basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE 12/*GRAAD 12*

MATHEMATICS P1/*WISKUNDE V1*

NOVEMBER 2023

MARKING GUIDELINES/*NASIENRIGLYNE*

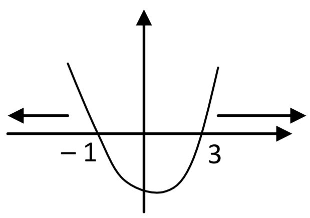
MARKS/*PUNTE*: 150

These marking guidelines consist of 17 pages.
Hierdie nasienriglyne bestaan uit 17 bladsye.

NOTE:
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking memorandum.

LET WEL:
- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord, merk slegs die EERSTE poging.*
- *Volgehoue akkuraatheid is DEURGAANS op ALLE aspekte van die memorandum van toepassing.*

QUESTION 1/*VRAAG* 1

1.1.1	$x^2+x-12=0$ $(x-3)(x+4)=0$ $x=3$ or $x=-4$	✓factors/formula ✓answer ✓answer (3)
1.1.2	$3x^2-2x=6$ $3x^2-2x-6=0$ $x=\dfrac{2\pm\sqrt{(-2)^2-4(3)(-6)}}{2(3)}$ $x=1{,}79$ or $x=-1{,}12$	✓ standard form ✓ substitution into correct formula ✓ answer ✓ answer (4)
1.1.3	$\sqrt{2x+1}=x-1$ $2x+1=(x-1)^2$ $2x+1=x^2-2x+1$ $x^2-4x=0$ $x(x-4)=0$ $x=0$ or $x=4$ $x\neq 0$ or $x=4$	✓ squaring both sides ✓ standard form ✓ both x-values ✓ valid answer (4)
1.1.4	$x^2-2x>3$ $x^2-2x-3>0$ $(x-3)(x+1)>0$ CV's: $x=-1$; $x=3$  $x<-1$ or $x>3$	✓ standard form ✓ critical values/factors ✓✓answer (4)

1.2	$\frac{1}{x}+\frac{1}{y}=1 \quad \ldots \quad (1)$	
	$x+2=2y \quad \ldots \quad (2)$	
	$x=2y-2$	✓ $x=2y-2$
	$\frac{1}{2y-2}+\frac{1}{y}=1$	✓substitution
	$y+2y-2=2y^2-2y$	
	$2y^2-5y+2=0$	✓standard form
	$(2y-1)(y-2)=0$	
	$y=\frac{1}{2}$ or $y=2$	✓y-values
	$x=-1$ or $x=2$	✓x-values (5)
	OR/*OF*	**OR/*OF***
	$\frac{1}{x}+\frac{1}{y}=1 \quad \ldots \quad (1)$	
	$x+2=2y \quad \ldots \quad (2)$	
	$y=\frac{x}{2}+1$	✓ $y=\frac{x}{2}+1$
	$\frac{1}{x}+\frac{1}{\frac{x}{2}+1}=1$	✓substitution
	$\frac{1}{x}+\frac{2}{x+2}=1$	
	$x+2+2x=x^2+2x$	
	$x^2-x-2=0$	✓standard form
	$(x+1)(x-2)=0$	
	$x=-1$ or $x=2$	✓x-values
	$y=\frac{1}{2}$ or $y=2$	✓y-values (5)

1.3	$2^{m+1}+2^{m}=3^{n+2}-3^{n}$ $2^{m}(2+1)=3^{n}(3^{2}-1)$ $2^{m}(3)=3^{n}(8)$ $2^{m}(3)=3^{n}(2^{3})$ $\therefore m=3$ and $n=1$ $\therefore m+n=4$ **OR/*OF*** $2^{m+1}+2^{m}=3^{n+2}-3^{n}$ $2^{m}(2+1)=3^{n}(3^{2}-1)$ $2^{m}(3)=3^{n}(8)$ $2^{m}(3)=3^{n}(2^{3})$ $2^{m-3}=3^{n-1}$ Only true if $m-3=0$ and $n-1=0$ $\therefore m+n=4$	✓ factors ✓ $2^{m}(3)=3^{n}(2^{3})$ (same bases) ✓ $m=3$ and $n=1$ ✓ $m+n=4$ (4) **OR/*OF*** ✓ factors ✓ $2^{m}(3)=3^{n}(2^{3})$ (same bases) ✓ $m-3=0$ and $n-1=0$ ✓ $m+n=4$ (4)
		[24]

QUESTION 2/*VRAAG* 2

2.1.1	7 + 12 + 17 + ….. $T_n = a + (n-1)d$ $T_{91} = 7 + (91-1)(5)$ $T_{91} = 457$ **OR/*OF*** $d = 5$ $T_n = 5n + 2$ $T_{91} = 5(91) + 2$ $T_{91} = 457$	✓$d = 5$ ✓substitution into correct formula ✓answer (3) **OR/*OF*** ✓$d = 5$ ✓substitution $n = 91$ ✓answer (3)
2.1.2	$S_n = \frac{n}{2}[2a + (n-1)d]$ $S_{91} = \frac{91}{2}[2 \times 7 + (91-1)(5)]$ $S_9 = 21\ 112$ **OR/*OF*** $S_n = \frac{n}{2}(a + l)$ $S_{91} = \frac{91}{2}(7 + 457)$ $S_{91} = 21\ 112$	✓substitution into correct formula ✓answer (2) **OR/*OF*** ✓substitution into correct formula ✓answer (2)
2.1.3	$T_n = 7 + (n-1)(5)$ $5n + 2 = 517$ $5n = 515$ $n = 103$	✓substitution into correct formula ✓equate ✓answer (3)
2.2.1	$T_1 = 3\,;\, T_2 - T_1 = 9$ and $T_3 - T_2 = 21$ 3 12 33 66 111 9 21 33 45 12 12 12 $\therefore T_5 = 3 + 9 + 21 + 33 + 45 = 111$ **OR/*OF*** $2a = 12$ $a = 6$ $3(6) + b = 9$ $b = -9$ $6 - 9 + c = 3$ $T_5 = 6(5)^2 - 9(5) + 6 = 111$	✓ constant second diff = 12 ✓ first differences : 33 and 45 (2) **OR/*OF*** ✓ constant second diff = 12 ✓substitute 5 (2)

2.2.2	$2a = 12$ $a = 6$ $3(6) + b = 9$ or $5\times 6 + b = 21$ $b = -9$ $6 - 9 + c = 3$ $c = 6$ $T_n = 6n^2 - 9n + 6$	✓ $2a = 12$ ✓ $3(6) + b = 9 / 5\times 6 + b = 21$ ✓ $6 - 9 + c = 3$ (3)
2.2.3	$T_n^{/} = 12n - 9 > 0$ $n > \frac{3}{4}$ $\therefore$ T_n is increasing for $n \in N$ **OR/*OF*** $n = -\frac{b}{2a} = -\frac{-9}{2(6)}$ $n = \frac{3}{4}$ $\therefore$ min at $n = 1$ for $n \in N$ $\therefore$ T_n is increasing for $n \in N$	✓ $T_n^{/} = 12n - 9$ ✓ $n > \frac{3}{4}$ ✓ increasing for $n \in N$ (3) **OR/*OF*** ✓ $n = -\frac{b}{2a} = \frac{9}{2(6)}$ ✓ $n = \frac{3}{4}$ ✓ increasing for $n \in N$ (3)
		[16]

QUESTION 3/*VRAAG* 3

3.1.1	$T_n = ar^{n-1}$ $T_n = 3(2)^{n-1}$	✓ $T_n = 3(2)^{n-1}$ (1)
3.1.2	$\sum_{p=1}^{k} \frac{3}{2}.2^p = 98\ 301$ $\sum_{p=1}^{k} \frac{3}{2}.2^p = 3+6+12+...$ $n = k$ $\frac{3\left[(2)^k - 1\right]}{2-1} = 98\ 301$ $(2)^k = 32\ 768$ $2^k = 2^{15}$ **OR/*OF*** $k = \log_2 32\ 768$ $\therefore k = 15$	✓expansion ✓ $n = k$ ✓substitution into correct formula ✓$k = 15$ (4)
3.2	$S_{22} = \frac{22}{2}\left[2a + 21(3)\right]$ $S_{22} = 22a + 693$ $S_\infty = \frac{a}{1-\frac{1}{3}}$ $= \frac{3a}{2}$ $\therefore 22a + 693 = \frac{3a}{2} + 734$ $44a + 1386 = 3a + 1468$ $41a = 82$ $a = 2$	✓ substitution into S_n ✓ $S_{22} = 22a + 693$ ✓substitution into S_∞ ✓ $S_{22} = S_\infty + 734$ ✓ answer (5)
		[10]

QUESTION 4/*VRAAG* 4

4.1	$y = -4$	✓ $y = -4$ (1)
4.2	x – intercept: $0 = 2^x - 4$ $4 = 2^x$ $x = 2$ $\therefore B(2;0)$	✓$y = 0$ ✓$x = 2$ (2)
4.3	$y = 2^0 - 4 = -3$ $\therefore A(0;-3)$ $y = mx + c$ $m = \frac{3}{2}$ $k(x) = \frac{3}{2}x - 3$	✓$y = -3$ ✓gradient ✓equation (3)
4.4	$k(1) = \frac{3}{2}(1) - 3 = \frac{-3}{2}$ $f(1) = 2^1 - 4 = -2$ Vertical distance $= -\frac{3}{2} - (-2) = \frac{1}{2}$ units	✓$k(1)$ ✓$f(1) = -2$ ✓answer (3)
4.5	$g(x) = f(x) + 4$ $g(x) = 2^x\ ;\ x \in [-2\ ;4)$	✓ $g(x) = 2^x$ (1)
4.6	Range of g: $y \in \left[\frac{1}{4};16\right)$ Domain of $g^{-1}: x \in \left[\frac{1}{4};16\right)$ or/*of* $\frac{1}{4} \le x < 16$	✓✓$x \in \left[\frac{1}{4};16\right)$ (2)
4.7	$g: y = 2^x$ $g^{-1}: x = 2^y$ $g^{-1}(x) = \log_2 x,\ x \in \left[\frac{1}{4};16\right)$	✓swop x and y ✓ equation (2)
		[14]

QUESTION 5/*VRAAG* 5

5.1	$(1\,;8)$	✓ $x=1$ ✓ $y=8$ (2)
5.2	$y=-\frac{1}{2}(0-1)^2+8$ $=7\frac{1}{2}$ $C\left(0\,;\frac{15}{2}\right)$	✓$x=0$ ✓ answer (2)
5.3	$8=\frac{d}{1}$ $\therefore d=8$	✓substitution $(1\,;8)$ (1)
5.4	$y\in R\,;\,y\neq 0$	✓ $y\neq 0$ (1)
5.5	$-3\leq x<0$ or $x\geq 5$ **OR/*OF*** $x\in[-3\,;\,0)\cup[5\,;\infty)$	✓✓ $-3\leq x<0$ ✓ $x\geq 5$ (3)
5.6	$-2x+k=\frac{8}{x}$ $-2x^2+kx-8=0$ $\Delta=(k)^2-4(-2)(-8)$ $k^2-64<0$ $CV: k=8\ ;\ k=-8$ $\therefore -8<k<8$ **or/of** $k\in(-8\,;8)$ **OR/*OF*** $g'(x)=h'(x)$ $-\frac{8}{x^2}=-2$ $-8=-2x^2$ $x=\pm 2$ $y=\pm 4$ $\therefore$ B(2 ; 4) and A(– 2 ; – 4) For tangents: $h(x)=-2x+k$ or $h(x)=-2x+k$ $4=-2(2)+k$ $-4=-2(-2)+k$ $k=8$ $k=-8$ $\therefore -8<k<8$ or/*of* $k\in(-8\,;8)$	✓ $-2x+k=\frac{8}{x}$ ✓ standard form ✓substitution into Δ ✓ $\Delta<0$ or $\Delta=0$ ✓inequality (5) **OR/*OF*** ✓ $-\frac{8}{x^2}$ ✓ $=-2$ ✓ x-values ✓ y-values ✓inequality (5)

5.7	$h(x)=-2x+8$ $-2x+8=\frac{8}{x}$ $-2x^2+8x=8$ $-2x^2+8x-8=0$ $x^2-4x+4=0$ $(x-2)^2=0$ $\therefore x=2$ $f(2)=\frac{15}{2}$ $h(2)=4$ $4=\frac{15}{2}+t$ $\therefore t=-\frac{7}{2}$ **OR/*OF*** $f(2)=\frac{15}{2}$ Tangent point of contact (2 ; 4) $\therefore 4=-\frac{1}{2}(2-1)^2+8+t$ $4=\frac{15}{2}+t$ $\therefore t=-\frac{7}{2}$ **OR/*OF*** $g(x)=8x^{-1}$ $g^{/}(x)=-8x^{-2}$ $-2=-8x^{-2}$ $\frac{1}{4}=\frac{1}{x^2}$ $x=2$ $y=\frac{8}{2}=4$ $R(2\,;4)$ $y=-\frac{1}{2}(x-1)^2+8+t$ $4=-\frac{1}{2}(2-1)^2+8+t$ $t=-\frac{7}{2}$	✓ $x=2$ ✓ $f(2)$ ✓ $h(2)$ ✓answer (4) **OR/*OF*** ✓ $x=2$ ✓ $f(2)$ ✓ $h(2)$ ✓answer (4) **OR/*OF*** ✓ $x=2$ ✓ $h(2)$ ✓ $f(2)$ ✓answer (4)
5.7		**[18]**

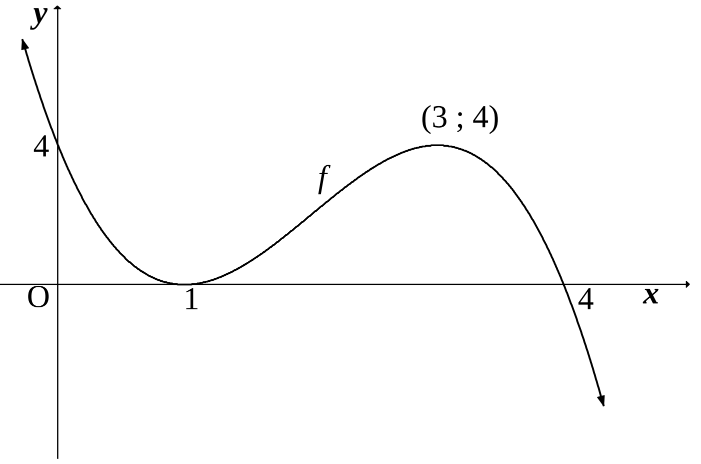
QUESTION 6/*VRAAG* 6

6.1.1	$A = P(1+i)^n$ $19\ 319{,}48 = 18\ 500\left(1+\dfrac{r}{1200}\right)^6$ $\left(1+\dfrac{r}{1200}\right) = \sqrt[6]{1{,}04429...}$ $\dfrac{r}{1200} = 0{,}00725...$ $r = 8{,}7\%$	✓ $n = 6$ ✓substitution into correct formula ✓answer (3)
6.1.2	$1+\dfrac{i}{100} = \left(1+\dfrac{8{,}7}{1200}\right)^{12}$ $r = 9{,}06\%$	✓substitution into correct formula ✓answer (2)
6.2.1	$A = P(1-in)$ $0 = 10\ \ 000(1-0{,}2n)$ $n = 5$	✓substitution into correct formula ✓answer (2)
6.2.2	$F = \dfrac{x\left[(1+i)^n - 1\right]}{i}$ $20\ 000 = \dfrac{x\left[\left(1+\dfrac{8{,}7}{1200}\right)^{60} - 1\right]}{\dfrac{8{,}7}{1200}}$ $x = \text{R}267{,}26$	✓ i ✓ n ✓substitution into correct formula ✓answer (4)
6.3	$P = \dfrac{x\left[1-(1+i)^{-n}\right]}{i}$ $1\ 600\ 000 = \dfrac{20\ 000\left[1-\left(1+\dfrac{0{,}112}{12}\right)^{-n}\right]}{\dfrac{0{,}112}{12}}$ $\dfrac{56}{75} = 1-\left(1+\dfrac{0{,}112}{12}\right)^{-n}$ $\left(1+\dfrac{0{,}112}{12}\right)^{-n} = \dfrac{19}{75}$ $-n = \log_{\left(1+\frac{0{,}112}{12}\right)}\left(\dfrac{19}{75}\right)$ $-n = -147{,}80$ Tino will make 147 withdrawals of R20 000	✓ i ✓substitution into correct formula ✓correct use of logs ✓ $-n = -147{,}80$ ✓ $n = 147$ (5)
		[16]

QUESTION 7/*VRAAG 7*

7.1	$f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{-4(x+h)^2-(-4x^2)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{-4x^2-8xh-4h^2+4x^2}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{-8xh-4h^2}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{h(-8x-4h)}{h}$ $f'(x)=\lim\limits_{h\to 0}(-8x-4h)$ $f'(x)=-8x$	✓substitution into correct formula ✓ $f(x+h) = -4x^2-8xh-4h^2$ ✓simplification ✓common factor ✓answer (5)
	OR/*OF* $f(x+h)=-4(x+h)^2=-4x^2-8xh-4h^2$ $f(x+h)-f(x)=-4x^2-8xh-4h^2-(-4x^2)$ $=-8xh-4h^2$ $f'(x)=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{-8xh-4h^2}{h}$ $f'(x)=\lim\limits_{h\to 0}\dfrac{h(-8x-4h)}{h}$ $f'(x)=\lim\limits_{h\to 0}(-8x-4h)$ $f'(x)=-8x$	**OR/*OF*** ✓ $f(x+h) = -4x^2-8xh-4h^2$ ✓simplification ✓substitution into correct formula ✓common factor ✓answer (5)
7.2.1	$f(x)=2x^3-3x$ $f'(x)=6x^2-3$	✓ $6x^2$ ✓ -3 (2)
7.2.2	$D_x\left[7.\sqrt[3]{x^2}+2x^{-5}\right]$ $D_x\left[7.x^{\frac{2}{3}}+2x^{-5}\right]$ $=\dfrac{14}{3}x^{-\frac{1}{3}}-10x^{-6}$	✓ $x^{\frac{2}{3}}$ ✓derivative with rational exp ✓ $-10x^{-6}$ (3)
7.3	$-6x^2+8>0$ $x^2<\dfrac{8}{6}$ CV's: $x=-\dfrac{2}{\sqrt{3}}$ or $x=\dfrac{2}{\sqrt{3}}$ Positive for : $-\dfrac{2}{\sqrt{3}}<x<\dfrac{2}{\sqrt{3}}$	✓CV's: $x=\pm\dfrac{2}{\sqrt{3}}$ ✓✓answer (3)
		[13]

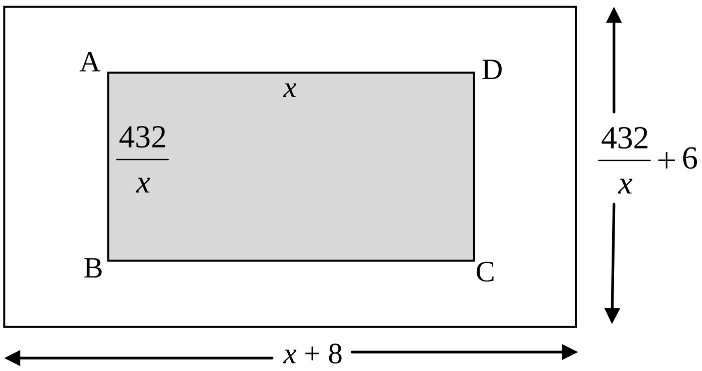
QUESTION 8/*VRAAG 8*

8.1	$f^{/}(x)=-3x^2+12x-9$ $-3x^2+12x-9=0$ $x^2-4x+3=0$ $(x-3)(x-1)=0$ $\therefore x=3$ or $x=1$ $f(3)=-(3)^3+6(3)^2-9(3)+4=4$ $f(1)=-(1)^3+6(1)^2-9(1)+4=0$ $\therefore$ turning points are: (3 ; 4) and (1 ; 0)	✓ $f^{/}(x)=-3x^2+12x-9$ ✓ $f^{/}(x)=0$ ✓ both x-values ✓ both y-values (4)
8.2		✓ y-intercept ✓ both x-intercepts ✓ both turning points ✓ shape (4)
8.3	$0<k<4$ or/*of* $k\in(0;4)$	✓✓k between y-values of turning points (2)

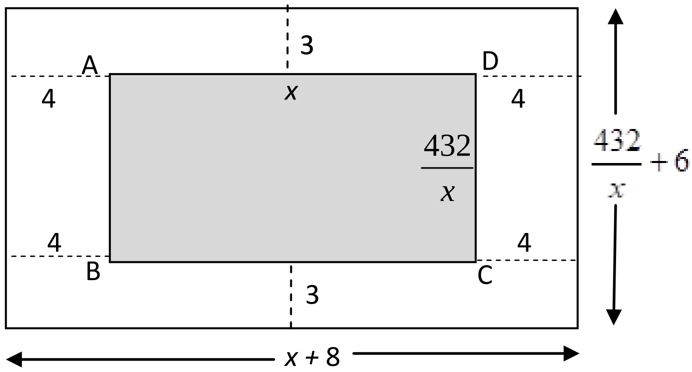
8.4	$f^{//}(x)=-6x+12=0$
	$x=2$
	Max at (2 ; 2)
	$f^{/}(2)=3$
	$\therefore y-2=3(x-2)$ or $2=3(2)+c$
	$g(x)=3x-4$ $g(x)=3x-4$
	OR/*OF*
	Point of inflection: $x=\frac{3+1}{2}$
	$x=2$
	Max at (2 ; 2)
	$f^{/}(2)=3$
	$\therefore y-2=3(x-2)$ or $2=3(2)+c$
	$g(x)=3x-4$ $g(x)=3x-4$
		✓ $f^{//}(x)=-6x+12$
		✓ $f^{//}(x)=0$
		✓ x-value
		✓ y-value
		✓gradient at x-value
		✓equation of tangent
		(6)
		OR/*OF*
		✓✓ $\frac{3+1}{2}$
		✓ x-value
		✓ y-value
		✓gradient at x-value
		✓equation of tangent (6)

8.5	$\tan\theta=3$ $\therefore\theta=71{,}57°$	✓ gradient of g ✓ answer (2)
		[18]

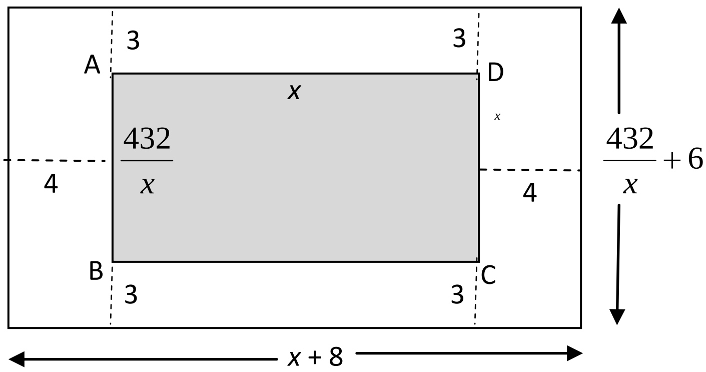
QUESTION 9/*VRAAG* 9



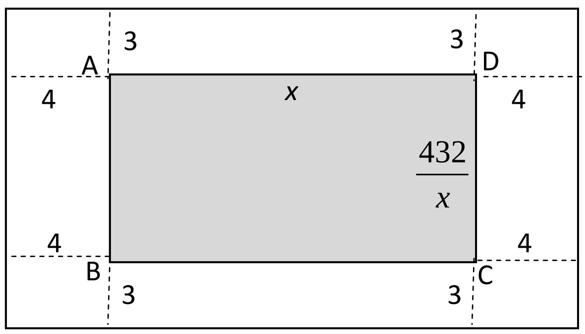
9.1	$432 = xb$ $\therefore b = \dfrac{432}{x}$ $A(x) = (x+8)\left(\dfrac{432}{x}+6\right)$ $A(x) = 432 + 6x + \dfrac{3456}{x} + 480$ $A(x) = \dfrac{3456}{x} + 6x + 480$	✓ $b = \dfrac{432}{x}$ ✓ $(x+8)$ ✓ $\left(\dfrac{432}{x}+6\right)$ (3)
9.2	$A(x) = 3456x^{-1} + 6x + 480$ $A^{/}(x) = -\dfrac{3456}{x^2} + 6$ $-\dfrac{3456}{x^2} + 6 = 0$ $3456 = 6x^2$ $\therefore x = \sqrt{576} = 24\text{ cm}$	✓ $3456x^{-1} + 6x + 480$ ✓ $A^{/}(x) = -\dfrac{3456}{x^2} + 6$ ✓answer (3)
		[6]



$$\text{total area} = 2(x+8)(3) + 2\left(\frac{432}{x}\right)(4) + \left(\frac{432}{x}\right)(x)$$

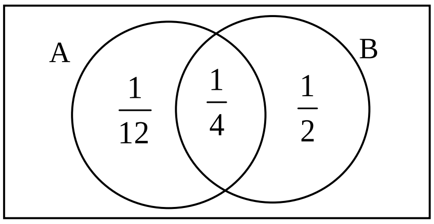
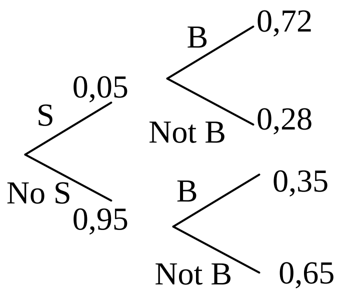


$$\text{total area} = 2(4)\left(\frac{432}{x}+6\right) + (x)\left(\frac{432}{x}+6\right)$$



$$\text{total area} = 4(4)(3) + 2(x)(3) + \left(\frac{432}{x}\right)(x) + 2\left(\frac{432}{x}\right)(4)$$

QUESTION 10/*VRAAG* 10

10.1.1	P(A and B) = P(A) × P(B) $= \frac{1}{3} \times \frac{3}{4}$ $= \frac{1}{4}$	✓ $\frac{1}{3} \times \frac{3}{4}$ ✓ $\frac{1}{4}$ (2)
10.1.2	P(A or B) = P(A) + P(B) – P(A and B) $= \frac{1}{3} + \frac{3}{4} - \frac{1}{4}$ $= \frac{5}{6}$ **OR/*OF***  P(A or B) = $\frac{1}{12} + \frac{1}{4} + \frac{1}{2} = \frac{5}{6}$	✓ substitution ✓ answer (2) **OR/*OF*** ✓ substitution ✓ answer (2)
10.2.1		✓ branch 1 with probabilities ✓ branch 2 with probabilities ✓ branch 3 with probabilities (3)
10.2.2	P(NOT below 0^o) = P(S; NOT below 0^o) + P(NS ; NOT below 0^o) = 0,05 × 0,28 + 0,95 × 0,65 = 0,6315	✓ value of P(S; NOT below 0^o) ✓ value of P(NS ; NOT below 0^o) ✓ answer (3)
10.3.1	n(S) = 10!	✓ 10! (1)

10.3.2	4 Options; **2** ×8 ×7 × 6 ×5 ×4 ×**1** ×3 ×2 ×1 = 80 640 8 ×**2** ×7 × 6 ×5 ×4 ×3 ×**1** ×2 ×1 = 80 640 8 ×7 ×**2** × 6 ×5 ×4 ×3 ×1 ×**1** ×1 = 80 640 8 ×7 × 6 ×**2** ×5 ×4 ×3 ×2 ×1 ×**1** = 80 640 Total number of possibilities = 322 560 P(5 learners in between) = $\frac{322\ 560}{10!}=\frac{4}{45}$ **OR/*OF*** **2** ×8 ×7 × 6 ×5 ×4 ×**1** ×3 ×2 ×1 4 possible starting positions $\therefore 4(2\times8!\times1)=322\ 560$ $8(8!)=322\ 560$ P(5 learners in between) = $\frac{322\ 560}{10!}=\frac{4}{45}$	✓ $(2\times8!)$ ✓✓4(2 × 8!) or 322 560 ✓ $\frac{322\ 560}{n(S)}$ (4) **OR/*OF*** ✓ $(2\times8!)$ ✓✓4(2 × 8!) or 322 560 ✓ $\frac{322\ 560}{n(S)}$ (4)
		[15]

TOTAL/*TOTAAL*: 150