Nonstationary Extremes, II
Heteroscedastic Extremes in Practice

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Plan for this session

1. Overview of available sofware
2. Implement univariate methods in R:
   (a) Block maxima approach
   (b) r-largest observations appraoch
   (c) Peaks over threshold
   (d) Heteroscedastic extensions
3. Discuss assumptions, diagnostics and model selection criteria

All code to implement the methods is available at

github.com/manueleleonelli.

Why R?

Three main reasons:

- R is free!
- R is open-source
- Largest set of functions for extreme value analysis (EVA)

Other sofware is available:

- In Matlab: `EVIM` and `WAFO`
- GUI software: `Xtremes` and `EXTREMES`
- Specialized software: `HYFRAN`, `GLSNet`

E. Gilleland, M. Ribatet, A.G. Stephenson (2013) A software review for extreme value analysis. *Extremes* 16:103-119.

R packages

Most functions for EVA are stored in R packages to perform a wide array of analyses:

- Univariate: `ismev`, `extRemes`, `evmix`, `POT` ...
- Bayesian: `evdbayes`, `revdbayes`, `MCMC4extremes` ...
- Multivariate: `evd`, `copula`, `extremis`, `texmex` ...
- Spatial: `spatialExtremes`, `spatialADAI` ...

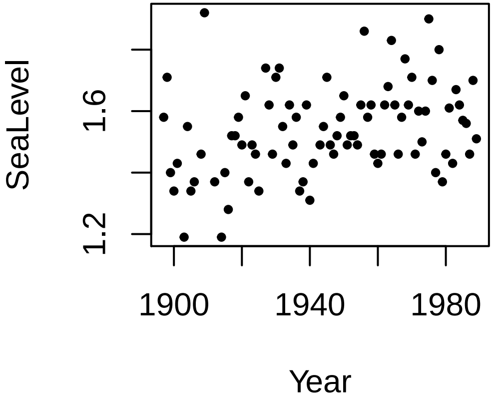
See this guide.

All these require at least some programming, as usual in R. One notable exception is in2extremes.

Let's get started!

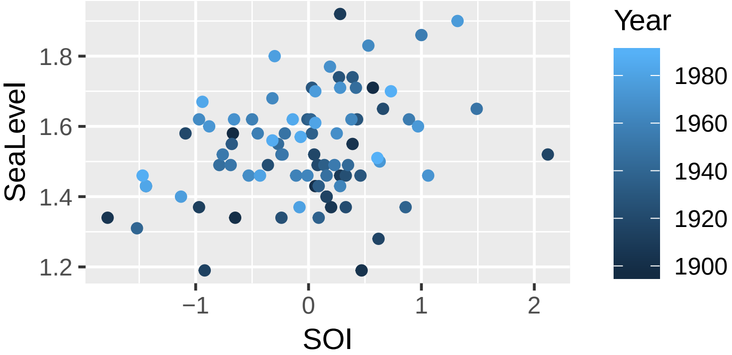
```
> library(ismev)
> data(fremantle)
> head(fremantle)
  Year SeaLevel   SOI
1 1897     1.58 -0.67
2 1898     1.71  0.57
3 1899     1.40  0.16
4 1900     1.34 -0.65
5 1901     1.43  0.06
7 1903     1.19  0.47
```

```
> plot(fremantle[,1:2], pch=19,cex=0.5)
```



Southern Oscillation Index (SOI)

```
> ggplot(fremantle,aes(x=SOI,y=SeaLevel,
+                       color=Year))+geom_point()
```



Fitting GEV distributions

```
> library(extRemes)
> ?fevd
```

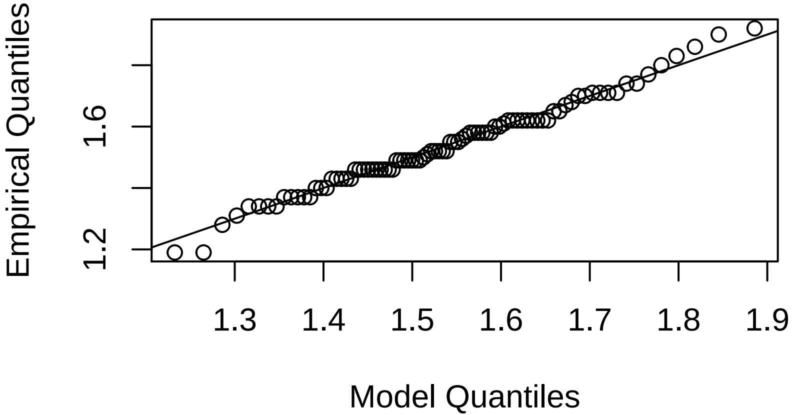
Let's look at the help

Another possibility is to use the `ismev` package.

E. Gilleland, R.W. Katz (2016) extRemes 2.0: An extreme value analysis package in R. *Journal of Statistical Software* 72:1-39.

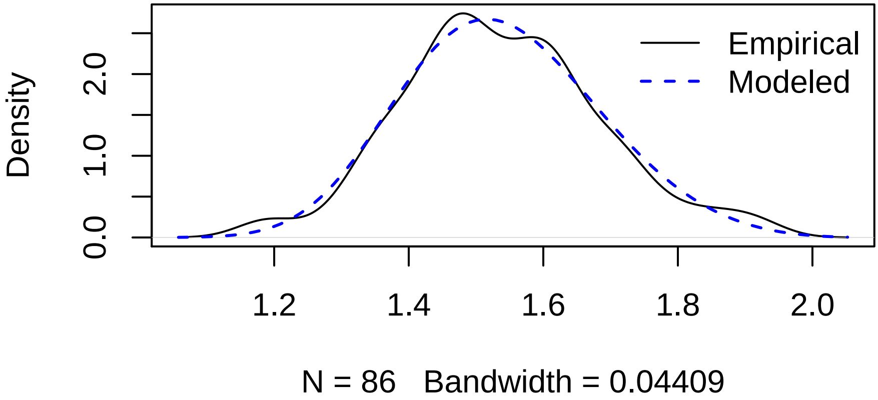
```
> fremantle_fit <- fevd(fremantle$SeaLevel,type="GEV")
> summary(fremantle_fit)
fevd(x = fremantle$SeaLevel, type = "GEV")

[1] "Estimation Method used: MLE"


 Negative Log-Likelihood Value:  -43.56663


 Estimated parameters:
  location      scale      shape
 1.4823417  0.1412723 -0.2174282

 Standard Error Estimates:
  location      scale      shape
0.01672527 0.01149706 0.06378114
```

```
> plot(fremantle_fit,type=c("qq"),main="")
```



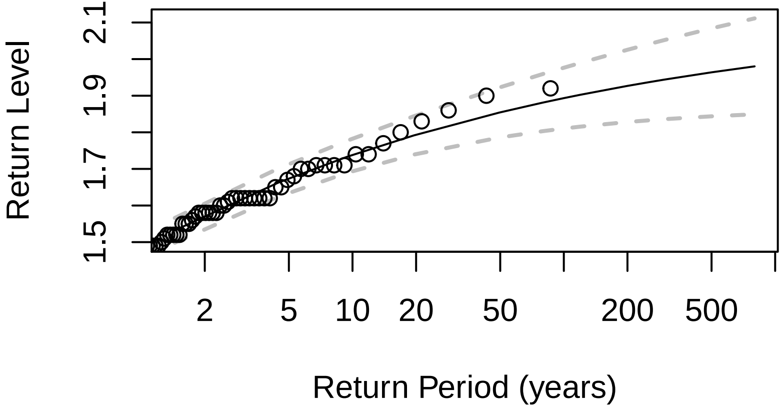
Diagnostics: densities

```
> plot(fremantle_fit,type=c("density"),main="")
```



Diagnostics: return levels

```
> plot(fremantle_fit,type=c("rl"),main="")
```



Inference: confidence intervals

```
> ci(fremantle_fit,type="parameter")
fevd(x = fremantle$SeaLevel, type = "GEV")

[1] "Normal Approx."

         95% lower CI   Estimate 95% upper CI
location    1.4495608  1.4823417   1.51512265
scale       0.1187385  0.1412723   0.16380615
shape      -0.3424370 -0.2174282  -0.09241948
```

Inference: return levels

```
> return.level(fremantle_fit,return.period=
+               c(2,10,20,50,100,500),do.ci=TRUE)
fevd(x = fremantle$SeaLevel, type = "GEV")

[1] "Normal Approx."

                     95% lower CI Estimate 95% upper CI
2-year return level      1.498259 1.532110     1.565962
10-year return level     1.689876 1.733753     1.777631
20-year return level     1.739108 1.791463     1.843817
50-year return level     1.785590 1.853927     1.922265
100-year return level    1.810194 1.893106     1.976017
500-year return level    1.843731 1.963815     2.083900
```

Bayesian GEV fitting

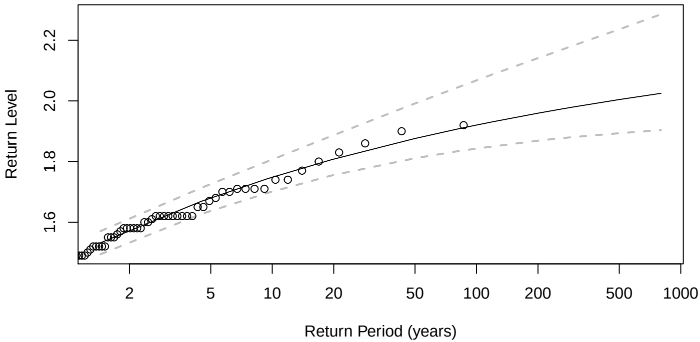
Let's look at the help again.

```
>  fremantle_fit_Bayes <- fevd(fremantle$SeaLevel,
+                                     type="GEV",method="Bayesian")
> summary(fremantle_fit_Bayes)
```

```
[1] "Quantiles of MCMC Sample from Posterior Distribution"

              2.5% Posterior Mean       97.5%
location  1.4432050      1.4806897  1.51878531
scale     0.1225513      0.1451410  0.17234688
shape    -0.3333519     -0.2014116 -0.05050336
```

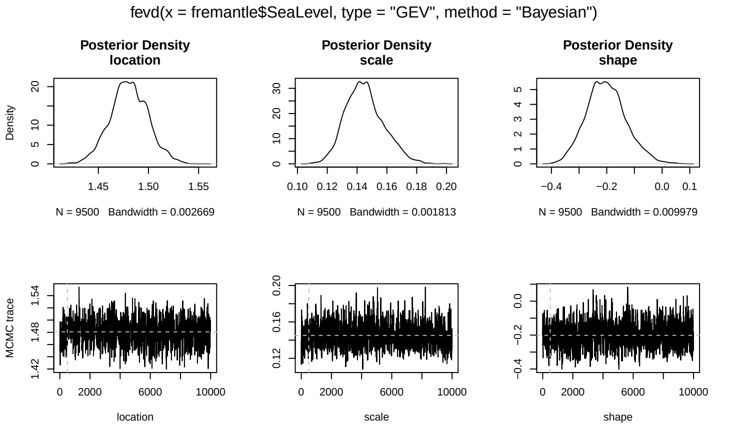
Bayesian return levels

```
> plot(fremantle_fit_Bayes,type="rl",main="")
```



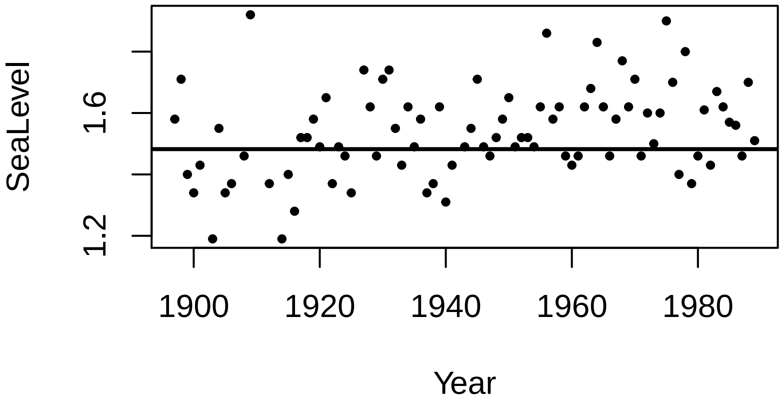
Convergence checking

```
> plot(fremantle_fit_Bayes,type="trace")
```



Estimated location of maximum Sea Level

```
> plot(fremantle[,1:2],pch=19,cex=0.5)
> abline(h=fremantle_fit$results$par[1],lwd=2)
```



Non-stationary fitting

```
> fremantle$Time <- fremantle$Year-min(fremantle$Year)
> fremantle_fit_time <- fevd(SeaLevel,data=fremantle,
+                              location.fun = ~Time)
> summary(fremantle_fit_time)
```

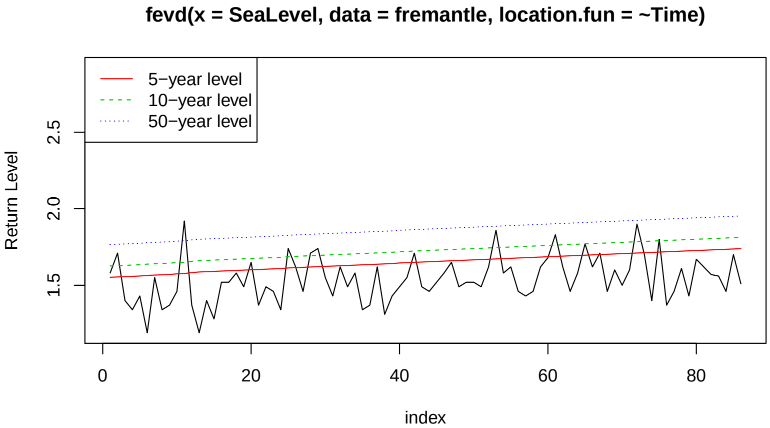
```
Estimated parameters:
        mu0          mu1        scale        shape
 1.382225781  0.002032151  0.124326256 -0.125309664

 Standard Error Estimates:
        mu0          mu1        scale        shape
0.0288458801 0.0004982185 0.0103749710 0.0682389684
```

Is there a time effect?

```
> ci(fremantle_fit_time,type="parameter")
fevd(x = SeaLevel, data = fremantle, location.fun = ~Time)

[1] "Normal Approx."

      95% lower CI     Estimate 95% upper CI
mu0     1.32568889  1.382225781  1.438762667
mu1     0.00105566  0.002032151  0.003008641
scale   0.10399169  0.124326256  0.144660825
shape  -0.25905558 -0.125309664  0.008436257
```

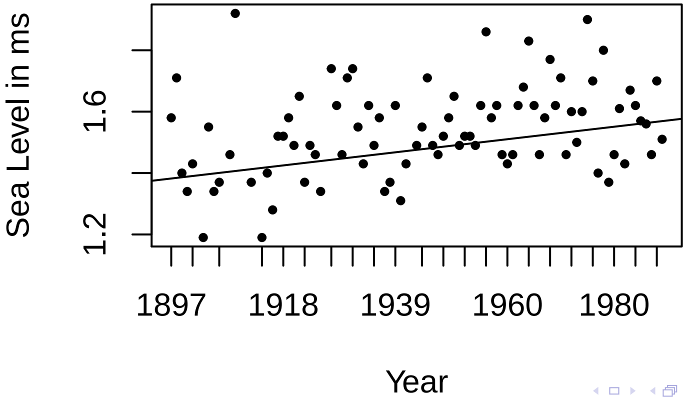
Time-varying return levels

```
> plot(fremantle_fit_time,"rl",rperiods=c(5,10,50))
```



Time-varying location of Sea Level

```
> plot(fremantle[,1:2],pch=19,cex=0.5)
> abline(a=fremantle_fit_time$results$par[1],
+                    b=fremantle_fit_time$results$par[2])
```

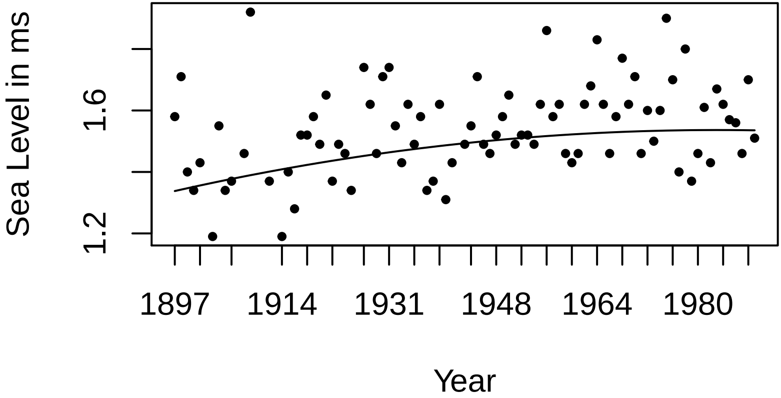


Model selection via hypothesis testing

```
> dev <- 2*(-fremantle_fit_time$results$value
+           +fremantle_fit$results$value)
> dev > qchisq(0.95,1)
[1] TRUE
> 1-pchisq(dev,1)
[1] 0.0003671508
```

Model	AIC	BIC
Stationary	-81.1	-73.8
Non-Stationary	-91.8	-82.0

A quadratic time effect?



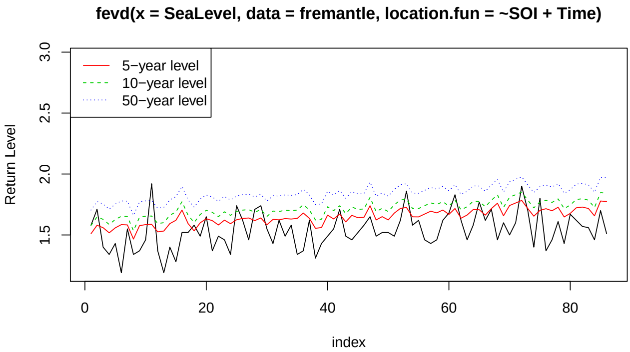
Deviance test has p-value 0.2231935 - *No significant quadratic effect*

Adding SOI to the model

```
> fremantle_fit_SOI <- fevd(SeaLevel,data=fremantle,
+                           location.fun = ~SOI+Time)
> ci(fremantle_fit_SOI,type="parameter")
fevd(x = SeaLevel, data = fremantle, location.fun = ~SOI + Tim

[1] "Normal Approx."

      95% lower CI     Estimate 95% upper CI
mu0    1.328079958  1.384328499   1.44057704
mu1    0.016019518  0.054519871   0.09302022
mu2    0.001141007  0.002114038   0.00308707
scale  0.101075964  0.120733366   0.14039077
shape -0.276802887 -0.149992773  -0.02318266
```

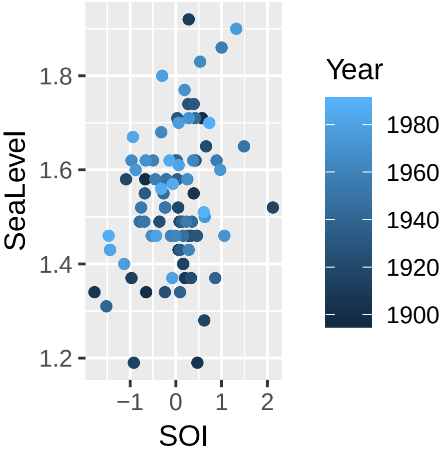
Time and SOI dependent returns

```
> plot(fremantle_fit_SOI,type="rl",rperiod=c(5,10,50))
```



Deviance test has p-value 5.47286e-06 - *Significant SOI effect*

What about an interaction?



We can test this easily using `location.fun = SOI+Time`.

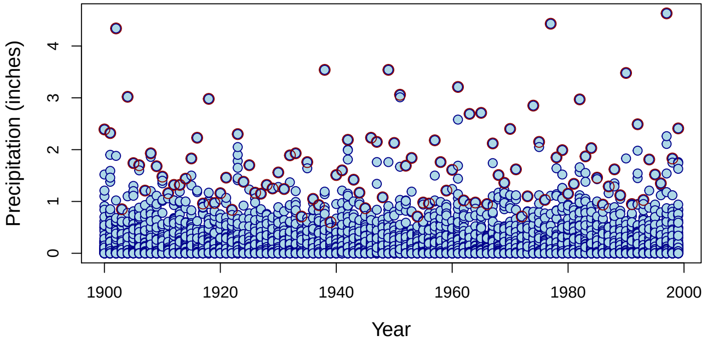
However the deviance test has p-value 0.6162236, thus no interaction effect.

Extracting maxima

```
> data(Fort)
> head(Fort)
  obs tobs month day year Prec
1   1    1     1   1 1900    0
2   2    2     1   2 1900    0
3   3    3     1   3 1900    0
4   4    4     1   4 1900    0
5   5    5     1   5 1900    0
6   6    6     1   6 1900    0
```

Extracting maxima

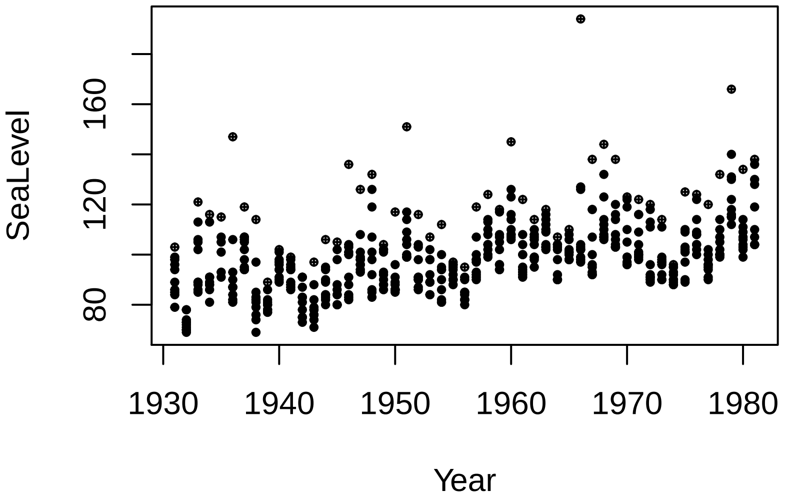
```
> bmFort <- blockmaxxer(Fort,
+                 blocks = Fort$year, which="Prec")
```



Venice sea levels

```
> data(venice)
> head(venice)
  Year  r1  r2  r3  r4  r5 r6 r7 r8 r9 r10
1 1931 103  99  98  96  94 89 86 85 84  79
2 1932  78  78  74  73  73 72 71 70 70  69
3 1933 121 113 106 105 102 89 89 88 86  85
4 1934 116 113  91  91  91 89 88 88 86  81
5 1935 115 107 105 101  93 91 NA NA NA  NA
6 1936 147 106  93  90  87 87 87 84 82  81
```

Venice sea levels



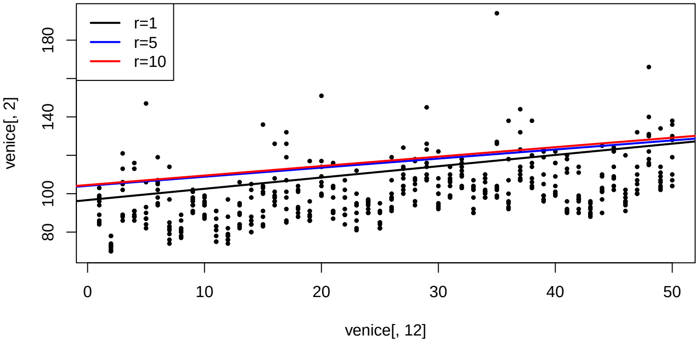
Fitting r-largest observations

```
> venice <- venice[-5,] #omit NA
> venice$time <- seq(1:nrow(venice)) #add time variable
> venice_fit_time <-fevd(r1,type="GEV",
+                         data = venice,location.fun = ~time)
> # Using the package ismev
> venice_fit_time_5 <- rlarg.fit(venice[,2:6],
+                                 ydat=venice,mul=12)
> venice_fit_time_10 <- rlarg.fit(venice[,2:11],
+                                 ydat=venice, mul=12)
```

r	β_0	β_1	σ	ξ
1	96.7 (4.3)	0.59 (0.14)	14.6 (1.6)	-0.022 (0.084)
5	104.1 (2.0)	0.47 (0.06)	12.3 (0.8)	-0.033 (0.043)
10	104.54 (1.7)	0.49 (0.04)	11.75 (0.7)	-0.06 (0.028)

Time-dependent location

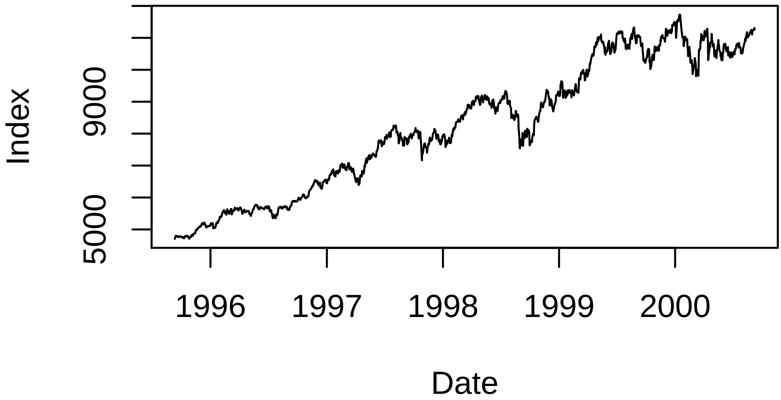
Various diagnostics can be plotted using `rlarg.diag`



Perhaps a “seasonal” effect?

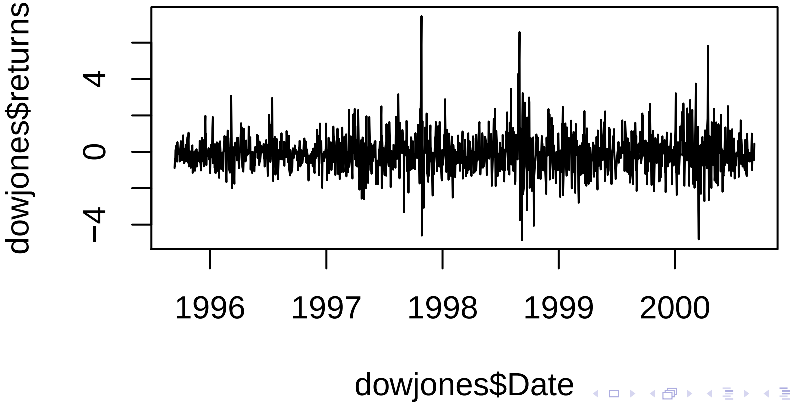
A financial application: the Dow Jones Index

```
> data(dowjones)
> plot(dowjones,type='l')
```



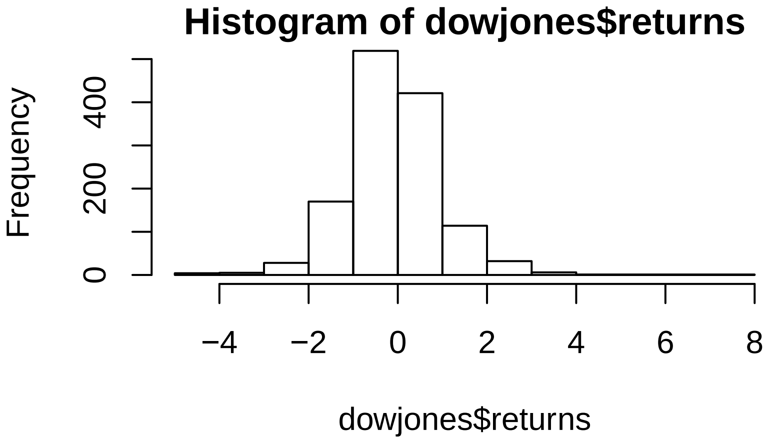
The negative log-returns

```
> dowjones <- data.frame(dowjones[-1,],
+                    returns=-100*diff(log(dowjones$Index)))
> plot(dowjones$Date,dowjones$returns,type="l")
```



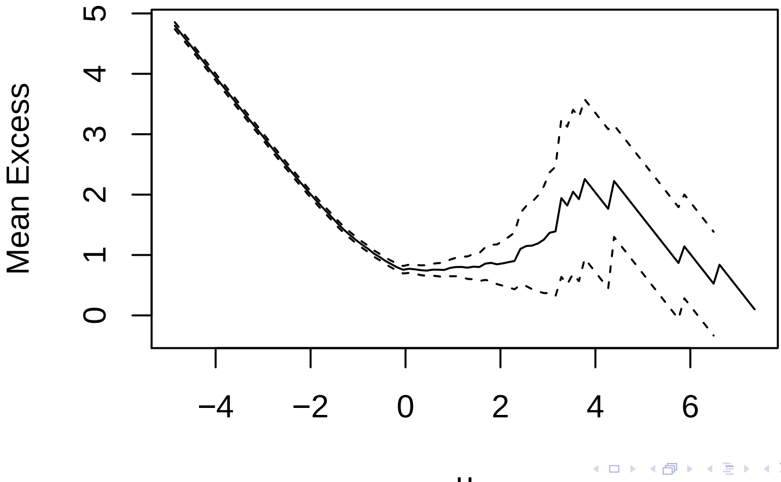
The returns' distribution

```
> hist(dowjones$returns)
```



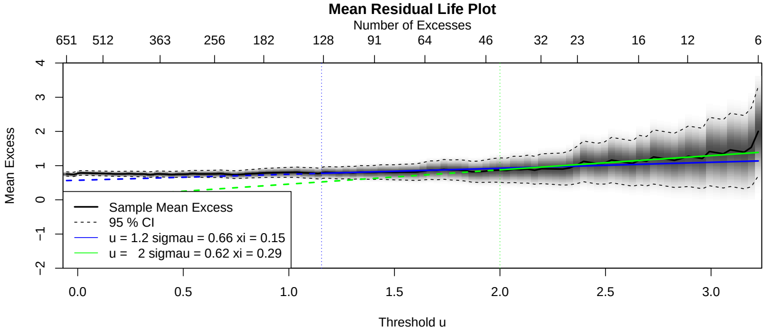
Choosing the threshold: MRL plot

```
> mrl.plot(dowjones$returns)
```



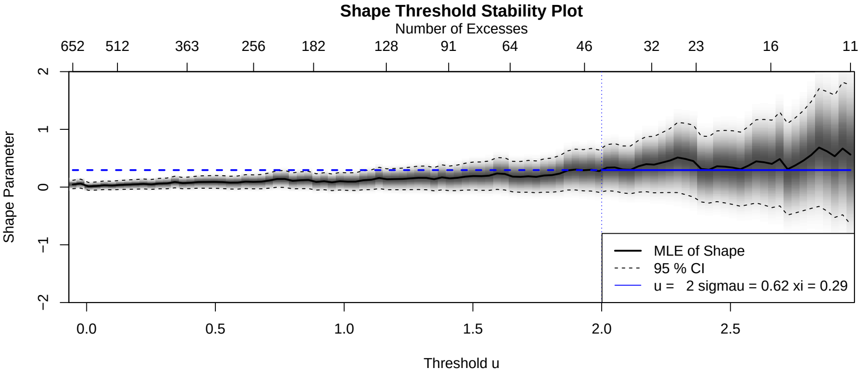
Choosing the threshold: MRL plot

```
> library(evmix)
> mrlplot(dowjones$returns,
+         try.thresh=c(quantile(dowjones$returns,0.9),2),
+         ylim = c(-2,4))
```



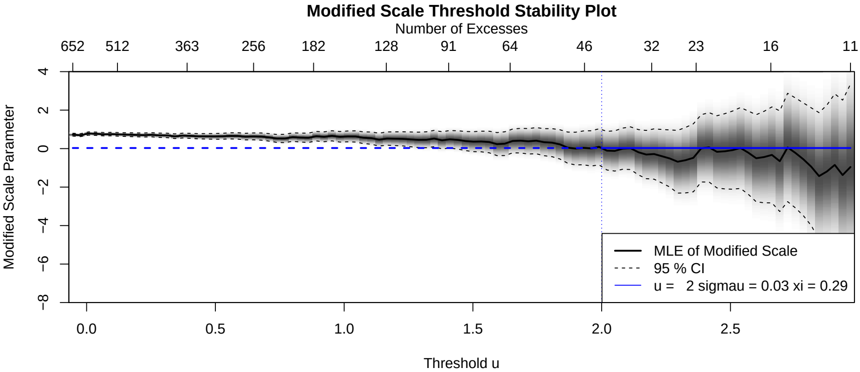
Choosing the threshold: stability plots

```
> tshapeplot(dowjones$returns,ylim=c(-2,2),try.thresh = 2)
```



Choosing the threshold: stability plots

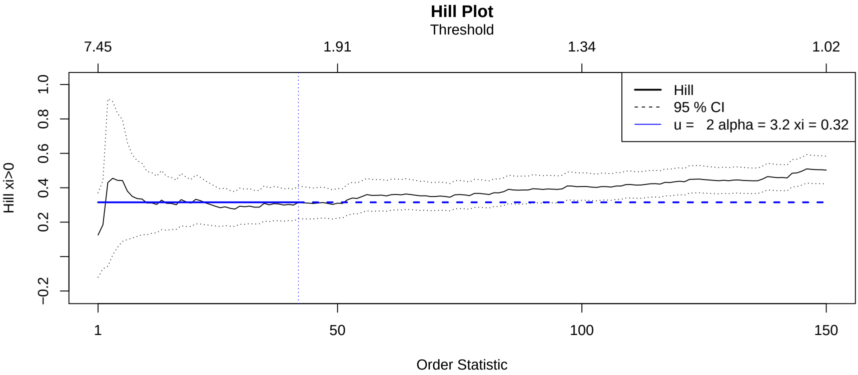
```
> tscaleplot(dowjones$returns,ylim=c(-8,4),try.thresh = 2)
```



Choosing the threshold: hill plot

This is only valid for heavy-tailed distributions $\xi > 0$

```
> hillplot(dowjones$returns,orderl=c(1,150),
+           try.thresh= 2,xlab="Order Statistic")
```



Fitting the GPD to exceedances

```
> dow_fit <- fevd(dowjones$returns,type="GP",threshold = 2)
> summary(dow_fit)
```

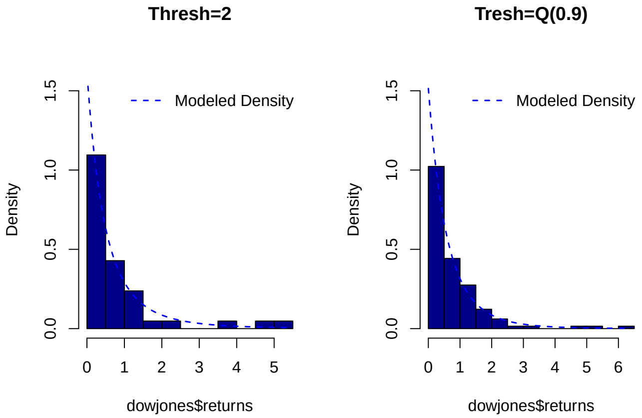
```
Estimated parameters:
    scale     shape
0.6183804 0.2941935

 Standard Error Estimates:
    scale     shape
0.1496571 0.1918915
```

Fitting the GPD to exceedances

```
> dow_fit2 <- fevd(dowjones$returns,type="GP",
+                   threshold=quantile(dowjones$returns,0.9))
> summary(dow_fit2)
```

```
Estimated parameters:
    scale     shape
0.6593380 0.1490148

 Standard Error Estimates:
     scale      shape
0.08408249 0.09380125
```

Estimated densities



Estimated parameters and return levels

Thresh = 2

	lower	mean	upper
scale	0.325	0.618	0.911
shape	-0.081	0.294	0.670

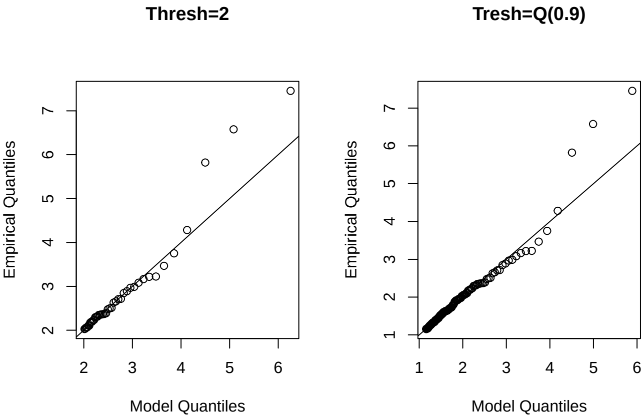
	lower	mean	upper
2-year	3.43	5.22	7.00
20-year	1.35	10.4	19.40
100-year	-5.74	16.7	39.20

Thresh = q(0.9)

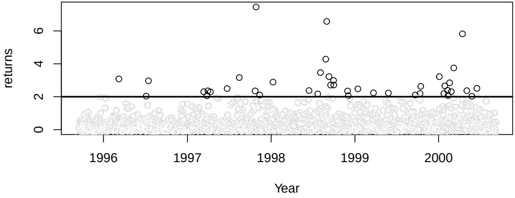
	lower	mean	upper
scale	0.494	0.659	0.824
shape	-0.034	0.194	0.333

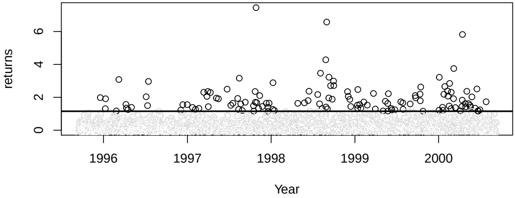
	lower	mean	upper
2-year	3.77	5.22	7.00
20-year	4.25	8.56	12.86
100-year	3.56	11.7	19.97

Quantile plots



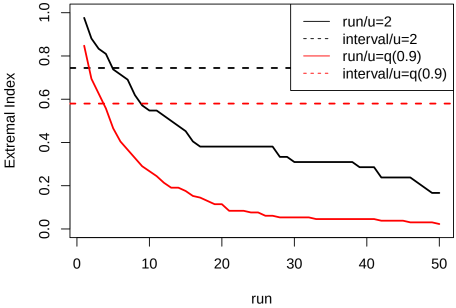
Are returns independent?





Extremal index

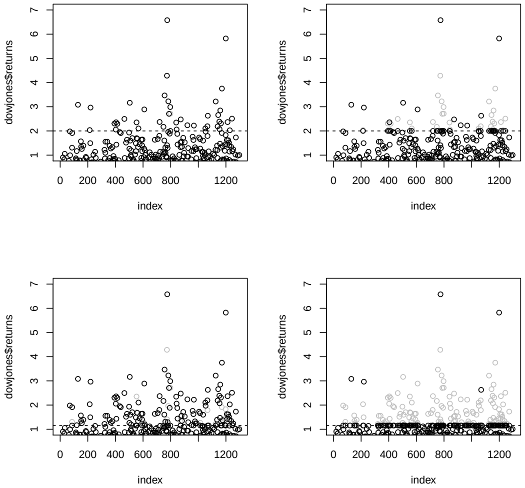
The extremal index θ measures the level of clustering of extreme events ($\theta = 1$ corresponding to independence)



C.A.T. Ferro,J. Segers (2003) Inference for clusters of extreme values . *Journal of the Royal Statistical Society Series B* 65:545-556.

Declustering

We can use the function `decluster` to extract the cluster maxima.



Results after declustering

Thresh.	r	σ	ξ	Ext.Index
2	0	0.62 (0.15)	0.29 (0.19)	0.74
2	5	0.61 (0.15)	0.31 (0.20)	0.76
2	7	0.79 (0.23)	0.24 (0.22)	1
q(0.9)	0	0.66 (0.08)	0.15 (0.09)	0.57
q(0.9)	4	0.72 (0.10)	0.12 (0.09)	0.70
q(0.9)	7	1.10 (0.22)	0.07 (0.14)	1

Heterоscedasticity

- Extremes may also not be identically distributed.
- The scedasis function gives a representation of tail risk.
- To compute this we need to restrict ourselves to the case $\xi > 0$ and constant.
- The `extremis` package gives an easy implementation for the scedasis.

J.H.J Einmahl, L. de Haan, C. Zhou (2016) Statistics of heteroscedastic extremes. *Journal of the Royal Statistical Society Series B* 78:31-51.

Testing for constant ξ

Here we implement the following test

1. Compute the Hill estimator $\hat{\xi}$ over the full time series using the k highest observations
2. Divide the time series into m blocks of equal length
3. Compute the Hill estimator $\hat{\xi}_l$ over each block using the $\lfloor k/m \rfloor + 1$ highest observations
4. Compute the test statistics

$$T = \frac{1}{m} \sum_{l=1}^{m} \left(\frac{\hat{\gamma}_l}{\hat{\gamma}} - 1 \right)^2$$

5. Compare kT with the quantile $\chi^2_{m-1}(1-\alpha)$ for a confidence level α

Results for the Dow Jones series

The values k were chosen to match the threshold used till now.

m	k	p-value
4	42	0.99
4	130	0.67
3	42	0.87
3	130	0.99

The assumption of constant ξ looks tenable.

Fitting the scedasis function

We can use the function `cdensity` to estimate the scedasis.

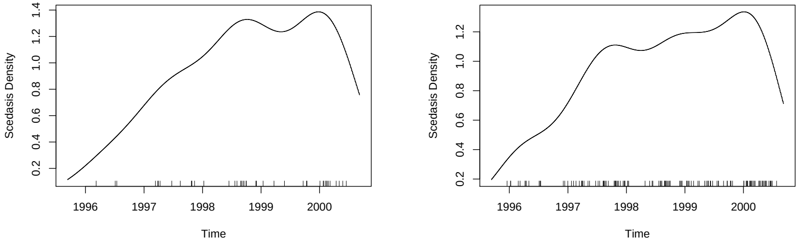


Figure: Scedasis function: threshold=2 (left), threshold=q(0.9) (right)

Fitting the scedasis cdf

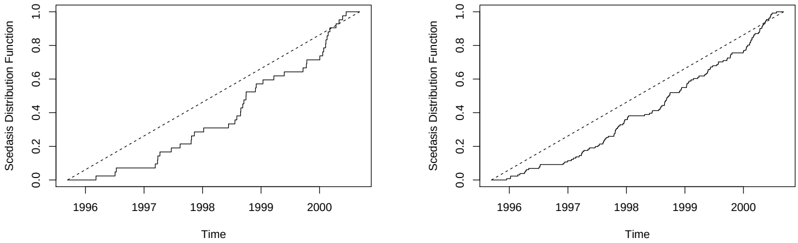


Figure: Scedasis cdf: threshold=2 (left), threshold=q(0.9) (right)

Conclusions

- R provide a set of packages to carry out a large number of inferential routines for extremes, including non-stationary, heteroscedastic extensions.
- Tomorrow we will look at the implementation of multivariate methods
- On Saturday you will actively fit extreme models to data.