1. 1.22 from KNNL.

2. 2.16 from KNNL.

3. Suppose $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$ for $i = 1, \ldots, n$, where $x_1, \ldots, x_n$ are known constants and

$$E\left(\epsilon_i\right) = 0, \quad i = 1, \ldots, n$$

$$\mathrm{Cov}\left(\epsilon_i, \epsilon_j\right) = \begin{cases} \sigma^2 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}, \quad i, j = 1, \ldots, n.$$

Let $\hat{\beta}_0$ and $\hat{\beta}_1$ be the ordinary least squares estimates for β_0 and β_1 and define

$$\hat{\epsilon}_i = y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i, \quad i = 1, \ldots, n.$$

(a) Prove that the vector of $\hat{\epsilon}$'s is orthogonal to the design. That is, prove

$$\sum_{i=1}^{n} \hat{\epsilon}_i = \sum_{i=1}^{n} \hat{\epsilon}_i x_i = 0.$$

(b) Let $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$, $\hat{\bar{\epsilon}} = n^{-1} \sum_{i=1}^{n} \hat{\epsilon}_i$ and $\hat{\bar{y}} = n^{-1} \sum_{i=1}^{n} y_i$. Using part (a), show that

$$\sum_{i=1}^{n} \left(\hat{\epsilon}_i - \hat{\bar{\epsilon}}\right)\left(\hat{y}_i - \hat{\bar{y}}\right) = 0.$$

(c) Define $\hat{\sigma}^2 = (n-2)^{-1} \sum_{i=1}^{n} \hat{\epsilon}_i^2$. Show that $E\left(\hat{\sigma}^2\right) = \sigma^2$.

4. You just started working for a company that is interested in looking at how radio advertising affected their sales. The data consist of the amount of sales Y_i in \$100 and the amount of radio advertising time X_i in hours for the ith month for $i = 1, \ldots, 24$.

The following R code was run:

```
#Radio is the number of radio advertisements for each month, in hours
#sales is the profit for each month, in dollars
fit <- lm(sales ~ Radio, data = Sales)
```

The output was collated into the following tables:

```
------------------------------------------------------------------------------
                    Number of Observations          24

                         Analysis of Variance
                         Sum of           Mean
Source              DF   Squares          Square       F Value    Pr(>F)
Model                1   1952.65488       1952.65488   712.51     <.0001
Error               22   60.29179         2.74054
Corrected Total     23   2012.94667

        Root MSE               1.65546      R-Square     0.9700
        Dependent Mean       114.88255      Adj R-Sq     0.9687
        Coeff Var              1.44100

                         Parameter Estimates
                 Parameter   Standard        t
Variable    DF   Estimate      Error     Value  Pr(>|t|)
Intercept    1   101.57570   0.60225    168.66   <.0001
Radio        1   1.15806     0.04338     26.69   <.0001
```

(a) Suppose conditional on the variable Radio, sales from each month are independent. Write down the model assumed when running the above R code. Clearly state any assumptions and define all notations.

(b) Assume that the model in part (a) is appropriate. Provide a point estimate and a 95% confidence interval for the change in expected monthly amount of sales if monthly radio advertisement increases by 10 hours.

(c) Assume that the model in part (a) is appropriate, and that the residuals in part (a) are normally distributed. The manager plans to have no radio advertisement next month.

  (i) Provide him with a point estimate and a 95% prediction interval for the amount of sales next month.

  (ii) Why is the normality assumption for the prediction interval important, and why is it not critical we make that assumption in part (b)? Please give an intuitive explanation rather than fancy mathematical derivations.

(d) Perform a test to study whether radio advertisement tends to increase the amount of sales. State clearly the hypothesis, the testing statistic, the p-value and your conclusion.