Maths for Computing

Lecture 1: Introduction and Set Theory

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Course's Objectives

Introduce basic mathematical concepts that are fundamental in modern computing applications

- Set Theory
- Functions
- Linear Algebra
- Calculus
- Combinatorics
- Graph Theory

We will (try to) motivate their use through practical applications.

Today's Objective

- Introduction and presentation of the course
- Synchronous vs asynchronous classes
- Introduction to set theory

Assessment

- Participation 20%
  - Attendance in synchronous classes;
  - Participation in the asynchronous classes;
- Quizzes 20%
  - 4 multiple choice quizzes consisting of 10 questions each.
  - 24hrs of time to complete it in the online campus.
- Exam 60%: multiple choice questions in the online campus (last lecture)
- Although there is no fixed rule and decisions will be made case-by-case a rule of thumb is that the overall grade should be above 50% and the exam grade above 35%. (The module is graded but no curve is applied).

Synchronous vs Asynchronous

- The module consists of 12 synchronous face2face classes and 3 asynchronous classes.
- For asynchronous classes you will be asked to read some documents discussing the use of mathematical techniques in practical applications and to contribute to a forum discussion.
- For the calendar of events refer to the online campus.

Rules

- Attendance in class (physical) is mandatory, unless there is a justification.
- Please follow the seating chart.
- I don't have any other strict rule, just follow common sense!

Sets

A *set* is an unordered collection of objects, called *elements* or *members* of the set. A set is said to *contain* its elements. We write $a \in A$ to denote that a is an element of the set A. The notation $a \notin A$ denotes that a is not an element of the set A.

- The set V of all vowels is $V = \{a, e, i, o, u\}$;
- The set O of odd positive integers less than 10 is $O = \{1, 3, 5, 7, 9\}$;
- The set of positive integers less than 100 is $\{1, 2, 3, \dots, 98, 99\}$.
- This way of describing sets is called *roster method*.

Sets

Another way to describe a set is to use the *set builder notation*:

- $O = \{x | x \text{ is an odd positive integer less than } 100\}$;
- $O = \{x \in \mathbb{Z}_+ | x \text{ is odd and } x < 100\}$

Some famous sets:

- $\mathbb{N} = \{0, 1, 2, 3, \dots\}$: the set of *natural numbers*;
- $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$: the set of *integers*;
- $\mathbb{Z}_+ = \{1, 2, 3, \dots\}$': the set of *positive integers*;
- $\mathbb{Q} = \{p/q | p, q \in \mathbb{Z} \text{ and } q \neq 0\}$: the set of *rational numbers*;
- $\mathbb{R}$: the set of *real numbers*;
- $\mathbb{R}_+$: the set of positive real numbers.

Intervals

Recall the notation for *intervals* of real numbers. When $a, b \in \mathbb{R}$ with $a < b$, we write

- $[a, b] = \{x \in \mathbb{R} | a \leq x \leq b\}$;
- $[a, b) = \{x \in \mathbb{R} | a \leq x < b\}$;
- $(a, b] = \{x \in \mathbb{R} | a < x \leq b\}$;
- $(a, b) = \{x \in \mathbb{R} | a < x < b\}$;

Note that $[a, b]$ is called a *closed interval* from a to b and (a, b) is called an *open interval* from a to b.

Two sets are *equal* if they have the same elements.

- $\{1,3,5\}$ and $\{3,5,1\}$ are equal;
- $\{1,3,5\}$ and $\{1,3,3,5,5,5\}$ are equal;

Sets can have other sets as members: for instance $\{\mathbb{N},\mathbb{Z},\mathbb{Q}\}$ is a set.

- A set with no elements is called *empty set* or *null set* and is denoted by $\emptyset$.
- A set with one element is called a *singleton set*. Notice that $\emptyset \neq \{\emptyset\}$.

Venn Diagrams

Sets can be represented graphically using Venn diagrams.

- The set of all objects under considerations, the universal set U, is represented by a rectangle;
- Inside this rectangle, circles or other geometrical shapes are used to represent sets.
- Sometimes points are used to represent particular elements of a set.

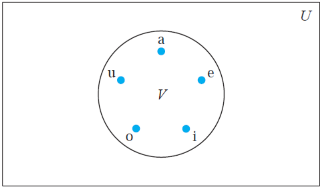

FIGURE 1 **Venn Diagram for the Set of Vowels.**

Subsets

The set A is a subset of B if and only if every element of A is also an element of B. We use the notation $A \subseteq B$ to indicate that A is a subset of B.

For every set S:

- $\emptyset \subseteq S$;
- $S \subseteq S$

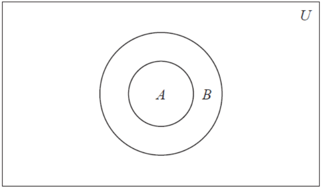

FIGURE 2 Venn Diagram Showing that A Is a Subset of B.

Subsets

When we wish to emphasize that a set A is a subset of a set B but that $A \neq B$ we write $A \subset B$ and say that A is a *proper subset* of B.

- The set of all odd positive integers less than 10 is a proper subset of the set of all positive integers less than 10;
- The set of rational numbers is a proper subset of the set of real numbers;
- The set of all people in China is a subset (not proper) of the set of all people in China.

Size of a Set

Let S be a set. If there are exactly n distinct elements in S where n is a non-negative integer, we say that S is a *finite set* and that n is the *cardinality* of S, denoted by $|S|$.
A set is said to be *infinite* if it is not finite.

- Let A be the set of odd positive integers less than 10. Then $|A| = 5$
- Let S be the set of letters in the English alphabet. Then $|S| = 26$
- The set of positive integers is infinite.

The Power Set

Given a set S, the *power set* of S is the set of all subsets of the set S. The power set of S is denoted by $\mathcal{P}(S)$.

What is the power set of $\{0, 1, 2\}$?

$$\mathcal{P}(\{0, 1, 2\}) =$$

If a set has n elements, then its power set has 2^n elements.

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The Power Set

Given a set S, the *power set* of S is the set of all subsets of the set S. The power set of S is denoted by $\mathcal{P}(S)$.

What is the power set of $\{0, 1, 2\}$?

$$\mathcal{P}(\{0,1,2\}) = \{\emptyset, \{0\}, \{1\}, \{2\}, \{0,1\}, \{0,2\}, \{1,2\}, \{0,1,2\}\}$$

If a set has n elements, then its power set has 2^n elements.

Cartesian Products

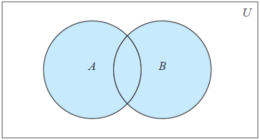
The *ordered n-tuple* $(a_1, a_2, \ldots, a_n)$ is the ordered collection that has a_1 as its first element, a_2 as its second element, ..., and a_n as its nth element.

Let A and B be sets. The *Cartesian product* of A and B denoted by $A \times B$ is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$. Hence $A \times B = \{(a, b) | a \in A \text{ and } b \in B\}$. We use the notation A^2 to denote $A \times A$.

Let A represent the set of all students at a university and B represent the set of all courses offered at the university. What is the Cartesian product $A \times B$ and how can it be used?

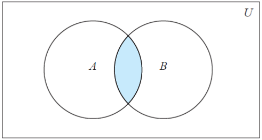
Let A and B be sets.

- The *union* of the sets A and B denoted by $A \cup B$ is the set that contains those elements that are either in A or in B or in both, that is $A \cup B = \{x | x \in A \text{ or } x \in B\}$
- The *intersection* of the sets A and B denoted by $A \cap B$ is the set containing those elements in both A and B, that is $A \cap B = \{x | x \in A \text{ and } x \in B\}$.



$A \cup B$ is shaded.

FIGURE 1 **Venn Diagram of the Union of A and B.**

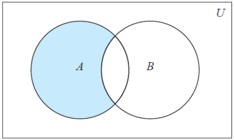


$A \cap B$ is shaded.

FIGURE 2 **Venn Diagram of the Intersection of A and B.**

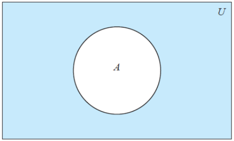
Let A and B be sets.

- The *difference* of the sets A and B denoted by $A - B$ is the set containing those elements that are in A but not in B, that is $A - B = \{x | x \in A \text{ an } x \notin B\}$
- The *complement* of the set A denoted by $\bar{A}$ is $U - A$, that is $\bar{A} = \{x | x \in U \text{ and } x \notin A\}$.



$A - B$ is shaded.

FIGURE 3 Venn Diagram for the Difference of A and B.



$\bar{A}$ is shaded.

FIGURE 4 Venn Diagram for the Complement of the Set A.

Set Operations

Let $A = \{1, 3, 5\}$, $B = \{1, 2, 3\}$ and $U = \{1, 2, 3, 4, 5\}$.

- $A \cap B =$
- $A \cup B =$
- $A - B =$
- $B - A =$
- $\bar{A} =$
- $\bar{B} =$

Set Operations

Let $A = \{1, 3, 5\}$, $B = \{1, 2, 3\}$ and $U = \{1, 2, 3, 4, 5\}$.

- $A \cap B = \{1, 3\}$
- $A \cup B = \{1, 2, 3, 5\}$
- $A - B = \{5\}$
- $B - A = \{2\}$
- $\bar{A} = \{2, 4\}$
- $\bar{B} = \{4, 5\}$

Two sets are called *disjoint* if their intersection is the empty set.

The *principle of inclusion-exclusion* states that

$$|A \cup B| = |A| + |B| - |A \cap B|$$