Advanced Autonomous Systems - Task #0

This task is composed by problems which are useful as a training session, for preparing you for the tasks and projects during subsequent lab sessions in AAS.

Problem 1

Given the following approximate model of a pendulum,

$$\ddot{\theta}(t) = -A \cdot \sin(\theta(t)) - B \cdot \dot{\theta}(t) + C \cdot u(t)$$

$$A = 100 \cdot \frac{rad}{s^2}, \quad B = 2 \cdot \frac{1}{s}, \quad C = 1 \cdot \frac{rad}{s^2 \cdot volt}$$

where $\theta(t)$ is the angular position (expressed in radians) of the pendulum, and $u(t)$ is the voltage (expressed in volts) controlling the pendulum's electric motor.

a) Obtain a state space representation for this system, in continuous time.

b) Obtain an approximate discrete time model (using Euler's approximation), for a sample time dt=1ms.

c) Implement a program (in plain Matlab language), for simulating the model proposed in (b).

Test your program simulating the following cases:

c.1) The pendulum is released, at time =0, having angular velocity =0 and angle = 60 degrees. The voltage of the electric motor is assumed to be constantly 0 volts (no torque being applied by the motor).

c.2) Similar to (c.1) but having the electric motor controlled with at voltage which follows the following time function:

$$u(t) = \begin{cases} 10 & \forall t \in [1,2] \\ 20 & \forall t \in [3,4] \\ 0 & otherwise \end{cases}$$

In both cases, perform the simulation for an interval of time from 0 to t=7 seconds.

Plot the results (position and angular velocity) in a figure.

Problem 2

Given the following discrete time model,

$$\mathbf{X}(k+1) = \mathbf{A} \cdot \mathbf{X}(k) + \mathbf{B} \cdot u(k), \quad u \in \mathbb{R}^1$$

$$\mathbf{A} = \begin{bmatrix} 0.9 & 0.1 & 0.1 \\ 0.1 & 0.9 & 0.3 \\ 0 & -0.3 & 0.9 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Implement a program (in plain Matlab programming language) for simulating this discrete-time model.

Consider the following cases:

a) Initial condition $\mathbf{X}_0 = [0 \quad 0 \quad 0]^T$ and input $u(k)$ defined as follows,

$$u(k) = \begin{cases} 1 & \forall k \in [10,20] \\ 2 & \forall k \in [21,30], \\ 0 & otherwise \end{cases}$$

b) Initial condition is $\mathbf{X}_0 = \begin{bmatrix} 10 & 20 & 15 \end{bmatrix}^T$ and input $u(k) = 0, \quad \forall k$.

In both cases, perform the simulation for an interval of time from *k=0* to *k=500*.
Show the evolution of the individual states in three (3) separate plots, against the time index (k).
Show the evolution of the states in the state space (hint: for plotting 3D curves, you may use Matlab's function *plot3*.)

Problem 3

Show the following PDFs, by plotting them as functions of the variable x.
All the PDF functions are Gaussian and have the same expected value but different variance.

$$p_x(x) = \mathbb{N}\left(x; \quad \hat{x}, \sigma_x^2\right), \quad \hat{x} = 1,$$

The notation $\mathbb{N}\left(x; \quad \hat{x}, \sigma_x^2\right)$ means that it is a Gaussian function (of the variable *x*) whose expected value and variance are $\hat{x}$ and σ_x^2 respectively.

Consider the following cases (different variances),

a) $\sigma_x^2 = 2$, b) $\sigma_x^2 = 16$, c) $\sigma_x^2 = 0.2$, d) $\sigma_x^2 = 0.01$, e) $\sigma_x^2 = 10000$,

Problem 4

Given the following simplified 3DoF kinematic model (of a car-like wheeled platform),

$$\dot{x}(t) = v(t) \cdot \cos\left(\theta(t)\right)$$
$$\dot{y}(t) = v(t) \cdot \sin\left(\theta(t)\right)$$
$$\dot{\theta}(t) = \tan\left(\alpha(t)\right) \cdot {}^{v(t)}\!\big/_{L}$$

a) Obtain an approximated discrete-time version of the model, assuming a small discrete step, e.g. dt=0.01 seconds. Consider the case of a vehicle that has ***L=2m***.

b) Implement a program for simulating the system in (a). Run it under different steering actions (sequences of steering angles $\alpha(k)$) and assume constant speed, $v(k) = 3$ m/s, $\forall k$.

c.1) See what happen if you keep the steering angle set at a constant value.
c.2) Try to generate a path having an 8-shape (define a proper sequence of control actions to achieve it).
c.3) Propose a controller (e.g. a Proportional controller) for controlling the heading of the platform (the controller set the steering angle in order to achieve the required heading).
c.4) Apply a small modification on the model (e.g. a small change in parameter L) and see how the result is affected, for a long-term simulation (for cases c.1 and c.2). Plot, jointly, both models' trajectories using different colors, to appreciate the different responses.

Note: There is not deadline / demonstration associated to this task. The purpose of this task is to give the students some initial training, before the actual projects are released. No marking is involved in this task. This task is intended to be solved during weeks 1 and 2.