Projection of (non-)smooth functions using mesh adaptivity

Problem

Consider the domain $\Omega = [0,1]^2$. Our objective is to project

$$f(\mathbf{x}) = \frac{1}{2}\left[1 - \tanh\left(\frac{\sqrt{(x-0.5)^2 + (y-0.5)^2} - 0.2}{0.005}\right)\right]$$

to the finite element space

$$V_h = \{v \in C^0(\Omega) : v|_K \in \mathbb{P}^p(K)\},$$

where $p = 1$, and to use mesh adaptivity.

Weak formulation

The discrete weak formulation is given as follows. Find $f_h(\mathbf{x}) \in V_h$ such that

$$\int_\Omega f_h(\mathbf{x})\varphi(\mathbf{x})d\mathbf{x} = \int_\Omega f(\mathbf{x})\varphi(\mathbf{x})d\mathbf{x}, \quad \forall \varphi(\mathbf{x}) \in V_h.$$

Since $f_h(\mathbf{x}) \in V_h$, $f_h(\mathbf{x}) = \sum_j F_j\varphi_j(\mathbf{x})$. Therefore,

$$\sum_j F_j \underbrace{\int_\Omega \varphi_i(\mathbf{x})\varphi_j(\mathbf{x})d\mathbf{x}}_{=m_{ij}} = \underbrace{\int_\Omega f(\mathbf{x})\varphi_i(\mathbf{x})d\mathbf{x}}_{=r_i}.$$

Numerical results

The solution for different grids is shown in Figure 1.

References

- For the baseline code, see
  https://github.com/manuel-quezada/AMCS_394E_Comp_with_FEM/tree/main/dealii/projection
- For the local mesh refinement, see
  https://www.dealii.org/9.0.0/doxygen/deal.II/step40.html.
  https://www.dealii.org/current/doxygen/deal.II/group__numerics.html
  https://www.dealii.org/current/doxygen/deal.II/classKellyErrorEstimator.html
  https://www.dealii.org/current/doxygen/deal.II/namespaceGridRefinement.html
  https://www.dealii.org/current/doxygen/deal.II/namespaceparallel_1_1distributed_1_1GridRefinement.html

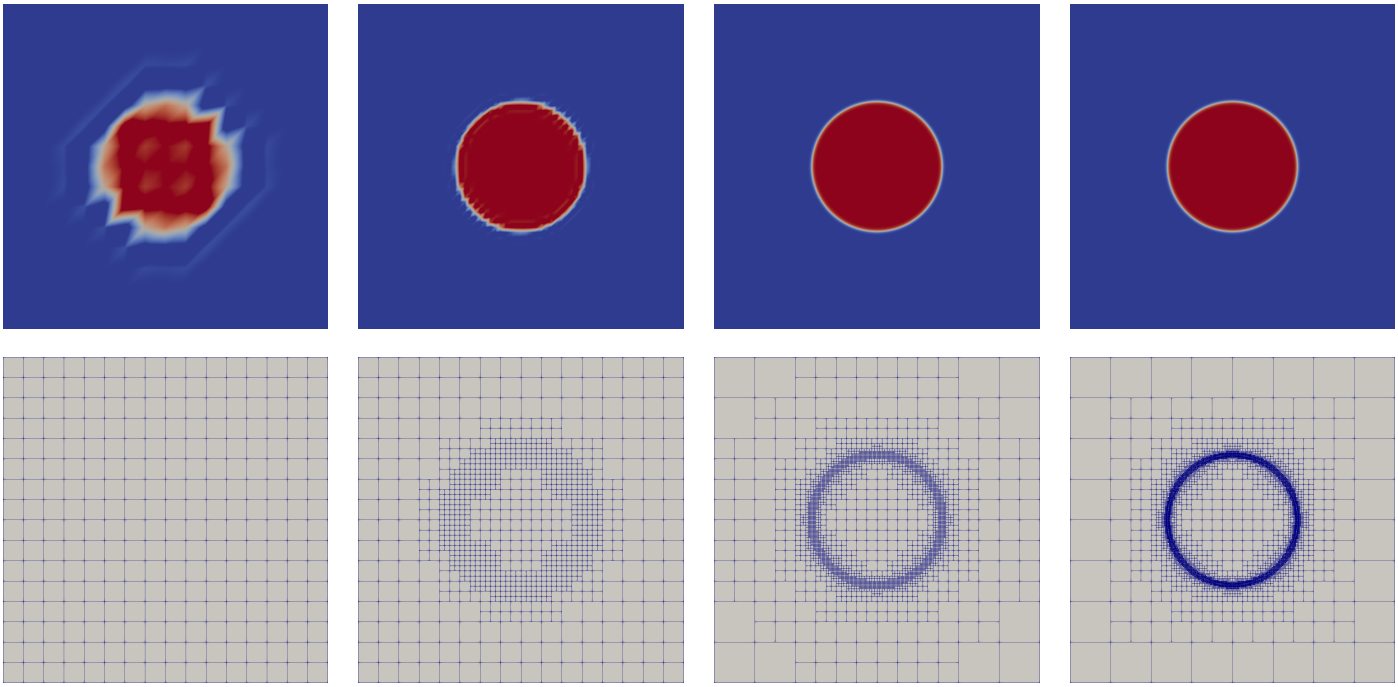

Figure 1: Projected function and the corresponding grids.