basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

FEBRUARY/MARCH 2018

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages and 1 information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 - 6x - 16 = 0$ (3)

1.1.2 $2x^2 + 7x - 1 = 0$ (correct to TWO decimal places) (4)

1.2 List all the integers that are solutions to $x^2 - 25 < 0$. (4)

1.3 Solve for x and y:

$-2y + x = -1$ and $x^2 - 7 - y^2 = -y$ (6)

1.4 Evaluate: $\dfrac{3^{2018} + 3^{2016}}{3^{2017}}$ (2)

1.5 Given: $t(x) = \dfrac{\sqrt{3x - 5}}{x - 3}$

1.5.1 For which values of x will $\dfrac{\sqrt{3x-5}}{x-3}$ be real? (3)

1.5.2 Solve for x if $t(x) = 1$. (4)

[26]

QUESTION 2

2.1 Given the following geometric sequence: $30 \ ; \ 10 \ ; \ \dfrac{10}{3} \ ; \ldots$

2.1.1 Determine n if the n^{th} term of the sequence is equal to $\dfrac{10}{729}$. (4)

2.1.2 Calculate: $30 + 10 + \dfrac{10}{3} + \ldots$ (2)

2.2 Derive a formula for the sum of the first n terms of an arithmetic sequence if the first term of the sequence is a and the common difference is d. (4)

[10]

QUESTION 3

The first three terms of an arithmetic sequence are -1 ; 2 and 5.

3.1 Determine the n^{th} term, T_n, of the sequence. (2)

3.2 Calculate T_{43}. (2)

3.3 Evaluate $\sum_{k=1}^{n} T_k$ in terms of n. (3)

3.4 A quadratic sequence, with general term T_n, has the following properties:

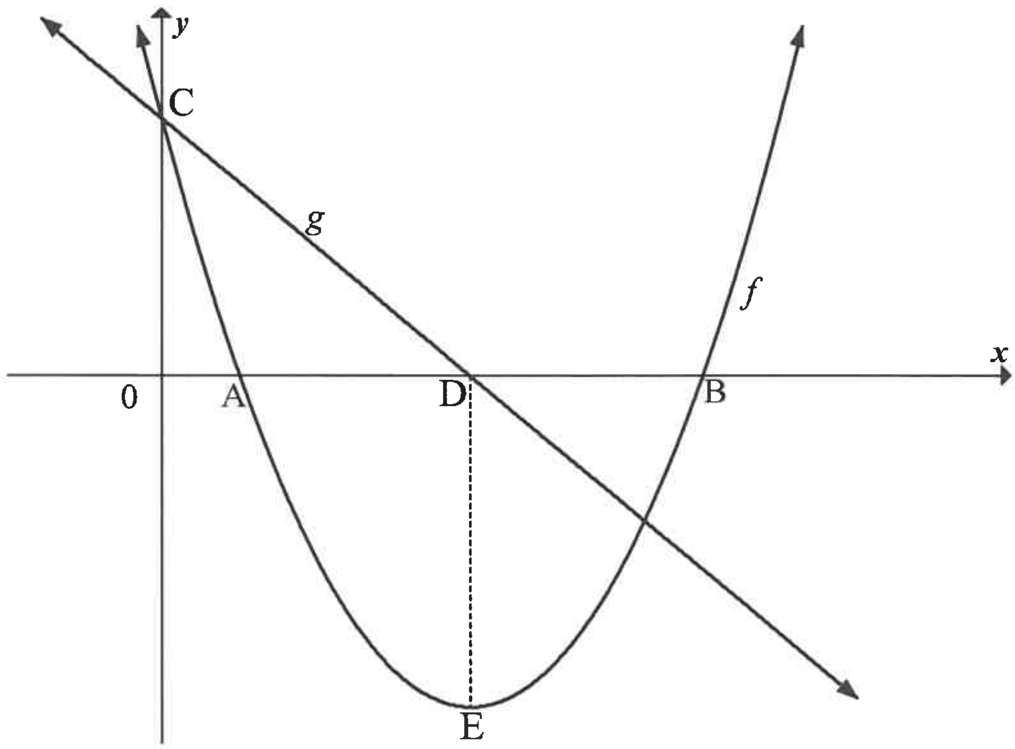
- $T_{11} = 125$
- $T_n - T_{n-1} = 3n - 4$

Determine the first term of the sequence. (6)
[13]

QUESTION 4

Below are the graphs of $f(x) = (x-4)^2 - 9$ and a straight line g.

- A and B are the x-intercepts of f and E is the turning point of f.
- C is the y-intercept of both f and g.
- The x-intercept of g is D. DE is parallel to the y-axis.



4.1 Write down the coordinates of E. (2)

4.2 Calculate the coordinates of A. (3)

4.3 M is the reflection of C in the axis of symmetry of f. Write down the coordinates of M. (3)

4.4 Determine the equation of g in the form $y = mx + c$. (3)

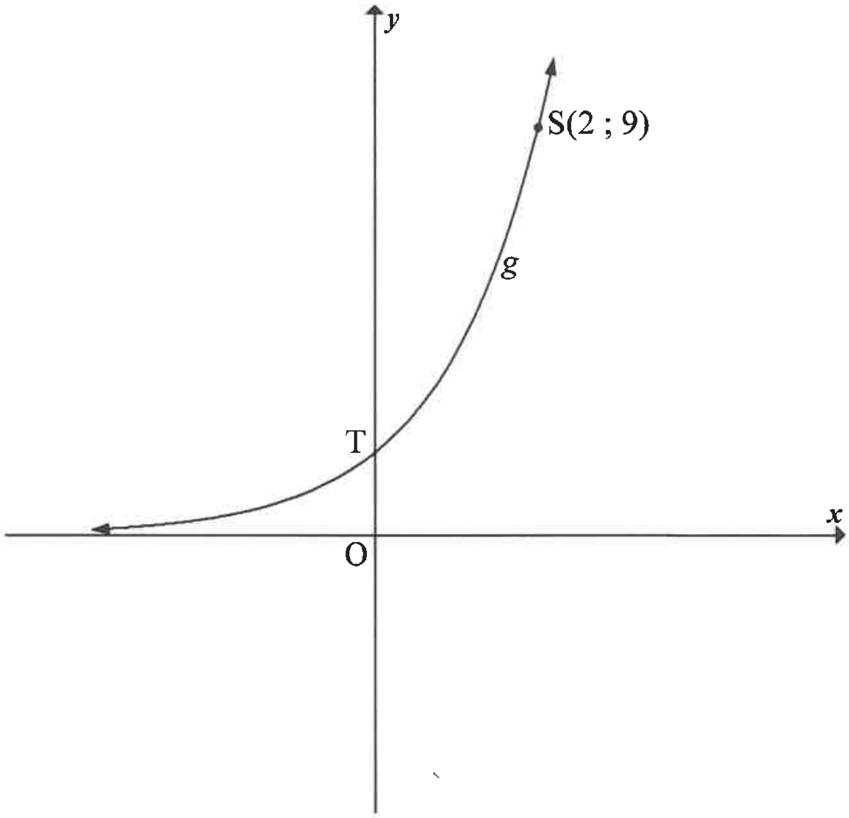
4.5 Write down the equation of g^{-1} in the form $y = \ldots$ (3)

4.6 For which values of x will $x(f(x)) \leq 0$? (4)

[18]

QUESTION 5

The graph of $g(x) = a^x$ is drawn in the sketch below. The point S(2 ; 9) lies on g. T is the y-intercept of g.



5.1 Write down the coordinates of T. (2)

5.2 Calculate the value of a. (2)

5.3 The graph h is obtained by reflecting g in the y-axis. Write down the equation of h. (2)

5.4 Write down the values of x for which $0 < \log_3 x < 1$. (2)
[8]

QUESTION 6

The function f, defined by $f(x)=\dfrac{a}{x+p}+q$, has the following properties:

- The range of f is $y \in R,\ y \neq 1$.
- The graph f passes through the origin.
- $\text{P}\left(\sqrt{2}+2;\sqrt{2}+1\right)$ lies on the graph f.

6.1 Write down the value of q. (1)

6.2 Calculate the values of a and p. (5)

6.3 Sketch a neat graph of this function. Your graph must include the asymptotes, if any. (4)
[10]

QUESTION 7

7.1 On 30 June 2013 and at the end of each month thereafter, Asif deposited R2 500 into a bank account that pays interest at 6% per annum, compounded monthly. He wants to continue to deposit this amount until 31 May 2018.

Calculate how much money Asif will have in this account immediately after depositing R2 500 on 31 May 2018. (3)

7.2 On 1 February 2018, Genevieve took a loan of R82 000 from the bank to pay for her studies. She will make her first repayment of R3 200 on 1 February 2019 and continue to make payments of R3 200 on the first of each month thereafter until she settles the loan. The bank charges interest at 15% per annum, compounded monthly.

7.2.1 Calculate how much Genevieve will owe the bank on 1 January 2019. (3)

7.2.2 How many instalments of R3 200 must she pay? (5)

7.2.3 Calculate the final payment, to the nearest rand, Genevieve has to pay to settle the loan. (5)
[16]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x)=4x^2$. (5)

8.2 Determine:

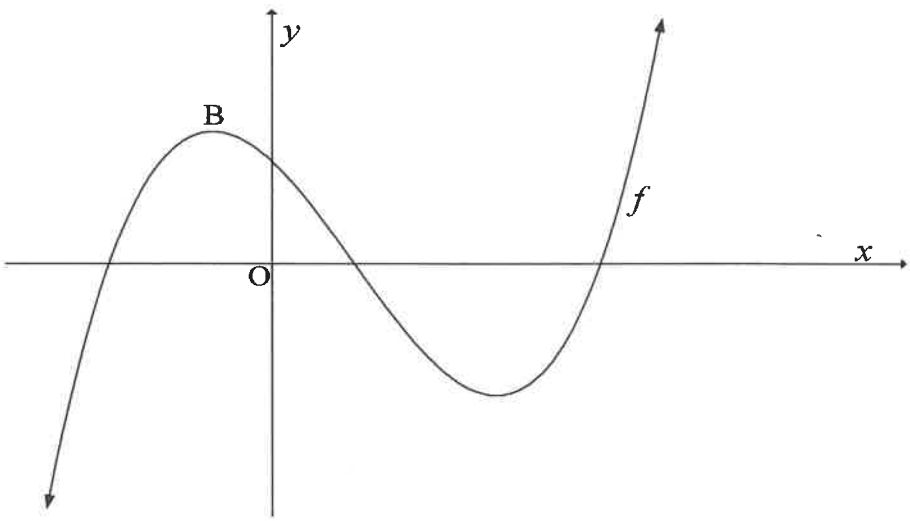
8.2.1 $D_x\left[\dfrac{x^2-2x-3}{x+1}\right]$ (3)

8.2.2 $f''(x)$ if $f(x)=\sqrt{x}$ (3)

[11]

QUESTION 9

The sketch below represents the curve of $f(x)=x^3+bx^2+cx+d$. The solutions of the equation $f(x)=0$ are -2 ; 1 and 4.



9.1 Calculate the values of b, c and d. (4)

9.2 Calculate the x-coordinate of B, the maximum turning point of f. (4)

9.3 Determine an equation for the tangent to the graph of f at $x=-1$. (4)

9.4 In the ANSWER BOOK, sketch the graph of $f''(x)$. Clearly indicate the x- and y-intercepts on your sketch. (3)

9.5 For which value(s) of x is $f(x)$ concave upwards? (2)

[17]

QUESTION 10

Given: $f(x)=-3x^3+x$.

Calculate the value of q for which $f(x)+q$ will have a maximum value of $\frac{8}{9}$. **[6]**

QUESTION 11

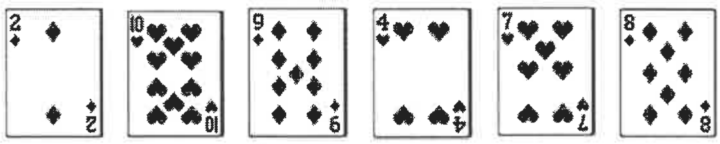
11.1 Veli and Bongi are learners at the same school. Some days they arrive late at school. The probability that neither Veli nor Bongi will arrive late on any day is 0,7.

11.1.1 Calculate the probability that at least one of the two learners will arrive late on a randomly selected day. (1)

11.1.2 The probability that Veli arrives late for school on a randomly selected day is 0,25, while the probability that both of them arrive late for school on that day is 0,15. Calculate the probability that Bongi will arrive late for school on that day. (3)

11.1.3 The principal suspects that the latecoming of the two learners is linked. The principal asks you to determine whether the events of Veli arriving late for school and Bongi arriving late for school are statistically independent or not. What will be your response to him? Show ALL calculations. (3)

11.2 The cards below are placed from left to right in a row.



11.2.1 In how many different ways can these 6 cards be randomly arranged in a row? (2)

11.2.2 In how many different ways can these cards be arranged in a row if the diamonds and hearts are placed in alternating positions? (3)

11.2.3 If these cards are randomly arranged in a row, calculate the probability that ALL the hearts will be next to one another. (3)
[15]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad$ $A = P(1-ni)$ $\quad$ $A = P(1-i)^n$ $\quad$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}\ ; r \neq 1$ $\quad$ $S_\infty = \frac{a}{1-r}\ ; -1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad$ $P = \frac{x\left[1 - (1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

$In\ \Delta ABC:\ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\quad$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$