E5 Kern- und Teilchenphysik
WiSe 17/18 – Excercise 3

Discussion: 21.11.2017 bis 27.11.2017

Students studying "Lehramt Gymnasium" please solve the exercises 1 and 2. All other students please solve all exercises

1. Age Determination with the Radiocarbon (^{14}C) Method (State examination assignment)

According to a chemical analysis, a wooden splinter from an archaeological find contains 1.2 g of carbon. A counting tube is used to register 845 decays / hour. The aim of this task is to determine the age of the wood chip.

Use the following:

i, At the time of death of the tree from which the splinter originates the atmospheric ratio was

$$\frac{\text{Number of } ^{12}\text{C atoms}}{\text{Number of } ^{14}\text{C atoms}} = 7.85 \times 10^{11}\ .$$

ii, The half-life of ^{14}C is $t_{1/2} = 5730\,\text{a}$.

iii, The molar mass of ^{12}C is by definition 12 g/mol.

iv, The counting tube completely encloses the wood chippings and has an efficiency of 100%, i.e it registers all decays in the splinter.

(a) Establish the differential equation for radioactive decay and derive the radioactive decay law.

(b) Obtain the relationship between the decay constant λ and the half-life time $t_{1/2}$.
Gewinnen Sie daraus den Zusammenhang zwischen der Zerfallskonstanten λ und der Halbwertszeit $t_{1/2}$.

(c) Explain in words the principle of the ^{14}C method.

(d) Calculate the number of ^{12}C atoms in the fragment. Then calculate the number of ^{14}C atoms present in the wood chip when the tree dies, and the number of these that is now measured with the counting tube and finally the age of the wood chip.

2. β-decay of Tritium

With $B(^3_1\text{H}) = 8.4818\,\text{MeV}$, Tritium $^3_1$H is bound stronger than $^3_2$He with $B(^3_2\text{He}) = 7.7181\,\text{MeV}$.

(a) Why is it still possible for $^3_1$H to transition into $^3_2$He through β-decay?

(b) Determine the β cut off energy E_0 and the maximum recoil energy of $^3_2$He for the case of a vanishing neutrino mass $m_\nu = 0$! Why can you calculate non-relativistically?
(Solution: $E_0 = 18.6\,\text{keV}$, $E(^3_2\text{He}) = 3.3\,\text{eV}$)

(c) How does E_0 change when the electron neutrino ν_e has a mass $m_\nu > 0$ hat?

Note: Using the so-called Curie representation, one tries to determine the electron neutrino mass from the β-spectrum of Tritium. The current upper limit obtained with this method (at 95% confidence) is $m_\nu < 2.2\ \text{eV/c}^2$.

3. α-Decay of ^{210}Po (Optional for Lehramtsstudierende)

Consider the α-decay of ^{210}Po to ^{206}Pb:

$$^{210}_{84}\text{Po} \rightarrow {}^{206}_{82}\text{Pb} + \alpha.$$

(a) Calculate the energy Q_α released in this decay.

(b) What is the kinetic energy of the α-particle taking into account the nuclear recoil?

Note:
$M(^{210}_{84}Po) = 209.9828741$ u, $M(^{11}_{82}Pb) = 205.9744657$ u, $M(^{4}_{2}He) = 4.0026033$ u, 1 u = 931.494 MeV/c^2.
$M(\dots)$ denotes the respective **atomic mass**.