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Remaining useful life estimation – A review on the statistical data driven approaches

Xiao-Sheng Si[a,c], Wenbin Wang[b,d,e,*], Chang-Hua Hu[a], Dong-Hua Zhou[c,*]

[a] Department of Automation, Xi'an Institute of Hi-Tech, Xi'an 710025, Shaanxi, China
[b] Salford Business School, University of Salford, Salford M5 4WT, UK
[c] Department of Automation, TNLIST, Tsinghua University, Beijing 100084, China
[d] School of Economics and Management, Beijing University of Science and Technology, China
[e] PHM Centre of City University of Hong Kong, Hong Kong

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ABSTRACT

Remaining useful life (RUL) is the useful life left on an asset at a particular time of operation. Its estimation is central to condition based maintenance and prognostics and health management. RUL is typically random and unknown, and as such it must be estimated from available sources of information such as the information obtained in condition and health monitoring. The research on how to best estimate the RUL has gained popularity recently due to the rapid advances in condition and health monitoring techniques. However, due to its complicated relationship with observable health information, there is no such best approach which can be used universally to achieve the best estimate. As such this paper reviews the recent modeling developments for estimating the RUL. The review is centred on statistical data driven approaches which rely only on available past observed data and statistical models. The approaches are classified into two broad types of models, that is, models that rely on directly observed state information of the asset, and those do not. We systematically review the models and approaches reported in the literature and finally highlight future research challenges.

1. Introduction

The remaining useful life (RUL) of an asset or system is defined as the length from the current time to the end of the useful life. The concept of the RUL has been widely used in operational research, reliability and statistics literature with important applications in other fields such as material science, biostatistics and econometrics. However, there are many definitions as what is regarded as the useful life. In 'Businessdictionary.com', it defines the useful life 'the period during which an asset or property is expected to be usable for the purpose it was acquired'. However, in accounting, it is defined as 'the expected period of time during which a depreciating asset will be productive'. The key word here is 'usable' or 'productive' which is again upon individual explanations. Clearly the definition of the useful life depends on the context and operational characteristics. In this paper we will assume that the definition of the useful life is known to the owner of the asset and our main interest is to investigate the literature on the modeling methods for RUL estimation given condition and health monitoring information.

RUL estimation is one of the key factors in condition based maintenance (CBM) (Cui et al., 2004; Lee et al., 2006; Wang, 2007a,b; Wang and Zhang, 2008), and prognostics and health management (Dong and He, 2007a; Pecht, 2008; Pecht and Jaai, 2010; Gašperin et al., 2011). It is critically important to assess the RUL of an asset while in use since it has impacts on the planning of maintenance activities, spare parts provision, operational performance, and the profitability of the owner of an asset (Jardine et al., 2006; Altay and Green, 2006; Elwany and Gebraeel, 2008; Wang et al., 2009; Kim and Kuo, 2009; Papakostas et al., 2010). RUL estimation has also an important role in the management of product reuse and recycle which has strategic impacts on energy consumption, raw material use, pollution and landfill (Mazhar et al., 2007). The reused products must have sufficient long lives left among others to be able to be reused. This puts the importance of the estimation of RUL beyond CBM and prognostics and health management because of the green issues associated.

The RUL of an asset is clearly a random variable and it depends on the current age of the asset, the operation environment and the observed condition monitoring (CM) or health information. Define X_t as the random variable of the RUL at time t (age or usage), then the probability density function (PDF) of X_t conditional on Y_t is

* Corresponding authors at: Salford Business School, University of Salford, Salford M5 4WT, UK. Tel.: +44 0161 2954124; fax: +44 0161 2954947 (W. Wang); tel.: +86 010 62794461; fax: +86 010 62786911 (D.-H. Zhou).

E-mail addresses: W.Wang@salford.ac.uk (W. Wang), zdh@mail.tsinghua.edu.cn (D.-H. Zhou).

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denoted as $f(x_t|Y_t)$ where Y_t is the history of operational profiles and CM information up to t. We are mainly interested in estimating $f(x_t|Y_t)$ or $E(X_t|Y_t)$. If Y_t is not available then the estimation of $f(x_t|Y_t)$ is trivial since

$$f(x_t|Y_t) = f(x_t) = \frac{f(t + x_t)}{R(t)}, \tag{1}$$

where $f(t + x_t)$ is the PDF of the life at $t + x_t$ and $R(t)$ is the survival function at t. The availability of Y_t will certainly provide more information about the RUL of an asset. However, it is a nontrivial task to incorporate Y_t into the estimation of X_t. This has lead to a large body of literature in the past decades on the formulation and estimation of $f(x_t|Y_t)$. In the follows, we refer the estimation of the RUL of an asset to the formulation and estimation of $f(x_t|Y_t)$ or $E(X_t|Y_t)$. For the property of $f(x_t)$ or $E(X_t)$ without the influence of Y_t, see Banjevic (2009), and Navarro and Rychlik (2010).

Throughout this paper, we focus on statistical data driven approaches for RUL estimation, which rely only on available past observed data and statistical models to estimate the RUL in terms of $f(x_t|Y_t)$ or $E(X_t|Y_t)$ in a probabilistic way. Statistical data driven approaches construct the RUL estimation models by fitting the model to available data under a probabilistic model without relying on any physics or engineering principle. They have certain advantages over other methods in that some nice mathematical properties can be analysed regarding to the estimated RUL. Generally, the data collected for RUL estimation can be categorised into two main types: the so-called event data and CM data. By event data we mean past recorded failure data. As analyzed by Jardine et al. (2006), to achieve a well estimated RUL using statistical data-driven methods, collecting and storing useful data (information) from targeted assets is necessary. However to some critical assets, they are not allowed to run to failure and thus event data may be scarce. As a result, CM data is an important source of information. Here the CM data is defined in a broad sense of any data which may have a connection with the estimation of the RUL such as monitored CM information, operational, performance, environmental information, and degradation signals. Such data are very versatile, such as vibration data, oil analysis data, temperature, pressure, moisture, humidity, loading, speed and environmental data, etc. The data can be objective or subjective depending on the nature of the data and the collection method.

Clearly, statistical data driven approaches rely on the availability of data and the nature of the data. Towards this point, it is desirable to classify the available CM data into categories so that the review can follow in the same manner. Based on our review on existing literature and our experience, we classify the observed CM data into direct CM and indirect CM (Wang and Christer, 2000). Direct CM data is the data which can describe the underlying state of the system directly so that the prediction of the RUL is actually the prediction of the CM data to reach a predefined threshold level. Examples such as wear and crack sizes are typical types of this kind of data if they can be observed. Indirect CM data is the data which can only indirectly or partially indicate the underlying state of the system so failure event data may be needed in additional to CM data for an RUL estimation purpose. The data obtained from vibration and oil based monitoring are examples of this type of data. From this line, we can then classify our review of papers into two broad categories, namely, models based on the directly observed state processes and models based on the indirectly observed state processes. The latter are also called as the partially observed state process models since there is a stochastic relationship between the observed CM processes and the unobservable state.

Because there have been several review papers published related to this topic scattered among operational research, reliability modeling, optimal maintenance, fault diagnosis and prognosis, and survival analysis literature, we will first give a brief, but yet complete review on these papers and point out the different focuses of these published reviews. The main point is to show that our paper covers an area that has not been previously comprehensively reviewed.

The remaining of the paper is organised as follows. Section 2 presents a brief survey of the review papers related to RUL. Section 3 is on the RUL estimation models based on the directly observed state processes. Section 4 reviews the RUL estimation models based on the indirectly observed state processes and Section 5 concludes the paper with an emphasis on future research challenges.

2. A brief survey over the existing review papers in relation to RUL

There have been excellent review papers over the last few decades on maintenance related issues, maintenance optimisation and modeling techniques and fault diagnostics, see for example Cho and Parlar (1991), Reinertsen (1996), Scarf (1997), Wang (2002a), and Zio (2009) to name a few. These reviews papers are extensive and cover many aspects of maintenance and reliability problems. However, they share one thing in common in that they little discussed RUL and the associated modeling techniques, though some of the modeling techniques they reviewed can be adopted for RUL estimation.

Along with the rapid advance in CM and prognostic health management, recently, several review papers specifically on CBM and associated RUL issues have appeared, see Kothamasu et al. (2006), Jardine et al. (2006), Heng et al. (2009), Pecht (2008), Gorjian et al. (2009a,b), van Noortwijk (2009), Dragomir et al. (2009) and Peng et al. (2010). These reflect the increasing importance of the topic. We noted that Kothamasu et al. (2006) only discussed managerial problems in health monitoring and prognostics without discussing the modeling issues of the problems. Both Jardine et al. (2006) and Heng et al. (2009) are comprehensive to some extent and well structured. But they focused on rotating machinery prognostics with limited discussions on statistical based prognostic methods. It is well known that prognostic techniques are not limited to rotating machinery since many industrial equipment items such as electronics and civil structures that do not have rotating components, but they are subject to deterioration and require RUL estimations. In Pecht (2008), he put the prognostic techniques under the categories of physics of failure, data driven and fusion. His review was largely from an electronics point of view, so physics of failure models play an important role. Physics-based failure models rely on the physics of the underlying degradation process to be able to predict the onset of failures. Data-driven approaches attempt to derive models directly from collected CM and event data. In this type, there are machine learning and statistics based approaches. The fusion approaches are the combination of the physics of failure and data driven models, but very few studies have been reported (Cheng and Pecht, 2009; Pecht and Jaai, 2010). Gorjian et al. (2009a) presented a state-of-the-art review of the existing literature on covariate-based models in reliability modeling. Gorjian et al. (2009b) presented a review of the existing literature on commonly used degradation models in reliability analysis. However, covariates are only part of the CM data and in many case there may be no such covariates present. Degradation models are important but again limited to those plant items which can have observed degradation signals. In many CBM practices, the CM data observed may not be degradation signals at all. van Noortwijk (2009) is very statistical, but biased towards Gamma processes which are only part of the degradation based approaches in RUL estimation. Dragomir et al.'s (2009) review was brief and limited with only 54 referred papers. Peng et al. (2010) recently provided an extended review on machine prognostics in the context of CBM based on the review of Jardine et al. (2006). Similarly, they placed a special emphasis on artificial intelligence methods, and with less discussion on statistical based prognostic methods.

From the above brief review on review papers in relation to RUL estimation, we can see that there is no such comprehensive review on statistical based data driven approaches for RUL estimation. This paper fills the gap and only focuses on statistical based data driven approaches for RUL estimation. Our review attempts to present a comprehensive review under a broad sense of statistical based approaches in RUL estimation. We do not address particular types of machines or assets and we do not only pay attention to covariates or degradation based models. Rather we only look at the published work from a statistical point of view in terms of the nature of the data and models. This will provide a coherent and unified point for references in statistical based RUL researches. As mentioned previously, we classify the reviewed methods into models based on the direct CM data and indirect CM data. In models based on the direct CM data, we review regression-based, Wiener processes, Gamma processes and Markovian-based models. For models based on the indirect CM data, we include stochastic filtering-based models, covariate-based hazard models and hidden Markov model (HMM) and hidden semi-Markov models (HSMM). These models cover all the statistical data driven models reported in the literature. The taxonomy of the statistical data driven approaches for the RUL estimation is shown in Fig. 1.

The arrow from indirect CM to direct CM implies that it is possible that indicators that are able to represent the health states of the system directly can be extracted or computed from indirect CM data using some signal processing techniques (e.g. Fourier and wavelet transform, etc.). In this case, the models based on direct CM data can also be used. In literature, such process is also known as feature extraction. For example, Qiu et al. (2006) used a wavelet filter for feature extraction and they demonstrated its potential in rolling element bearing prognostics. However, their paper did not discuss how to develop the RUL modeling approach. In the most recent, Gašperin et al. (2011) applied an envelop analysis to extract feature signals and then used them to predict the RUL. In fact, Jardine et al. (2006) classified feature extraction as an important step in data processing step in a CBM program. Since Jardine et al. (2006) have given a good review on this subject, we will not touch these techniques again. It is worth noting that the definition of the RUL in many current methods is often threshold-based. In the case using feature extraction, it may be difficult to determine such threshold for the extracted feature. This poses challenges for further research. In the following, according to Fig. 1, we start first with the RUL estimation models based on directly observed state processes.

3. RUL estimation models based on directly observed state processes

Traditional methods based on failure time analysis for estimating the component lifetime rely only on the failure event data (Kalbfleisch and Prentice, 2002; Lawless, 2002). It is noted however that failure data is scarce in reality or non-existent at all. As an alternative roadmap, the RUL estimation models based on directly observed state processes have been developed to estimate the RUL. As mentioned previously, CM data describe the underlying state of the asset directly in these kinds of models so that the estimation of the RUL is really the estimation of the CM data to reach a predefined threshold level. If the CM paths can be modeled properly, one can then estimate the RUL directly without the need of failure data. This is clearly an advantage than those models which may require failure event data. However, both a threshold and a model to describe the CM data are required to determine the RUL. The set-up of the threshold is often a decision based on the engineering experience, the analysis of past data or the recommended standards. Therefore in this paper our attention is on the models developed to describe the CM data subject to a predefined threshold level.

Currently, there are two popular approaches used for modeling RUL based on the directly observed state processes. The first approach models the state evolution as a continuous process while the second approach assumes the state following a discrete space. Therefore, we review such two approaches separately. For the continuous process approaches, we consider the following three: regression, Brownian motion, and Gamma processes. The models with discrete states are mainly Markov chain-based models with a discrete or continuous time space.

3.1. Regression-based models

Regression-based methods are commonly used in industry and also in academic fields for lifetime estimation due to their simplicity (Meeker and Escobar, 1998; Li and Nilkitsaranant, 2009; Li et al., 2000). A simple linear regression model is perhaps the simplest model to use for trending the CM path (Usynin and Hines, 2007). The fundamental principle of these methods is that the health of the systems under study can be mapped by some key CM variables and then the RUL can be estimated by monitoring, trending, and predicting these CM variables with a predefined threshold w. To date, two of the most popular methods of these models are machine learning and random coefficient regression methods. Machine learning approaches use observed data and some statistical techniques such as least squares, but they do not have a probabilistic orientation, and therefore, no PDF of the RUL is available. It is noted however that such PDF of the RUL is essential for risk analysis and maintenance decision making (Wang and

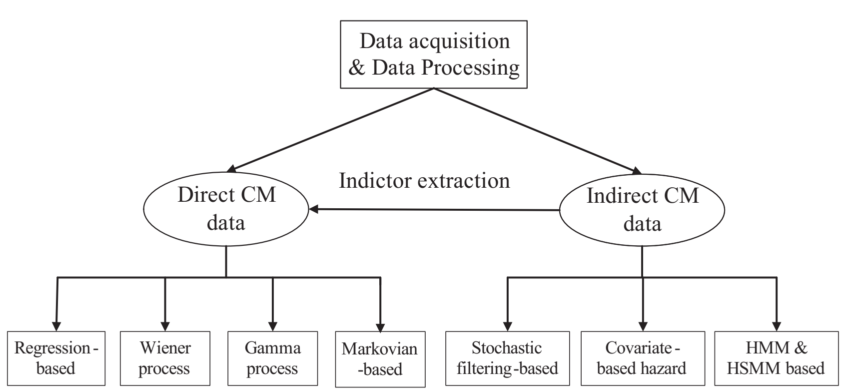

Fig. 1. Taxonomy of statistical data driven approaches for the RUL estimation.

Christer, 2000). In this sense they do not belong to the statistical methods for RUL estimation as we defined in this paper, and we exclude them in this review. However, Jardine et al. (2006) and Heng et al. (2009) have given a detailed review on these methods for RUL estimation.

Random coefficient regression methods use the CM data to depict the CM path and then infer the lifetime distribution. Lu and Meeker (1993) were the first authors to present a general nonlinear regression model to characterize the degradation path of a population of units. Their general degradation model, given the observed sample degradation $Y(t)$ at time t, can be represented as $Y(t) = D(t;\boldsymbol{\varphi},\boldsymbol{\theta}) + \varepsilon(t)$, where $D(t;\boldsymbol{\varphi},\boldsymbol{\theta})$ is the actual path at time t, $\boldsymbol{\varphi}$ is the fixed effect regression coefficients, common for all units, $\boldsymbol{\theta}$ is the random effect representing individual unit characteristics, and $\varepsilon(t)$ is the random error term described by $N(0,\sigma_\varepsilon)$. Usually, $\boldsymbol{\theta}$ and $\varepsilon(t)$ are assumed to be independent of each other. Using this model, the RUL can be defined as $X_{t_i} = \{x_{t_i} : D(t_i + x_{t_i};\boldsymbol{\varphi},\boldsymbol{\theta}) \geqslant w | D(t_i;\boldsymbol{\varphi},\boldsymbol{\theta}) < w\}$ at sampling time t_i. Along the line of Lu and Meeker (1993), many extensions and applications have appeared in literature, such as Upadhyaya et al. (1994), Tseng et al. (1995), Lu et al. (1997), Meeker and Escobar (1998), Zuo et al. (1999) and Robinson and Crowder (2000). Based on the critical analysis of the previous methods and case studies involved, Wang (2000) enumerated the underlying assumptions of the random coefficients model as follows: (i) the condition of the device deteriorates with operating time and the level of deterioration can be observed at any time; (ii) the device being monitored comes from a population of devices, each of which exhibits the same degradation form; and (iii) the distribution of the random term across the population of devices is known. We note that both Lu and Meeker (1993) and Wang (2000) assumed that the error in the degradation signal was independent and identically distributed across the population of devices. The other work followed such assumptions with some minor variations. Bae and Kvam (2004) presented nonlinear random coefficients models to characterize the degradation of vacuum fluorescent displays. Recently, Bae et al. (2007) analyzed the link among various degradation models with mix-nonlinear coefficients and then pointed out that the degradation implied the lifetime distribution.

Since 2005, based on the general assumptions summarized by Wang (2000) and the general model of Lu and Meeker (1993), Gebraeel (2006), Gebraeel and Pan (2008), Gebraeel et al. (2005, 2009) presented several random coefficient models for RUL estimation. Gebraeel et al. (2005) developed some improved models similar to the closed-form models presented by Lu and Meeker (1993), but incorporated some differences. Whereas Lu and Meeker (1993) developed methods to compute life distributions for a population of components, Gebraeel et al. (2005) focused on computing a RUL distribution for a single operating device using sensor based monitoring signals. As a result, it was stated that a linkage between the historical data and real-time CM information of the individual equipment was constructed to continuously update the parameters of the random coefficient model under a Bayesian framework. Furthermore, they used a random coefficient model that assumed a Brownian motion (Wiener) error process instead of the independent normal random error used in Lu and Meeker (1993). Gebraeel (2006) further generalized the conclusions to the case that the prior distribution of the two parameters in the random coefficient model was a joint s-normal distribution. Recently, many variants have been developed along this line through taking time-varying environments and absence of prior knowledge into account by Gebraeel and Pan (2008) and Gebraeel et al. (2009). Other extensions and applications can be found in Chakraborty et al. (2009), Elwany and Gebraeel (2009), and Kaiser and Gebraeel (2009).

Almost all the work of Gebraeel and his co-authors are based on the original work in Gebraeel et al. (2005). The most different characteristics of Gebraeel et al. (2005) is that Brownian motion was incorporated into the random coefficient regression model, which enhanced the dynamics of the original regression methods. It is worth noting that the Brownian motion was just used as an error term in their random coefficient models and the availability of the explicit first passage time (FPT) from the Brownian motion with drift was not utilized. As such, the results of RUL estimation are approximations.

In contrast with statistical learning methods, random coefficient regression models can provide a PDF of the RUL, but a closed-form of such PDF is only available in some special cases. In most cases, a stepwise approximation or simulation has to be used for finding an approximated RUL. Otherwise, a monotonic assumption has to be used to simplify the formulation (Park and Bae, 2010). While we are not sure that the degradation process is always monotonic, applying these kinds of methods may generate a conservative estimation. Another problem with the random coefficient regression models is that it cannot model the temporal variability in RUL estimation (Pandey et al., 2009). This is largely due to the fact that random coefficient models have no FPT motivation. A final limitation is the common use of an independent and identical Gaussian noise term which partially contributed to the inability of capturing the temporal structure and also restricted the model to be non-monotonic.

3.2. Brownian motion with drift (Wiener processes)

Strictly speaking Wiener processes are types of regression models. However they have specific properties so we review them separately in this section. In general, a Wiener process $\{Y(t), t \geqslant 0\}$ can be represented as $Y(t) = \lambda t + \sigma B(t)$, where λ is a defined drift parameter, $\sigma > 0$ is a diffusion coefficient, and $B(t)$ is the standard Brownian motion. The definition of the RUL at time t_i can be represented by the first passage time (FPT) of $\{Y(t), t \geqslant t_i\}$ crossing threshold w as $X_{t_i} = \inf\{x_{t_i} : Y(t_i + x_{t_i}) \geqslant w | Y(t_i) < w\}$. It is known in literature that the PDF of the first passage time of the Wiener process is the inverse Gaussian distribution. Therefore, the PDF of the RUL at time t_i can be written as, Cox and Miller (1965),

$$f_{X_{t_i}}(x_{t_i}) = \frac{w - Y(t_i)}{\sqrt{2\pi x_{t_i}^3 \sigma^2}} \exp\left(\frac{(w - Y(t_i) - \lambda x_{t_i})^2}{2x_{t_i}\sigma^2}\right). \tag{2}$$

Wiener processes for degradation modeling are appropriate for the case that the degradation processes vary bi-directionally over time with Gaussian noises. Modeling degradation processes with Wiener processes has certain mathematical advantages. The most important one is that the distribution of the FPT can be formulated analytically, known as the inverse Gaussian distribution (Tweedie, 1945, 1957; Cox and Miller, 1965), which has many merits and has been applied in reliability and lifetime analysis widely since 1970s, see Chhikara and Folks (1977). Such explicit FPT distribution is derived utilizing the property that the sum of Gaussian variables are Gaussian again. Doksum and Hoyland (1992) applied Brownian motion with drift to a variable-stress accelerated life testing experiment. They made a transformation of a non-stationary Wiener process to a stationary Wiener process. The time-scale transformation was successfully applied by Doksum and Normand (1995) in a biomarker process and by Whitmore and Schenkelberg (1997) in a degradation model of self-regulating heating cables. Whitmore (1995) incorporated measurement errors into a degradation model. The measurement errors were assumed to be independent and identically distributed random quantities having normal distributions and they were independent of the degradation process. Both Liao and Elsayed (2006) and Tseng et al. (2003) used a simple Wiener process to determine the lifetime for the light intensity of LED lamps of contact image scanners. However, a major disadvantage of the Wiener process-based models is that they only use information contained in the current degradation data, and ignore any information given by the entire sequence of observations. To remedy such a weakness, Tseng and Peng (2004) proposed an integrated Wiener process to model the cumulative degradation path of a product's quality characteristics. Lee and Tang (2007) handled the failure time prediction problem under a time-censored degradation test, in which a modified Expectation and Maximization (EM) algorithm was used to estimate the mean failure time. Recently, Lee and Whitmore (2006) and Balka et al. (2009) reviewed some methods of cure rate models based on the FPT using Wiener processes. Though their review was mainly on cure rate modeling in the biostatistics field but the principle is the same to RUL modeling. For some applications in survival analysis in relation to lifetime modeling, see Whitmore and Su (2007), Lee et al. (2009), and Pennell et al. (2010).

Along the line of the work by Whitmore (1986, 1995), Peng and Tseng (2009) incorporated the random effect of a drift coefficient and measurement errors into a Wiener process-based degradation process for lifetime assessment. Similarly, in a recent paper, Wang (2010) also considered a Wiener process with the random effect, but without measurement errors for bridge beams lifetime modeling. Park and Padgett (2005a,b, 2006) applied Brownian motion and the geometric Brownian motion to model the initial damage under accelerated testing and to further infer the lifetime. In addition, Wiener processes have been applied in accelerated life testing to make inference of the lifetime under the actual operating condition, see Padgett and Tomlinson (2004) and Liao and Tseng (2006). Although many degradation phenomena are described by Wiener processes which have been widely applied in both reliability engineering and biostatistics in relation to estimating the lifetime. The matter of whether a Wiener process is a suitable model for degradation processes, however, deserves a few comments.

Firstly, in physics, a Wiener process aims at modeling the movement of small particles in fluids and air with tiny fluctuation. A characteristic feature of this process in the context of reliability is that the plant's performance of interest can increase or decrease, similar to the random walk of small particles in fluids and air. For this reason, a Wiener process is inadequate in modeling deterioration which is monotone. Secondly, a Wiener process is a time homogeneous process but not all degradation processes have this property. For example, a fatigue crack can grow faster or slower during the course of crack propagation, which produces time heterogeneity. In general, for a time heterogeneous stochastic process, the FPT distribution is related to solving the Fokker–Planck–Kolmogorov (FPK) equation with boundary constraints (Cox and Miller, 1965) and this equation is difficult to solve analytically. Thirdly, modeling a degradation process by a Wiener process implies that the degradation process, given its current state, evolves to a future state independently of its past behavior, referred to as its Markov property. While the Markov property is a valid assumption in many applications, it does not always hold in general. Fourthly, the variance of the noise term in the Wiener process is proportional to the time length it measured, which is a strong requirement not many state processes can possess. Finally, one underlying assumption in most of the work with Wiener processes so far is that the mean degradation path is linear or can be linearized. For example, Ray and Tangirala (1996) and Ray and Phoha (1999) used a first-order linear shape filter to model fatigue crack dynamics, in which the degradation process was modeled by a Wiener process and an extended Kalman filter was applied to estimate the RUL instead of solving FPK equation. The main limitation of these studies is that only the point estimate of the RUL can be obtained. It is well-known that nonlinearity exists extensively in practice and the linear model cannot trace the dynamics of such degradation processes. Therefore, for a model to be realistic it should incorporate a nonlinear structure. However, these ideas and challenges remain to be solved with an exception of the work of Tseng and Peng (2007) and Wang and Carr (2010). Tseng and Peng (2007) addressed this problem through using a stochastic differential equation for degradation modeling of LED with a restrictive assumption that the proportion between the expectation and variance of the derivative of the degradation process was a constant. In contrast, Wang and Carr (2010) also tried to solve this problem by allowing the drifting parameter to be modeled by a Kalman filter, which made the Wiener process become adaptive to past data. As a result, the RUL can be estimated recursively using the Brownian motion. In their model the nonlinearity can be handled via the updating of the drifting parameter which is linear in one step but nonlinear across the whole path. Note that a common characteristic shared by these two studies is that certain assumptions must be satisfied, but are difficult to justify. In addition, Ebrahimi (2005) discussed some theoretical problems of assessing system reliability using diffusion processes. Generally speaking, the difficulties encountered are mainly two types. Firstly, calculating the distribution of RUL is equivalent to solve the FPK equation with constraints, which is rather difficult to solve under the nonlinear case in particular. Secondly, the available closed-form results of the PDF of the FPT for nonlinear degradation are limited for a few special cases (Mehr and McFadden, 1965; Buonocore et al., in press). Since the available numerical algorithms often require a long computation time and a large memory storage space and are not appropriate for online CBM decision support (Nardo et al., 2001), numerical solutions or analytic approximation methods are needed to be developed in future. One final comment related to both regression and Wiener process based models is whether the definition of the FPT as the RUL is appropriate or not since the degradation process can go back after the first time crossed the threshold level. This matter has not been thoroughly treated yet.

3.3. Gamma processes

Sometimes, degradation processes are monotonic and evolving only in one direction, as in wear processes or fatigue crack propagation for examples. In such cases, a Gamma process is a natural model for the degradation processes in which the deterioration is supposed to take place gradually over time in a sequence of tiny positive increments. In theory, a Gamma process $\{Y(t), t \geqslant 0\}$ has the following three properties (Abdel-Hameed, 1975): (i) The increment $Y(t_i) - Y(t_{i-1})$ for a given time interval $\Delta = t_i - t_{i-1}$ has a Gamma distribution $Ga(\nu(t_i) - \nu(t_{i-1}),\sigma)$ with shape function $\nu(t) > 0$ and scale parameter $\sigma > 0$; (ii) The increments for any set of disjoint time intervals are independent random variables having the distributions described in property (i); (iii) $Y(0) = 0$ almost surely. Using a Gamma process, the RUL at time t_i can be defined in a similar way as the Wiener process, i.e. $X_{t_i} = \inf\{x_{t_i} : Y(t_i + x_{t_i}) \geqslant w | Y(t_i) < w\} = \{x_{t_i} : Y(t_i + x_{t_i}) \geqslant w | Y(t_i) < w\}$. The last term is derived from the monotonic property of the Gamma processes. From the definition of the RUL using a Gamma process, we can see that the calculation is straightforward and can be obtained using the properties of the Gamma process described above. van Noortwijk (2009) presented an excellent review on the Gamma process in the context of maintenance. In his review, the rich theoretical aspects of the Gamma processes, such as the statistical properties, methods for estimation, approximation, and simulation were reviewed as well as successful maintenance applications. Therefore in our paper, we only consider the results in literature so far with relation to RUL modeling. For specific details of the Gamma processes, see the review by van Noortwijk (2009).

Gamma processes use the Gamma distribution which has a nice property that the sum of Gamma distributed increments is again a

Gamma variable. Gamma processes have been proven to be useful in determining the optimal inspection and maintenance decisions (Dieulle et al., 2003; Crowder and Lawless, 2007; Ebrahimi, 2009; Nicolai et al., 2009; Pandey et al., 2009). Singpurwalla (1995) considered the Gamma process as a model for degradation in a dynamic environment. Subsequently, Singpurwalla and Wilson (1998) addressed the issues of the two scales encountered in reliability and survival analysis: time and usage. The authors applied a Gamma process to model a usage process for inferring the lifetime distribution and used a Monte Carlo technique to simulate the developed model. Bagdonavicius and Nikulin (2000) used a Gamma degradation process that allowed covariates, but did not use the concept of the FPT to a damage threshold for lifetime modeling. Wang et al. (2000) proposed a model to predict the distribution of the residual lifetimes of three critical water pumps at a large soft-drinks manufacturing plant. In their paper, the authors assumed that the hazard rate was not deterministic but instead followed a Gamma process. Lawless and Crowder (2004) constructed a Gamma process-based degradation model incorporating covariates and a random effect to characterize the different rates among the different individuals. One of its advantages is that the failure time distribution can be formulated in a tractable way and thus the RUL estimation can be derived correspondingly. Park and Padgett (2005a,b, 2006) and Tseng et al. (2009) considered Gamma processes in accelerated degradation modeling and then inferred the lifetime distribution at the use condition. Some recent extensions were presented by van Noortwijk et al. (2007) and Kuniewslci et al. (2009) by combining a Gamma process with a Poisson process for modeling the degradation and the initial defect in order to determine the lifetime distribution.

The advantage of the Gamma process for RUL estimation is that the required mathematical calculations are relatively straightforward and the physical meaning is easy to understand. In contrast to the random coefficient regression methods, Gamma process-based degradation models can take the temporal variability into account as argued by Pandey et al. (2009). This property can also be applied to the aforementioned Wiener process based models and the forthcoming Markov based models since they are all based on those stochastic processes that can capture the dynamics of the degradation processes. For this reason, they have certain advantages over random coefficient regression methods. Nevertheless, we should note that the Gamma process seems only appropriate to represent degradation by a strictly monotonic process. This may restrict the application of the Gamma processes for degradation modeling. Similar to Wiener processes, modeling degradation as a Gamma process implies that the degradation process, given its current state, evolves to a future state independently of its past behavior due to its independent increment property. As a result, modeling degradation with a Gamma process implied a special Markov process with a continuous state and time space allowing transitions in one direction. Further, the noise in the Gamma process must be a Gamma distribution of a specified parameter structure, and therefore no room for other distributions including a general Gamma distribution. For these reasons, some modified Gamma processes should be incorporated into RUL modeling in further studies to enhance their modeling ability.

3.4. Markovian-based models

The underlying assumptions of Markovian-based models are twofold. One is that the future degradation state of the item depends only on the current degradation state, which is often termed as being memoryless. The other is that the system's state can be revealed directly by the observed CM information. In general, it is assumed that the degradation process $\{Y_n, n \geqslant 0\}$ evolves on a finite state space $\Phi = \{0,1,\ldots,N\}$ with 0 corresponding to the perfect healthy state and N representing the failed state of the monitored system. The RUL at time instant n can be defined as $X_n = \inf\{x_n : Y_{n+x_n} = N | Y_n \neq N\}$. Assuming that the transition probability matrix is $\mathbf{P}$, then the matrix can be written as

$$\mathbf{P} = \begin{pmatrix} \widetilde{\mathbf{P}} & \mathbf{P}_0 \\ \mathbf{0} & 1 \end{pmatrix} \quad \text{with } \mathbf{P}_0 = (\mathbf{I} - \widetilde{\mathbf{P}})\mathbf{e}. \tag{3}$$

where $\widetilde{\mathbf{P}}$ is the transition matrix for transient states $\Phi \backslash \{N\}$, $\mathbf{I}$ is the identity matrix and $\mathbf{e} = (1,\ldots,1)^T$ is a column vector with dimensions $N-1$. In principle, RUL estimation using Markovian-based models can be captured by computing the FPT which is the amount of time the process will take to transit from the current state to the absorbing state N for the first time. Usually, formulating the probabilistic property of the RUL is involved to calculate the Phase-Type (PH) distribution (Delia and Rafael, 2006, 2008). An excellent review on the PH distribution in Markov chains is provided by Aalen (1995). It follows that the distribution and expectation of the RUL can be obtained as,

$$\Pr(X_n = k) = \boldsymbol{\alpha}_n \widetilde{\mathbf{P}}^{k-1}(\mathbf{I} - \widetilde{\mathbf{P}})\mathbf{e}, \quad E(X_n) = \boldsymbol{\alpha}_n (\mathbf{I} - \widetilde{\mathbf{P}})^{-1}\mathbf{e}, \tag{4}$$

where $\boldsymbol{\alpha}_n = (\alpha_n(0), \alpha_n(1), \ldots, \alpha_n(N-1))$ represents the distribution of the current degradation status such as $\alpha_n(i) = \Pr(Y_n = i)$, $i \in \Phi \backslash \{N\}$.

The above representation only considers the simplest case. In literature, Kharoufeh and his co-authors have made many work in this framework. Kharoufeh (2003) considered the reliability of a single-unit system whose cumulative damage over time was a continuous wear process that depended on an external environment process. The external process was characterized as a time-homogeneous Markovian environment with continuous time. The key development was to calculate the PH distribution. In order to lower the dimension and reduce the computation load, a one dimension Laplace–Stieltjes transform was used to calculate the RUL distribution in a closed-form by Kharoufeh and Sipe (2005). Further, Kharoufeh and Cox (2005) extended such work by considering two kinds of sensors data, environment observations and degradation measures in their stochastic failure models to numerically compute the RUL distributions and their moments. However, the number of the states in a Markovian model was estimated by a K-mean clustering algorithm which implied that there must be sufficient data on failures and degradation to estimate the model parameters. Recently, Kharoufeh and Mixon (2009) extended the work of Kharoufeh and Cox (2005) by proving several limit theorems related to a time-scaled version of the degradation process and a space-scaled version of the unit's random lifetime. Although their models were mathematically appealing and easy to implement, they lacked the flexibility to account for the environment state sojourn times or shock inter-arrival times which may not be exponentially distributed. Additionally, Lee et al. (2010) incorporated the Markov property into a regression model and presented a new model for the survival analysis called Markov Threshold Regression, in which the subject's health followed a stochastic process and failure occurred when the process first reached a failure state.

In summary, we can learn that Markovian-based models have been widely applied to RUL estimation and to maintenance decision making support. The main reason is that the plant operation condition can be divided into several meaningful states, such as "*Good*", "*OK*", "*Minor defects only*", "*Maintenance required*", "*Unserviceable*", so that the state definition is closer to what is used in industry than other stochastic models, and therefore is easy to understand. On the other hand, there is a strong mathematical basis in the Markov theory underpinning the model for reliability analysis and RUL estimation. However, it is worth noting that RUL estimation with Markov models suffers some limitations. First, the central property of Markov models is the conditionally independent or memoryless assumption. This is a relatively strong assumption and thus may lead to an approximation for the true process. However, it lacks practical justifications in many cases, and therefore, not many case studies have been reported. It is interesting to note that there is no available method for testing the Markov property for a general system, though the Markov theory has been presented over the past hundred years. Second, the system health state sojourn time is assumed to be exponentially distributed in the Markov models and thus may be inappropriate for some cases. However we are aware that semi-Markov-chain-based models can partially resolve this issue. For a recent extension to semi-Markov chains for RUL estimation, see Kharoufeh et al. (2010). Finally, the transition probability among the system states in Markov models is often determined by empirical knowledge or by a large number of samples, which is not always available. Suffice to say that there is still room for more advanced and practical methods either in theory or in practice, and thus much work still needs to be carried out under a Markov framework for RUL estimation.

The above reviewed models are almost all based on the assumption of a directly observed degradation process which can reveal the state variable directly. However, in many CBM practices, the CM data observed can only indicate the likelihood of the underlying state process partially or indirectly. In such circumstances, the aforementioned methods may be invalid. To remedy this weakness, models based on indirectly observed state processes have been developed and reported in literature. This is the topic of the next section.

4. RUL estimation models based on indirectly observed state processes

There are three main types of models which have been used for RUL estimation fallen into this category. The first is the filtering type of models developed in Wang and Christer (2000) (see e.g. Wang, 2002b; Batzel and Swanson, 2009; Orchard and Vachtsevanos, 2009; Tang et al., 2010), and the second one is the covariate-based hazard model, such as, proportional hazard model (PHM) and its variants (Makis and Jardine, 1991; Banjevic and Jardine, 2006; Ghasemi et al., 2010; Vlok et al., 2004). Since 2000, as the third kind of methods, hidden Markov model (HMM) based methods have been applied to address RUL estimation problems due to its concept clarity (Bunks and McCarthy, 2000; Baruah and Chinnam, 2005; Camci and Chinnam, 2010). Further, an extension to HMM, hidden semi-Markov models (HSMM), have also been used for estimating the RUL since 2006 (Dong et al., 2006; Dong and He, 2007a,b). In the following, we will survey these three main models in this category used for RUL estimation specifically.

4.1. Stochastic filtering-based models

To our best knowledge, Sarma et al. (1978) first used the Kalman filtering technique for establishing a decision model for health monitoring of aero-engines. Christer et al. (1995) proposed a CM model using stochastic filtering, in which the conditional residual life is defined as the time lapse from any time point that monitoring information is obtained to the time that it may be declared to be failed given no other maintenance involved. In Wang et al. (1997), a state space model for the unobserved condition and observed condition related variables was established to model the conditional residual time. Such a model, in its most simple form, considers both the unobserved condition, x_t, and the observed CM data, y_t, as non-stationary stochastic processes such that, $x_t = \alpha x_{t-1} + \varepsilon_t$ and $y_t = \beta x_t + \eta_t$ where ε_t and η_t are Gaussian noises (disturbance factors) and α and β are the parameters of the state space model (in general α and β are time varying non-scalars). The Kalman filter was used to predict x_{t+k} given CM information to date, $Y_t = \{y_1, \ldots, y_t\}$. Christer et al. (1997) applied this method to a case study relating to the monitoring of the refractory thickness in an inductor furnace used in the refining of copper. For decision modeling purposes, the method used in Christer et al. (1997) depends on setting a critical level, C, for x_t. One difficulty lies in relating the unobserved condition to the CM variable(s). Another difficulty is the selection of C when the condition variable x_t is unobservable. The Kalman filtering approach overcomes the problem using only the last CM readings in most published models since it uses the history information to time t. It is noted, however, that the linear and Gaussian assumptions restrict the application of the Kalman filtering approach. Similarly, Batzel and Swanson (2009) presented a RUL estimation method based on the Kalman filter for aircraft power generators. In their work, it was assumed that the relationship between the RUL and the estimated state followed a time-dependent function. Hence, the RUL estimation was achieved by minimizing the difference between the value of such function and a pre-determined state threshold. Then, only a point estimate of the RUL can be obtained. Recently, Tang et al. (2010) also used a stochastic filter based method to achieving RUL estimation under several special cases, such as the Benes filter, particle filter and multiple-model filter. They illustrated how to use these filters to estimate the PDF of the RUL through some case studies. However, the PDF of the RUL was dependent on the selected threshold and can only be approximated by numerical simulation technique in most cases. In addition, Luo et al. (2008) used a multiple-model filter to estimate the mean and variance of the RUL without considering the distribution of RUL explicitly. Similar idea can also be found in Phelps et al. (2007). Orchard and Vachtsevanos (2009) presented a failure prognostic model to predict the evolution in time of the fault indicator and compute the PDF of the RUL of the faulty subsystem. They used a non-linear state-space model (with unknown time-varying parameters) and a particle filtering algorithm that can update the current state estimate. For a good comparison between particle filtering and statistical learning for RUL estimation, see Saha et al. (2009). All of the above filtering-based models have one drawback in common in that they need a threshold level. The subsequent stochastic filtering-based models do not need the threshold as we will observe.

In order to overcome the limitation of the Kalman filtering approach in Christer et al. (1997), Wang and Christer (2000) further developed the RUL estimation models using a semi-stochastic, non-Gaussian and non-linear filtering technique. Conceptually, the RUL of an asset is unknown but one thing we do know is that over an interval of time, the RUL is just an interval shorter at the end of the interval than at the beginning of the interval if nothing happened during that interval. In the mean time we may observe an increasing or decreasing trend of the monitored CM information. Based on these observations, in their paper, the problem can be formulated as follows with a simple and intuitive form. If we define x_t as the RUL at time t, the current monitoring check point, and the relationship between x_t and y_t, x_{t-k} can be described as $x_t = x_{t-k} - (t-k)$, if $x_{t-k} > t-k$ and $y_t = g(x_t, \eta_t)$ where g is a function to be determined, η_t is a noise term, k is the time of the last monitoring check and $t-k$ is the interval between the current and the last check. In this kind of models, a concept called a floating scale parameter was used to model the relationship between x_t and y_t which was usually modeled by a Weibull distribution. The idea was to let the scale parameter of y_t be a function of x_t, which enabled a updating mechanism of the scale parameter.

Under the above framework, the key for RUL estimation was to formulate the relationship between x_t and the condition monitoring history $Y_t = \{y_1, \ldots, y_t\}$. It is shown that this relationship can be established by recursive filtering as follows (Jazwinski, 1970; Wang and Christer, 2000)

$$p_t(x_t|Y_t) = \frac{p(y_t|x_t)p_{t-k}(x_t + t - k|Y_{t-k})}{\int_0^\infty p(y_t|x_t)p_{t-k}(x_t + t - k|Y_{t-k})dx_t}. \quad (5)$$

In Wang and Christer (2000), they applied this technique to the same data set of the furnace case described in Christer et al. (1997). Instead of predicting the thickness of the refractory, a RUL model was used and no such a critical level C was required. This may be one of the most appealing features in applications since the threshold is difficult to obtain. In addition, the effectiveness of preventive maintenance was also considered, which required an additional function to be established to describe the relationship between the RULs before and after the preventive maintenance actions. This function can be easily incorporated into the semi-stochastic filtering framework, which significantly improved the flexibility for RUL modeling and for further scheduling maintenance strategy. It is of interest to note that Myötyri et al. (2006) demonstrated a similar idea to Wang and Christer (2000). The difference is that Myötyri et al. (2006) proposed a real-time lifetime prediction method based on stochastic filtering but considered the uncertainties in both the degradation process and CM measures. However the degradation process was modeled as a discrete time finite Markov chain. As for the application of RUL estimation in CBM, see Wang (2002b, 2003) for example. Carr and Wang (2008) provided a specific case comparison of a semi-stochastic filter and other approaches for RUL estimation using oil-based CM information in CBM. Since the semi-stochastic filter uses a casual relationship between the observed CM data and the underlying RUL often observed in practice and also the full history of CM information, it is not surprising that the filter-based approach outperforms the other methods in this particular case.

Other extensions of semi-stochastic filtering for RUL estimation include four aspects. Wang (2007b) combined the delay time model (DTM) (Baker and Christer, 1994), with semi-stochastic filtering for RUL estimation. The successful use of the DTM concept in RUL depends heavily upon how accurately the initiated point of the failure delay stage can be detected and how well the failure delay time distribution can be estimated from the CM information. In his study, Wang (2007b) overcame the above two difficulties under a Bayesian framework and a probability model was presented to achieve these aims. As the second extension, Wang and Zhang (2008) further incorporated expert judgments into the previously developed model. Expert judgment information is a valuable source of information to be used in RUL estimation and maintenance decision making. It can be seen as complementary information to the limited CM data in some practices. Particularly, in safety–critical systems, a large number of CM data is not available in general. The last two extensions considered the influences of external variables (Wang and Hussian, 2009) and multiple failure modes (Carr and Wang, 2010) respectively.

The main problems in the abovementioned semi-stochastic filtering-based models are: (1) One dimensional input of $y_t|x_t$ was assumed or otherwise a multiple dimension PDF or some kind of dimension reduction technique must be used. This is difficult especially when there are a large number of observed CM variables. (2) It was assumed that there were no other actions during the CM check interval, so a deterministic relationship between x_t and x_{t-k} can be obtained, which greatly simplified the subsequent modeling process. However, it is likely that preventive maintenance actions can take place or even the operating environment can change during the interval so the deterministic relationship proposed as $x_t = x_{t-k} - (t - k)$, if $x_{t-k} > t - k$ may not be held. The problem can be partially solved by using a deterministic function, say $x_t = f(x_{t-k}, t - k)$ to incorporate some of the changes happened during the interval. However if such changes are stochastic then a closed form of the required RUL prediction model does not exist unless in very few special cases, Tang et al. (2010). (3) In formulating $y_t = g(x_t, \eta_t)$ it assumes a one way relationship between y_t and x_t and in some cases a reversed relationship between some y_t and x_t may also exist. Wang and Hussian (2009) partially addressed this problem, but still more research is required. (4) The last one which is common to most models discussed in this section is that failure event data is required for model parameter estimation, which may be scarce in reality.

4.2. Covariate based hazard models

In many practical situations, such as the wear-out of a mechanical component or the deterioration of an electronic device, the degradation process is caused by one or more factors that are called covariates. For example, a wear-out process can be affected by temperature, material properties and running speed. Some researchers even considered the CM information as covariate information (Scarf, 1997) depending on the definition of the CM information. These covariates change stochastically and may influence and/or indicate the lifetime. Therefore, it is important to incorporate these covariates in lifetime modeling (Leemis, 1987; Lawless and Crowder, 2004).

As one of the most reported covariate-based models, the proportional hazards model (PHM) has been very popular since Cox's pioneering work in lifetime analysis (Cox, 1972). The paper by Cox (1972) that proposed the PHM is one of the top most referenced papers in statistical sciences due to its generality, flexibility, and simplicity. As a result, PHMs have been widely used to relate the system's CM variables and external factors to the failure of a system and hence have been applied in different areas of life data analysis and CBM (Love and Guo, 1991; Jardine et al., 1997; Kumar and Westberg, 1997; Lugtigheid et al., 2008; Zhao et al., 2010). The number of applications of the PHM in literature is increasing every year. An excellent review of the literature on the PHM can be found in Kumar and Klefsjö (1994). The most important advantage of the PHM with time-dependent covariates over the other statistical approaches is that covariate information can be easily combined with a baseline hazard function. Thus, the effect of different covariates on the total hazard can be easily evaluated.

The conventional PHM assumes that the hazard rate of a system at time t consists of two multiplicative factors, a baseline hazard function and a function of covariates, that is, $h(t|\mathbf{z}(t)) = h_0(t)c(\boldsymbol{\beta}\mathbf{z}(t))$ where $\boldsymbol{\beta}$ is a column vector consisting of the corresponding regression coefficients and $\mathbf{z}(t)$ is the covariate vector. In this model, the baseline hazard rate $h_0(t)$ can be either nonparametric or parametric. When the baseline function is nonparametric, it can be estimated using the failure event data and censored data, such as the Kaplan–Meier estimator (Kaplan and Meier, 1958) and the Nelson–Aalen method (Aalen et al., 2008). In the case that the baseline hazard $h_0(t)$ is specified to a parametric distribution, the Weibull distribution is widely used as the baseline function for PHM (Vlok et al., 2002; Jardine et al., 2008; Lin et al., 2006).

Let $\mathbf{Z}(t)$ denote the entire covariate information history up to time t, that is, $\mathbf{Z}(t) = \{\mathbf{z}(s), 0 \leqslant s \leqslant t\}$, and $\mathbf{z}(s)$ is the information obtained at time s. Then, based on the PHM, the RUL can be defined as $X_t = \{x_t\colon\ T - t|T > t, \mathbf{Z}(t)\}$, where T denotes the lifetime. Therefore, the PDF of the RUL at time t can be easily formulated as follows,

$$f_{x_t}(x_t|\mathbf{z}(t)) = \frac{f(t + x_t|\mathbf{z}(t))}{R(t|\mathbf{z}(t))} = h(t + x_t|\mathbf{z}(t))\frac{R(t + x_t|\mathbf{z}(t))}{R(t|\mathbf{z}(t))} \quad (6)$$

with $R(t|\mathbf{z}(t)) = P(T > t|\mathbf{Z}(t)) = \exp\left\{-\int_0^t h(s|\mathbf{z}(s))ds\right\}$. The detailed treatment can be found in Banjevic (2009) and Zuashkiani et al. (2009).

From the above formulation, we observe that a PHM needs the event data such as failure and censored data as well as CM information to predict the RUL without an exact failure threshold. Liao et al. (2006) addressed a problem of modeling multiple degradation features and hard failures in the health monitoring of critical individual units using a PHM. An interesting case study incorporating some extensions can be found in Wang and Zhang (2005). Two main extensions are that the RUL was directly proportional to the observed covariates and the principal component analysis (PCA) has been adopted to reduce the dimension of the original data set. Similar to Wang and Zhang (2005) and Lin et al. (2006) also used PCA to reduce the number of covariates in the PHM, as well as to eliminate possible collinearity between the covariates.

A version of the PHM proposed by Makis and Jardine (1992) was used where the evolution of the CM information was modeled using a discrete-state Markov process and the transition rates were estimated from the available data. Among many other studies, the PHM has been used in a hazard alarm setting in Vlok et al. (2002) and in a RUL estimation context in Banjevic and Jardine (2006). Elsayed (2003) showed a PHM application using the Weibull distribution for furnace tube failures with the scale parameter depending on the operating temperature. In his work, the covariate (temperature) was time independent, which may be restrictive. As such, Banjevic and Jardine (2006) considered a more complex situation than Elsayed (2003). The failure process along with the covariate process was represented by a discrete non-homogeneous Markov chain. They discussed how to infer the RUL from such double processes and presented some approximation algorithms for calculating the RUL. The appealing point is that the estimated RUL can incorporate the dynamics of the plant's operating condition. As a result, such RUL estimation considered the real-time CM information and thus overcame the drawback of traditional PHM based RUL estimation methods in which the time dependent and stochastic behavior of covariates are not fully considered. Descriptions of an attempt to make the model more available to industrial applications can be found in Banjevic et al. (2001) and Jardine et al. (1997, 1999), in which an applicable software, EXAKT, has been developed based on the PHM proposed to calculate the optimal maintenance or replacement time interval utilizing the failure history (both event and CM data) as well as cost data. One of three main functions of the EXAKT is to calculate the RUL based on the PHM with Markov changing covariates. Banjevic (2009) discussed some practical considerations for the RUL in theory and practice. He proved that the limiting distribution of a standardized RUL is exponential. As such, he argued that a point prediction of the RUL is relatively inaccurate and may not be very useful since the variability of the RUL is relatively large. However, we note that he only considered the case that the hazard function is free from the effect of covariates. When the covariate information is incorporated, it has been observed that the variability of the RUL was substantially reduced (Wang and Zhang, 2005). Banjevic (2009) also presented an insightful discussion on some practical issues when the CM information was incorporated into the calculation of RUL. Recently, Ghasemi et al. (2010) proposed a model to calculate the mean RUL of a piece of equipment. A PHM was used to model the failure and the mean RUL while the unobservable degradation state was modeled by a hidden Markov model. But this model assumed that the transition matrix was prior known and the observations followed a discrete distribution. This is sometimes not always available for a practical case. Recently, Zhao et al. (2010) presented a CBM model through PHM with environment covariates modeled by a homogeneous Markov chain. You et al. (2010) developed a two-zone PHM model to predict equipment RUL, in which the lifecycle was divided into two zone, i.e. a stable zone and a degradation zone. The modeling concept in this work is similar to the two-stage model in Wang (2007a) but with different statistical models. One critical point is that the accuracy of the RUL estimation is dependent in part on the selection or detection of the change-point from the stable zone to the degradation zone.

In the framework of covariate based models, there are some other variants based on the basic PHM to achieve hazard modeling, such as proportional intensities model (Vlok et al., 2004; Percy and Alkali, 2007), proportional covariate models (Sun et al., 2006), additive hazard models (Aalen et al., 2008), etc. For an extensive discussion on practical problems of covariate based models for reliability modeling, see Gorjian et al. (2009a). It is noted, however, that those variants have fewer applications than PHM. Even if other variants have relaxed some assumptions in PHM and indeed have potentials for RUL estimation in terms of the theoretical basis and feasibility in concept, there is still some way to go before they become accepted by practitioners.

The main problems of using covariate-based hazard models for RUL estimation are: (1) The models mixed the casual relationship of different covariates. Some may impact on the hazard and some may be influenced by the hazard. For instance, in the oil analysis programme of an engine, metal concentrations are good indicators of an engine's wear. However, the contaminants in the engine oil can impact on the wear rather than indicate the wear. So these two types of elements in an engine's oil are functionally different towards the engine' wear and should be modeled differently. Wang and Hussian (2009) called metal concentrations as internal variables and contaminants as external variables and modeled them differently. (2) When the evolution of covariates is stochastic, another process (mostly a Markov chain) has to be used for describing the covariate process, which is an added burden to the model (Banjevic and Jardine, 2006; Ghasemi et al., 2010). (3) From the definition of a PHM, we have $h(t|\mathbf{z}_1(t)\ \ )/h(t|\mathbf{z}_2(t)) = c(\boldsymbol{\beta}\ \ z_1(t))/c(\boldsymbol{\beta} z_2(t))$ which is proportional to the difference of covariates at any given time instance, sometimes known as the proportionality assumption. This assumption imposes a severe limitation in which the survival or reliability curves, for engineering assets with different covariates but the same base-line hazard, will never cross (Gorjian et al., 2009a). (4) The parameters in the baseline hazard function and those $\boldsymbol{\beta}$s must be estimated together at the same time because of the multiplicative effect between them. This requires sufficient failure event data and associated CM information which may not always be available in practice. This contrasts with the semi-stochastic filtering based models where the parameters in the initial life distribution, $p(x_0)$, can be estimated separately, which may require less event data than the PHM-based models. These problems pose some challenges for future research.

4.3. Hidden Markov model (HMM) and hidden semi-Markov model (HSMM) based methods

HMM is composed of two stochastic processes, a hidden Markov chain $\{Z_n,\ n \geqslant 0\}$, which is unobservable and represents the real state of the deterioration, and an observable process $\{Y_n,\ n \geqslant 0\}$, which is the observed CM information from monitoring and tests. Similar to Markovian-based models in Section 3.4, it is assumed that the degradation process $\{Z_n,\ n \geqslant 0\}$ evolves according to a Markov chain on a finite state space. Generally, a conditional probability measure $P(Y_n|Z_n = i),\ i \in \Phi\backslash\{N\}$ is used to represent the relationship between $\{Y_n,\ n \geqslant 0\}$ and $\{Z_n,\ n \geqslant 0\}$. As such the RUL at time instant n can be defined as $X_n = \inf\{x_n : Z_{n+x_n} = N|Z_n \neq N, Y_j, 0 \leqslant j \leqslant n\}$. It is worth noting that calculating the distribution of X_n is non-trivial and challenging.

For the stochastic filtering-based RUL models in Section 4.1, two stochastic processes are proposed, one is the unobserved state evolution process and the other is the observed CM information process. To a broad sense those models are the continuous state HMMs. Since most of the literature we encountered in HMMs for

RUL estimation used Markov chains so our review is centred on hidden Markov chain-based RUL models. This makes the stochastic filtering-based RUL models different from the HMM-based RUL models in that the former used a continuous state variable of x_t which may be the RUL directly, while the HMMs we reviewed used discrete state variables. A HMM is composed of two stochastic processes, a hidden Markov chain, which is unobservable and accounts for the real state of the deterioration, and an observable process, which accounts for observation obtained from monitoring and tests. As a result, a conditional probability measure is used to represent the relationship between the observations and the hidden state.

At the beginning of the new century, Bunks and McCarthy (2000) first described a case to apply a HMM-based technology to the problem of CBM and applied the model developed to Westland helicopter gear box data. They argued in their paper that two features of the HMM were particularly useful for the monitoring of machine health. The first was that computationally efficient methods existing for computing the likelihoods using HMMs and secondly there existed efficient techniques which can be used for system identification using HMMs Rabiner (1989). However, parameters estimation for HMMs may not be that easy and efficient, especially for online identification. Many researchers have pointed out that the major limitations of HMMs' parameters estimation are of heavy computation and need a large memory, Azimi et al. (2005). Another point worth noting is that, in Bunks and McCarthy (2000), each state was associated with an HMM. As a result, this required a significant effort for online parameter estimation for each HMM. Lin and Makis (2002, 2003) adopted a continuous time hidden Markov chain with a finite state space to describe the state process in a stochastic model with partially observed working states which were represented by CM variables. An EM algorithm was applied to obtain the mean RUL given the observations up to a certain time. Similarly, Wang (2007b) used the concept of an HMM and stochastic filtering in modeling the initial point of a random defect, and then the time to failure. However, in the work of Lin and Makis (2002, 2003), the discrete approximations that result from choosing discrete state and observation processes to represent the continuous CM and state information may result in the loss of information provided in the original data. To remedy this weakness, Wang (2007a) presented a prognosis model for wear prediction based on a HMM and stochastic filtering, in which a continuous random variable was used to represent the unobservable state instead of using a discrete approximation. However, computation time increased substantially.

Along the lines of the above work, Baruah and Chinnam (2005) presented a novel method for employing a HMM to carry out both diagnostic as well as prognostic activities for metal cutting tools. The method employed a HMM for modeling sensor signals emanating from the machine, and in turn, identified the health state of the cutting tool as well as facilitated RUL estimation. From Baruah and Chinnam's modeling process, one can learn that the accuracy of such a method depends highly on the sample size. The previous mentioned methods for RUL estimation considered homogeneous Markov chains only. As for a further extension, Wang (2006) reported on a HMM-based model to assess the current and future states of a monitored system based on measured condition monitoring information to date, where the transition of the system state followed a time-dependent Markov chain. The delay-time-model was used to model the transition probability of the system state.

Chinnam and his co-authors recently extended the work in Baruah and Chinnam (2005) in two ways. One was to achieve autonomous yet effective diagnostics and prognostics by employing an unsupervised competitive learning algorithm (Chinnam and Baruah, 2009), and the other was to estimate online the health-state and RUL by introducing both hierarchical HMMs and dynamic Bayesian networks (Camci and Chinnam, 2010). In fact, Bunks and McCarthy (2000) first mentioned the potential that a HMM can be used in a hierarchical fashion where the state of a HMM can contain another entire HMM for prognostics. In Camci and Chinnam (2010), the authors used this concept, and health-states were represented as distinct nodes at the top of the hierarchy. Further, RUL estimation was achieved by formulating the state transition probabilities derived from a hierarchical HMM. The two extensions were demonstrated by a drilling process and it seemed that these were effective and outperformed other HMM methods from literature based on such a specific case study. However, those papers mentioned above paid little attention to parameters estimation in HMM and the transition probability matrix was specified using a prior knowledge of the case involved.

Some other useful references on HMMs in failure prognosis and RUL estimation are Ocak et al. (2007), Tai et al. (2009) and Zhou et al. (2010). We note firstly that one of the main characteristics in such methods is that it is assumed that the condition transition of the system obeys the Markov property. However, since the true state transition is unobservable, and hence the models based on the Markov property can be considered as an approximation. As a result, from an application or a physical interpretation point of view there is a clear need to test the validity of the Markov property and analyze the influence on the estimated results if the Markov property is not satisfied. Secondly, due to the non-zero probability of self-transition of a non-absorbing state, the state duration of a HMM is implicitly a geometric distribution. This restricts the application of HMMs to RUL estimation, especially in maintenance decision making. Thirdly, due to its independence of the past history, caused by its Markov property, a HMM has a limited power in modeling the temporal structures of the prediction problems. The fourth and the final limitation is that only the mean and variance of the RUL can be estimated so far and it is difficult to obtain an explicit distribution of the RUL in a closed-form. Therefore, much additional work is required for extending this kind of models for RUL estimation.

As a generalization of HMM, HSMMs are useful in many engineering applications, such as signal estimation, diagnostics and prognostics, and many others. For such an extension, a HSMM is traditionally defined by allowing the unobserved state process to be a semi-Markov chain. As indicated by Guédon (1999), hidden semi-Markov chains possess both the flexibility of hidden Markov chains for approximating complex probability distributions and the flexibility of semi-Markov chains for representing temporal structures. Recently, Yu (2009) presented an excellent tutorial on the principle, estimation algorithms, developments and applications of HSMM.

Although a HSMM is appealing in concept and flexibility, and has been successfully applied in speech recognition (Yu, 2009), the literature of HSMMs in RUL estimation is very limited. Since 2006, the HSMM has attracted some attentions in health state prognostics. Dong et al. (2006) showed a case study of a HSMM based method on equipment health diagnostics and prognostics as well as RUL estimation of an UH-60A Blackhawk main transmission. Through the experimental studies, the authors argued that HSMMs offered several significant advantages over HMMs in equipment health diagnostics and prognostics. Further, Dong and He (2007a) first presented a HSMM based framework and methodology for multi-sensor equipment diagnosis and prognosis. The duration of a health state was modeled by an explicit Gaussian probability function. The estimated state duration probability distributions can then be used to predict the RUL of a component. Subsequently, Dong and He (2007b) presented a statistical modeling methodology for performing both diagnostics and prognostics in a unified framework. The methodology was developed based on a segmental HSMM. In the proposed framework, for each health state of components, a HSMM was built and trained. Each health state of a component corresponded to a segment of the HSMM. For prognostics, another HSMM was used to model a component's life cycle. The estimated state duration probability distributions combined with a state-changing point detection can then be used to predict the RUL of a system. Although the results from HSMMs are promising, a HSMM has the same drawback as a HMM, that is, it is assumed that observations are independent of its history. Along the lines of these works, two recent extensions have been developed. One incorporated an auto-regressive structure in a HSMM (AR-HSMM) modeling process to avoid the Markov chain's memoryless property (Dong, 2008). The other considered the aging factors in the HSMM to characterize the deterioration of the system, and then developed an improved RUL computation method (Peng and Dong, 2011). However, the aging factors were determined by a prior knowledge. It is noted also that the AR-HSMM used only partial history information and it was not completely free from the memoryless assumption. On the other side, one of the main drawbacks is, the same to HMMs, HSMM-based methods can only conduct mean RUL estimation and its variance, and cannot provide a PDF of the RUL. Although a HSMM is more powerful than a HMM in modeling RUL estimation, a HSMM leads to more complex parameters identification problems (Yu, 2009; Azimi et al., 2005). In addition, for specific applications in RUL estimation, little attention has been paid on parameters identification algorithms in HSMMs as well as HMMs, though this is a key element for successfully implementing the proposed methods, especially when the size of the samples is small. Therefore, any new contribution on such problems seems to be valuable in view of the potential applications to health state modeling and RUL as well as further maintenance planning and scheduling.

Besides the above discussed models, there are some other works which also considered the fact that the degradation cannot be observed directly sometimes by instruments or sensors and thus is latent or hidden (Xu et al., 2008; Li et al., 2010). One interesting extension is the bivariate Wiener process of Whitmore et al. (1998) and Lee et al. (2000) in which the latent processes of the degradation and an observable process are combined to estimate the lifetime. Recently, two extensions were conducted along this line. One considered the threshold was unknown or random (Singpurwalla, 2006) and the other assessed the lifetime using the intermediate data which were obtained from degradation processes over certain non-failure thresholds (Tang and Su, 2008). However, due to the complicated stochastic relationship between the hidden degradation process and the observed CM process, RUL modeling gets more difficult than the models based on directly observable state processes. Nevertheless, the idea behind this kind of RUL modeling is more applicable and closer to engineering applications, and thus seems promising and deserves further research.

5. Conclusions and future challenges

In this paper, we have attempted to review and summarize the recent modeling developments for estimating the RUL. The review is centred on statistical data driven approaches which use only available past observed data and statistical models. We first review the existing review papers in relation to RUL estimation and show that there is a need for this comprehensive review. The reviewed RUL estimation models are classified into two broad types of models, that is, the RUL estimation models based on the directly observed state processes, and those cannot be observed directly. Various techniques, models, algorithms and their modeling principles have been discussed with extensive references quoted. We critically review the pros and cons of the approaches reported in the literature. Specifically, there are a number of challenges and practical problems to be further studied before valid and robust models can be applied to practical systems. These challenges and problems also have theoretical implications which are not well dealt with in the current literature. We summarize them as follows.

(1) It is desirable to develop a RUL estimation model based on very few or no data situations. This is typical for newly commissioned systems where no observed failure data and CM information exist (Silver and Fiechter, 1995; Percy, 2002). In this case, the quantity and completeness of data was insufficient to fit the full statistical models discussed previously. This may render the need of physics-based models with the help of subjective expert knowledge from design and manufacturing. However, how to establish a physics-based RUL model and how to best use subjective judgmental information in RUL estimation remain a challenge. Pecht (2008) discussed in depth of some physics-of-failure models in electronics prognostics. For more details and discussions on collecting and processing subjective information, see van Noortwijk et al. (1992), Wang (1997), Zuashkiani et al. (2009), Doyle et al. (2009). Up to now, however, few researches consider the effect of subjective information in RUL estimation.
(2) The second challenge lies in data fusion where multi-dimensional CM input data must be dealt with. Typically we can see in CM practice that multi-channel CM data may be available, which may be correlated and not on the same scale and measurement units. This will also impose a severe challenge to threshold-based models which were mostly established under a single threshold level. In this case how to effectively use all available information to estimate the underlying RUL is a question not being well answered.
(3) The third challenge is how to model the influence of external environmental variables such as speed or loading on RUL estimation. This is a complicated issue since those variables will impact on the observed CM variables which in turn will influence the RUL estimation. If it is not done properly, overfitting can occur, which may reduce the robustness of the developed estimation model.
(4) The fourth challenge is the development of a model which can deal with multiple failure modes for a single component, which is again a common scenario observed in CM practice. Obviously this will further complicate the issue since for some CM techniques, the observed signals are heavily influenced by the underlying failure modes, and therefore the RUL is failure mode dependent.

In addition, some of aforementioned models are simply theoretical formulations without real application focuses. Therefore, much more work should be done to speed up the progress of engineering-oriented studies. As such, we can state that the pace of the development of RUL estimation models for maintenance is very quick in recent years but there is still a long way to go.

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