11N501

Roll No. ____________

Total No. of Pages: 3

11N501

B. Tech. I - Sem. (New Scheme) (Main) Exam., May - 2023

All Branch

1FY1 – 01 Engineering Mathematics – I

Common to all Branches

Time: 3 Hours **Maximum Marks: 70**

Instructions to Candidates:

*Part – A: Short answer questions (up to 25 words) 10 × 2 marks = 20 marks. All **ten** questions are compulsory.*

*Part – B: Analytical/Problem solving questions 5 × 4 marks = 20 marks. Candidates have to answer **five** questions out of **seven**.*

*Part – C: Descriptive/Analytical/Problem Solving/Design questions 3 × 10 marks = 30 marks. Candidates have to answer **three** questions out of **five**.*

Schematic diagrams must be shown wherever necessary. Any data you feel missing may suitably be assumed and stated clearly. Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. NIL 2. NIL

PART – A

Q.1 What is the range of x for which the function - [2]

$y = x^4 - 6x^3 + 12x^2 + 5x + 7$

is concave upwards?

Q.2 Find the radius of curvature at the origin for the curve $x^3 + y^3 - x^2 + y = 0$ [2]

Q.3 Find the asymptotes of $y^2 (x - b) = x^3 + a^3$. [2]

Q.4 Provide order and classification of the partial differential equation - [2]

$x (y^2 + z^2 q^2) = z p y^2$.

Q.5 If $y_1 = x^2$ is one of the independent solutions of the differential equation - [2]

$$x^2 \frac{d^2y}{dx^2} - 2y = 0$$

Then, find the second linearly independent solution.

Q.6 Solve the initial value problem - [2]

$(1 + y) dx + (x + 2y) dy = 0, y (0) = 1$

Q.7 Solve $\frac{dy}{dx} = \text{Cos}(x + y) + \sin(x + y)$. [2]

Q.8 Write a short note on the symmetry of Cartesian curves. [2]

Q.9 Solve $(D^4 + a^4) y = 0, a > 0$. [2]

Q.10 If $u = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$, Show that [2]

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$$

PART – B

Q.1 If $\theta = t^n \exp\left(\frac{-r^2}{4t}\right)$, find the value of n for which the relation [4]

$$\frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}$$

is true.

Q.2 Trace the polar curve $r = 3+2 \cos \theta$. [4]

Q.3 Find the equation of the cubic which has the same asymptotes as the curve [4]

$x^3 - 6x^2y + 11 x y^2 - 6y^2 + x + y + 1 = 0$

and which touches the axis of y at origin and passes through the point (3, 2).

Q.4 If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $\left(x\frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}\right) = \frac{1}{2}\tan u$. [4]

Q.5 Find the solution of the differential equation - [4]

$$\frac{d^3y}{dx^3} - 9\frac{dy}{dx} = 10 \cos x.$$

Q.6 Solve the differential equation - [4]

$$(x+a)^2\frac{d^2y}{dx^2}-4\,(x+a)\frac{dy}{dx}+6y=x, a>0.$$

Q.7 Reduce the partial differential equation - [4]

$y + 2zq = q\,(4xp + yq)$

To Clairaut's form and hence solve it.

PART – C

Q.1 Find the minimum value of the function $x + y + z$ subject to the condition [10]

$\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1.$

Q.2 Solve the differential equation - [10]

$$\cos x\frac{d^2y}{dx^2}+\sin x\frac{dy}{dx}-2y\cos^3x=2\cos^3x.$$

Q.3 Show that the coordinates (α,β) of the center of curvature of the curve $27ay^2 = 4x^3$ at (x, y) are given by $3a\,(\alpha + x) + 2x^2 = 0$ and $\beta = 4y + \frac{9ay}{x}$. [10]

Q.4 Solve by method of variation of parameters - [10]

$$\frac{d^2y}{dx^2}+(1-\cot x)\frac{dy}{dx}-y\cot x=\sin^2 x.$$

Q.5 Write Charpit's equations. Find a general solution of $p^2 x + q^2 y = u$, $u =(x, y)$ using Charpit's equations. [10]
