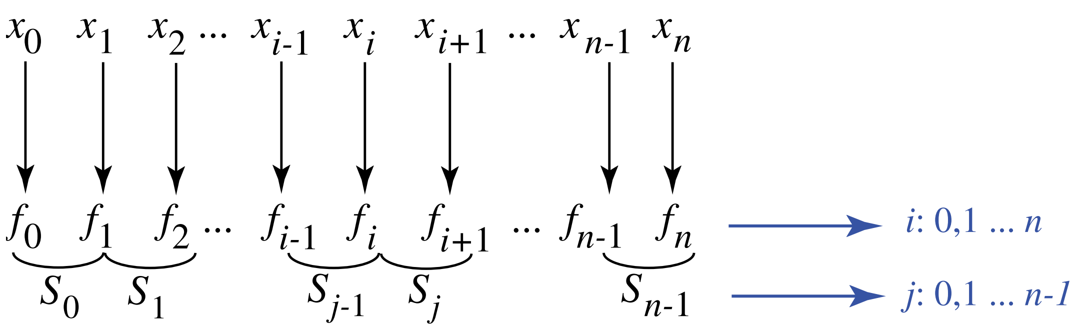
Resumen *splines*



$$S_i(x) = a_i(x - x_i)^3 + b_i(x - x_i)^2 + c_i(x - x_i) + d_i \quad (i = 0, 1 \ldots n - 1)$$

$$h_i \equiv x_{i+1} - x_i \tag{1}$$

$$\boxed{d_i = f_i} \tag{2}$$

$$\boxed{a_{i-1} = \frac{b_i - b_{i-1}}{3(x_i - x_{i-1})}} \tag{8}$$

$$\boxed{c_i = \frac{d_{i+1} - d_i}{x_{i+1} - x_i} - \frac{(b_{i+1} + 2b_i)}{3}(x_{i+1} - x_i)} \tag{7}$$

$$\boxed{b_{i-1}h_{i-1} + 2\, b_i(h_i + h_{i-1}) + b_{i+1}h_i = \frac{3}{h_i}(d_{i+1} - d_i) - \frac{3}{h_{i-1}}(d_i - d_{i-1})}$$

Condiciones *naturales* de contorno:

$$b_0 = 0; \quad b_n = 0 \tag{6}$$

Cambio de variables:

$$\begin{cases} u_i \equiv b_i \quad \text{(incógnitas)} \\ r_i \equiv \dfrac{3}{h_i}(d_{i+1} - d_i) - \dfrac{3}{h_{i-1}}(d_i - d_{i-1}) \\ A_i \equiv h_{i-1} \\ B_i \equiv 2(h_i + h_{i-1}) \\ C_i \equiv h_i \end{cases} \tag{3}$$

[Téngase en cuenta que para todas estas variables, $i = 1, 2 \ldots n-1$, excepto para A_i: $(i = 2, 3 \ldots n-1)$ y para C_i: $(i = 1, 2 \ldots n-2)$].

Sistema de ecuaciones *tridiagonal*:

$$\begin{pmatrix} B_1 & C_1 & & & & \\ A_2 & B_2 & C_2 & & & \\ & A_3 & B_3 & C_3 & & \\ & & \ddots & \ddots & \ddots & \\ & & & A_{n-2} & B_{n-2} & C_{n-2} \\ & & & & A_{n-1} & B_{n-1} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_{n-2} \\ u_{n-1} \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ r_{n-2} \\ r_{n-1} \end{pmatrix}$$

Sustitución/eliminación gaussiana:

$$\begin{cases} B_1 & \longrightarrow B_1 \\ r_1 & \longrightarrow r_1 \end{cases}$$

$$(i = 2, 3 \ldots n-1) \qquad \begin{cases} B_i & \longrightarrow B_i - \dfrac{C_{i-1} A_i}{B_{i-1}} \\ r_i & \longrightarrow r_i - \dfrac{r_{i-1} A_i}{B_{i-1}} \end{cases} \tag{4}$$

$\Longrightarrow$ Solución del sistema:

$$\begin{cases} u_{n-1} & = \dfrac{r_{n-1}}{B_{n-1}} \\ u_i & = \dfrac{r_i - C_i u_{i+1}}{B_i} \quad (i = n-2, n-3 \ldots 1) \end{cases} \tag{5}$$