E5 Kern- und Teilchenphysik
WiSe 17/18 – Excercise 2

Discussion: 07.11.2017 bis 13.11.2017

Students studying "Lehramt Gymnasium" please solve the exercises 1 a and 2. All other students please solve all exercises

1. Rutherford-Scattering

Consider the scattering of α-particles with an energy of 10 MeV on a thin gold foil (Ladungszahl Z=79, Massenzahl A=197).

(a) Show that the integrated cross-section for collision/impact parameter $b < b_0$ is given by $\sigma = \pi b_0^2$. On the one hand, calculate this explicitly from the differential cross-section $\mathrm{d}\sigma/\mathrm{d}\Omega$ for Rutherford scattering, on the other hand by means of geometrical observation!

(b) **Optional for Lehramtsstudierende:**
Calculate the cross section for the scattering of α-particles on a gold core for scattering angles greater than 10° and 20°.

2. Bethe-Weizsäcker Formula (State examination assignment)

In 1935, C.F. von Weizsäcker created a phenomenological formula for describing the core masses, which represents empirical correlations in the observation of isotopes. Then the mass M of a nucleus with the mass number A and the charge number Z is:

$$M(A,Z) = N m_n + Z m_p + \left(-a_V A + a_O A^{2/3} + a_C \frac{Z^2}{A^{1/3}} + a_A \frac{(N-Z)^2}{A} + a_P \frac{\delta}{A^{1/2}}\right) \frac{1}{c^2}\,, \tag{1}$$

with: $a_V = 15.67\,\mathrm{MeV}$, $a_O = 17.23\,\mathrm{MeV}$, $a_C = 0.71\,\mathrm{MeV}$, $a_A = 23.29\,\mathrm{MeV}$, $a_P = 11.2\,\mathrm{MeV}$

$$\text{and } \delta = \begin{cases} -1 & \text{for } gg \text{ nuclei: even (\textbf{g}erade) number of protons + even number of neutrons} \\ 0 & \text{for } ug/gu \text{ nuclei: odd (\textbf{u}ngerade) number of protons + even number of neutrons} \\ & \text{or even number of protons + odd number of neutrons} \\ 1 & \text{für } uu \text{ nuclei} \end{cases}$$

Further, $N = A - Z$, m_n is the neutron mass and m_p the proton mass.

(a) Indicate which terms represent the binding energy B. Sketch the course of the binding energy per nucleon as a function of the mass number A.

(b) Explain the physical meaning of the last 5 terms of the given mass formula. Are the respective contributions attractive or repulsive? Justify the dependence on the mass number A as well as the nuclear charge number Z.

(c) Give a formula for calculating the most stable isobar to a given A by deriving the binding energy per nucleon to and, thus determining the extrema of B/A as a function of Z. In this case, you simply assume δ to be constant.

(d) For $Z = 22$ the binding energy for $A = 48$ becomes maximal. Show this by calculating the values B/A for the $Z = 21, 22$ and 23 using the Weizsäcker mass formula.

(e) In fact, the $^{48}_{20}$Ca nuclide is stable, but not $^{48}_{21}$Sc. Explain this behavior, which is different from the Weizsäcker mass formula.