Multi-agent discrete-time dynamical system to simulate distribution networks evolution

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I. Introduction

This documentation introduces a comprehensive mathematical formalisation of a multi-agent discrete-time dynamical system of the interaction between electricity prosumers, the electricity distribution system operator, and the technological (generation, storage) and regulatory (tariff structure, incentive schemes) environment. Such a dynamical system may be used to, for any type of environment, simulate the evolution of the deployment of distributed electricity generation, as well as the evolution of the cost of distribution. The system relies on the assumption that individual electricity consumers behave statistically as rational agents, who may invest in optimised distributed renewable energy installations, if they are cost-efficient compared to the retail tariff. By modelling the cost recovery scheme of the distribution system operator, the simulator then computes the evolution of the retail tariff in response to a change in the aggregated consumption and generation of the consumers in the distribution network, induced by the deployment of distributed generation. Simulations related with several environments are provided. The multi-agent discrete-time dynamical system consists of three distinct components: (i) the *optimisation of DRE units*, (ii) the *investment decision process*, and (iii) the *computation of the distribution tariff*. In the following, the three mathematical formalisation of the three components is presented.

II. Problem Formalization

A. Optimisation of DRE units

The goal of this section is to describe a method to size DRE installations so that the associated LCOE is minimised over a given period of time. This method relies on an optimisation framework instantiated in the form of a linear program. The optimisation horizon is set to $Y \in \mathbb{N}$ years which are divided into 8760 time-steps ($Y \times 8760$):

- $\mathcal{T} = \{0, \ldots, T-1\}$, with $T = 8760$, represents a time discretisation of one year (in hours).
- $\mathcal{Y} = \{0, \ldots, Y-1\}$, represents the years of the optimisation.

The linear program requires several parameters as inputs. Let G denote a tuple gathering these inputs:

$$G = (P, \Pi, F, H, U) \in \mathcal{G}, \quad \text{with} \tag{1}$$
$$\mathcal{G} \subset \left(\mathbb{R}^+\right)^2 \times \left(\mathbb{R}^+\right)^2 \times \left(\mathbb{R}^+\right)^2 \times \left(\mathbb{R}^+\right)^4 \times \left(\mathbb{R}^+\right)^{2\times T}$$

where:

- $P = \left(P^{(pv)}, P^{(bat)}\right)$ corresponds to the price of PV modules per kWp $P^{(pv)}$, and the price of batteries per kWh $P^{(bat)}$, at the time the DRE installation is deployed.
- $\Pi = \left(\Pi^{(in)}, \Pi^{(out)}\right)$ represents the electricity prices. $\Pi^{(in)}$ is the network tariff at which the users buy the electricity, and $\Pi^{(out)}$ is the price at which DRE owners sell the electricity.
- $F = \left(F^{(-)}, F^{(+)}\right)$ are scalars where $F^{(+)}$ (resp. $F^{(-)}$) constraints the ratio of energy that can flow into (resp. out) of the battery, divided by the battery capacity.
- $H = \left(\eta^{(c)}, \eta^{(d)}, dod, B\right)$ defines the battery parameters: efficiency of charging ($\eta^{(c)}$), efficiency of discharging ($\eta^{(d)}$), depth of discharge (dod), and battery lifetime in years ($B > 0$).
- $U = (U_0, \ldots, U_{T-1})$, where $\forall t \in \mathcal{T}$, $U_t = \left(U_t^{(d)}, U_t^{(p)}\right)$ represents the user consumption profile $U_t^{(d)}$ (in terms of hourly energy consumption), and production profile $U_t^{(p)}$ (in kW produced per kWp installed). The latter only applies for DRE installations.

Let us introduce the variables of the optimisation problem. Let $\mathcal{A} = \left\{(p, b) \,|\, p \in [0, \bar{p}]\,; b \in [0, \bar{b}]\right\}$ denote the space of sizing variables containing: PV size (p) in kWp and battery size (b) in kWh. Then, let i_0 represent the investment costs as a function χ_0, which is a mapping from $(\mathcal{A} \times \mathcal{G})$ to $\mathbb{R}$. We assume a linear growth of the investment costs according to the sizing variables A.

$$i_0 = \chi_0\left(A, G\right) = \left(p \cdot P^{(pv)} + \frac{Y}{B} \cdot b \cdot P^{(bat)}\right) \tag{2}$$

The yearly costs of operation are represented by ξ_y. To compute them, we resort to different equations, depending on the compensation mechanism used. In our work two mechanisms are considered: net-metering (*NM*) and net-purchasing (*NP*):

a) NM: the imports and the exports of a DRE are recorded with a single meter that runs forwards when importing electricity, and backwards when exporting electricity. In practice this supposes that the electricity exported is paid for at the same price as the electricity consumed (network tariff). At the end of a billing period, the balance is evaluated, and the customers pay for the difference between imports and exports (in NM this is the net consumption of the users). However, if the meter features a negative balance (larger exports than imports), the excess is not paid to the customer. The resulting total yearly costs within this compensation mechanism framework depend on the electricity costs c_y^{NM}, and the operation and maintenance costs m_y:

$$\xi_y^{NM} = c_y^{NM} + m_y \quad \forall y \in \mathcal{Y} \tag{3}$$

b) NP: two distinct meters are associated to two different prices. The first one records imports (in NP this is the net consumption of the users) which are paid for at network tariff. The second records exports which are remunerated at a different price. With this mechanism, there is no upper bound to the amount of electricity the DRE owner may sell. The resulting total yearly costs within this compensation mechanism framework depend on the electricity costs c_y^{NP}, the operation and maintenance costs m_y, and the revenues v_y:

$$\xi_y^{NP} = c_y^{NP} + m_y - v_y \quad \forall y \in \mathcal{Y} \tag{4}$$

Determining the electricity costs requires information about the volumes of electricity imported and exported from and to the grid, at every time step t. Let $\rho_t^{(-)}$ denote the volume of electricity imported from the grid, and $\rho_t^{(+)}$ the volume exported to the grid. The costs are computed as:

$$c_y^{NM} = \max\left\{0, \sum_{t=0}^{T-1} \left(\rho_t^{(-)} - \rho_t^{(+)}\right) \cdot \Pi^{(in)}\right\} \quad \forall y \in \mathcal{Y} \tag{5}$$

$$c_y^{NP} = \sum_{t=0}^{T-1} \rho_t^{(-)} \cdot \Pi^{(in)} \quad \forall y \in \mathcal{Y} \tag{6}$$

The operation and maintenance costs can be represented as a function χ_m that maps $\mathcal{A} \to \mathbb{R}$. Its computation is the same for both compensation mechanisms. It is assumed to depend linearly on the sizing variables A [15]:

$$m_y = \chi_m(A) = \frac{1}{200} \cdot p + \frac{1}{100} \cdot b \quad \forall y \in \mathcal{Y} \tag{7}$$

As seen in equation (5), in the electricity cost computation of NM, the revenues are subtracted from the costs. Thus, to obtain an equivalent electricity cost for NP, the revenues must also be computed and subtracted from the costs. The revenues for year y are represented by v_y:

$$v_y = \sum_{t=0}^{T-1} \rho_t^{(+)} \cdot \Pi^{(out)} \quad \forall y \in \mathcal{Y} \tag{8}$$

The energy balance of the system depends on the imports $\rho_t^{(-)}$, exports $\rho_t^{(+)}$, the electricity produced by the PV array k_t, the demand $U_t^{(d)}$, the maximum production (kW/kWp) $U_t^{(p)}$, the energy flow into the battery $j_t^{(-)}$, and the energy flow out of the battery $j_t^{(+)}$. The energy flows into and out of the battery also depend on the variation of the state of charge between $t-1$ and t ($soc_t - soc_{t-1}$). Thus, the following equations control the energy balance, taking into account the state of charge of the batteries:

$$\rho_t^{(+)} - \rho_t^{(-)} = k_t - U_t^{(d)} - j_t^{(-)} + j_t^{(+)} \quad \forall t \in \mathcal{T}, \tag{9}$$

with:

$$k_t = p \cdot U_t^{(p)} \quad \forall t \in \mathcal{T} \tag{10}$$

$$j_t^{(-)} \leq b \cdot F^{(-)} \quad \forall t \in \mathcal{T} \tag{11}$$

$$j_t^{(+)} \leq b \cdot F^{(+)} \quad \forall t \in \mathcal{T} \tag{12}$$

$$b \cdot dod \leq soc_t \leq b \quad \forall t \in \mathcal{T} \tag{13}$$

$$soc_t = \begin{cases} soc_{t-1} - \dfrac{j_t^{(+)}}{\eta^{(d)}} + j_t^{(-)} \cdot \eta^{(c)} & \forall t \in \mathcal{T} \setminus \{0\} \\ soc_0 \in [b \cdot dod, b] \text{ for } t = 0 \end{cases} \tag{14}$$

Let $LCOE$ denote the general objective function that represents the levelized cost of electricity. This objective function is a mapping from $(\mathcal{G} \times \mathcal{A})$ to $\mathbb{R}$. For a given pair $(G, A) \in (\mathcal{G}, \mathcal{A})$, $LCOE\,(G, A)$ is defined as follows:

$$LCOE\,(G, A) = \frac{i_0 + \sum_{y=0}^{Y-1} \dfrac{\xi_y - \mu_y}{(1+r)^y}}{\sum_{y=0}^{Y-1} \dfrac{d_y}{(1+r)^y}} \tag{15}$$

where the yearly demand of the system is defined as $d_y = \sum_{t=0}^{T-1} U_t^{(d)}$, r represents the discount rate, and μ_y stands for the potential yearly subsidy (quota) the user may receive (depending on the regulation). We define $\widehat{LCOE}_G$ as the minimum value of the objective function (subject to the previous constraints):

$$\widehat{LCOE}_G = \min_{\substack{A \,\in\, \mathcal{A} \\ s.t.(2)-(14)}} LCOE\,(G, A) \tag{16}$$

Furthermore, for every $G \in \mathcal{G}$, there is at least one configuration $A^* \in \mathcal{A}$ that leads to the minimum $LCOE$:

$$A_G^* \in \underset{\substack{A \,\in\, \mathcal{A} \\ s.t.(2)-(14)}}{\arg\min}\, LCOE\,(G, A) \tag{17}$$

To represent the diversity of customers within a DN, we introduce a set of $I \in \mathbb{N}$ users $\{G_1, \ldots, G_I\} \subset \mathcal{G}$. Each user G_i is characterised by specific consumption and production profiles $U_i = (U_{i,0}, \ldots, U_{i,T-1})$ (see equation (1)), where $U_{i,t} = \left(U_{i,t}^{(d)}, U_{i,t}^{(p)}\right) \in \mathbb{R}^{+(2\times T)}$ $\forall i \in \mathcal{I}$, with $\mathcal{I} = \{1, \ldots, I\}$. Consequently, one has:

$$G_i = (P, \Pi, F, H, U_i) \in \mathcal{G} \tag{18}$$

Following equations (16) and (17), each user G_i is associated with a minimal LCOE value $\widehat{LCOE}_{G_i}$, and an optimal sizing configuration $A_{G_i}^*$:

$$\widehat{LCOE}_{G_i} = \min_{\substack{A \,\in\, \mathcal{A} \\ s.t.(2)-(14)}} LCOE\,(G_i, A) \tag{19}$$

$$A_{G_i}^* \in \underset{\substack{A \,\in\, \mathcal{A} \\ s.t.(2)-(14)}}{\arg\min}\, LCOE\,(G_i, A) \tag{20}$$

B. Investment decision process

This part of the simulator requires the vector $\widehat{LCOE}_{G_{i,n}}$ and the network tariff $\Pi_n^{(in)}$ to simulate the interaction between the different actors of the multi-agent model.

For every potential DRE installation, we can determine a price ratio $\Gamma_{i,n}$ between the levelized cost of electricity $\widehat{LCOE}_{G_{i,n}}$ and network tariff $\Pi_n^{(in)}$, thus:

$$\forall n \in \mathcal{N}, \forall i \in \mathcal{J}_n, \ \ \Gamma_{i,n} = \frac{\widehat{LCOE}_{G_{i,n}}}{\Pi_n^{(in)}} \tag{21}$$

This price ratio will take a value between 0 and 1, since $\widehat{LCOE}_{G_{i,n}}$ cannot be greater than $\Pi_n^{(in)}$, due to optimality constraints. To establish whether a non-DRE owner decides to deploy -or not- a DRE installation, we make use of a Bernoulli random variable whose parameter $p_{i,n}$ is a function of $\Gamma_{i,n}$. We assume that:

$$\begin{aligned} \forall n \in \mathcal{N}, \forall i \in \mathcal{J}_n, \ \ \exists f_{i,n} : [0,1] \rightarrow [0,1]\,, \\ p_{i,n} = f_{i,n}\left(\Gamma_{i,n}\right) \end{aligned} \tag{22}$$

For simplicity, in the following we assume that all the previously defined mappings are equal to a unique linear mapping, i.e.:

$$\exists \alpha : \forall n \in \mathcal{N}, \forall i \in \mathcal{J}_n, \ \ p_{i,n} = \min\left\{1, \alpha \cdot \Gamma_{i,n}\right\} \tag{23}$$

Then, the random variable $\beta_{i,n}$, that controls the decision of investing or not in a DRE installation of size $A^*_{G_{i,n}}$, is drawn from the Bernoulli distribution $B(1, p_{i,n})$:

$$\forall i \in \mathcal{J}_n, \forall n \in \mathcal{N}, \ \ \beta_{i,n} \sim B(1, p_{i,n}) \tag{24}$$

The decision for every owner $\theta_{i,n}$ is given by:

$$\theta_{i,n} = 1 - \beta_{i,n} \quad \forall i, n \in \mathcal{J}_n \times \mathcal{N} \tag{25}$$

where $\theta_{i,n} \in \{0, 1\}$ by definition of the Bernoulli distribution. If $\theta_{i,n} = 1$, a DRE installation of size $A^*_{G_{i,n}}$ is deployed by the agent i. This agent is then removed from the set of users $\mathcal{J}_n$. In this way, when a DRE installation of size $A^*_{G_{i,n}}$ is deployed, the user i is prevented from investing in the future. If $\theta_{i,n} = 0$, the simulation continues, and another opportunity will be given to user i in the following step $n + 1$. The set $\mathcal{J}_{n+1}$ can thus be computed as follows:

$$\mathcal{J}_{n+1} = \mathcal{J}_n \setminus \{i | \theta_{i,n} = 1\} \tag{26}$$

C. Computation of the distribution tariff

The prevailing manner of calculating distribution tariffs requires three elements for every tarification period: the estimation of costs, the estimation of total demand, and the imbalance between the estimated revenues recovered and the actual revenues recovered from the previous period. We introduce the concept of tarification period as the time between two different distribution tariffs updates. Let $n \in \mathcal{N}$ denote the time index of the tarification period, with $\mathcal{N} = \{0, \ldots, N-1\}$. Some characteristics of the users may evolve with n. Thus, each user is associated with an n-dependent tuple:

$$G_{i,n} = (P_n, \Pi_n, F, H, U_i) \tag{27}$$

In this work, for the sake of simplification, we assign a one-year period to a given tariff, though it can be adjusted to a different time frame. At every year n, we compute the distribution component of the network tariff[1], denoted by $\Pi_n^{(dis)}$. From the distribution tariff, it is possible to compute the network tariff $\Pi_n^{(in)}$ using the following expression:

$$\Pi_n^{(in)} = \Pi_n^{(dis)} + \lambda_n \quad \forall n \in \mathcal{N} \tag{28}$$

where λ_n represents the costs of energy, transmission, and taxes. We assume that λ_n is constant over time. For a given network tariff $\Pi_n^{(in)}$, and maintaining constant inputs of the user tuples $G_{i,n}$, a fraction of the users of the DN might decide to deploy a DRE installation. We denote by $\mathcal{J}_n$ the subset of $\mathcal{I}$ corresponding to users that have not yet invested in a DRE installation at time n. This deviation in the generation paradigm of the DN induced by new DRE owners (given by the set $\mathcal{J}_n \setminus \mathcal{J}_{n+1}$) will also induce a shift (typically upwards) of the distribution price $\Pi_{n+1}^{(dis)}$, and since $\lambda_{n+1} = \lambda_n$, the network tariff $\Pi_{n+1}^{(in)}$ will be shifted similarly. Furthermore, following this reasoning, a new network tariff $\Pi_{n+1}^{(in)}$ may cause even more deployment of DRE installations during the subsequent time steps.

We assume that the initial situation at $n = 0$ is balanced, in a sense that the costs incurred by the DSO are equal to the revenues recovered by it, i.e. $R_0 = C_0$. Furthermore, we also assume that the DN costs remain constant over the entire simulation, thus $C_0 = C = C_n$, $\forall n \in \mathcal{N}$. In addition, we may compute the total demand of the system at every time n as the overall demand of all the users of the DN: the ones that belong to $\mathcal{I}$, represented by the net consumption of the users in $\mathcal{I}$, which depends on n, and also the ones that will never deploy a DRE installation due to technical and/or economic constraints, represented by a constant consumption σ, which does not depend on n, and is assumed constant. The net consumption Ω_n^{NM} (NM setting) and Ω_n^{NP} (NP setting) are computed the following way:

$$\Omega_n^{NM} = \max\left\{0, \sum_{i=0}^{I} \sum_{t=0}^{T-1} \left(\rho_{i,t}^{(-)} - \rho_{i,t}^{(+)}\right)\right\} \tag{29}$$

$$\Omega_n^{NP} = \sum_{i=0}^{I} \sum_{t=0}^{T-1} \rho_{i,t}^{(-)} \tag{30}$$

where $\rho_{i,t}$ is defined according to equation (9), which is applied to every user $i \in \mathcal{I}$. Since the number of DRE installations may evolve with n, so does the computation of Ω_n^{NM} and Ω_n^{NP}. Let us denote by Ω_n the net consumption corresponding to the chosen setting (NM or NP). The overall demand is then calculated as follows:

$$D_n = \Omega_n + \sigma \quad \forall n \in \mathcal{N} \tag{31}$$

From this initial state, the simulation starts by estimating the electricity demand at time $n+1$. Let $\widehat{D}_{n+1}$ denote the estimation of the electricity demand at time $n + 1$, this is computed

[1]The network tariff is divided in four main components: energy, transmission, distribution, and taxes.

by assuming that, in equilibrium conditions, the demand will remain constant from n to $n+1$. Thus, the estimated electricity demand for the following period corresponds to the registered demand from the previous period:

$$\widehat{D}_{n+1} = D_n \qquad \forall n \in \mathcal{N} \tag{32}$$

Then, the revenues R_{n+1} that will be collected at time $n+1$ can be estimated as a function of the estimated demand $\widehat{D}_{n+1}$ and the distribution tariff $\Pi_n^{(dis)}$; note that the distribution tariff is adjusted at the end of the period, not at the beginning. The estimated revenues are computed as:

$$\widehat{R}_{n+1} = \widehat{D}_{n+1} \cdot \Pi_n^{(dis)} \quad \forall n \in \mathcal{N} \tag{33}$$

When the transition from n to $n+1$ is completed, the overall actual demand can be computed similar to equation (25). Consequently, the actual revenues recovered by the DSO are determined as:

$$R_{n+1} = D_{n+1} \cdot \Pi_n^{(dis)} \quad \forall n \in \mathcal{N} \tag{34}$$

If the estimation of revenues and the actual revenues do not match, an imbalance Δ_{n+1} emerges, which depends on the estimation of revenues at period $n+1$, and the actual revenues at period $n+1$. Assuming the period has already been realised, we change the indices from $n+1$ to n:

$$\Delta_n = \widehat{R}_n - R_n \quad \forall n \in \mathcal{N} \tag{35}$$

Finally, the distribution tariff for the following period $\Pi_{n+1}^{(dis)}$ is computed as a function of the DSO costs C, the total estimated electricity demand of the DN $\widehat{D}_{n+1}$, and the imbalance Δ_n from the previous period:

$$\Pi_{n+1}^{(dis)} = \frac{C + \Delta_n}{\widehat{D}_{n+1}} \quad \forall n \in \mathcal{N} \tag{36}$$

where $\Pi_{n+1}^{(dis)}$ is transformed back into $\Pi_{n+1}^{(in)}$ by the computation of equation (22).

Finally, to initialise the simulation, an initial network tariff must be assumed. By feeding the DRE simulator with $\pi_0^{(in)}$ the deployment of DRE installations might be induced, leading to a change in the composition of users of the DN, and introducing the first imbalance Δ_0 in the system.

D. *Dynamics of the multi-agent system*

The resulting discrete-time system dynamics can be summarised as follows: first, $\Pi_0^{(in)}$ is assumed from a reference DN. Then, we iterate over the following steps, until a stopping criterion is reached (either the simulation horizon is reached, or there is no more evolution of decentralised production):

(i) At every iteration $n \geq 0$, for all $i \in \mathcal{I}_n$, given a network price $\Pi_n^{(in)}$, we compute the optimal sizing configuration of user i, denoted by $A^*_{G_{i,n}}$, and the associated levelized cost of electricity $\widehat{LCOE}_{G_{i,n}}$. See equations (1) to (20).

(ii) Then, we compute $\Gamma_{i,n}$ and θ_i. DRE installations corresponding to $\theta_i = 1$ are deployed. See equations (31) to (36).

(iii) An imbalance $\left(\widehat{R}_n - R_n\right)$ emerges, caused by the DRE installations that were not accounted for in the estimation of revenues $\widehat{R}_n$. See equations (21) to (29).

(iv) The network tariff $\Pi_n^{(in)}$ is updated in order to compensate for the imbalance according to equation (30).