Prof. Dr. Stefan Hofmann Winter term 2017/18

Exercises on General Relativity TVI TMP-TC1

Problem set 4, due November 20th

Exercise 1 – Geodesic equation

In the lecture the field Γ was introduced as

$$\Gamma^{\alpha}_{\mu\nu} = \tfrac{1}{2} g^{\alpha\beta} \left(\partial_\nu g_{\mu\beta} + \partial_\mu g_{\nu\beta} - \partial_\beta g_{\mu\nu} \right) , \tag{1}$$

together with the geodesic equation

$$\ddot{x}^\mu + \Gamma^{\mu}_{\alpha\beta} \dot{x}^\alpha \dot{x}^\beta = 0 \, , \tag{2}$$

where $\dot{x}^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\lambda}$.

(i) Show that the geodesic equation is equivalent to the following expression:

$$\dot{x}^\alpha \left(\partial_\alpha \dot{x}^\mu + \Gamma^{\mu}_{\alpha\beta} \dot{x}^\beta \right) =: \dot{x}^\alpha \nabla_\alpha \dot{x}^\mu = 0 \, . \tag{3}$$

(ii) Consider the metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and evaluate the geodesic equation in the Newtonian limit; therefore, Φ is sufficiently small, time independent and geodesics are considered in the non-relativistic limit $|\dot{x}^0| \gg |\dot{x}^i|$ with $i \in I(3)$. Use for $h_{\mu\nu}$ the result from problem set 3 for a point mass ($h_{\mu\nu} = -2\Phi\delta_{\mu\nu}$ with the gravitational potential Φ).

Exercise 2 – First step to a gauge invariant self-interaction

(i) Recall the Fierz-Pauli action from sheet 3 and rewrite it in the following useful form:

$$S_{FP}[h] = (8\bar{G}_N)^{-1} \int_{M_4} \partial_\alpha h_{\mu\nu} M^{\alpha\mu\nu\beta\rho\sigma} \partial_\beta h_{\rho\sigma} . \tag{4}$$

(ii) Derive the energy-momentum tensor (EMT) T_{FP} of h using your results from problem sheet 1 and the functional (4).
Is this EMT gauge invariant?

(iii) In order to couple the h field to itself, we introduce a source term $S_I[h]$ to the Fierz-Pauli action,

$$S_I[h] = \int_{M_4} h_{\mu\nu} T^{\mu\nu}_{FP} \, . \tag{5}$$

How does this change the EMT of the theory $\Big($EMT(S_{FP}) = EMT$(S_{FP}+S_I)$ or EMT$(S_I) = 0$ $\Big)$?

(iv) Recall why this is a problem in order to find a self-consistent completion of the Fierz-Pauli theory.

(Hint: You do not need to find the complete expression for the gauge transformation or the new contribution to the EMT, but it is sufficient to show that at least one of the many terms does not cancel.)

Exercise 3 – Expansion of the measure function

Expand, to third order in $\frac{1}{\zeta}$, the integral measure function $\sqrt{-\det(g)}$ by using the expansion $g \to \eta + \frac{h}{\zeta}$, η being the Minkowski metric and h a small fluctuation around η. The mass dimension $[\zeta]$ is equal to $[h]$.

General information

The lecture takes place on Monday at 14:00-16:00 and on Friday at 10:00 - 12:00 in A348 (Theresienstraße 37).

Presentation of solutions:
Monday at 16:00 - 18:00 in B 138

There are six tutorials:
Monday at 12:00 - 14:00 in A 249
Thursday at 16:00 - 18:00 in A 449
Friday at 14:00 - 16:00 in B 139, C 113 and A 249
Friday at 16:00 - 18:00 in A 249

The webpage for the lecture and exercises can be found at

`www.physik.uni-muenchen.de/lehre/vorlesungen/wise_17_18/tvi_tc1_gr/index.html`