Multi-Objective Optimization for Portfolio Decarbonization: A Hybrid Framework with TOPSIS Selection Process

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1 Introduction

The transition toward sustainable finance has created increasing demand for investment strategies that balance financial performance with environmental, social, and governance (ESG) objectives. Traditional portfolio theory emphasizes the trade-off between return and risk, but modern investors must also account for externalities such as carbon emissions and corporate responsibility. This multidimensional setting raises the central question: how can portfolios be constructed that are both competitive in financial markets and aligned with sustainability goals?

In practice, portfolio design under sustainability constraints requires the integration of heterogeneous signals. Expected returns and downside risks capture the performance dimension, while ESG scores and carbon intensity represent the sustainability dimension. Treating these signals in isolation risks either neglecting financial efficiency or overlooking environmental impact. A unified framework is needed to reconcile these objectives in a transparent and auditable manner.

In this paper we propose a hybrid pipeline that addresses this challenge in two stages. First, we apply a multi-criteria decision-making method (TOPSIS) to rank the investment universe according to four criteria: low carbon intensity, high ESG score, high expected return, and low downside risk. This produces a shortlist of candidates that balance sustainability and performance. Second, we allocate capital within this shortlist using three complementary approaches: a parametric mean–variance optimizer, a tail-risk–sensitive CVaR optimizer, and a data-driven LSTM forecaster converted to portfolio weights. This combination allows us to compare classical risk–return theory, coherent risk measures, and modern machine learning within a common evaluation protocol.

Our contributions are threefold. First, we provide a reproducible data engineering layer that enforces consistent coverage and removes unreliable observations. Second, we formalize a variant of TOPSIS adapted to sustainability-aware investing through explicit cost-to-benefit transformations. Third, we benchmark three allocation backends under identical constraints and evaluation metrics, demonstrating how portfolio performance varies under different optimization philosophies. Results on a ten-year S&P 500 sample show that CVaR optimization delivers the highest absolute returns, mean–variance achieves superior risk-adjusted performance, and LSTM allocations remain close to the equal-weight benchmark.

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2 Data and Preprocessing

2.1 Files and universe screen

The code and the needed CSV files can be found in the following repository: Portfolio Decarbonization Github. The pipeline expects three CSV files in a working directory:

- `prices.csv`: a daily close price panel $P_{t,i}$ indexed by trading date t and instrument i.
- `fundamentals_esg_snapshot.csv`: instrument–level fundamentals including Revenue, EBIT, and an ESG score for the 2024.
- `spx500_carbon_emissions_snapshots.csv`: total CO_2-E emissions per instrument.

We require instruments to be *present by* 2015–06–01 and to maintain sufficient continuity thereafter. Specifically, we forward–fill gaps up to 10 consecutive trading days and compute per–asset coverage as the fraction of non–missing rows after 2015–06–01. Assets below the coverage threshold are dropped.

Let $\mathcal{I}$ denote the surviving instrument set and $\{t_0, \dots, t_T\}$ the aligned date index. We form simple returns

$$r_{t,i} \;=\; \frac{P_{t,i}}{P_{t-1,i}} - 1, \qquad t = 1, \dots, T,\; i \in \mathcal{I}. \tag{1}$$

Daily averages and covariances are computed over a designated training window (Section 4).

2.2 Engineered signals

Annualized mean return. For each i, define $\bar{r}_i = \frac{1}{T}\sum_{t=1}^{T} r_{t,i}$ and the annualized estimator

$$\hat{\mu}_i \;=\; 252\,\bar{r}_i. \tag{2}$$

Downside risk (annualized). Let $r_{t,i}^- = \min(r_{t,i}, 0)$ and $s_i^- = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(r_{t,i}^-)^2}$. The annualized downside deviation is

$$\hat{\sigma}_i^{\downarrow} \;=\; \sqrt{252}\; s_i^-. \tag{3}$$

This penalizes negative fluctuations only, aligning with capital preservation priorities.

Carbon intensity and ESG. Let Em_i denote total metric tons CO_2-E emissions and Rev_i revenue expressed in millions. We define carbon intensity

$$\mathrm{CI}_i \;=\; \frac{\mathrm{Em}_i}{\mathrm{Rev}_i}, \tag{4}$$

and take the LSEG-provided ESG score as S_i^{ESG}. Observations with nonpositive or missing revenue are excluded to avoid pathological intensities.

3 Selection via TOPSIS

We construct a four–criterion decision matrix $X \in \mathbb{R}^{n\times 4}$ over $n = |\mathcal{I}|$ instruments:

(1) LowCarbon, (2) ESG, (3) Return, (4) LowDownside.

To harmonize scales and convert cost criteria into benefits, we apply the *vector normalization* step of classical TOPSIS. For criterion j with raw values $\{x_{ij}\}_{i=1}^{n}$, the normalized entries are

$$z_{ij} \;=\; \frac{x_{ij}}{\sqrt{\sum_{k=1}^{n} x_{kj}^2}}. \tag{5}$$

For cost criteria (CarbonIntensity and DownsideRisk), we negate before normalization so that lower raw values map to higher normalized scores:

$$z_{i,\mathrm{LC}} = \frac{-\,\mathrm{CI}_i}{\sqrt{\sum_{k=1}^{n} \mathrm{CI}_k^2}}, \qquad z_{i,\mathrm{ESG}} = \frac{S_i^{\mathrm{ESG}}}{\sqrt{\sum_{k=1}^{n} (S_k^{\mathrm{ESG}})^2}}, \tag{6}$$

$$z_{i,\mathrm{Ret}} = \frac{\hat{\mu}_i}{\sqrt{\sum_{k=1}^{n} \hat{\mu}_k^2}}, \qquad z_{i,\mathrm{LD}} = \frac{-\,\hat{\sigma}_i^{\downarrow}}{\sqrt{\sum_{k=1}^{n} (\hat{\sigma}_k^{\downarrow})^2}}. \tag{7}$$

Stacking, $Z = [z_{ij}]$ with columns ordered as above.

Let $w \in \mathbb{R}^4$ be a nonnegative weight vector with $\sum_j w_j = 1$. In the code we use

$$w = \big[\,0.30,\ 0.30,\ 0.10,\ 0.30\,\big]^\top, \tag{8}$$

balancing climate impact, ESG quality, expected return, and downside risk.

Define the weighted normalized score matrix $V = [v_{ij}]$ by elementwise scaling

$$v_{ij} \;=\; w_j\, z_{ij}. \tag{9}$$

The ideal best/worst for criterion j are

$$v_j^+ \;=\; \max_i v_{ij}, \qquad v_j^- \;=\; \min_i v_{ij}, \tag{10}$$

and the Euclidean distances of asset i to the ideals are

$$d_i^+ \;=\; \sqrt{\sum_{j=1}^{4} (v_{ij} - v_j^+)^2}, \qquad d_i^- \;=\; \sqrt{\sum_{j=1}^{4} (v_{ij} - v_j^-)^2}. \tag{11}$$

The TOPSIS *closeness coefficient* is

$$\mathrm{CC}_i \;=\; \frac{d_i^-}{d_i^+ + d_i^-} \in [0,1], \tag{12}$$

with larger values preferred. We rank by CC_i and retain the top K names (with $K \le 100$ depending on screening). This shortlist feeds the allocation stage.

4 Optimization Techniques

4.1 Train–test protocol and metrics

Let $R \in \mathbb{R}^{T\times N}$ be the return matrix for the N shortlisted assets with rows $r_t^\top$. We split the history into train and test: by default, the last two years (about 504 daily observations) form the test set if the history is sufficiently long; otherwise, we reserve roughly 20% (bounded below by 60 trading days).

Let $w \in \mathbb{R}^N$ denote portfolio weights, long–only with $\sum_i w_i = 1$ and per–asset cap $0 \le w_i \le u$ (typically $u = 0.05$). Given a daily return series $r_t^{(p)} = r_t^\top w$, we report:

$$\widehat{\mu} = 252 \cdot \frac{1}{T} \sum_t r_t^{(p)}, \qquad \widehat{\sigma} = \sqrt{252} \cdot \sqrt{\frac{1}{T-1} \sum_t (r_t^{(p)} - \bar{r}^{(p)})^2}, \tag{13}$$

$$\widehat{\mathrm{SR}} = \widehat{\mu}/\widehat{\sigma}, \qquad \widehat{\mathrm{MDD}} = \min_t \left(\frac{\prod_{s\le t}(1 + r_s^{(p)})}{\max_{u\le t} \prod_{s\le u}(1 + r_s^{(p)})} - 1 \right). \tag{14}$$

Where, $\widehat{\mathrm{SR}}$ represent the Sharpe Ratio and $\widehat{\mathrm{MDD}}$ is the Maximum Drowdown. We simulate wealth paths from an initial 10,000 euro, applying a one–shot fixed cost of 1 euro per active asset at the test start; continuous full–window plots exclude costs for comparability. The transaction costs are assumed equal to the ones of platforms like "Trade Republic", where investors can buy assets with 1 euro commission.

4.2 Mean–Variance (MV)

The classical mean–variance framework of Markowitz formulates portfolio choice as a trade–off between expected return and risk, measured by variance. Let $\mu \in \mathbb{R}^N$ denote the vector of expected asset returns and $\Sigma \in \mathbb{R}^{N\times N}$ their covariance matrix. For a given risk–aversion parameter $\gamma > 0$, the investor solves

$$\max_{w\in\mathbb{R}^N} \; w^\top \mu - \tfrac{\gamma}{2} w^\top \Sigma w, \qquad \sum_i w_i = 1,\; 0 \le w_i \le u, \tag{15}$$

where the constraints impose a fully invested, long–only allocation with per–asset cap u. This problem balances the portfolio's expected return with its variance penalty, and the optimal γ governs the aggressiveness of the allocation along the efficient frontier. In our application γ is selected to maximize the Sharpe ratio during the training period.

4.3 Conditional Value at Risk (CVaR)

Variance captures overall dispersion but does not distinguish between mild fluctuations and severe losses. A coherent risk measure that directly controls downside exposure is Conditional Value at Risk (CVaR). Given portfolio losses $L_t(w) = -r_t^\top w$ and a confidence level α, CVaR corresponds to the expected loss in the worst $(1-\alpha)$ fraction of scenarios:

$$\mathrm{CVaR}_\alpha(w) \;=\; \mathbb{E}\big[\, L_t(w) \;\big|\; L_t(w) \text{ in worst } (1-\alpha) \,\big]. \tag{16}$$

Minimizing CVaR yields allocations that explicitly limit tail risk, while optional constraints (e.g. minimum expected return or allocation caps) ensure practical feasibility. The formulation is convex and lends itself to efficient optimization. In our application we selected $\alpha = 20\%$.

4.4 Long Short–Term Memory (LSTM)

While MV and CVaR rely on historical means and covariances, modern approaches attempt to exploit temporal patterns in the data. Recurrent neural networks, and in particular Long Short–Term Memory (LSTM) architectures, are well suited for sequential modeling. An LSTM can process a sliding window of past returns to predict the next–day returns across multiple assets simultaneously. Formally, the network learns a nonlinear mapping

$$f_\theta : \; (R_{t-L}, \ldots, R_{t-1}) \;\mapsto\; \hat{R}_t,$$

where $R_t \in \mathbb{R}^N$ denotes the vector of asset returns at time t. By minimizing prediction error over the training set, the model captures both temporal dependencies and cross–asset interactions. The forecasts $\hat{R}_t$ are then interpreted as *alphas*: positive signals are retained, while negative ones are discarded or down–weighted, and the resulting signal vector is converted into a feasible portfolio through the same capped–simplex projection as in mean–variance optimization.

4.5 Baselines and reporting

We benchmark against the equal–weight (EW) portfolio $w_i = 1/N$. For each method (MV, CVaR, LSTM), we report train/test/full annualized statistics, and cumulative wealth in euro. Fixed costs are charged once at test inception proportional to active names.

5 Results

We report results on a ten–year sample; additional evidence for the five–year window is provided in Appendix A. The investable universe is first ranked using TOPSIS on four criteria—low carbon intensity, ESG score, expected return, and low downside risk—and the top $K \leq 100$ assets are retained for allocation. Figures 7 and 8 visualize the cross–sectional landscape behind the TOPSIS scores.

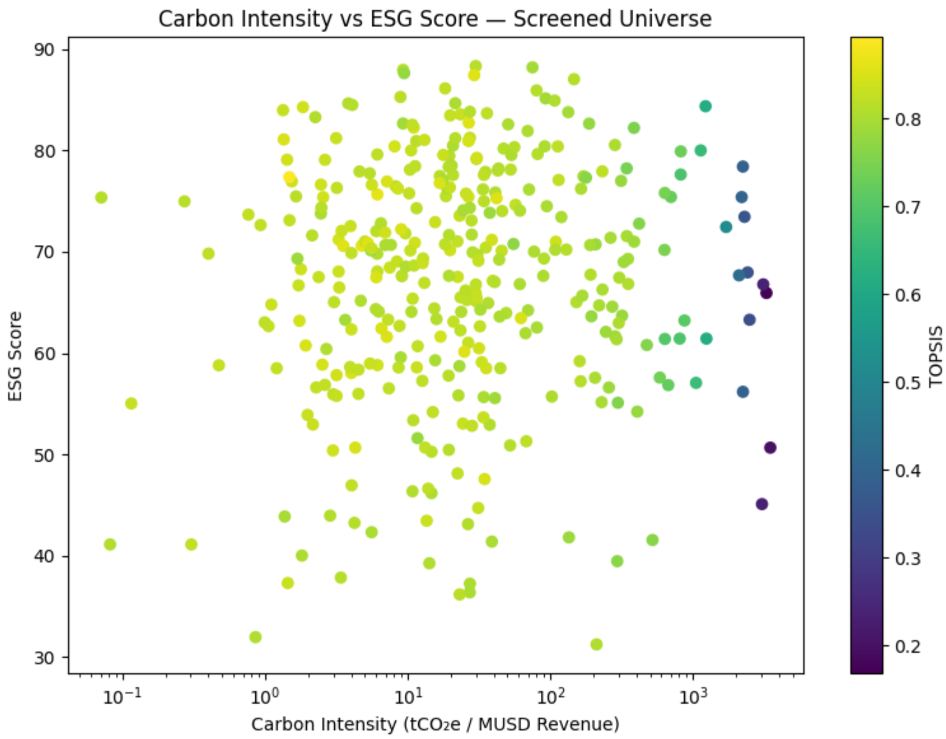

Figure 1: Log-scaled carbon intensity vs ESG score colored by TOPSIS closeness.

Train window. The optimization is calibrated on the first eight years. Figure 9 plots cumulative wealth from a one–shot investment at the beginning of the train period; Table 4 reports annualized performance statistics. Mean–Variance (MV) and CVaR both improve markedly over the equal-weight (EW) benchmark, with CVaR delivering the highest *absolute* return on train while MV achieves the highest *risk-adjusted* return (Sharpe). This pattern is consistent with the objectives of the two optimizers: CVaR explicitly shapes the loss tail and may accept somewhat higher variance in exchange for better protection against extreme losses; MV directly penalizes variance and therefore tends to produce allocations that are more efficient in Sharpe on samples with symmetric risk.

Test window. The trained weights are held out of sample for the last two years. As shown in Figure 10 and Table 5, performance differences are less pronounced over this short window—

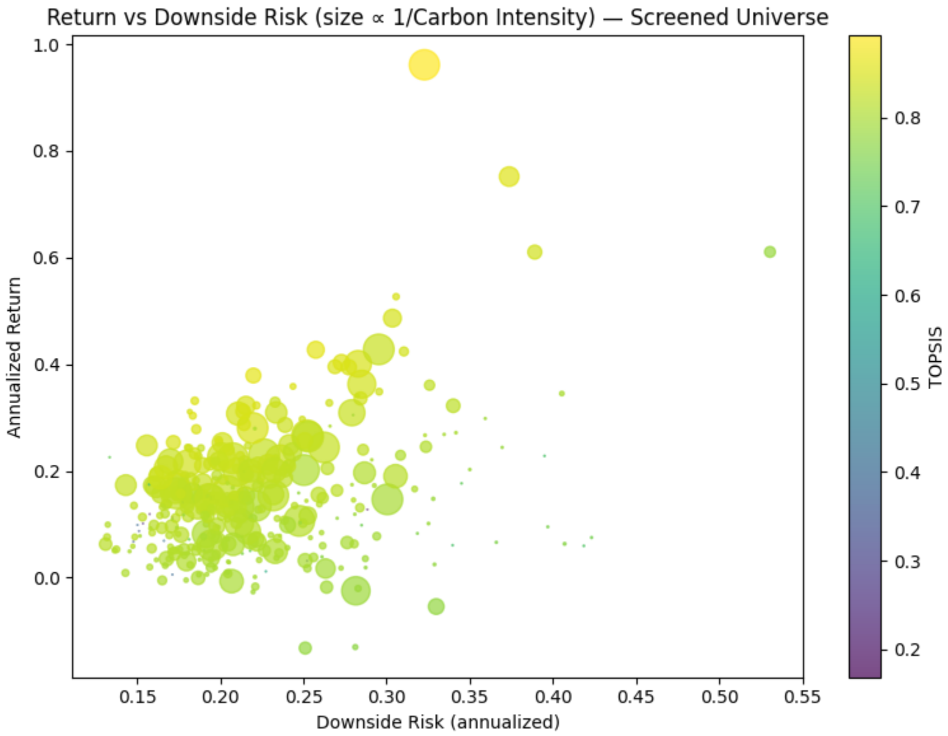

Figure 2: Annualized downside risk vs annualized return colored by TOPSIS closeness.

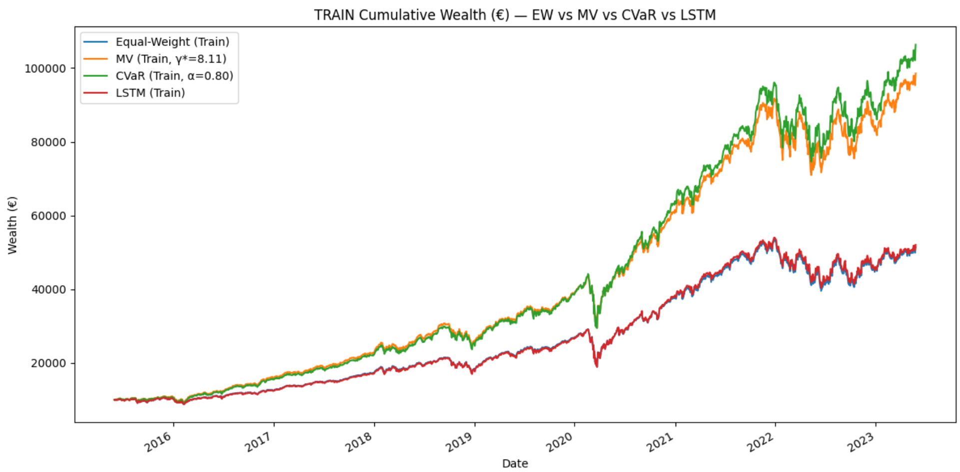

Figure 3: Cumulative wealth from investing at the start of the train window using optimized weights (01.06.2015–01.06.2023).

Table 1: Portfolio — TRAIN (EW vs MV vs CVaR vs LSTM)

Portfolio	**Ann.Return**	**Ann.Vol**	**Sharpe**	**MaxDD**
Equal-Weight	0.353	0.220	1.604	−0.202
Mean-Variance	0.476	0.213	2.234	−0.176
CVaR	0.448	0.199	2.249	−0.156
LSTM	0.351	0.220	1.600	−0.203

expected in a buy-and-hold setting with modest turnover. CVaR again delivers the highest *return*, whereas MV retains an edge in *Sharpe* due to slightly lower realized volatility.

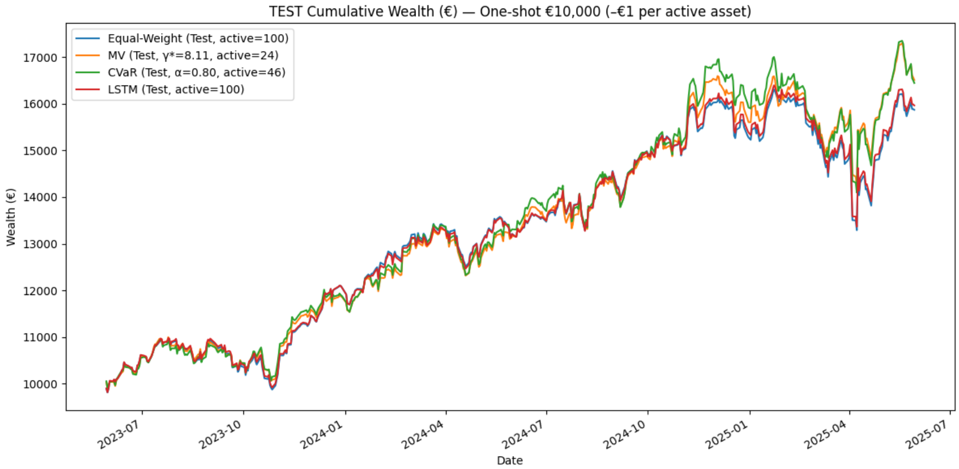

Figure 4: Cumulative wealth during the two-year test (01.06.2023–01.06.2025). A €1 per-stock transaction cost is applied at inception.

Table 2: Portfolio — Test (EW vs MV vs CVaR vs LSTM)

Portfolio	ActiveAssets	Upfront	InitialNet	FinalWealth	Ann.Return	Ann.Vol	Sharpe	MaxDD
Equal-Weight	100.000	100.000	9990.000	12 227.788	0.224	0.199	1.123	−0.180
Mean-Variance	24.000	24.000	9976.000	12 570.360	0.248	0.225	1.106	−0.192
CVaR	51.000	51.000	9949.000	12 432.775	0.234	0.193	1.212	−0.157
LSTM	100.000	100.000	9990.000	12 224.220	0.223	0.197	1.130	−0.179

Full horizon. Assuming a buy-and-hold at inception with optimized weights, the full ten-year cumulative wealth (Figure 11) confirms the pattern seen in sub-windows: CVaR attains the highest terminal value/return but with slightly higher volatility than MV, which translates into a marginally lower Sharpe (Table 6). The LSTM allocation tracks the equal-weight benchmark closely throughout. This similarity indicates that, given our features (daily returns only) and calibration, the LSTM does not extract a sufficiently strong predictive signal; alternatively, the network may be under-regularized/under-specified relative to the problem scale, causing its out-of-sample portfolio to regress toward a diversified baseline.

Table 3: Portfolio — Full Window (EW vs MV vs CVaR vs LSTM, no costs)

Portfolio	Ann.Return	Ann.Vol	Sharpe	MaxDD
Equal-Weight	0.327	0.216	1.154	−0.202
Mean-Variance	0.430	0.215	1.998	−0.192
CVaR	0.405	0.198	2.047	−0.157
LSTM	0.326	0.215	1.152	−0.203

Weights. Figure 12 compares the optimized allocations for the three methods. MV displays tighter concentration in names with strong mean–variance trade-offs; CVaR tilts toward profiles that reduce tail risk even when variance is higher; the LSTM portfolio is broadly diversified, echoing the EW baseline.

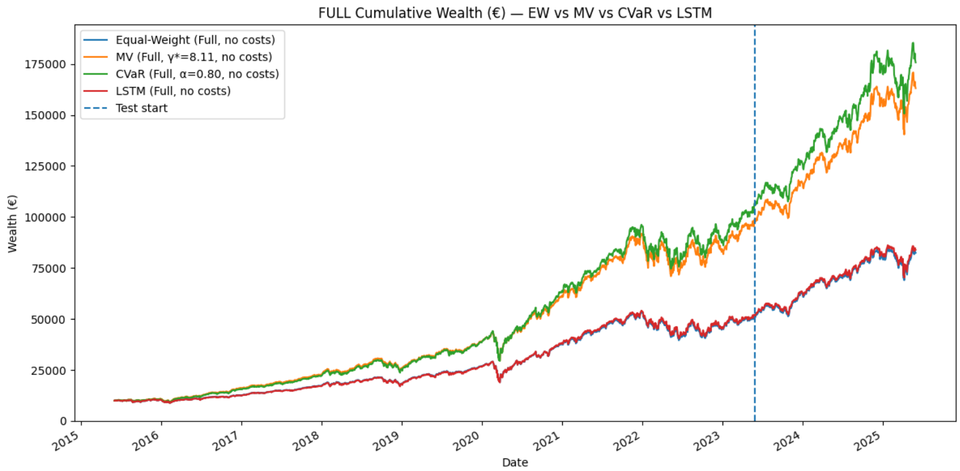

Figure 5: Cumulative wealth over the full ten-year window (01.06.2015–01.06.2025). The blue vertical line marks the train/test split.

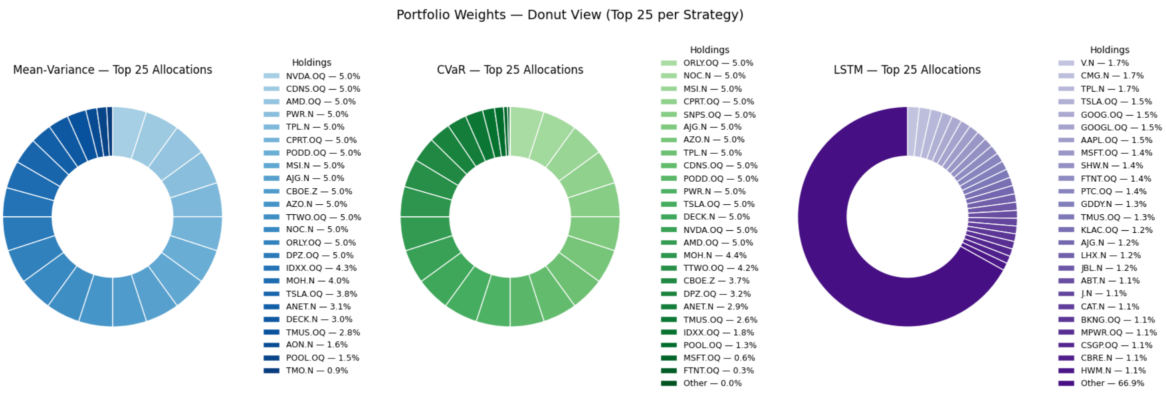

Figure 6: Optimized weights across the three allocation methods.

A Results for Five-Years Portfolio

Table 4: Portfolio — Train (EW vs MV vs CVaR vs LSTM)

Portfolio	Ann.Return	Ann.Vol	Sharpe	MaxDD
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Table 6: Portfolio — Full Window (EW vs MV vs CVaR vs LSTM, no costs)

Portfolio	Ann.Return	Ann.Vol	Sharpe	MaxDD
Equal-Weight	0.327	0.216	1.154	−0.202
Mean-Variance	0.430	0.215	1.998	−0.192
CVaR	0.405	0.198	2.047	−0.157
LSTM	0.326	0.215	1.152	−0.203

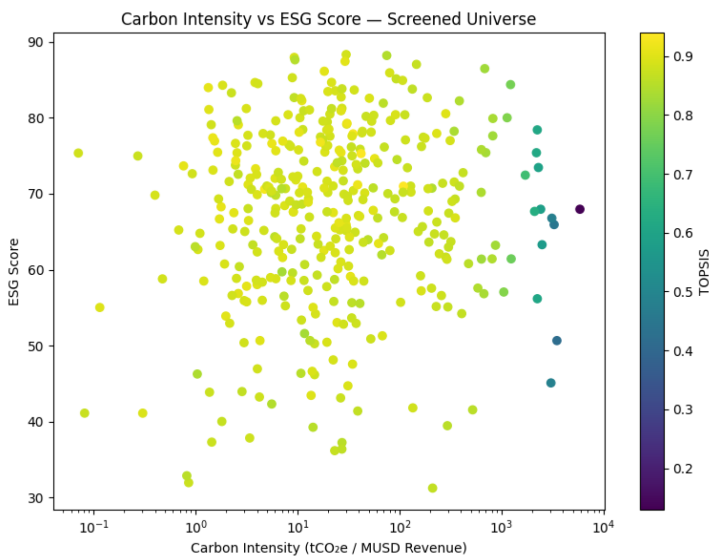

Figure 7: Log-scaled carbon intensity vs ESG scores in terms of TOPSIS results.

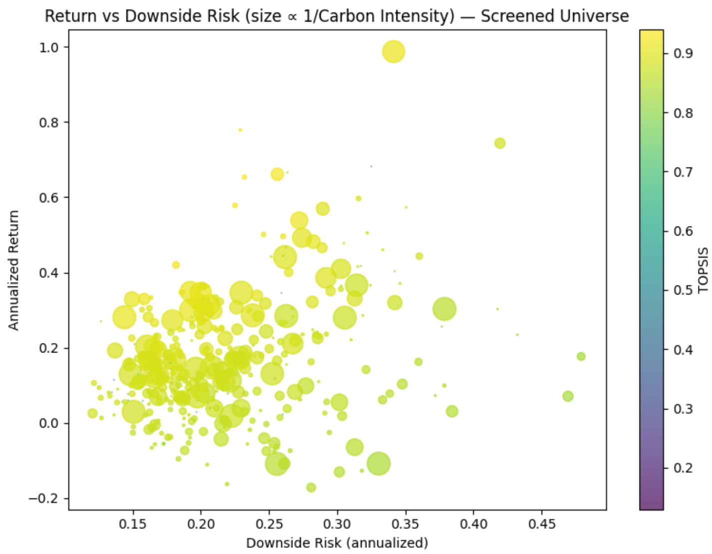

Figure 8: Annualized downside risk against annualized returns in terms of TOPSIS results.

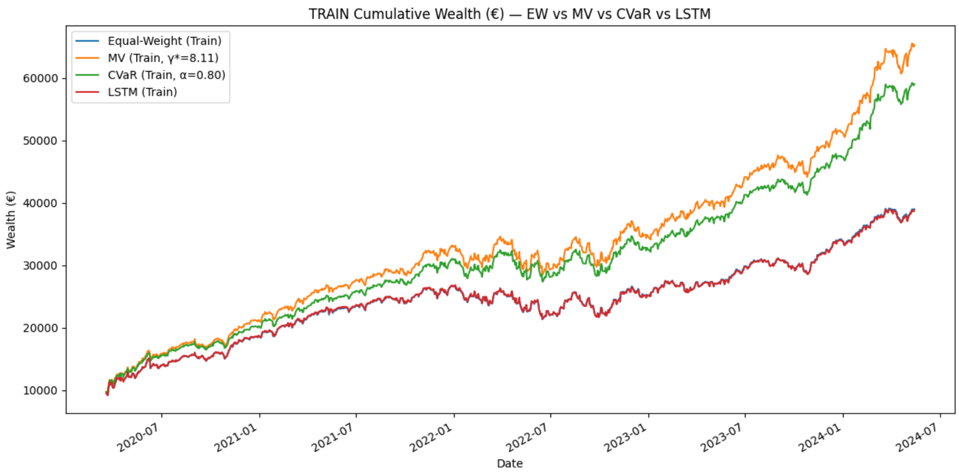

Figure 9: Cumulative wealth obtained by investing at the beginning of the period using the optimized weights over the first four years of the selected sample (01.06.2020 – 01.06.2024).

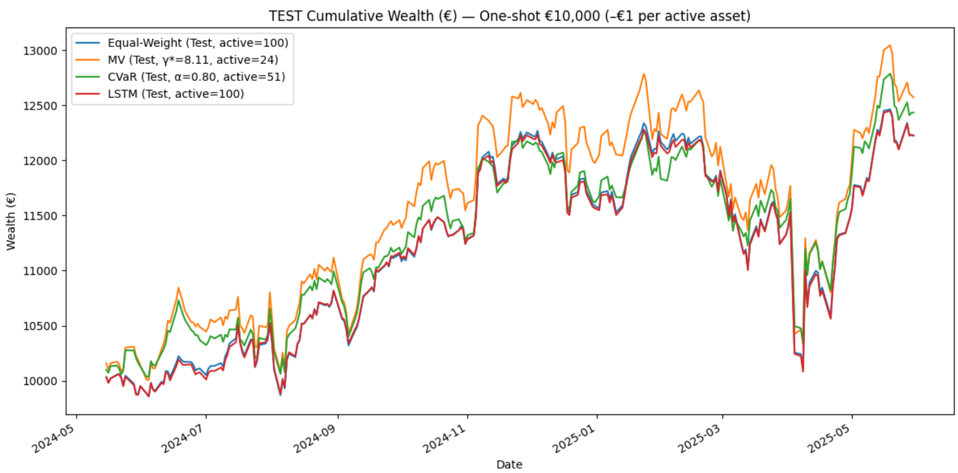

Figure 10: Cumulative wealth during the one-year test of the optimized weights (01.06.2024 – 01.06.2025). Transaction costs of €1 per stock are included at inception.

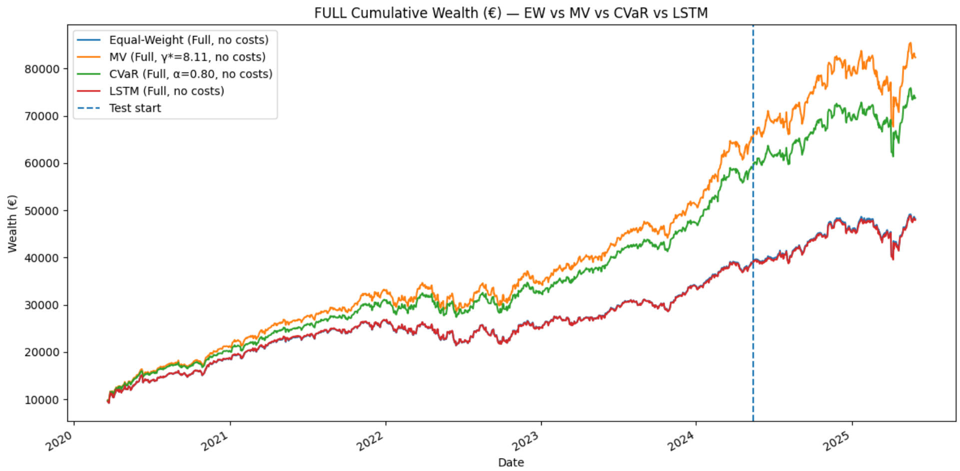

Figure 11: Cumulative wealth over the full five-year window (01.06.2020 – 01.06.2025).

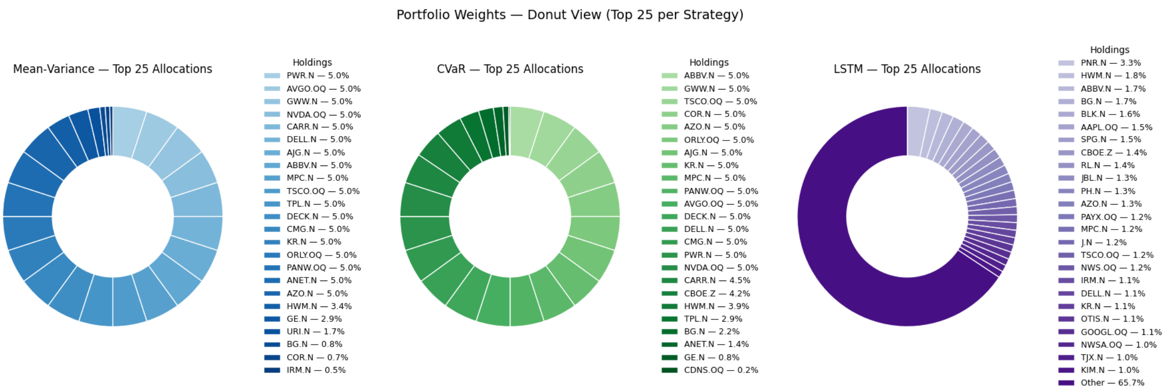

Figure 12: Comparison of the optimized weights for the three optimization techniques.