Probabilistic Robotics Course

Localization with Kalman Filters [Example Application]

G. Grisetti, B. Della Corte

{grisetti,dellacorte}@dis.uniroma1.it

Dept of Computer Control and Management Engineering
Sapienza University of Rome

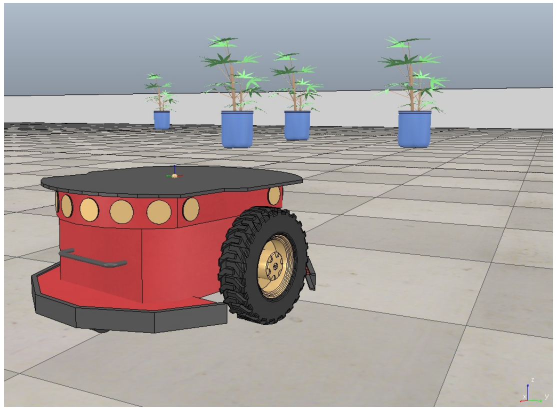
Outline

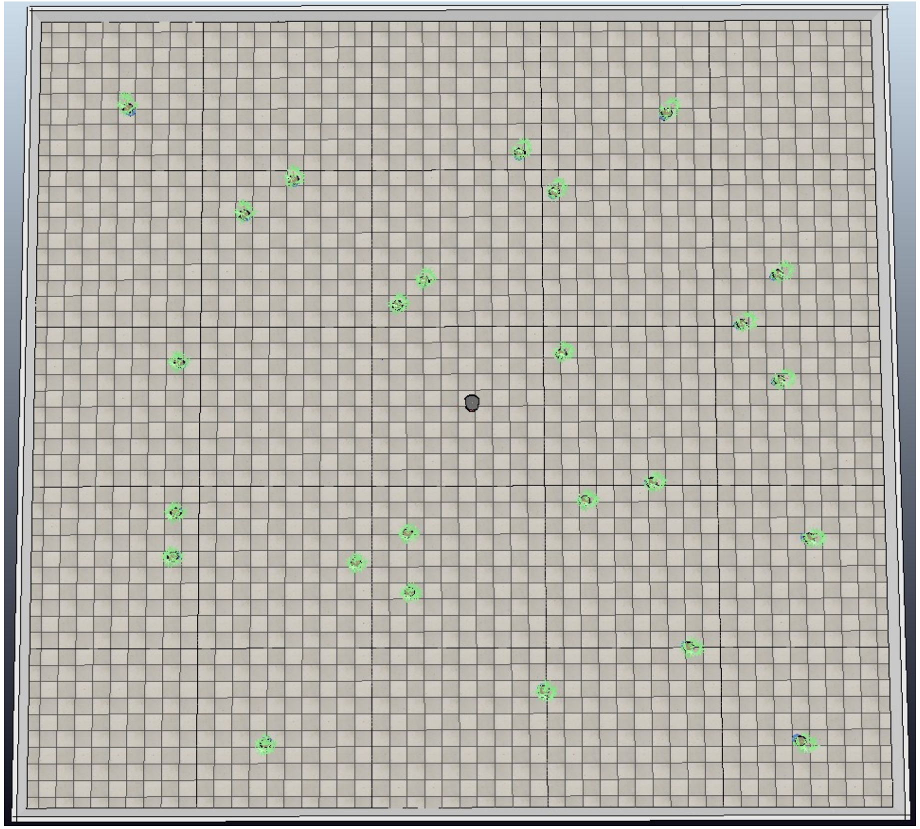
- Scenario
- Controls
- Observations
- Non-Linear Systems and Gaussian Noise
- Extended Kalman Filter
- Unscented Kalman Filter

Scenario

Orazio moves on a 2D plane

- Is controlled by translational and rotational velocities
- Senses a set of uniquely distinguishable landmarks through a “2D landmark sensors”
- The location of the landmarks in the world is known





Approaching the problem

We want to develop a KF based algorithm to track the position of Orazio as it moves

The inputs of our algorithms will be

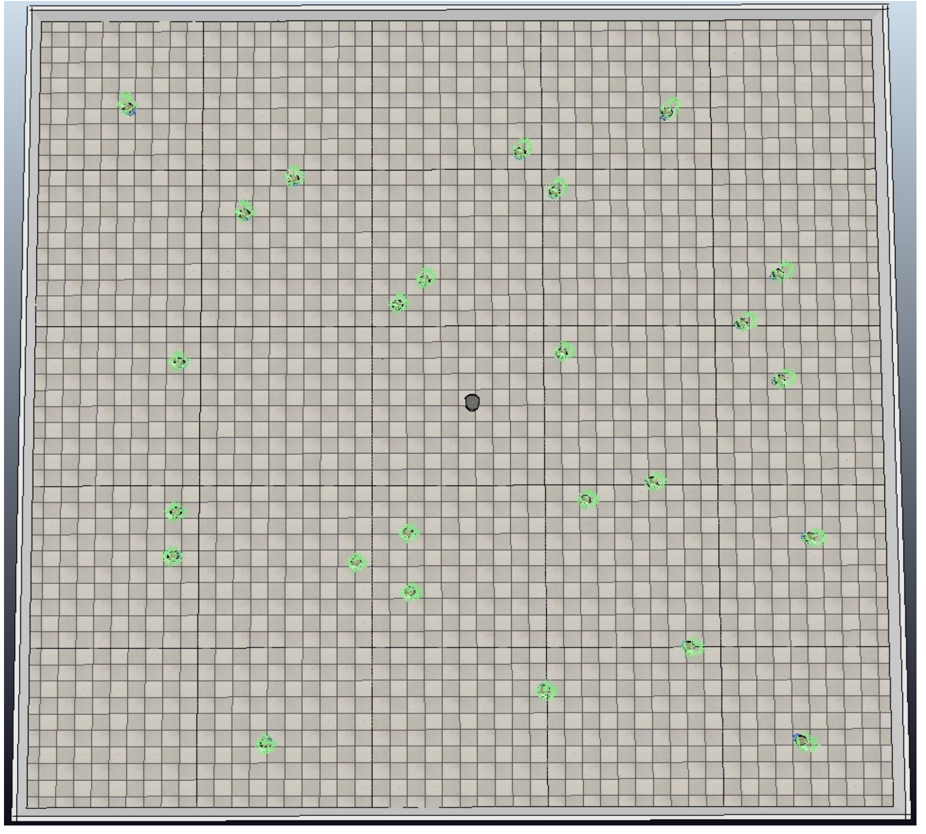
- velocity measurements
- landmark measurements

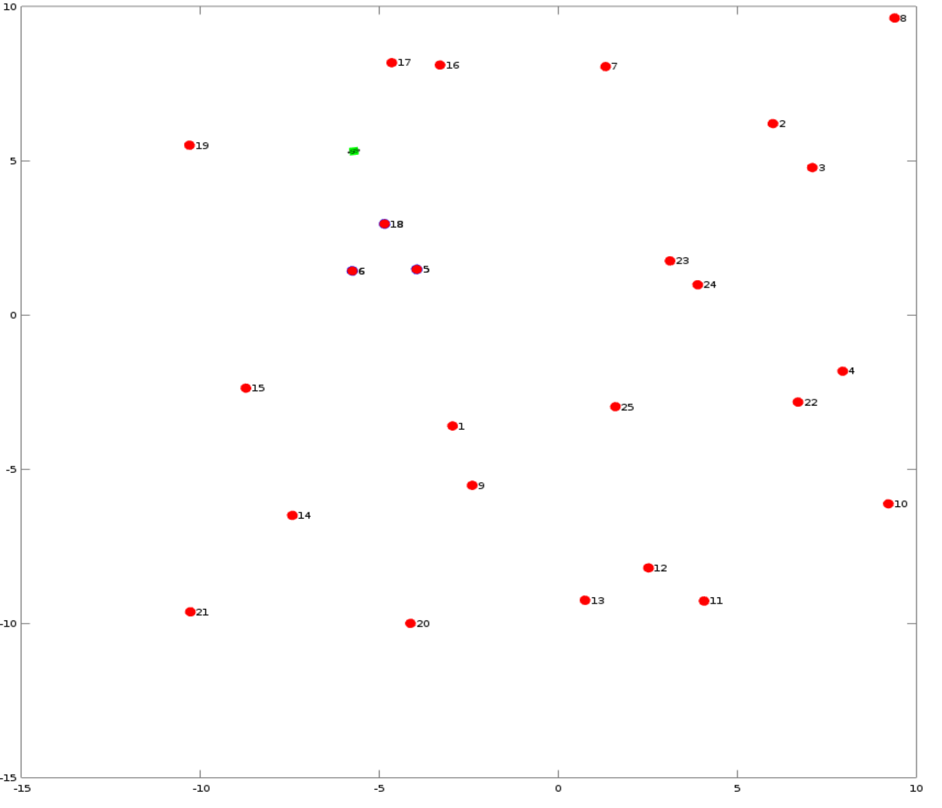
The prior knowledge about the map is represented by the location of each landmark in the world

Prior

The map is represented as a set of landmark coordinates

$$\mathbf{l}^{[i]} = \begin{pmatrix} x^{[i]} \\ y^{[i]} \end{pmatrix} \in \Re^2$$





Domains

Define

- state space

$$\mathbf{X}_t = [\mathbf{R}_t | \mathbf{t}_t] \in SE(2)$$

- space of controls (inputs)

$$\mathbf{u}_t \;=\; \begin{pmatrix} v_t \\ \omega_t \end{pmatrix} \in \Re^2$$

- space of observations (measurements)

$$\mathbf{z}_t^{[i]} \;=\; \begin{pmatrix} x_t^{[i]} \\ y_t^{[i]} \end{pmatrix} \in \Re^2$$

Domains

Find an Euclidean parameterization of non-Euclidean spaces

$$\mathbf{X}_t = [\mathbf{R}_t | \mathbf{t}_t] \in SE(2) \Rightarrow \mathbf{x}_t = \begin{pmatrix} x_t \\ y_t \\ \theta_t \end{pmatrix} \in \Re^3$$

- state space

poses are not Euclidean, we map them to 3D vectors

$$\mathbf{u}_t = \begin{pmatrix} v_t \\ \omega_t \end{pmatrix} \in \Re^2$$

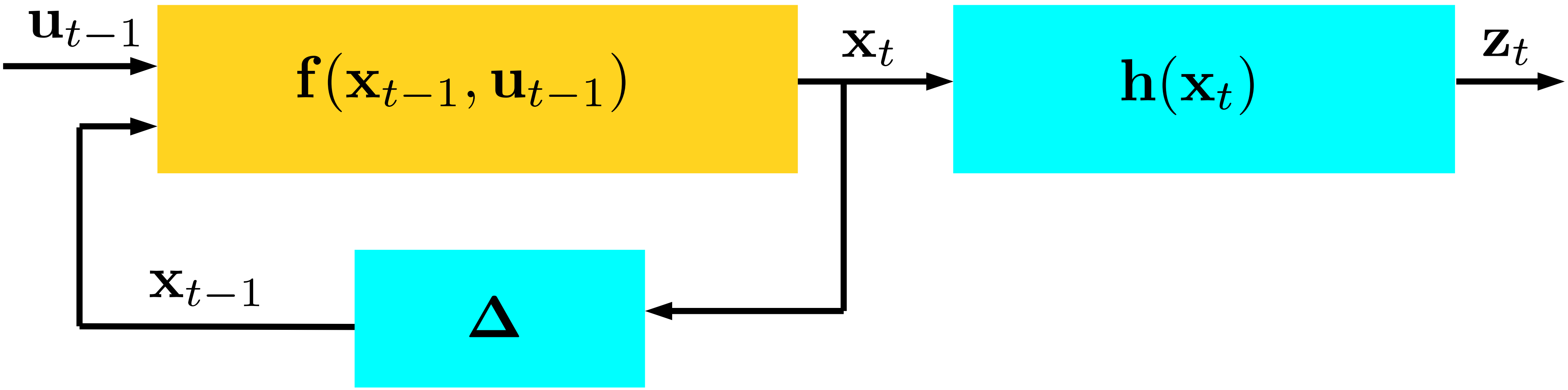
- space of controls (inputs)

measurement and control, in this problem are already Euclidean

$$\mathbf{z}_t = \begin{pmatrix} x_t^{[i]} \\ y_t^{[i]} \end{pmatrix} \in \Re^2$$

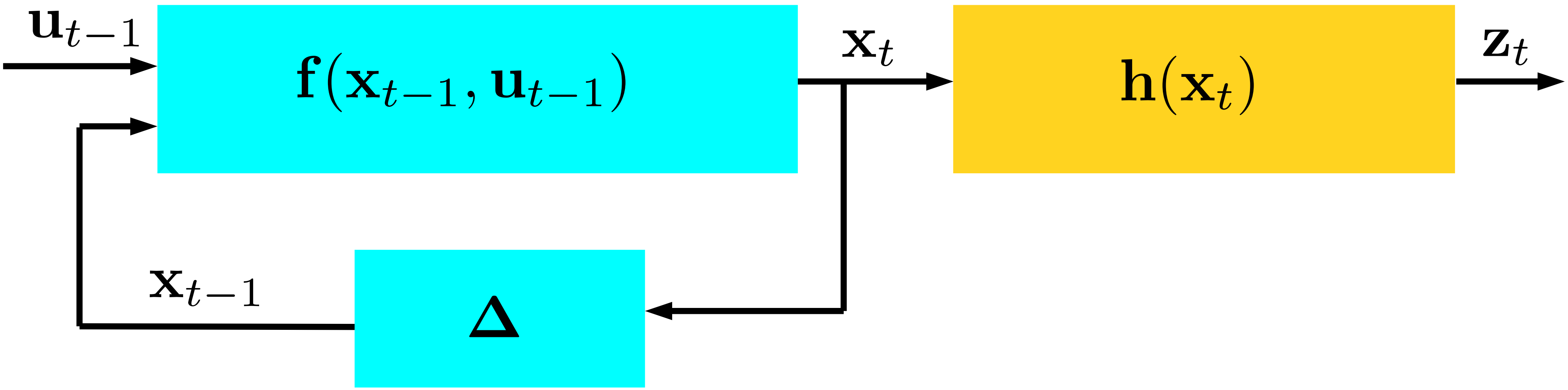
- space of observations (measurements)

Transition Function



$$\mathbf{x}_t = \mathbf{f}(\mathbf{x}_{t-1}, \mathbf{u}_{t-1}) = \begin{pmatrix} x_{t-1} + v_{t-1}\Delta t \cdot \cos(\theta_{t-1}) \\ y_{t-1} + v_{t-1}\Delta t \cdot \sin(\theta_{t-1}) \\ \theta_{t-1} + \Delta t \cdot \omega_{t-1} \end{pmatrix}$$

Measurement Function



We have [i] measurement functions, one per landmark

$$
\begin{aligned}
\mathbf{z}_t^{[i]} &= \mathbf{h}^{[i]}(\mathbf{x}_t) \\
&= \mathbf{R}_t^T(\mathbf{l}^{[i]} - \mathbf{t}_t) \\
&= \begin{pmatrix} \cos\theta_t(x^{[i]} - x_t) + \sin\theta_t(y^{[i]} - y_t) \\ -\sin\theta_t(x^{[i]} - x_t) + \cos\theta_t(y^{[i]} - y_t) \end{pmatrix}
\end{aligned}
$$

relative position of the l^{th} landmark w.r.t the robot at time t

$$
\mathbf{R}_t = \begin{pmatrix} \cos\theta_t & -\sin\theta_t \\ \sin\theta_t & \cos\theta_t \end{pmatrix}
$$

rotation matrix of theta

Control Noise

We assume the velocity measurements are effected by a Gaussian noise resulting from the sum of two aspects

- a constant noise
- a velocity dependent term whose standard deviation grows with the speed
- translational and rotational noise are assumed independent

$$\mathbf{n}_{u,t} \sim \mathcal{N}\left(\mathbf{n}_{u,t}; \mathbf{0}, \begin{pmatrix} v_t^2 + \sigma_v^2 & 0 \\ 0 & \omega_t^2 + \sigma_\omega^2 \end{pmatrix}\right)$$

Measurement Noise

We assume it is zero mean, constant

$$\mathbf{n}_z \sim \mathcal{N}\left(\mathbf{n}_z; \mathbf{0}, \left(\begin{array}{cc} \sigma_z^2 & 0 \\ 0 & \sigma_z^2 \end{array}\right)\right)$$

Jacobians!

At each time step our system will need to compute the derivatives of transition and measurement functions

$$\mathbf{f}(\mathbf{x}, \mathbf{u}) = \begin{pmatrix} x + v\Delta t \cdot \cos(\theta) \\ y + v\Delta t \cdot \sin(\theta) \\ \theta + \Delta t \cdot \omega \end{pmatrix}$$

$$\mathbf{A}_t = \frac{\partial \mathbf{f}(\cdot)}{\partial \mathbf{x}} = \begin{pmatrix} 1 & 0 & -v\Delta t \cdot \sin(\theta) \\ 0 & 1 & +v\Delta t \cdot \cos(\theta) \\ 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{B}_t = \frac{\partial \mathbf{f}(\cdot)}{\partial \mathbf{u}} = \begin{pmatrix} \Delta t \cdot \cos(\theta_{t-1}) & 0 \\ \Delta t \cdot \sin(\theta_{t-1}) & 0 \\ 0 & \Delta t \end{pmatrix}$$

Jacobians (cont)

We will have *n* measurement functions, one for each landmark

$$\mathbf{h}^{[i]}(\mathbf{x}_t) \quad = \quad \mathbf{R}_t^T(\mathbf{l}^{[i]} - \mathbf{t}_t)$$

this is a column vector!!!

$$\mathbf{C}_t^{[i]} = \frac{\partial \mathbf{h}^{[i]}(\cdot)}{\partial \mathbf{x}} = \begin{pmatrix} -\mathbf{R}_t^T & \frac{\partial \mathbf{R}_t^T}{\partial \theta_t}(\mathbf{l}^{[i]} - \mathbf{t}_t) \end{pmatrix}$$

derivative of rotation matrix w.r.t. theta

$$\frac{\partial \mathbf{R}_t}{\partial \theta_t} = \begin{pmatrix} -\sin\theta_t & -\cos\theta_t \\ \cos\theta_t & -\sin\theta_t \end{pmatrix}$$

Hands on!

g2o Wrapper

Load your Vrep acquired dataset

```
[land, pose, transition, obs] = loadG2o('my_dataset.g2o');
```

It returns 4 Struct-Array(Landmark, Poses, Transitions, Observations), *i.e.* :

```
land =

  1x25 struct array containing the fields:

    id
    x_pose
    y_pose
```

```
pose =

  1x137 struct array containing the fields:

    id
    x
    y
    theta
```

```
transition =

  1x136 struct array containing the
fields:

    id_from
    id_to
    v
```

```
obs =

  1x136 struct array containing the fields:

    pose_id
    observation
```

EKF Localization

```
% load your own dataset dataset
[landmarks, poses, transitions, observations] = loadG2o('dataset.
    g2o');
mu = rand(3,1)*20-10; % init mean
mu(3) = normalizeAngle(mu(3));

sigma = eye(3)*0.001; % init covariance

%simulation cycle
for i=1:length(transitions)
    % predict with transitions
    [mu, sigma] = ekf_prediction(mu, sigma, transitions(i));
    % correct with observations
    [mu, sigma] = ekf_correction(mu, sigma, landmarks,
   observations(i));

    plot_state(landmarks, mu, sigma, observations(i));
endfor
```

EKF Localization

```
% load your own dataset dataset
[landmarks, poses, transitions, observations] = loadG2o('dataset.g2o');
mu = rand(3,1)*20-10; % init mean
mu(3) = normalizeAngle(mu(3));

sigma = eye(3)*0.001; % init covariance

%simulation cycle
for i=1:length(transitions)
    % predict with transitions TODO
    [mu, sigma] = ekf_prediction(mu, sigma, transitions(i));
    % correct with observations TODO
    [mu, sigma] = ekf_correction(mu, sigma, landmarks, observations(i));

    plot_state(landmarks, mu, sigma, observations(i));
endfor
```