Nonstationary Multivariate Extremes, II
Computing & Applications

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Plan for this session

- We will study the extreme dependence of maxima temperatures in Switzerland in the open field and under the forest canopy.
- We will carry out all the steps of the analysis, including data preparation.
- We will assess how the extreme dependence varies according to a covariate.
- We will use a variety of estimators to measure it formally.

The data


Neunkirch
Othmarsingen
Vordemwald
Bettlachstock
Schänis
Beatenberg
Nationalpark
Celerina
Lausanne
Chironico
Jussy
Visp
Isone
Novaggio

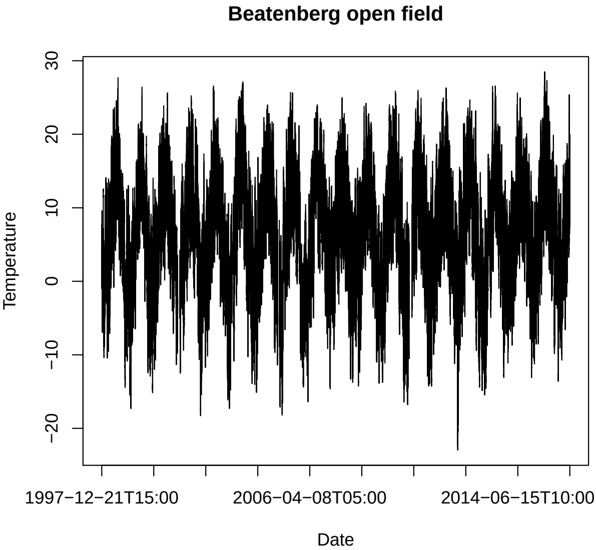
The data

	station	type	altitude	longitude	latitude	slope
1	Beatenberg	forest	1533	7.76392	46.69990	66
2	Beatenberg	field	1567	7.77215	46.70048	66
3	Bettlachstock	forest	1116	7.41732	47.22440	66
4	Bettlachstock	field	1077	7.41080	47.22227	66
5	Chironico	forest	1416	8.81281	46.44623	35
6	Chironico	field	1483	8.81822	46.44087	35
7	Celerina	forest	1883	9.88881	46.49213	34
8	Celerina	field	1764	9.87587	46.50608	34
9	Isone	forest	1214	9.00769	46.12506	58
10	Isone	field	1152	9.00731	46.12608	58
11	Jussy	forest	517	6.28694	46.22848	3
12	Jussy	field	502	6.29315	46.23129	3
13	Lausanne	forest	835	6.65802	46.58396	7
13	Lausanne	field	797	6.64817	46.55885	7

Preprocessing

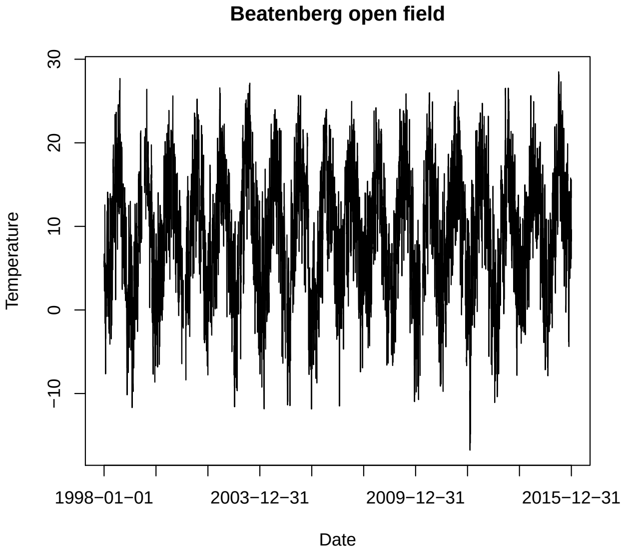
	Station	Altitude	Longitude	Latitude	Slope
1	Beatenberg	1533	7.76392	46.69990	66
2	Bettlachstock	1116	7.41732	47.22440	66
3	Chironico	1416	8.81281	46.44623	35
4	Celerina	1883	9.88881	46.49213	34
5	Isone	1214	9.00769	46.12506	58
6	Jussy	517	6.28694	46.22848	3
7	Lausanne	835	6.65802	46.58396	7
8	Nationalpark	1893	10.23047	46.66164	11
9	Neunkirch	588	8.53455	47.68406	58
10	Novaggio	900	8.83428	46.02162	68
11	Othmarsingen	497	8.22631	47.39852	27
12	Schaenis	735	9.06665	47.16594	60
13	Visp	694	7.85708	46.29691	80
14	Vordemwald	500	7.88686	47.27417	14

The data

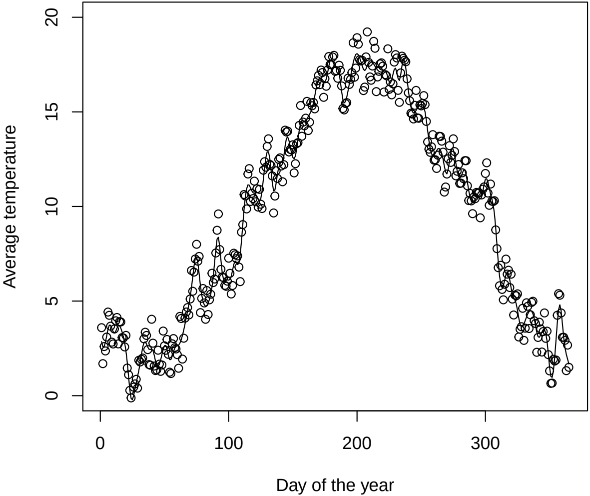


Beatenberg open field
Temperature
30
20
10
0
−10
−20
1997−12−21T15:00
2006−04−08T05:00
2014−06−15T10:00
Date

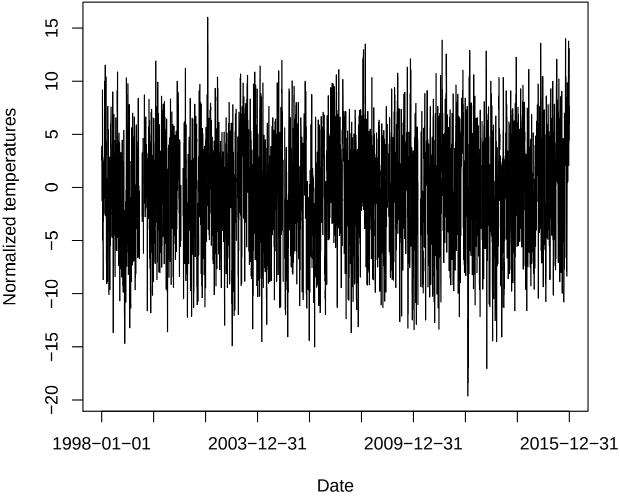
Preprocessing



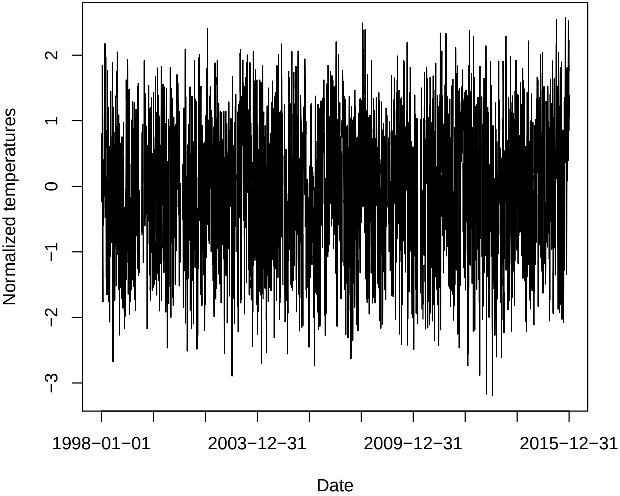
Preprocessing



Preprocessing



Preprocessing

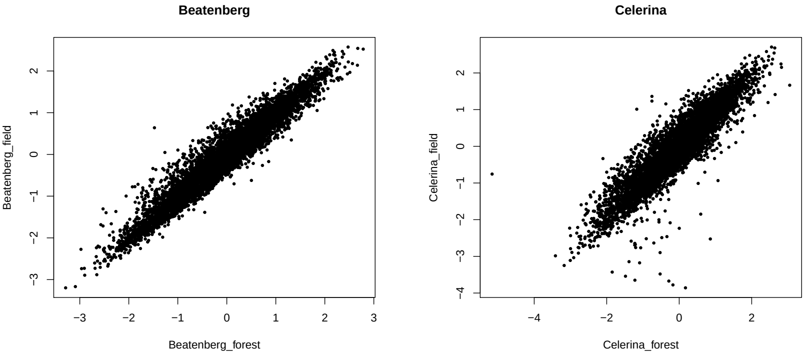


Preprocessing

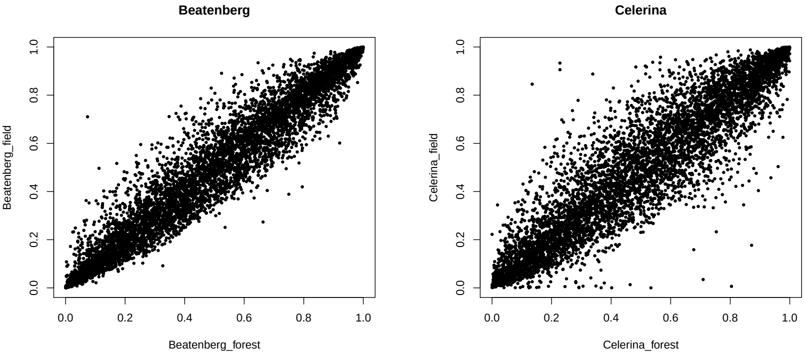
- We (almost) repeated the steps of Ferrez et al. (2011) to obtain a stationary time series.
- They also fornally showed that there is no long-range dependence between the maxima by fitting an AR model.
- The same steps need to be repeated for all the locations and open field vs canopy cover.

J. Ferrez, A.C. Davison, M. Rebetez (2011) Extreme temperature analysis under forest cover compared to an open field. *Agricultural and Forest Meteorology* 151:992-1001

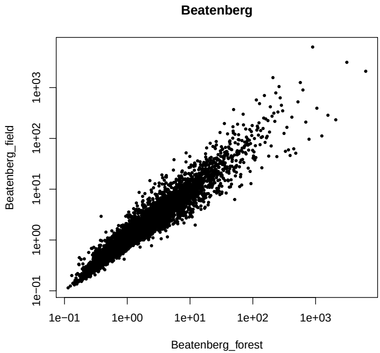
Bivariate depedendence

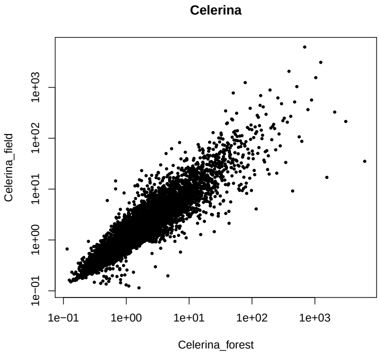


Bivariate dependence



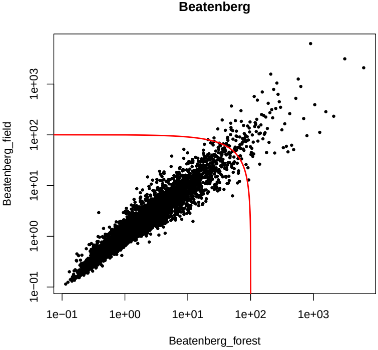
Bivariate dependence

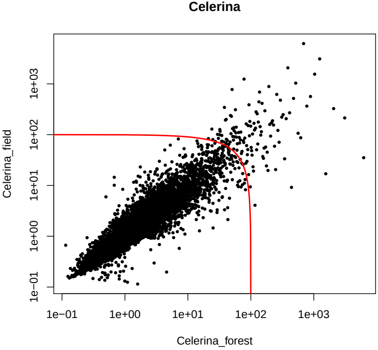




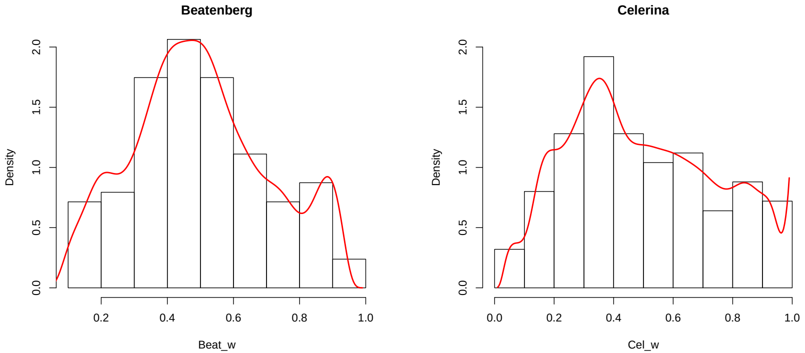
Standard modeling

First we are going to analyse these data independently.





Angular densities



Drawbacks

- We could fit the angular density at each location and study the dependence;
- However it would be wiser to fit a joint model borrowing strength from the information at all locations;
- Formal way to assess the effect of covariates.

Fitting covariate-dependent estimators

To fit these more general models, we need to first create a sample from each spectral density given a level of the covariate.

Thus for this example, we extract the exceedances over a certain radius (98%) and merge these exceedances into a unique data frame.

Formatting the data

	Date	Spec	Altitude	Longitude	Latitude	Slope
1	12/01/1998	0.877932756	1533	7.76392	46.69990	66
2	16/02/1998	0.855883696	1533	7.76392	46.69990	66
3	18/02/1998	0.861498954	1533	7.76392	46.69990	66
4	19/02/1998	0.846453404	1533	7.76392	46.69990	66
5	20/02/1998	0.846340250	1533	7.76392	46.69990	66
6	21/02/1998	0.900171830	1533	7.76392	46.69990	66
7	04/03/1998	0.918338421	1533	7.76392	46.69990	66
8	29/03/1998	0.800190525	1533	7.76392	46.69990	66
9	05/06/1998	0.734209495	1533	7.76392	46.69990	66
10	12/08/1998	0.428316960	1533	7.76392	46.69990	66
11	05/01/1999	0.890854241	1533	7.76392	46.69990	66
12	06/01/1999	0.904461500	1533	7.76392	46.69990	66
13	24/03/2001	0.769527590	1533	7.76392	46.69990	66
14	02/08/2001	0.652809307	1533	7.76392	46.69990	66
15	02/10/2001	0.606764573	1533	7.76392	46.69990	66

The SERR estimator

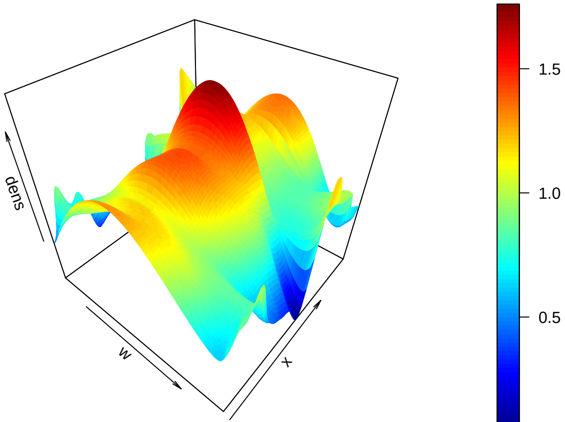
The spectral surface h_x is estimated by

$$\hat{h}_x(w) = \frac{\mathbb{K}_b(x - x_k)\tilde{h}_k(w)}{\sum_{k=1}^{K} \mathbb{K}_b(x - x_k)}$$

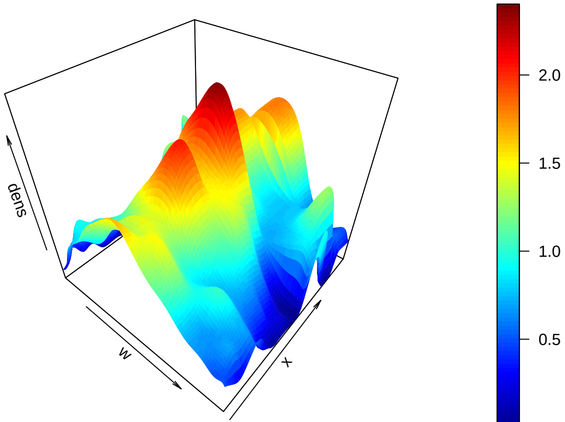
where

- $\mathbb{K}_b$ is a kernel density estimator
- b is the bandwidth
- $\tilde{h}_k$ is the Euclidean likelihood estimator

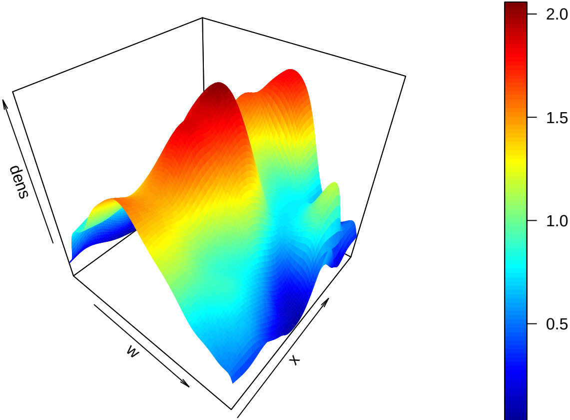
Altitude, nu=10, b=50



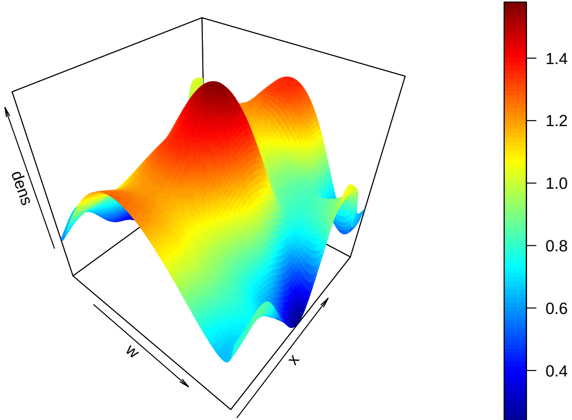
Altitude, nu=50, b=50



Altitude, nu=50, b=100



Altitude, nu=10, b=100



The AoAS estimator

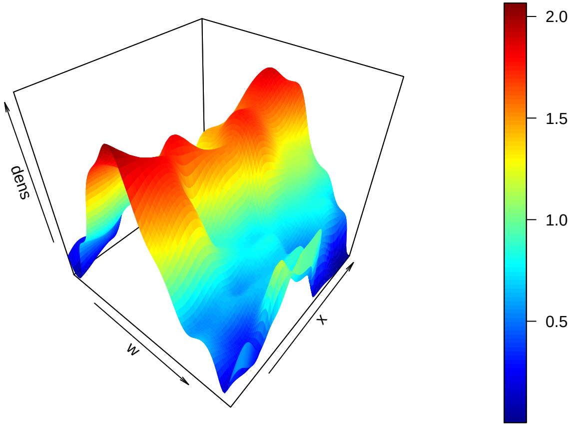
The spectral surface is estimated by

$$\hat{h}_x(w) = \sum_{i=1}^{n} \pi_{b,i}(x)\beta(w; \nu w_i\theta_b(x) + \tau, \nu(1 - w_i\theta_b(x)) + \tau),$$

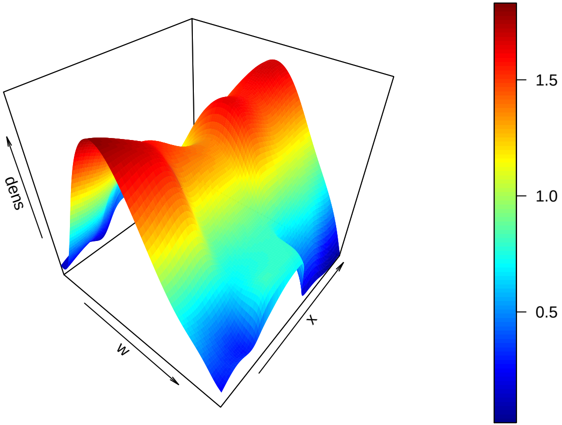
where

$$\theta_b(x) = \frac{1/2}{\sum_{i=1}^{n} \pi_{b,i}(x) w_i}, \qquad \pi_{b,i}(x) = \frac{\mathbb{K}_b(x - x_i)}{\sum_{j=1}^{n} \mathbb{K}_b(x - x_j)}$$

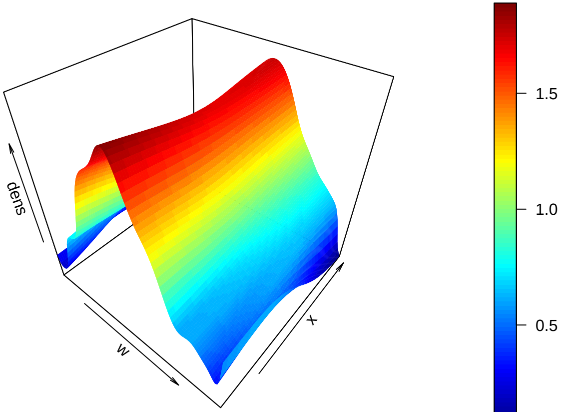
Slope, nu=100, b=5



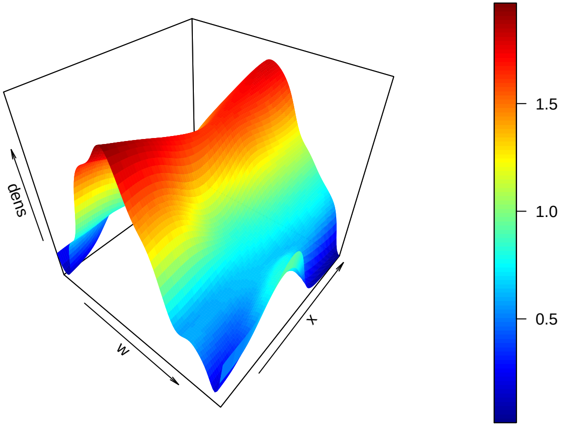
Slope, nu=20, b=5



Slope, nu=100, b=20



Slope, nu=100, b=10



Density ratio model

Density ratio models have been investigated in details yesterday and an application of these studied in details.

Code to implement these can be found here

Conclusions

We briefly discussed data pre-processing for the open-field vs under-canopy heat dependence and illustrated some of the estimators of the spectral density.

The code to implement these is given so that you can try and fit various spectral surfaces.