Prof. Dr. Stefan Hofmann Winter term 2017/18

# Exercises on General Relativity TVI TMP-TC1

## Problem set 0, due October 23rd

This problem set should be considered as a repetition of some topics introduced in the standard relativistic mechanics and classical electrodynamics courses. As there are no tutorials this week it will be discussed only in the central exercise. In this course we will use the mostly plus metric of Minkowski space-time, namely $\eta_{\mu\nu} = \mathrm{diag}(-,+,+,+)$.

## Exercise 1 – Most general action

Construct for the real scalar field $\phi(x)$ the most general action in four dimensional spacetime. In order to achieve this use only positive powers of the following objects: the real scalar field $\phi$, a constant $\Lambda$ with mass dimension $[\Lambda] = [\phi]$, and the partial derivative $\partial_\mu$.
*Hint:* You do not need to specify dimensionless constants and signs. Work in natural units for particle physics: $c = \hbar = k_B = 1$.

## Exercise 2 – Classical electrodynamics

The Lagrange density of classical electrodynamics (massless vector field) without external sources is the following:

$$\mathcal{L} = -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} \,, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{1}$$

Determine the equations of motion for the field $A_\mu$ by varying the action $S[A] = \int \mathrm{d}^4 x \, \mathcal{L}$.

## Exercise 3 – (Lorentz)Tensors

Consider the following expressions and explicitely determine whether they are components of Lorentz tensors and or tensors.

(i) $\partial_\mu \phi$, (ii) $\partial_\mu A_\nu$, (iii) $F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu$, (iv) $S_{\mu\nu} := \partial_\mu A_\nu + \partial_\nu A_\mu$

*Hint*: Bare in mind, that a proper tensor is a general geometrical quantity with components having the following transformation properties under coordinate transformations $\boldsymbol{x} \to \tilde{\boldsymbol{x}}(\boldsymbol{x})$:

$$\tilde{T}^{\tilde{\mu_1} \cdots \tilde{\mu_k}}_{\tilde{\nu_1} \cdots \tilde{\nu_l}} = T^{\mu_1 \cdots \mu_k}_{\nu_1 \cdots \nu_l} \frac{\partial \tilde{x}^{\tilde{\mu_1}}}{\partial x^{\mu_1}} \cdots \frac{\partial x^{\nu_l}}{\partial \tilde{x}^{\tilde{\nu_l}}} \tag{2}$$

## Exercise 4 – Wave equation

Solve the differential equation

$$\Box \psi = 0 \tag{3}$$

in Minkowski spacetime with three spatial dimensions in spherical coordinates.

## General information

The lecture takes place on Monday at 14:00-16:00 and on Friday at 10:00 - 12:00 in A348 (Theresienstraße 37).

Presentation of solutions:
Monday at 16:00 - 18:00 in B 138

There are six tutorials:
Monday at 12:00 - 14:00 in A 249
Thursday at 16:00 - 18:00 in A 449
Friday at 14:00 - 16:00 in B 139, C 113 and A 249
Friday at 16:00 - 18:00 in A 249

The webpage for the lecture and exercises can be found at

`www.physik.uni-muenchen.de/lehre/vorlesungen/wise_17_18/tvi_tc1_gr/index.html`