Discussion: 16.01.2018 bis 22.01.2018

For students studying "Lehramt Gymnasium" this exercise is ***completely voluntary****. All other students please solve all exercises.*

1. Form Factor and Mean Square Radius of Nucleons with Spherical-Symmetric Charge Distribution

Consider a nucleus with spherical charge density distribution $f(r) = \frac{1}{Z \cdot e} \cdot \varrho(r)$. Show that

(a) that the form factor is given by

$$F(q) = \frac{4\pi\hbar}{Zeq} \int \varrho(r) \sin\left(\frac{qr}{\hbar}\right) r \mathrm{d}r.$$

Do this by calculating $F(\vec{q}) = \int f(\vec{r}) e^{i\vec{q}\vec{r}/\hbar} \mathrm{d}V$ without using a Taylor series

(b) that the derivative $\mathrm{d}F(q)/\mathrm{d}q^2$ for $q = 0$ is given by

$$\frac{\mathrm{d}F(q)}{\mathrm{d}q^2}\Big|_{q=0} = -\frac{\langle r^2 \rangle}{6\hbar^2}.$$

Hint: Determine the Taylor series of the result from exercise 1a around $q = 0$.

(c) In the last attendance exercise we showed that the mean square radius $\langle r^2 \rangle$ of a nucleus with Gaussian charge distribution is given by $3/a^2$. Calculate $\langle r^2 \rangle$ again using the result of exercise 1b, that is using the gradient of the form factor

$$F(q) = \exp\left(-\frac{q^2}{2a^2\hbar^2}\right)$$

at $q = 0$.

2. Form Factor: Electron Scattering on Gold Nuclei

Electrons with an energy of $E = 500\,\mathrm{MeV}$ are scattered on gold nuclei.

(a) Calculate the form factor of the gold nucleus using the result from exercise 1a. Assume that the nucleus is a homogeneously charged sphere with radius R
(Solution: $F(q) = \frac{3\hbar^3}{R^3 q^3}\left[\sin\frac{qR}{\hbar} - \frac{qR}{\hbar}\cos\frac{qR}{\hbar}\right]$)

(b) Calculate the maximum value for $\alpha = \frac{|q|R}{\hbar}$
Hint: Use the approximate formula for the nucleus radius given in the lecture: $R \approx 1.2\,\mathrm{fm}\sqrt[3]{A}$.
(Solution: $\alpha_{\mathrm{max}} = 35.53$)

(c) How many minima would we see in the angular distribution when neglecting nuclear interactions?
Hint: The cross section is proportional to $|F(q)|^2$ which means the roots of $F(q)$ determine the position of the minima in the angular distribution.
(Solution: 10)

3. Kinematics of Electorn-Nucleon-Scattering (former examination exercise)

An electron with Energy $E = 25\,\mathrm{GeV}$ is scattered on a resting proton in an angle of $\theta = 10^\circ$. We neglect the electron mass.

- Elastic scattering:
  - (a) Sketch the diagram of the scattering process including incoming, outgoing and exchange particles. Define the corresponding four-vectors in the diagram (including momentum vectors)
  - (b) Show that for elastic scattering the energy of the scattered electron is given by $E' = E/[1 + \frac{E}{m_p c^2}(1 - \cos\theta)]$ (proton mass $m_p = 938\,\mathrm{MeV}/c^2$).
    Calculate E' and, with derivation, the four-momentum transfer Q^2. How large is the Bjorken scaling variable x?
- Inelastic scattering:
  - (c) Sketch the diagram of the scattering process and label the produced hadronic system in the diagram. Define the four-vector of the hadronic system
  - (d) Let the energy of the scattered electron be $E' = 10\,\mathrm{GeV}$.
    Calculate Q^2 and, with derivation, the invariant mass of the hadronic system.
    Calculate the Bjorken scaling variable x