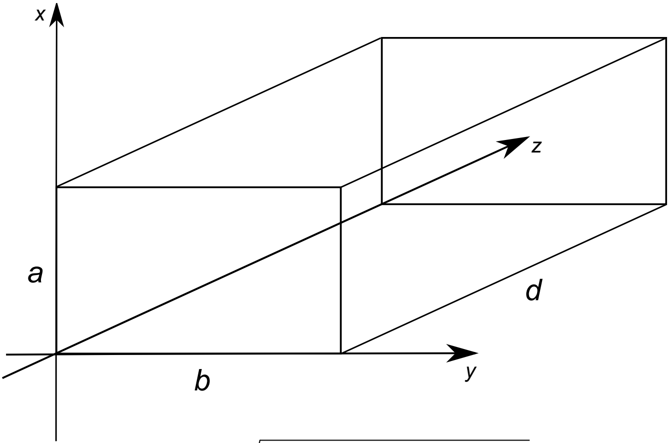
Reverberation chamber simulations based on numerical solutions of Maxwell's equations (FDTD, FEM, image theory,...) :

- are convenient to provide general trends (Ex. statistical uniformity vs losses)
- provide more data than measurements,
- do not allow a much better understanding of the cavity physics.

By using an analytical approach at the *mode level* we expect to :

- get a more precise view of the multi-modal field distribution in the chamber,
- understand how the number of independent observations is related to the successive states of multi-modal field distribution according to source stirring.

Rectangular cavity



$$f_{mnp} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}, \tag{1}$$

E-field wave guide equations [?, ?]

(similar equations may be derived for the H-field) For a propagation along z :

TM mode :

$$E_{x_{mnp}}^{TM^z} = \frac{-k_x k_z E_0}{k_{mnp}^2 - k_z^2} \cos k_x x \sin k_y y \sin k_z z \tag{2}$$

$$E_{y_{mnp}}^{TM^z} = \frac{-k_y k_z E_0}{k_{mnp}^2 - k_z^2} \sin k_x x \cos k_y y \sin k_z z \tag{3}$$

$$E_{z_{mnp}}^{TM^z} = E_0 \sin k_x x \sin k_y y \cos k_z z \tag{4}$$

TE mode :

$$E_{x_{mnp}}^{TE^z} = \frac{-j\omega_{mnp}\mu k_y H_0}{k_{mnp}^2 - k_z^2} \cos k_x x \sin k_y y \sin k_z z \tag{5}$$

$$E_{y_{mnp}}^{TE^z} = \frac{-j\omega_{mnp}\mu k_x H_0}{k_{mnp}^2 - k_z^2} \sin k_x x \cos k_y y \sin k_z z \tag{6}$$

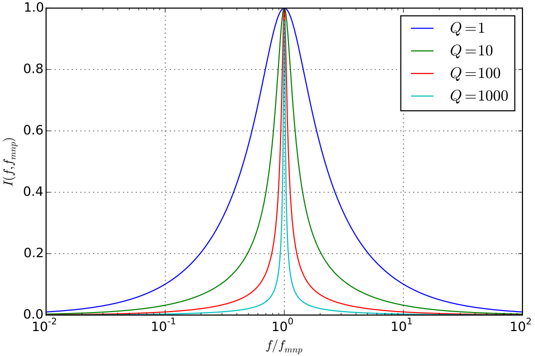
$$E_{z_{mnp}}^{TE^z} = 0 \tag{7}$$

$$\left(k_x = \frac{m\pi}{a},\ k_y = \frac{n\pi}{b},\ k_z = \frac{p\pi}{d},\ k_{mnp}^2 = k_x^2 + k_y^2 + k_z^2\right)$$

Universal resonant curve

At a frequency f, the amplitude of a mode with frequency , f_{mnp} can be computed from the effective/composite Q factor [**?**] :

$$I(f, f_{mnp}) = \frac{1}{\sqrt{1 + Q^2(f)\left(\frac{f}{f_{mnp}} - \frac{f_{mnp}}{f}\right)^2}}, \tag{8}$$



E-field (1/2)

The E-field observed at the frequency f is approximated by summing the E-fields produced by all modes in an adequate bandwidth $f \pm \Delta f$. The E-field along the x axis at the frequency f at the position (x, y, z) is given by :

$$E_x(f, x, y, z) = \sum_{f_{mnp} \in f \pm \Delta f} I(f, f_{mnp}) \left(E_{x_{mnp}}^{TM^i} + E_{x_{mnp}}^{TE^i} \right), \tag{9}$$

with $i = x, y, z$ if the TM or TE modes exist for the given triplet (m, n, p).

E-field (2/2)

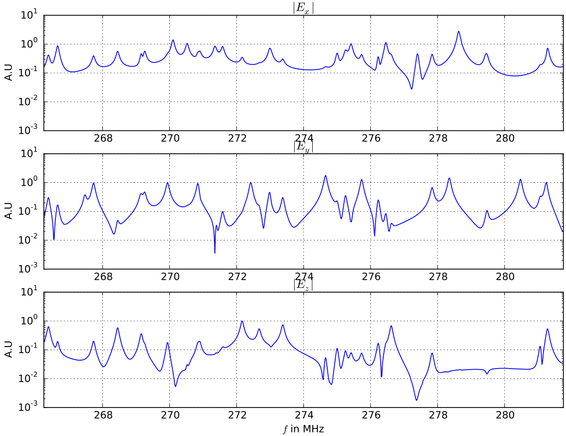


FIGURE: E-field computed at $M(3.5, 1.7, 1)$.

However, the modal response of the cavity is bounded to the source excitation.

Response to an isotropic and infinitely small EM source

- A infinitely small transmitter is placed at the position $A(x_e, y_e, z_e)$
- The mode amplitudes/coefficients of each TM or TE mode excited at the frequency f at this position A are given by : (For the E-field components on the x axis)

$$C_{x_{mnp}}^{TM^z} = I(f, f_{mnp}) \frac{-k_x k_z}{k_{mnp}^2 - k_z^2} \cos k_x x_e \sin k_y y_e \sin k_z z_e \qquad (10)$$

$$C_{x_{mnp}}^{TE^z} = I(f, f_{mnp}) \frac{-j\omega_{mnp}\mu k_y}{k_{mnp}^2 - k_z^2} \cos k_x x_e \sin k_y y_e \sin k_z z_e \qquad (11)$$

- By reciprocity, the E_x component of the E-field at the frequency f produced by this transmitter at a given position $M(x, y, z)$ is given by :

$$E_x(f, x, y, z) = \sum_{f_{mnp} \in f \pm \Delta f} \left(C_{x_{mnp}}^{TM^i} E_{x_{mnp}}^{TM^i} + C_{x_{mnp}}^{TE^i} E_{x_{mnp}}^{TE^i} \right), \qquad (12)$$

with $i = x, y, z$.

Source stirring

Source stirring is then performed analytically.

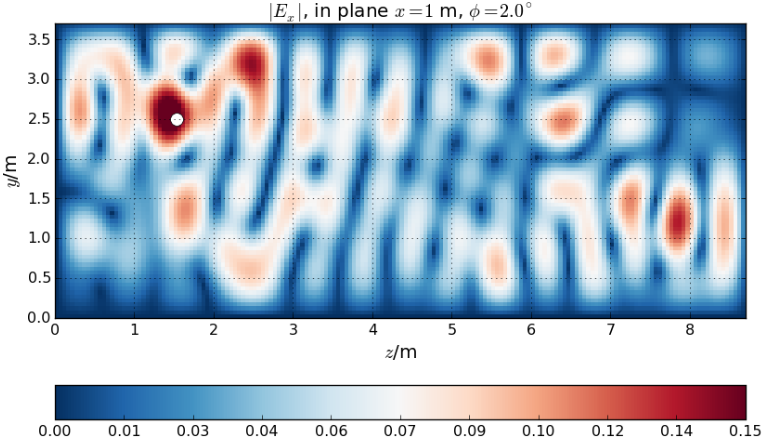
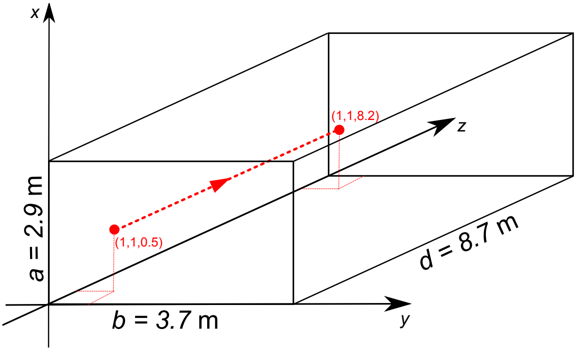


FIGURE: $|E_x|$ in V/m at $f = 274$ MHz, with a source moving along a circle in the plane $x = 1$ m.

Source stirring : linear path



- $N_s = 201$ steps, linear movement along the z axis
- The complex E_x field is computed at $N = 150$ arbitrary positions in the working volume for every step and every frequency

Counting the modes

- By moving the source in the working volume, different modes are excited.
- We compute the mode coefficients of every mode excited.
- We want to estimate the number of modes $N_{x_{modes}}$ that have a significant impact on the E-field in the chamber.

We define as a significant mode, a mode whose maximum magnitude over the source positions reaches at least 30% of the maximum magnitude of the mode with maximum magnitude.

Counting the modes - Example at 50 MHz (1/2)

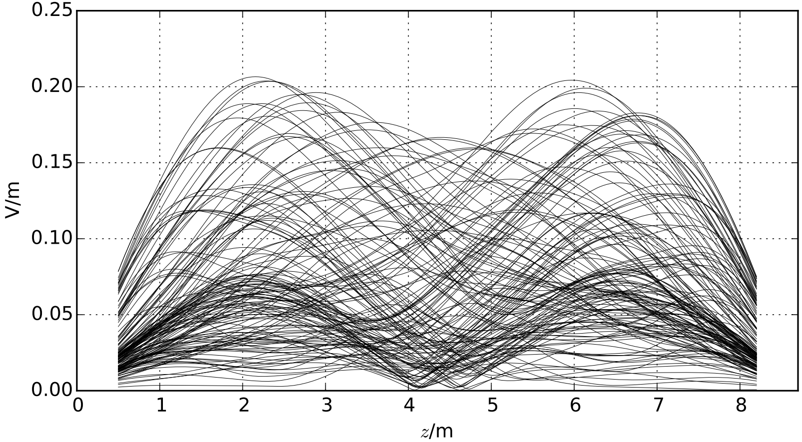


FIGURE: $|E_x|$ vs. transmitter position at $N = 150$ positions (50 MHz).

Counting the modes - Example at 50 MHz (2/2)

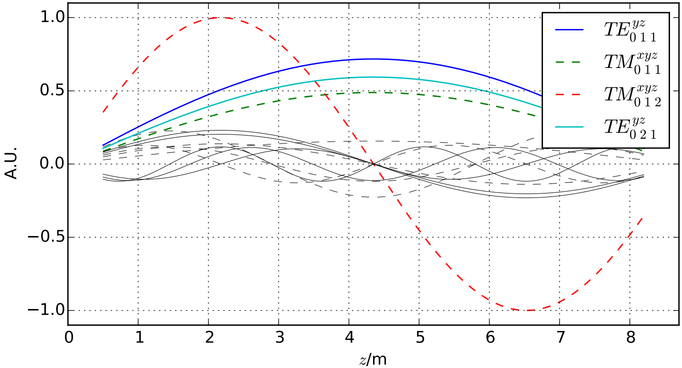


FIGURE: Modes coefficients vs. transmitter position at 50 MHz.

4 main modes. 3 of them with index $p = 1$ are clearly correlated. Number of independently excited modes :

$$N_{x_{modes}} = 2$$

Counting the modes - Example at 425 MHz (1/2)

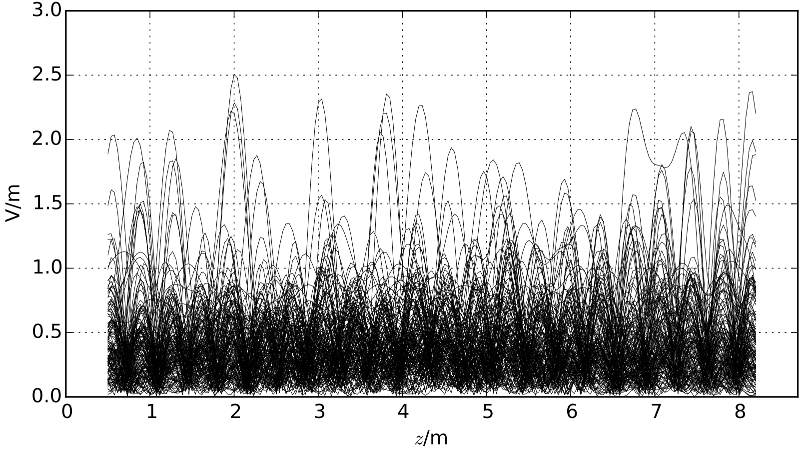


FIGURE: $|E_x|$ vs. transmitter position at $N = 150$ positions (425 MHz).

Counting the modes - Example at 425 MHz (2/2)

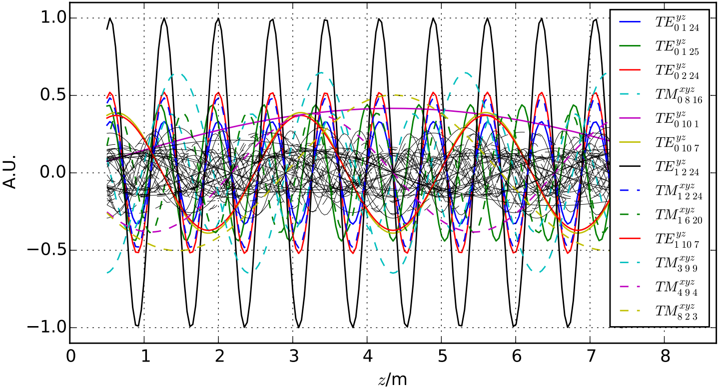


FIGURE: Modes coefficients vs. transmitter position at 425 MHz.

Groups of modes with same p indexes are correlated,
$N_{x_{modes}} = 9$ (with distinct p indexes)

Number of modes and number of independent positions

Entropy gives the number of independent positions $N_{x_{eff}}$ along the transmitter path (among N_s positions) the correlation matrix of complex E_x values [**?**] :

$$N_{x_{eff}} = \frac{N_s^2}{\sum_{i,j=1}^{N_s} |r_{x_{ij}}|^2}, \tag{13}$$

where r_x is the correlation matrix computed from the complex E_x values.

How $N_{x_{eff}}$ and $N_{x_{modes}}$ are related ?

$N_{x_{eff}}$ and $N_{x_{modes}}$

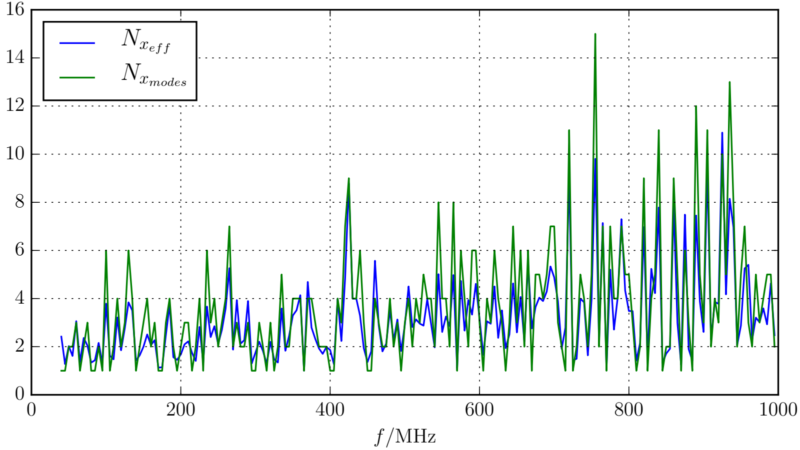


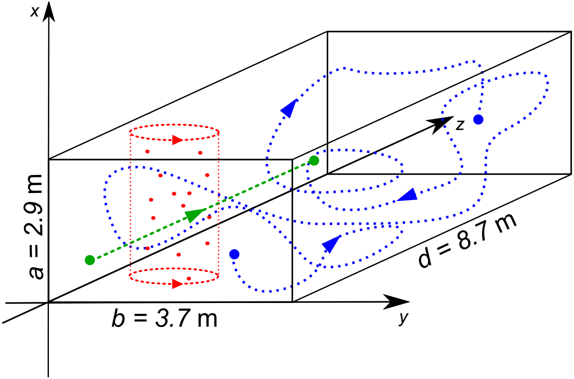
FIGURE: $N_{x_{eff}}$ and $N_{x_{modes}}$ vs. frequency

Partial conclusion

- $N_{x_{eff}}$ and $N_{x_{modes}}$ are in good agreement for an isotropic source stirring.
- The number of modes that are excited independently by moving linearly the transmitter is close to the number of independent positions derived from the entropy estimation [**?**].
- Source stirring implies independent mode excitations but also correlated excitation of groups of mode (linear path along z direction)

Different methods

- Linear movement along the z axis (as previously presented).
- Along a smoothed random curve in the whole working volume.
- Source(s) randomly placed in a cylinder volume (1 m radius) and rotating around a vertical axis :
  - 1 source
  - 6 sources
  - 100 sources



Linear path vs. random path

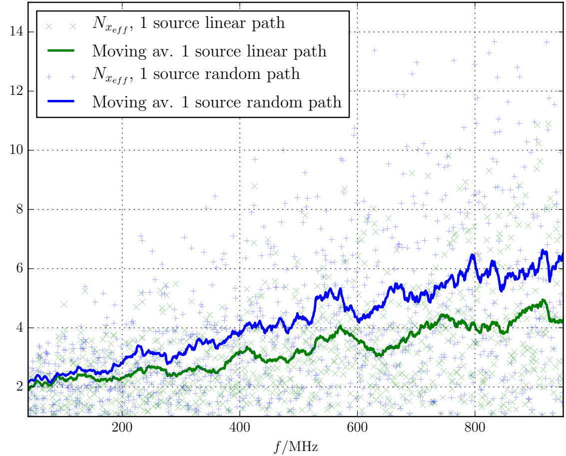


FIGURE: Independent source stirring positions vs. frequency for a linear source movement or a random movement.

Sources rotating around one axis

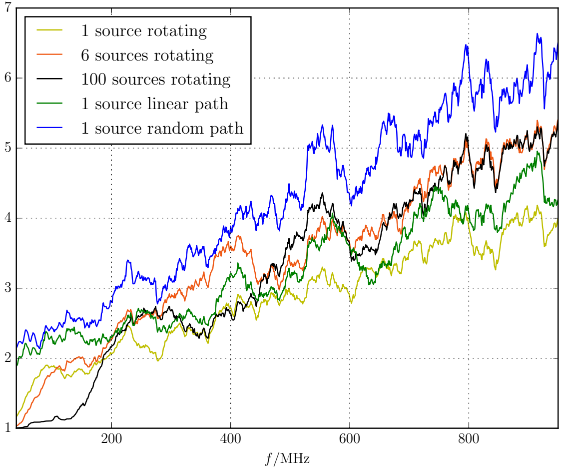


FIGURE: Independent source stirring positions vs. frequency for different numbers of sources rotating around one axis.

Partial conclusion

- At low frequencies moving a large number of sources reduce the number of independent positions, the modes are not selected individually
- At high frequencies, increasing the number of sources allows to get more independent positions.
- Moving one source randomly in the volume gives the best results.

Efficient source stirring is achieved by selecting a large number of modes individually.

Conclusion

- We presented a very simple and fast analytical model based on a modal combination of 3 wave guides field equations.
- This model allows an analysis of the behavior of a rectangular cavity at the mode level.
- The number of modes **independently excited** by a moving source along a path is related to the number of independent positions observed.
- The model allows to benchmark different source(s) stirring approaches.

[1] R. Harrington, *Time-Harmonic Electromagnetic Fields*.
New York : McGraw-Hill Book Company, 1961 (repr. 2001).

[2] D. A. Hill, *Electromagnetic Fields in Cavities : Deterministic and Statistical Theories*.
Hoboken, NJ, USA : John Wiley & Sons, Inc., 2009.

[3] F. E. Terman, *Radio Engineer's Handbook*.
McGraw-Hill Book Company, 1943.

[4] R. Pirkl, K. Remley, and C. Patane, "Reverberation chamber measurement correlation," *Electromagnetic Compatibility, IEEE Transactions on*, vol. 54, pp. 533–545, June 2012.

Presentation, source code and data available at :

`http://github.com/manuamador/IEEE-EMC2015`

Statistical Validation

Anderson Darling, GoF test :