Nuclear Physics B228 (1983) 205-215

ON THE DETERMINATION OF $\sin^2\theta_w$ IN SEMILEPTONIC NEUTRINO INTERACTIONS

C.H. LLEWELLYN SMITH

Department of Theoretical Physics, University of Oxford, 1 Keble Road, Oxford OX1 3NP, UK

Received 2 February 1983
(Revised 4 July 1983)

It is shown that if $\sin^2\theta_w$ is measured in semileptonic neutrino interactions then, contrary to a claim in the literature, the error due to unknown dynamical (higher-twist) corrections to the QCD parton model is small provided an isoscalar target is used. The largest contributions to σ_{nc} and σ_{cc} are related by isospin invariance alone. Neglecting heavy quarks and Kobayashi-Maskawa (KM) mixing, the parton model is only needed for very small terms and introduces an uncertainty in $\sin^2\theta_w$ which is probably less than 1%. There is a much larger theoretical error due to uncertainties in the element U_{cs} of the KM matrix and in the strange quark distribution. With the full range $0.80 \leqslant |U_{cs}| \leqslant 0.98$ which is allowed phenomenologically, these uncertainties give $\delta \sin^2\theta_w = \pm 0.008$. There is also an error of ± 0.004 due to uncertainties in $|U_{dc}|$ and $|U_{du}|$.

1. Introduction

Measurements of neutral current couplings are becoming sensitive to second-order electro-weak effects. This will make it possible to distinguish $SU(2)\times U(1)$ from other theories with similar leading order low-energy behaviour (e.g. non-gauge theories which are described by Bjorken's formalism [1] or supersymmetric $SU(2)\times U(1)$, in which the plethora of new particles changes the radiative corrections [2]) and determine whether it really is a renormalizable theory.

Recently it has been claimed [3] that measurements of $\sin^2\theta_w$ in semileptonic neutrino interactions suffer from essentially irreducible theoretical errors, due to higher-twist corrections to the QCD parton model, possibly as large as 10%, which would make more accurate measurements pointless. The main purpose of this paper is to point out that this conclusion is not correct if an isoscalar target is used. For $I=0$ targets the largest contributions to σ_{nc} and σ_{cc} are related by isospin invariance alone. The QCD parton model is only needed for relatively small terms and dynamical uncertainties introduce an error which is probably less than 1%.

There is a much larger theoretical uncertainty due to ignorance of the strange quark distribution and of the element U_{cs} of the Kobayashi-Maskawa matrix U_{ij}. Using the limits [4] $0.80 < |U_{cs}| < 0.98$, uncertainties in strange quark contributions give an error $\delta \sin^2\theta_w = \pm 0.008$. However, if the common belief that U_{ij} is almost diagonal is accepted, the error is reduced substantially. If, for example, we believe that

$$|U_{cb}|^2 + \sum_i |U_{ci}|^2 \leqslant (0.22)^2,$$

where i labels undiscovered quarks and $0.22 = \sin\theta_C$ is taken as a typical off-diagonal element, then unitarity gives $0.95 \leqslant |U_{cs}| \leqslant 0.98$, using [4] $0.20 \leqslant |U_{cd}| \leqslant 0.24$. With these bounds on $|U_{cs}|$, the error in $\sin^2\theta_w$ from strange quark contributions would only be ± 0.002. Other theoretical errors from the interpretation of the data are ± 0.004 or less. Thus, if it is accepted or could be shown that $|U_{cs}|$ is close to one, there would be no obstacle in principle or in practice [5] to measuring $\sin^2\theta_w$ to ± 0.005 in semileptonic neutrino interactions*.

This paper is organised as follows. In sect. 2 some results of calculations of radiative corrections are used to establish criteria for the accuracy to which it is desirable to measure $\sin^2\theta_w$. In sect. 3 formulae for the determination of $\sin^2\theta_w$ are derived for an ideal world with just $u, d, \bar{u}, \bar{d}$ quarks and zero Cabibbo angle, and the influence of corrections to the QCD parton model is analysed. In sect. 4 the effects of heavy quarks and Kobayashi-Maskawa mixing are discussed. Conclusions are summarised in sect. 5.

2. Criteria for accuracy

If $SU(2) \times U(1)$ is correct M_W, M_Z and all neutral current measurements (in $\nu, \bar{\nu}$, ed, μN, e^+e^-) must be described by a single parameter $\sin^2\theta_w$, in addition to α and G_F, to all orders in the couplings. Four examples indicate the accuracy needed to test second order electro-weak effects.

(i) An average of values of $\sin^2\theta_w$ deduced from

$$R_\nu = \frac{\sigma(\nu N \to \nu X)}{\sigma(\nu N \to \mu X)},$$

using the Born approximation for the electroweak interactions is 0.227 ± 0.015. Applying first-order corrections (W, Z propagator corrections, vertex corrections, wave-function renormalizations, WW, ZZ, Wγ, Zγ exchange, real bremsstrahlung) this becomes** $\sin^2\theta_{\overline{ms}}(M_W) = 0.215 \pm 0.015$ $\overline{ms}$ indicates that $\sin^2\theta$ is defined in the modified minimal subtraction scheme and a scale $\mu = M_W$ has been used; the

* See note added in proof.

** The radiative corrections quoted here and below are taken from ref. [6] (the corrections to $\sin^2\theta$ measured using both R_ν and $R_{\bar{\nu}}$ and the prediction for ρ^2 are derived from this reference where they are implicit); they are based on the assumption that there are three quark and lepton doublets with $M_t < M_W$ and $M_{Higgs} < 1$ TeV. Complete calculations of radiative corrections to R_ν, which agree with those in ref. [6], are also given in ref. [7]. For a review with references to other calculations see ref. [8]. The importance of testing $SU(2) \times U(1)$ to second order has been stressed particularly by Veltman.

numerical value of $\sin^2\theta$ depends on the scheme but physical predictions, e.g. for M_Z in terms of R_ν, only depend on the scheme used in the calculation to order beyond the one considered. The SLAC ed experiment [9], as interpreted by Kim et al. [10], gives $\sin^2\theta = 0.223 \pm 0.015$ which corresponds to $\sin^2\theta_{\overline{\mathrm{ms}}}(M_W) = 0.215 \pm 0.015$ when electroweak radiative corrections are applied. The fact that this agrees with the value from R_ν is a non-trivial test of the theory; the radiative corrections might have moved the values apart! However, for a real test we would like the experimental errors to be a factor of two or more smaller than the radiative corrections, say, $\delta \sin^2\theta_w = \pm 0.005$ or less in the case of R_ν.

(ii) R_ν is sensitive to the corrections to the W propagator whereas

$$R_{\nu e} = \frac{\sigma(\nu_\mu e \to \nu_\mu e)}{\sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e)} ,$$

is not. These corrections are particularly sensitive to large-mass splittings in SU(2) doublets [11,12] e.g. $M_t - M_b$ if $M_t > M_W$ or $M_L - M_{\nu_L}$ for a new lepton doublet with $M_L > M_W \gg M_{\nu_L}$. Thus, for example, $\sin^2\theta$ obtained from R_ν assuming $M_t < M_W$ in making radiative corrections would turn out to be larger than the value deduced from $R_{\nu e}$ by $2 \times 10^{-3}(M_t/M_W)^2$ if in fact M_t is very large. This example demonstrates explicitly the fact that different experiments are sensitive to different radiative corrections and provide complementary tests of $SU(2) \times U(1)$. It also shows that measurements of R_ν giving $\delta \sin^2\theta_w = \pm 0.005$, the accuracy apparently attainable [13] using $R_{\nu e}$, are very desirable.

(iii) Using the lowest-order theory, the value of R_ν gives $M_Z = 89.0^{+2.2}_{-2.0}$ GeV, $M_W = 78.2^{+2.7}_{-2.5}$ GeV. Including the electroweak radiative corrections and using the second-order mass formula these predictions become $M_Z = 93.8^{+2.5}_{-2.2}$ GeV, $M_W = 83.1^{+3.1}_{-2.8}$ GeV. An improvement of a factor of three or more in the errors would lead to a really clear distinction between the lowest order and the corrected predictions.

(iv) Suppose that R_ν and $R_{\bar{\nu}}$ are fitted to two parameters: $\sin^2\theta_{\mathrm{expt}}$ and ρ^2 (the usual NC/CC strength parameter) in the Born approximation for electroweak interactions. Radiative corrections give $\sin^2\theta_{\overline{\mathrm{ms}}}(M_W) = \sin^2\theta_{\mathrm{expt}} - 0.004$ in this case, independent of M_t as $\sin^2\theta$ is determined by $R_\nu/R_{\bar{\nu}}$. $SU(2) \times U(1)$ predicts $1 - 0.016 + 4 \times 10^{-3}(M_t/M_W)^2$ for ρ^2 defined in this way. This suggests that measurements of ρ^2 to 1% or better are very desirable, although very difficult.

We conclude that really significant tests of $SU(2) \times U(1)$ are possible if the experimental errors and theoretical uncertainties in measuring $\sin^2\theta$ can be reduced to ± 0.005 or less.

3. A formula for θ_w

Consider a world with only u, d, ū and d̄ quarks and zero Cabibbo angle. Cross sections appropriate to this hypothetical world will be denoted $\hat{\sigma}$. In sect. 4 we consider how $\hat{\sigma}$'s can be constructed from the real measured σ's. In $SU(2)\times U(1)$ the neutral current has the structure $I_3\gamma_\mu^L - zQ(\gamma_\mu^L + \gamma_\mu^R)$ where $z = \sin^2\theta_w$, $\gamma_\mu^{L,R} = \gamma_\mu(1 \mp \gamma_5)$. In terms of the strong isospin operator I_3 and hypercharge Y, this current can be decomposed into vector and axial parts thus

$$V = I_3(1-2z) - zY, \qquad A = -I_3,$$

using $Q = I_3 + \frac{1}{2}Y$. If we sum over all final states, there is no interference between the isovector (I_3) and isoscalar (Y) currents for an $I=0$ target. The cross section for neutrinos (antineutrinos) is therefore symbolically proportional to:

$$\tfrac{1}{2}\left[1+(1-2z)^2\right][V\cdot V + A\cdot A](I_3)^2 \underset{[+]}{-} (1-2z)2V\cdot A(I_3)^2$$
$$+\tfrac{1}{2}\left[(1-2z)^2-1\right][V\cdot V - A\cdot A](I_3)^2 + z^2Y^2V\cdot V. \tag{1}$$

The first two terms are proportional to the sum and difference respectively of charged current neutrino and antineutrino cross sections by isospin invariance alone *without any dynamical assumptions*. To deal with the third term we write

$$[V\cdot V - A\cdot A] = \tfrac{1}{2}\hat{\varepsilon}(x,Q^2)[V\cdot V + A\cdot A]. \tag{2}$$

If the constituent quark mass were zero then $\hat{\varepsilon}$ would vanish as there would be manifest γ_5 invariance. Thus $\hat{\varepsilon}$ behaves as m_q^2/Q^2 for large Q^2. To deal with the last term we use the QCD parton model, multiplying the result by $1+\varepsilon(x,Q^2)$ to indicate the uncertainties thus:

$$z^2Y^2V\cdot V = \tfrac{2}{9}z^2(I_3)^2[V\cdot V + A\cdot A](1+\varepsilon(x,Q^2)). \tag{3}$$

ε is a typical higher-twist term which behaves as $\langle p_T^2\rangle/Q^2$ for large Q^2. Combining eqs. (1)–(3) we obtain for an $I=0$ target:

$$\frac{d^2\hat{\sigma}_{nc}^{\nu(\bar\nu)}}{dx\,dy} = \left(\tfrac{1}{2} - z + \tfrac{5}{9}z^2\right)\frac{d^2\hat{\sigma}_{cc}^{\nu(\bar\nu)}}{dx\,dy} + \tfrac{5}{9}z^2\frac{d^2\hat{\sigma}_{cc}^{\bar\nu(\nu)}}{dx\,dy}$$
$$+\left(\tfrac{1}{18}\varepsilon z^2 + \tfrac{1}{4}[z^2 - z]\hat{\varepsilon}\right)\left(\frac{d^2\hat{\sigma}_{cc}^{\nu}}{dx\,dy} + \frac{d^2\hat{\sigma}_{cc}^{\bar\nu}}{dx\,dy}\right). \tag{4}$$

Note that for $z\to 0$ this formula reduces to $2d^2\hat{\sigma}_{nc}^{\nu(\bar\nu)} = d^2\hat{\sigma}_{cc}^{\nu(\bar\nu)}$, a result which follows immediately from the form of the weak currents and isospin invariance, and that ε and $\hat{\varepsilon}$ drop out of $d^2\sigma_{nc}^{\nu} - d^2\sigma_{nc}^{\bar\nu}$, for which eq. (4) becomes the Paschos-

Wolfenstein relation [14]. The formulae in ref. [3] do not respect either of these properties*.

Eq. (4) for σ_{nc} in terms of $\sigma_{cc}^{\nu,\bar{\nu}}$ would give $\sin^2\theta$ with a theoretical error

$$\delta z = -0.006\langle\varepsilon\rangle + 0.10\langle\hat{\varepsilon}\rangle\,, \tag{5}$$

for $\hat{\sigma}_{cc}^{\nu}/\hat{\sigma}_{cc}^{\bar{\nu}} = 2$, $z = 0.22$. At large Q^2, ε behaves as μ^2/Q^2, where $\mu^2 = \langle p_T^2\rangle \simeq 0.2$ GeV^2 might be a reasonable value. In typical SPS experiments $\langle Q^2\rangle \simeq 20$ GeV^2 and very roughly $\langle 1/Q^2\rangle = \frac{1}{10}$ GeV^{-2}. Thus even in the very extreme case of $\mu^2 = 1$ GeV^2 the error due to ε is of order 0.0006 or 0.3%. The second term behaves as m_q^2/Q^w which is of order 0.01 for $m_q = 350$ MeV and gives an error of order 0.001 or 0.5%. There could of course be a large numerical coefficient multiplying m_q^2 but since, as we shall see, the corrections involved in constructing $\hat{\sigma}$ from σ introduce errors of order 0.005 it seems that higher-twist corrections are not a serious problem. The errors due to ε and $\hat{\varepsilon}$ could be reduced to negligible levels by even a crude cut to reduce the contribution of low-Q^2 events, using only hadronic information to estimate Q^2 so that the cut is identical for $\mathrm{d}\sigma_{cc}$ and $\mathrm{d}\sigma_{nc}$.

The $\varepsilon, \hat{\varepsilon}$ contributions to eq. (4) can be cast in another form with a smaller coefficient by using electromagnetic structure functions. Writing the last two terms in eq. (1) as

$$2(z-z^2)(V\cdot V + A\cdot A)I_3^2 - 4(z-z^2)V\cdot VI_3^2 + z^2V\cdot VY^2\,,$$

the first piece is given by $\mathrm{d}\hat{\sigma}_{cc}^{\nu} + \mathrm{d}\hat{\sigma}_{cc}^{\bar{\nu}}$. The other terms can be evaluated using the quark parton model modified by an uncertainty factor $\tilde{\varepsilon}$ to write

$$V\cdot VY^2 = \tfrac{4}{9}V\cdot VI_3^2(1+\tilde{\varepsilon})\,,$$

giving

$$V\cdot VI_3^2 \simeq \tfrac{9}{10}f^{\text{e.m.}}\left(1 - \tfrac{1}{10}\tilde{\varepsilon}\right),$$

$$V\cdot VY^2 \simeq \tfrac{2}{5}f^{\text{e.m.}}\left(1 + \tfrac{9}{10}\tilde{\varepsilon}\right), \tag{6}$$

where

$$f^{\text{e.m.}} = V\cdot V\left(I_3^2 + \left(\tfrac{1}{2}Y\right)^2\right),$$

is measured in electromagnetic scattering on isoscalar targets. Using these relation-

* In ref. [3] the coherence of u- and d-quarks in an $I = 0$ target was ignored; the resulting erroneous formulae include a higher-twist correction to σ_{nc}/σ_{cc} which is not multiplied by z and would therefore give a large error in z. Properly evaluated, the diagrams considered in ref. [3] are of course consistent with eq. (4) and contribute to ε. Note that for a target with $I \neq 0$ we cannot relate σ_{nc} to σ_{cc} without the parton model even for $z = 0$ so higher-twist corrections may indeed be large.

ships we find

$$\frac{d^2\hat{\sigma}_{nc}^{\nu(\bar{\nu})}}{dx\,dy} = \left(\tfrac{1}{2} - \tfrac{1}{2}z\right)\frac{d^2\sigma_{cc}^{\nu(\bar{\nu})}}{dx\,dy} + \tfrac{1}{2}z\frac{d^2\sigma_{cc}^{\bar{\nu}(\nu)}}{dx\,dy}$$

$$+\frac{G^2ME_\nu}{\pi}\left[\left(1 - y - \frac{Myx}{2E_\nu}\right)F_2^{\text{e.m.}} + y^2x^2F_1^{\text{e.m.}}\right]\left[\tfrac{2}{5}(10z^2 - 9z) + \tfrac{9}{25}z\tilde{\varepsilon}\right]. \tag{7}$$

Asymptotically $\tilde{\varepsilon}, \varepsilon, \hat{\varepsilon} \to 0$ and we can use the relationship [15] $F_{1,2}^{\text{e.m.}} = \frac{5}{18}F_{1,2}^{\nu,\bar{\nu}}$ to check that eqs. (7) and (4) are equivalent. If eq. (7) integrated to the total cross section is used to obtain z, the error due to ignorance of $\tilde{\varepsilon}$ is $\delta z = 0.02\tilde{\varepsilon}$ which, conservatively, is less than 0.002.

In practice the combination of data from different experiments needed to use eq. (7) would presumably introduce unacceptable systematic errors. However, using the fact that to first order in ε (in which incidentally $\tilde{\varepsilon} = \varepsilon - \frac{1}{2}\hat{\varepsilon}$)

$$\frac{F_2^{\text{e.m.}}}{F_2^{\nu}} = \tfrac{5}{18} + \tfrac{1}{4}\left(\tfrac{1}{2}\hat{\varepsilon} + \tfrac{1}{9}\varepsilon\right), \tag{8}$$

we see that accurate tests of the $\frac{5}{18}$ relation can provide a partial check on the corrections in eq. (4), although logically it is possible that $\frac{1}{2}\hat{\varepsilon} - \frac{1}{9}(0.28\varepsilon)$, the combination which enters eq. (4), is large even if $\frac{1}{2}\hat{\varepsilon} + \frac{1}{9}\varepsilon = 0$.

4. Heavy quark contributions and mixing angles

To construct the $\hat{\sigma}$'s from measured cross sections we must remove the heavy-quark contributions and correct for the fact that Kobayashi-Maskawa matrix U_{ij} is not diagonal. For the present purpose of estimating the error in $\sigma - \hat{\sigma}$ (although not for calculating the absolute value in practice) we can safely ignore scaling violations in light-quark distributions and treat threshold factors in an approximate fashion. For charged currents then

$$\hat{\sigma}_{cc}^{\nu} \propto u + d + \tfrac{1}{3}(\bar{u} + \bar{d}), \tag{9a}$$

$$\sigma_{cc}^{\nu} - \hat{\sigma}_{cc}^{\nu} \propto \left(u + d + \tfrac{1}{3}(\bar{u} + \bar{d})\right)\left(|U_{ud}|^2 - 1\right) + (u + d)\xi_{dc}|U_{cd}|^2$$
$$+ 2s\left(|U_{us}|^2 + \xi_{sc}|U_{cs}|^2\right) + \tfrac{1}{3}(\bar{u} + \bar{d})|U_{us}|^2 + \tfrac{2}{3}\bar{c}\left(|U_{dc}|^2 + |U_{sc}|^2\right) \tag{9b}$$

$$\hat{\sigma}_{cc}^{\bar{\nu}} \propto \bar{u} + \bar{d} + \tfrac{1}{3}(u + d), \tag{10a}$$

$$\sigma^{\bar\nu}_{cc} - \hat\sigma^{\bar\nu}_{cc} \propto \left(\bar u + \bar d + \tfrac{1}{3}(u+d)\right)\left(|U_{ud}|^2 - 1\right) + \tfrac{1}{3}(u+d)|U_{us}|^2$$
$$+ (\bar d + \bar u)\xi_{\bar d \bar c}|U_{dc}|^2 + 2\bar s\left(|U_{us}|^2 + \xi_{\bar s \bar c}|U_{sc}|^2\right) + \tfrac{2}{3}c\left(|U_{dc}|^2 + |U_{sc}|^2\right), \tag{10b}$$

where

(i) for light quarks (u, d, s), $u \equiv \int xu(x, \langle Q^2\rangle)\,dx$ etc;

(ii) the fact that $u = d$, $\bar u = \bar d$ for an $I = 0$ target has been used;

(iii) the ξ factors represent threshold suppressions; we shall use $\xi_{dc} = \xi_{\bar d \bar c} = 0.84$ and $\xi_{sc} = \xi_{\bar s \bar c} = 0.65$ calculated by CDHS for the average of events with $E_\nu > 35$ GeV in their experiment [16];

(iv) the contributions from scattering on charmed quarks should be calculated using a fusion model as discussed below.

For neutral currents

$$\hat\sigma^{\nu}_{nc} \propto 0.158(u+d) + 0.065(\bar u + \bar d), \tag{11a}$$

$$\sigma^{\nu}_{nc} - \hat\sigma^{\nu}_{nc} \propto 0.196c + 0.250s, \tag{11b}$$

using $u = d$, $\bar u = \bar d$ where

(i) the normalization is such that a u-quark contributes

$$\left(\tfrac{1}{2} - \tfrac{2}{3}z\right)^2 + \tfrac{1}{3}\left(\tfrac{2}{3}z\right)^2;$$

(ii) it has been assumed that $s = \bar s$, $c = \bar c$; the error due to failure of this assumption for s-quarks, which is not necessarily negligible, is discussed below.

We will suppose that R_ν is used to measure $\sin^2\theta$. Using eq. (4) the contribution to R_ν proportional to $\hat\sigma^{\bar\nu}_{cc}/\hat\sigma^{\nu}_{cc}$ is very small and the error in z is approximately $\delta z = \tfrac{1}{2}\delta R/R$ for $z \simeq 0.22$. The only errors from constructing $\hat\sigma$ which are potentially contributions to σ_{cc} we use

$$|U_{cd}|^2 + |U_{cs}|^2 = 0.84 \pm 0.16\,.$$

The error in this quantity, combined with the 20% error which we attribute to the use of the fusion model, gives $\delta z \gtrsim 0.001$ for $c/(u+d) \gtrsim 0.01$, where the bound is no longer so conservative as $Z + g \to c + \bar{c}$ is suppressed by phase space relative to $W^+ + g \to c + \bar{s}(\bar{d})$.

The treatment of strange quark contributions is more problematic. The only information available about $s(x, Q^2)$ is derived from the contribution of $\nu s \to \mu^- c$, $c \to \mu^+ x$ to dimuon production. Combined with the average branching ratio [16] $B = (7.1 \pm 1.3)\%$ for the mixture of charmed particles produced, this gives the charm contribution $\sim s|U_{sc}|^2$ to σ^{ν}_{cc}. However, σ^{ν}_{nc} depends on $s + \bar{s}$ alone, which cannot be determined without independent knowledge of $|U_{sc}|$ (statements about the magnitude of $s + \bar{s}$ in the literature almost all involve explicit or implicit assumptions about U_{ij}).

The relevant data are

(i) $(|U_{cs}|^2/|U_{cd}|^2)2s/(u+d) = 1.19 \pm 0.09$ which is determined by using the x distribution of dimuon production to separate $d \to c$ from $s \to c$ and is independent of B [16];

(ii) $|U_{cd}| \leqslant 0.24$ from unitarity [4]; $|U_{cd}| > 0.20$ from the rate of dimuon production and B [16];

(iii) $|U_{cs}|^2 \leqslant 1 - |U_{cd}|^2$ from unitarity; $|U_{cs}| > 0.8$ from $\Gamma(D^+ \to \bar{K}^0 e^+ \nu_e)$ with a (not necessarily reliable) model for the form factors [4].

Combining this information we find

$$\frac{2s}{u+d} = 0.080 \pm 0.035\,.$$

In passing we note that using the additional information [18] that $\bar{\alpha} = 0.16 \pm 0.01$, where $\bar{\alpha}$ is the parameter that characterizes the shape of $d\sigma^{\bar{\nu}}_{cc}/dy$, we find

$$\frac{\bar{u} + \bar{d}}{u+d} = 0.15 \pm 0.03\,.$$

If we treated the s contributions (σ^s) to σ^{ν}_{cc} and σ^{ν}_{nc} separately, these data would give

$$\frac{\sigma^s_{cc}}{\hat{\sigma}^{\nu}_{cc}} \simeq \frac{2s}{u+d}\left(|U_{us}|^2 + 0.65|U_{cs}|^2\right)$$

$$= 0.043 \pm 0.012 \to \delta z = \pm 0.006,$$

$$\frac{\sigma^s_{nc}}{\hat{\sigma}^{\nu}_{nc}} \simeq 0.79\frac{2s}{u+d}$$

$$= 0.063 \pm 0.028 \to \delta z = \pm 0.014\,.$$

However, the errors are correlated and for $R^s = \partial R_\nu/\partial s|_{s=0}s$ we have

$$\frac{R^s}{R_\nu} \simeq \frac{2s}{u+d}|U_{cs}|^2\left(\frac{0.74}{|U_{cs}|^2} - 0.65\right)$$

$$= 0.021 \pm 0.016 \rightarrow \delta z = \pm 0.008, \tag{12}$$

which is still distressingly large. Furthermore note that if $(s-\bar{s})/2s = \eta \neq 0$, an analysis based on the assumption $\eta = 0$ would be in error by

$$\delta z = \frac{1}{2}\frac{2s}{u+d}0.79\eta \simeq 0.03\eta;$$

i.e. $\eta = 0.17$, corresponding to $\bar{s}/s = 0.66$, would give $\delta z = 0.005$. Although such a large $\bar{s}/s$ asymmetry would be unexpected* it cannot be excluded a priori and would be very hard to detect experimentally.

There are two ways in which this uncertainty could be reduced:

(i) By making a cut against small x thereby removing all sea contributions, in which case the only error from U_{ij} would be $\delta z = \pm 0.004$ from ignorance of $|U_{ud}|^2 + |U_{cd}|^2$. However, this is very difficult in practice as the errors in measuring x in NC reactions are uncontrollable for $x \rightarrow 0$. Alternatively we could measure z by using the Paschos-Wolfenstein quantity [13]:

$$\Delta = \frac{\sigma^{\nu}_{nc} - \sigma^{\bar{\nu}}_{nc}}{\sigma^{\nu}_{cc} - \sigma^{\bar{\nu}}_{cc}},$$

in which sea quarks cancel and the error from $|U_{ud}|^2 + |U_{cd}|^2$ is $\delta z = \pm 0.003$ (for a discussion of uncertainties in the analysis of Δ see ref. [19]). However, Δ is subject to systematic errors which are hard to control to this level of accuracy.

(ii) By improving our knowledge of U_{ij} and of $s(x, Q^2)$ and $\bar{s}(x, Q^2)$. Improved dimuon data plus, more importantly, better information about the average branching ratio B, or direct measurement of charm production in a holographic bubble chamber, would help. However, what is really needed is more information about U_{sc} or another way to measure s and $\bar{s}$. Measurement of $|U_{sc}|$ in charm decay is problematic as it must rely on a model and it seems unlikely that the lower bound we have used can be improved credibly. If $|U_{bc}|$ is found to be large, the upper bound on $|U_{sc}|$ would be improved while a small value of $|U_{bc}|$ would enhance the belief that $|U_{ic}|$ is small for further quarks and therefore $|U_{sc}|$ is close to one. As discussed in the introduction, where eq. (12) and the measured value of $2s|U_{cs}|^2([u+d]|U_{cd}|^2)^{-1}$ were used, the assumption that $|U_{bc}|^2 + \Sigma|U_{ic}|^2 \lesssim \sin^2\theta_C$

* As an example of an asymmetry generating mechanism, consider the virtual processes $N \leftrightarrow K\Sigma, K\Lambda$ which would give an s distribution harder than the $\bar{s}$ distribution.

reduces the error dramatically*. Direct measurements of s and $\bar{s}$ are possible in principle using the ν and $\bar{\nu}$ y-distributions for neutral currents and for charged currents with the contribution of charmed particle production subtracted, but this is not very sensitive to s, $\bar{s}$ e.g. the α parameters are need to $\pm 10\%$ to get $2s/(u+d)$ to ± 0.03! (Sehgal [20] has used this method to obtain a rough lower bound of $2s/(\bar{u}+\bar{d}) > 0.16$). Alternatively it is conceivable that s and $\bar{s}$ could be measured by studying strange particle production as a function of x in neutral current events or electroproduction.

5. Conclusions

We have shown that if $z = \sin^2\theta_w$ is measured in semileptonic neutrino interactions on isoscalar targets, the error due to dynamical corrections to the QCD parton model is probably less than $\delta z = \pm 0.002$. This conclusion is not affected by the recent observations [21] of dramatic differences between the structure functions of iron and deuterium provided that the origin of this effect is not associated with large isospin violations or with enormous higher-twist corrections to the parton model in nuclei. The largest uncertainty introduced by the analysis of semileptonic neutrino data is due to ignorance of $|U_{cs}|$ and of the strange quark distribution, which give $\delta z = \pm 0.008$. There is also an error of $\delta z = \pm 0.004$ from uncertainties in $|U_{dc}|$ and $|U_{ud}|$.

We see that the theoretically desirable goal of measuring $\sin^2\theta_w$ to an accuracy of ± 0.005 or better is at present unobtainable because of uncertainties in $|U_{cs}|$ and in the strange quark sea. Knowledge of these quantities will undoubtedly improve but it is hard to see how this source of error can be reduced dramatically in the near future. However, if the common view that $|U_{cs}|$ is close to one is accepted the uncertainty would be much less e.g. if it is *assumed* that $|U_{cs}| \geqslant 0.95$ the sum of the errors due to ignorance of $s(x)$, $|U_{ud}|$, $|U_{dc}|$ and $|U_{cs}|$ would be ± 0.006. The view that $|U_{cs}|$ is close to one will be strengthened if $|U_{cb}|$ turns out to be small but it will be hard to prove experimentally and should not be accepted uncritically without a deeper understanding of U_{ij}*.

This work was done in preparation for the SPS workshop while I was visiting CERN.

I am grateful to Jaap Panman for many discussions about semileptonic neutrino experiments and to J. Prentki and D. Haidt for comments on the first draft of this paper.

Note added in proof:

At the Cornell 1983 International Symposium on Lepton and Photon Interactions at High Energies, the MAC and MARK II groups reported a lifetime for the b quark of order 10^{-12} sec. This requires $|U_{cb}|$ to be very small and, if there are only six

* See note added in proof.

quarks, implies the $|U_{cs}| \geqslant 0.97$. If there are more than six quarks $|U_{cs}|$ could in principle be smaller but only if $|U_{cx}|$ is substantial for some new species of quark (x).

References

[1] J.D. Bjorken, Phys Rev. D19 (1979) 335
[2] W.J. Marciano and G. Senjanovic, Phys. Rev. D25 (1982) 3092
[3] M. Glück and E. Reya, Phys. Rev. Lett. 47 (1981) 1104
[4] S. Pakvasa, J. de Phys. 43 (1982) 234
[5] J. Panman, CERN yellow report 83-02, vol. 2 (1983) 146
[6] J.F. Wheater and C.H. Llewellyn Smith, Nucl. Phys. B208 (1982) 27
[7] W.J. Marciano and A. Sirlin, Nucl. Phys. B189 (1981) 442
[8] J.F. Wheater, J. de Phys. 43 (1982) 305
[9] C.Y. Prescott et al., Phys. Lett. 84B (1979) 524
[10] J.E. Kim et al., Rev. Mod. Phys. 53 (1981) 211
[11] M. Veltman, Nucl. Phys. B123 (1977) 89
[12] M.S. Chanowitz et al., Phys. Lett. 78B (1978) 285
[13] A. Blondel, F. Jacquet and P. Mine, CERN yellow report 83-02, vol. 2 (1983) 161;
K. Winter, CERN yellow report 83-02, vol. 2 (1983) 170;
[14] E.A. Paschos and L. Wolfenstein, Phys. Rev. D7 (1973) 91
[15] C.H. Llewellyn Smith, Nucl. Phys. B17 (1970) 277
[16] H. Abramowicz et al., Z. Phys. C15 (1982) 19
[17] J.J. Aubert et al., Nucl. Phys. B213 (1983) 31
[18] M. Jonker et al., Phys. Lett. 109B (1982) 133
[19] E.A. Paschos and M. Wirbel, Nucl. Phys. B194 (1982) 189
[20] L.M. Sehgal, J. de Phys. 43 (1982) 22
[21] J.J. Aubert et al., Phys. Lett. 123B (1983) 275;
A. Bodek et al., Phys. Rev. Lett. 50 (1983) 1431