## Functional linear model with shape constraints


### Abstract

In this thesis we study the linear model with functional covariates and scalar response under shape constraints on the weight function. These constraints may be, among others, positivity, monotonicity, convexity or beeing constantly zero after a certain point.

We study and develop theoretical tools for the convergence of random elements in a separable Hilbert space. We use a (semi) norm based on the covariance operator of the functional covariates and related to the mean squared prediction error. We demonstrate an Uniform Law of Large Numbers for its empirical version and obtain a result that links the convergence rate of the empirical (semi) norm to the (semi) norm of interest.

These results allow us to obtain convergence rates for a wide family of estimators in the unconstrained model and show their consistency under the norm induced by the inner product. These results are obtained with milder hypothesis than those commonly used in the functional data literature.

For the constrained model we propose a family of estimators that satisfy the shape constraints. Under certain conditions, we prove that their convergence rates are as good as or better than those obtained for the uncontstrained estimators. We perform a simulation study to compare the prediction errors for finite samples.

Finally we consider a special kind of constraint where the weight function is decreasing and equals zero after a certain point. We define an estimator for the change point, i.e. the first moment where the function is constantly zero. We prove the consistency of the proposed estimator and we conduct a simulation study in order to show its behaviour on different sample sizes.



*Palabras clave: Functional linear model, Constrained estimation, Penalization, Functional data. Historical Functional linear Model, Change point detection.*