BUILDING HEATER MODEL

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NOTATION

Sets

$\mathcal{T}$	Set of time steps t in the simulation's time horizon

Parameters

$T_t^{(a)}$ or $T_{\text{ambient}}(t)$	Ambient temperature at time t
T_{initial}	Initial temperature of the building in °C
$c(t)$	Price of electricity at timet in \$/MWh
α	Heating coefficient in °C/kWh
k	Cooling coefficient in 1/h

Variables

T_t or $T(t)$	Temperature of the building in °C
P_t or $P_{\text{heater}}(t)$	Power used by the heater in kW

1 INTRODUCTION

This technical report introduces a methodology to compute the optimal schedule of a heater used to maintain the temperature of a building within certain bounds.

The following assumptions are taken:

- the ambient temperature is known and fully predictable;
- the electricity prices are known and fully predictable;

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- the building temperature can be reasonably approximated using Newton's law of cooling.

2 PROBLEM STATEMENT

Let $\mathcal{T}$ denote the set of time-steps comprising a simulation horizon such that $\mathcal{T} = \{0, \ldots, T-1\}$, where $T = |\mathcal{T}|$ is the cardinality of the set.

Knowing Newton's law of cooling, which is given by:

$$\frac{dT(t)}{dt} = k\left(T_{\text{ambient}}(t) - T(t)\right) + \alpha P_{\text{heater}}(t), \tag{1}$$

where t is the time since initial conditions, T is the building temperature, T_{ambient} is the ambient temperature, P_{heater} is the power supplied to the heater, and k and α are the cooling and heating coefficients, respectively.

Assuming an infinitesimally small dt, this differential equation can be approximated and solved for $T(t)$:

$$T(t) = T_{\text{ambient}}(t) + \frac{\alpha}{k} P_{\text{heater}}(t). \tag{2}$$

The ambient temperature is given by:

$$T_{\text{ambient}}(t) = 15 - 10 \sin\left(\frac{\pi}{12}(t+4)\right), \tag{3}$$

and the cost per unit of energy (\$/MWh) is given by:

$$c(t) = 40 + 25 \sin^2\left(\frac{\pi}{12} t\right). \tag{4}$$

With these data, the problem to solve is how to compute the optimal power fed to the heater so as to maintain the building temperature within certain bounds. Indeed, the temperature can be kept within some bounds:

$$\underline{T} \leqslant T(t) \leqslant \overline{T}. \tag{5}$$

Likewise, the power fed to the heater must be kept within some bounds:

$$\underline{P} \leqslant P_{heater}(t) \leqslant \overline{P}. \tag{6}$$

3 PROBLEM FORMULATION

This problem can be formulated as a linear program, and solved using mathematical programming techniques.

The objective function is:

$$\min_{z \in \mathcal{Z}} \sum_{t=0}^{T-1} P_t \left[40 + 25 \left(\sin \left(\frac{\pi t}{12} \right) \right)^2 \right] \Delta t, \tag{7}$$

subject to:

$$T_t^{(a)} = 15 - 10 \sin \left(\frac{\pi}{12} (t + 4) \right), \quad \forall\ t \in \mathcal{T}, \tag{8}$$

$$T_t = T_t^{(a)} + \frac{\alpha}{k} P_t, \quad \forall\ t \in \mathcal{T} \setminus \{0\}, \tag{9}$$

$$T_t = T_{\text{initial}}, \quad \text{for } t = 0, \tag{10}$$

$$T_t \in [\underline{T}, \overline{T}], \tag{11}$$

$$P_t \in [\underline{P}, \overline{P}], \tag{12}$$

$$T_t^{(a)} \in \mathbb{R}. \tag{13}$$

In these equations, $z \in \mathcal{Z}$ is the vector of decision variables such that $z = (T_t, P_t) \in \mathcal{Z}$.