Prof. Dr. Stefan Hofmann Winter term 2017/18

Exercises on General Relativity TVI TMP-TC1

Problem set 12, due January 28th

Exercise 1 – Curvature of a 2-sphere

Calculate the Riemann tensor, Ricci tensor and Ricci scalar of the 2-sphere. Use the Christoffel symbols derived on previous sheets. Changing the coordinates, i.e. to different polar coordinates like in exercise 10.2.iii), how do these quantities change?

Exercise 2 – Geodesic deviation on a 2-sphere

Show that on the surface of a unit sphere two particles separated by initial distance d, starting from the equator and travelling north (i.e. on lines of constant φ) will have a separation s given by $s = d\sin(\vartheta)$.

Exercise 3 – Friedmann-Lemaître-Robertson–Walker spacetime

Consider the Friedmann-Lemaître-Robertson–Walker spacetime with squared line element

$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathrm{d}t^2 + a^2(t)\left(\frac{\mathrm{d}r^2}{1-kr^2} + r^2\mathrm{d}\Omega^2\right) , \tag{1}$$

with t the time coordinate, r the radial coordinate, $\mathrm{d}\Omega^2 = \left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right)$ with angles θ, φ and $a(t)$ the scaling parameter and $k \in \{+1, 0, -1\}$.

(i) Find the Ricci tensor and Ricci scalar by calculating the Christoffel symbols and by explicitly exploiting the properties of the Riemann tensor in order to only compute its 20 independent components. Algebraic programs which can do this computation for an arbitrary metric will be discussed in the tutorials.

(ii) Consider the case $k = 0$. Introducing a new time coordinate η with $\mathrm{d}t = a(t)\mathrm{d}\eta$, how does (1) change and what implications can you conclude?

General information

The lecture takes place on Monday at 14:00-16:00 and on Friday at 10:00 - 12:00 in A348 (Theresienstraße 37).

Presentation of solutions:
Monday at 16:00 - 18:00 in B 138

There are six tutorials:
Monday at 12:00 - 14:00 in A 249
Thursday at 16:00 - 18:00 in A 449
Friday at 14:00 - 16:00 in C 113 and A 249
Friday at 16:00 - 18:00 in A 249

The webpage for the lecture and exercises can be found at

`www.physik.uni-muenchen.de/lehre/vorlesungen/wise_17_18/tvi_tc1_gr/index.html`