Hw9_Ex_1

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```
library(MASS) # Contains function boxcox
setwd('/home/noble_mannu/Documents/PhD/First/STAT_2131_Applied_Statistical_Methods_I/HW9')
```

Homework 9

Exercise 1

Part (a)

Here we regress gamble onto the predictors sex, status, income and verbal and show the summary of the model.

```
Data.a <- read.table('Gambling.txt', header = TRUE)
linMod.a <- lm(gamble ~ ., data = Data.a)
summary(linMod.a)
```

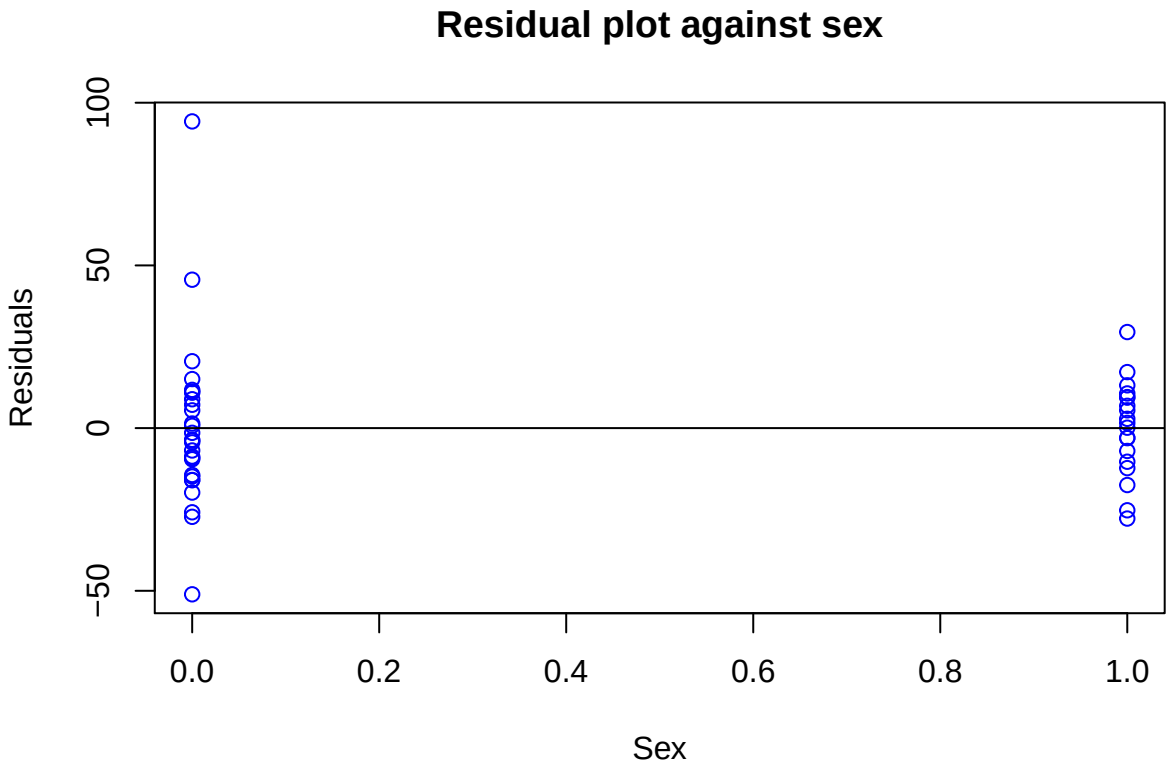
```
##
## Call:
## lm(formula = gamble ~ ., data = Data.a)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -51.082 -11.320  -1.451   9.452  94.252
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  22.55565   17.19680   1.312   0.1968
## sex         -22.11833    8.21111  -2.694   0.0101 *
## status        0.05223    0.28111   0.186   0.8535
## income        4.96198    1.02539   4.839 1.79e-05 ***
## verbal       -2.95949    2.17215  -1.362   0.1803
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 22.69 on 42 degrees of freedom
## Multiple R-squared:  0.5267, Adjusted R-squared:  0.4816
## F-statistic: 11.69 on 4 and 42 DF,  p-value: 1.815e-06
```

Next we plot the residual plots.

Residuals plots

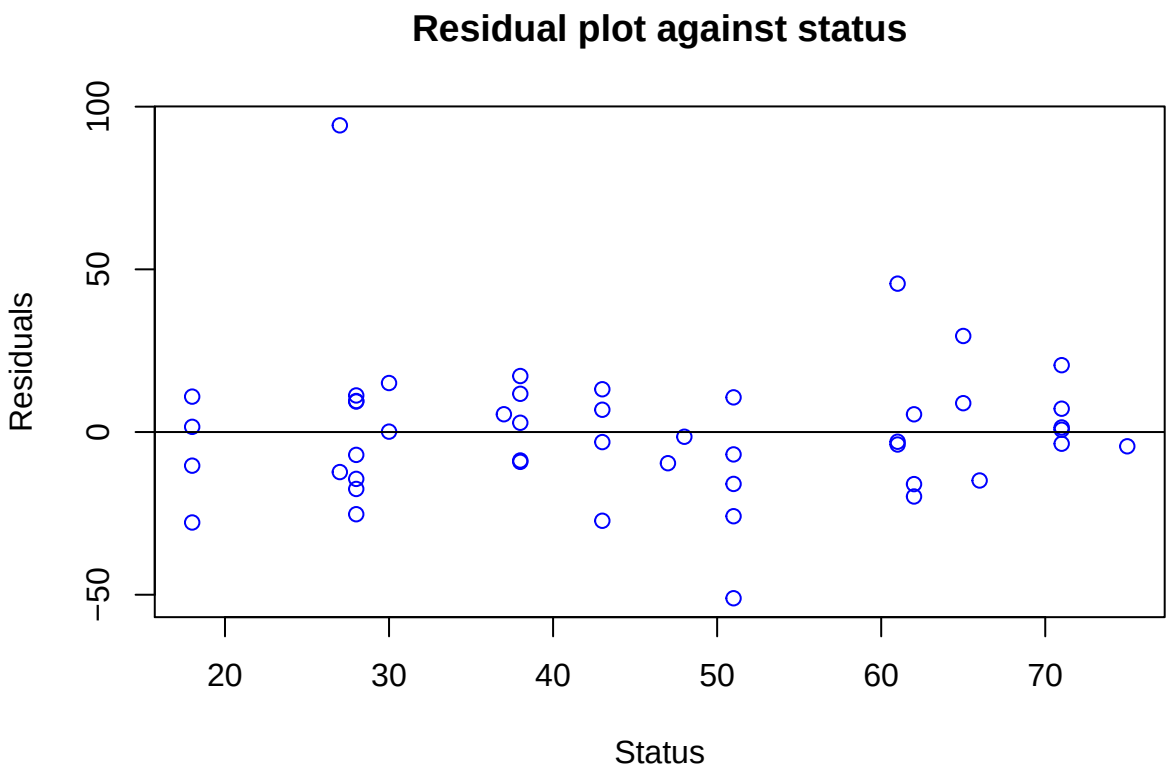
Plot the residuals against sex

```
plot(Data.a$sex, resid(linMod.a), xlab = "Sex", ylab = "Residuals",
     main = 'Residual plot against sex', col = 'blue')
abline(a=0, b=0)
```



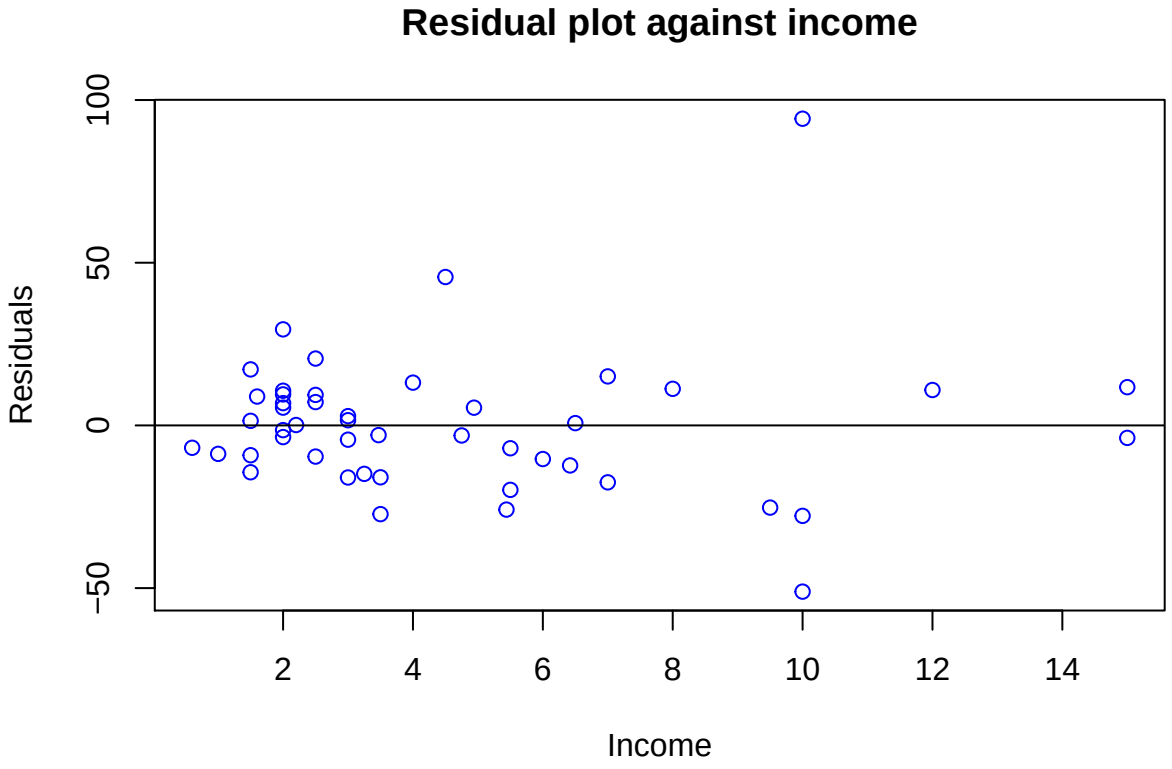
Plot the residuals against status

```
plot(Data.a$status, resid(linMod.a), xlab = "Status", ylab = "Residuals",
     main = 'Residual plot against status', col = 'blue')
abline(a=0, b=0)
```



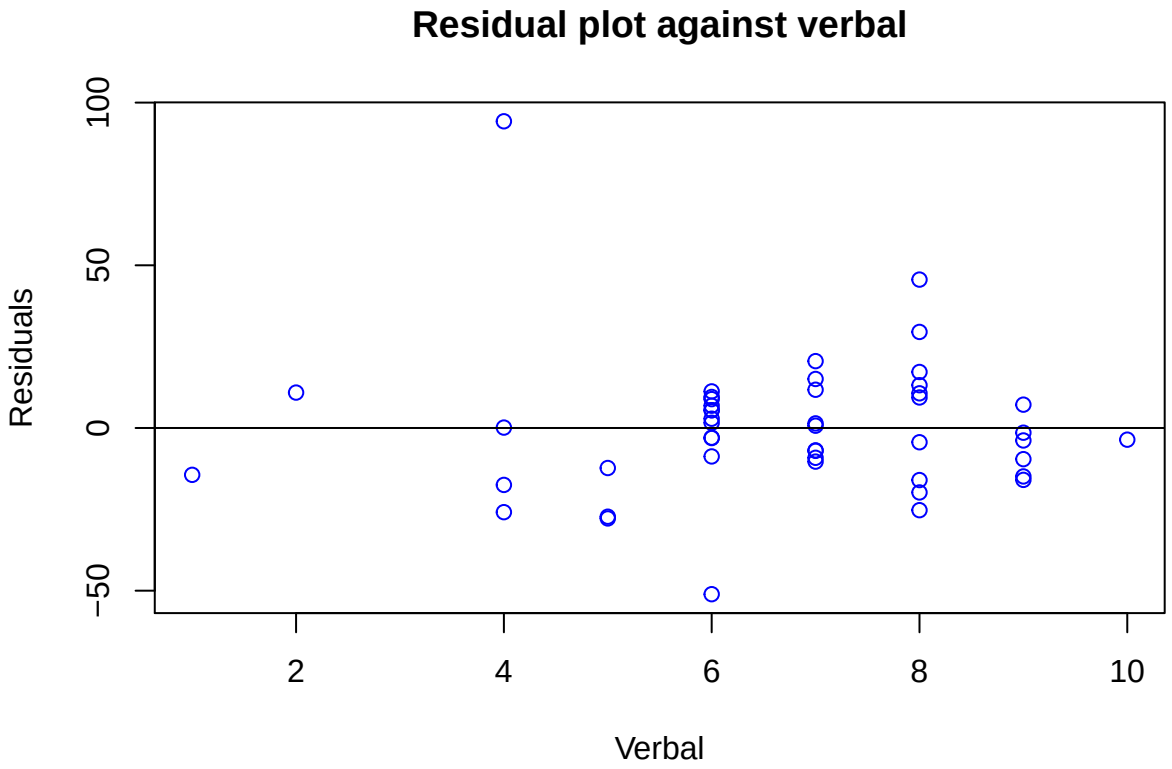
Plot the residuals against income

```
plot(Data.a$income, resid(linMod.a), xlab = "Income", ylab = "Residuals",
     main = 'Residual plot against income', col = 'blue')
abline(a=0, b=0)
```



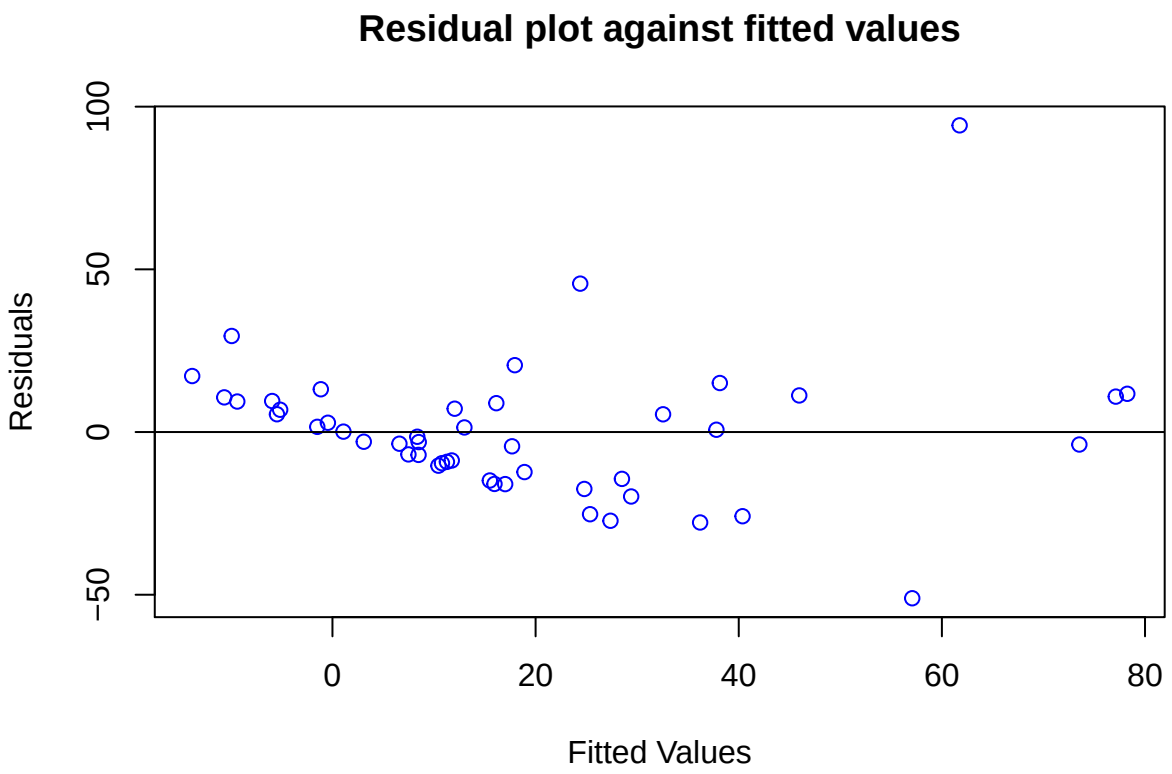
Plot the residuals against verbal

```
plot(Data.a$verbal, resid(linMod.a), xlab = "Verbal", ylab = "Residuals",
     main = 'Residual plot against verbal', col = 'blue')
abline(a=0, b=0)
```



Plot the residuals against fitted values

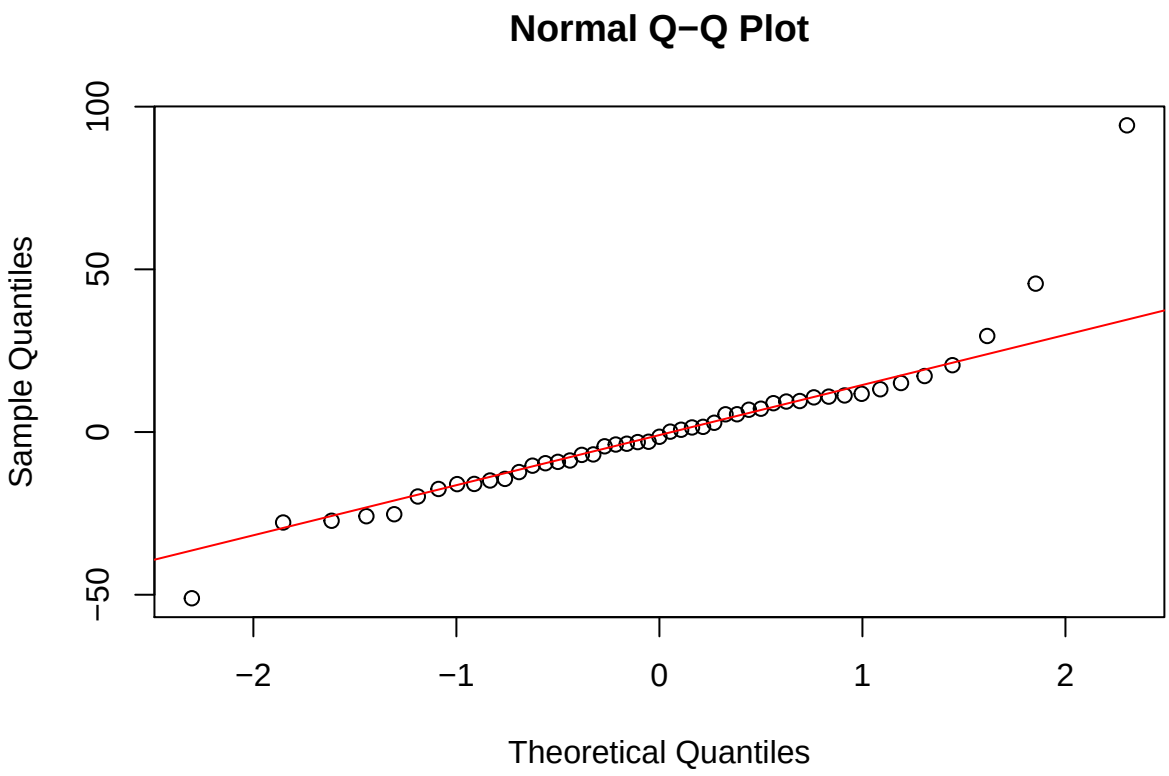
```
plot(linMod.a$fitted.values, resid(linMod.a), xlab = "Fitted Values",
     ylab = "Residuals", main = 'Residual plot against fitted values',
     col = 'blue')
abline(a=0, b=0)
```



By looking at the residual plots with 'income' and 'fitted values' I think that there might be evidence that the constant variance assumption is being violated. With 'income' there seems to be more spread while this variable gets bigger. In addition, I think that the distribution of the points in the plot vs fitted values leads to think that the variance is not constant.

Q-Q Plot

```
qqnorm(residuals(linMod.a))
qqline(residuals(linMod.a), col="red")
```



In the Q-Q Plot we can see that both tails deviate. I might be a little bit worried about this but I think the normality assumption is not violated here.

Part (b)

In this part we use Box Cox to suggest a transformation for the response variable. The transformed variable, $\tilde{Y}$ will satisfy the usual mean and variance assumptions.

NOTE: In order to use the boxcox() function we add a small value $\delta = 10^{-8}$ to the column *gamble* of our dataset to get such function to work.

```
Data.b <- data.frame(Data.a) # We make a copy of the original data
Data.b$gamble <- Data.b$gamble + 10^(-8) # We add 10^{-8} to column gamble
linMod.b <- lm(gamble ~ ., data = Data.b) # We run the regression model
summary(linMod.b) # To display the summary of the model
```

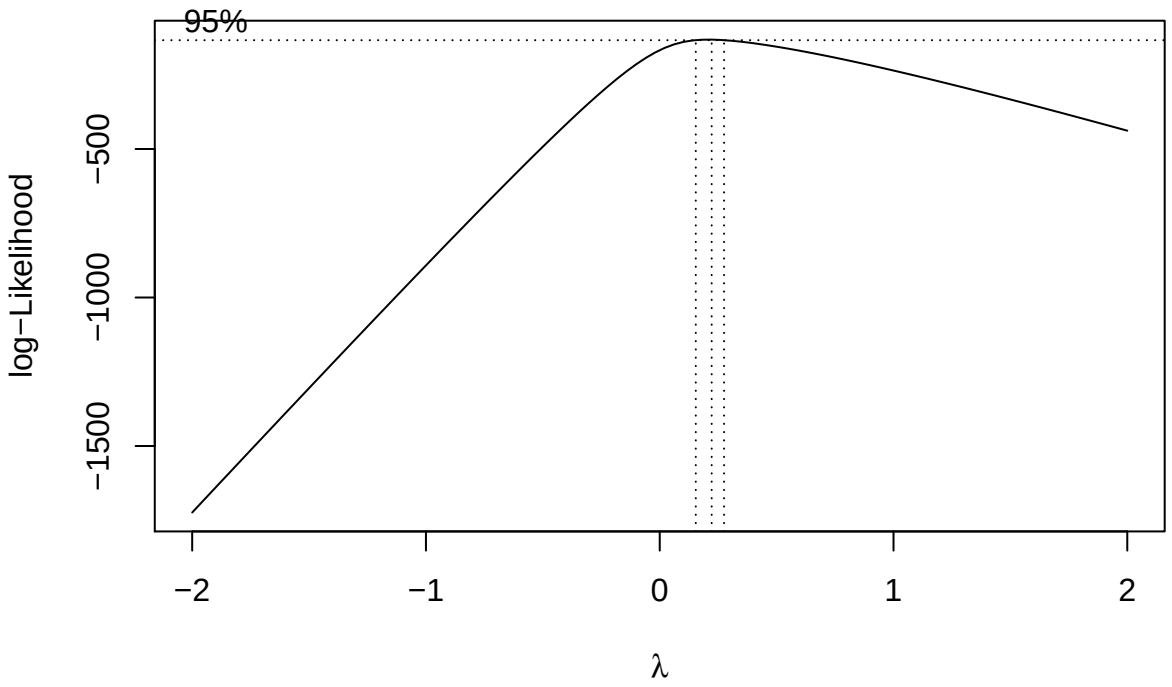
```
##
## Call:
## lm(formula = gamble ~ ., data = Data.b)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -51.082 -11.320  -1.451   9.452  94.252
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  22.55565   17.19680   1.312   0.1968
## sex         -22.11833    8.21111  -2.694   0.0101 *
## status        0.05223    0.28111   0.186   0.8535
## income        4.96198    1.02539   4.839 1.79e-05 ***
```

```
## verbal       -2.95949    2.17215  -1.362   0.1803
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 22.69 on 42 degrees of freedom
## Multiple R-squared:  0.5267, Adjusted R-squared:  0.4816
## F-statistic: 11.69 on 4 and 42 DF,  p-value: 1.815e-06
```

Next we'll run the boxcox() function to determine which value for λ is reasonable for the transformation. The transformed data will have the form:

$$\tilde{Y} = \frac{Y^{\lambda} - 1}{\lambda}$$

```
boxcox.b <- boxcox(linMod.b, plotit = T)  # Run boxcox()
```




```
argmax.b <- which.max(boxcox.b$y) # We get the index for argmax(log-likelihood)
lambda.b <- boxcox.b$x[argmax.b] # We get the value for the argmax
```

We get that the best value for λ is:

```
lambda.b
```

```
## [1] 0.2222222
```

Here we obtain $\tilde{Y}$ and we plot it against the residuals of the first model we ran in part(b). In addition, we run another regression model with $\tilde{Y}$ as response.

NOTE: I discussed this problem with the professor and he told me that for simplicity's sake (i.e. being able to explain the transformation if we were required to do so) I should go for $\lambda = 0.25$ instead of the value I got above. So I will use such value rather than $\lambda = 0.2222222$. The reason is that the transformation with the first value will be more easily understood.

$$Y^{0.25} = Y^{\frac{1}{4}} = \sqrt{\sqrt{Y}}$$

```r
lambda.b.1 <- 0.25 # We'll use the value suggested by the instructor
```

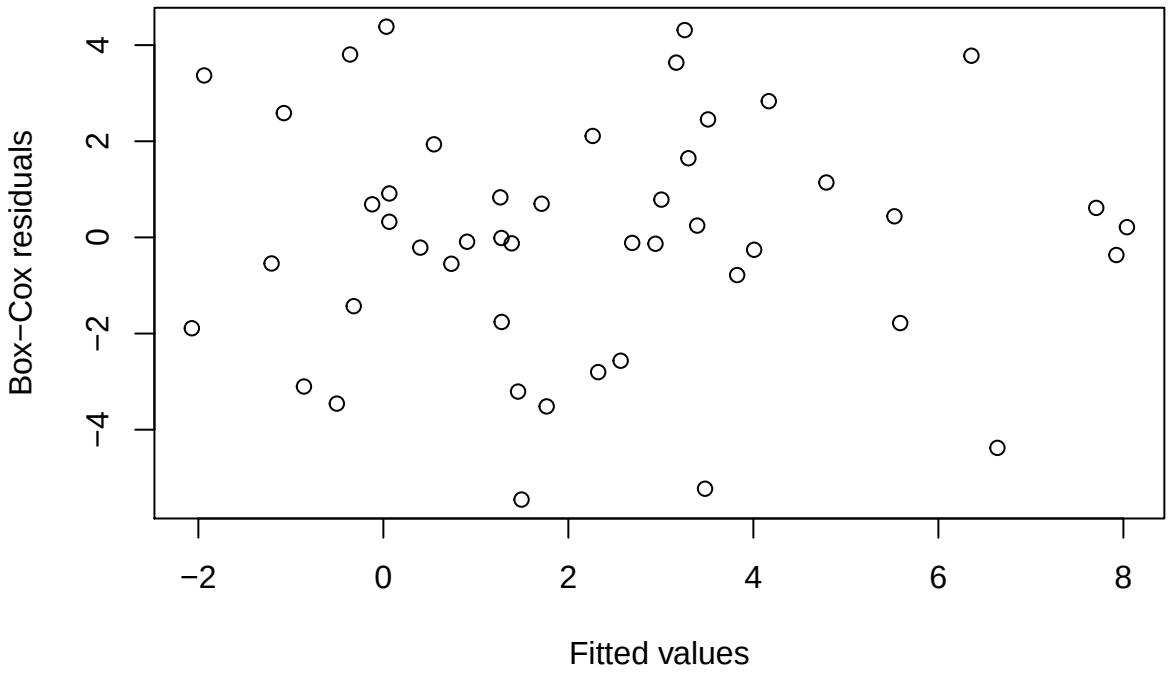
```r
Y.tilde.b <- (Data.b$gamble^lambda.b.1 - 1)/lambda.b.1 # We transform Y
# Run regression with Y_tilde as response
linMod.b.2 <- lm(Y.tilde.b ~ sex+status+income+verbal, data = Data.b)
summary(linMod.b.2)
```

```
##
## Call:
## lm(formula = Y.tilde.b ~ sex + status + income + verbal, data = Data.b)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.4536 -1.5945 -0.0122  1.3945  4.3834
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.09354    1.94384   1.077   0.2876
## sex         -2.21942    0.92814  -2.391   0.0213 *
## status       0.06365    0.03178   2.003   0.0516 .
## income       0.50381    0.11591   4.347 8.59e-05 ***
## verbal      -0.62329    0.24553  -2.539   0.0149 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.565 on 42 degrees of freedom
## Multiple R-squared:  0.5235, Adjusted R-squared:  0.4782
## F-statistic: 11.54 on 4 and 42 DF,  p-value: 2.078e-06
```

Residual plots under the new model

Here we plot the fitted values vs the estimated residuals.

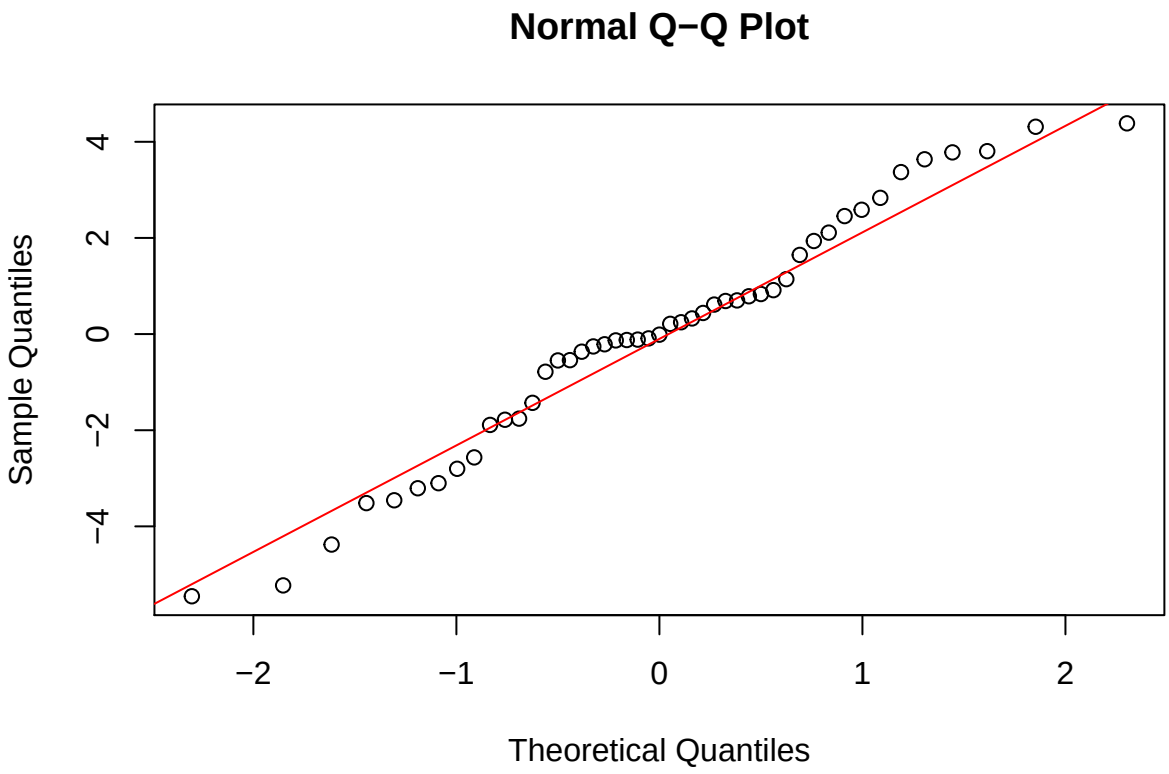
```r
plot(linMod.b.2$fitted.values, linMod.b.2$residuals, xlab="Fitted values",
     ylab="Box-Cox residuals")
```



We can notice right away that the residual plot with the fitted values for the model with the transformed data $\tilde{Y}$ looks a lot better than that of our original model. Here we can say that the constant variance assumption holds.

Q-Q Plot under the new model

```
qqnorm(residuals(linMod.b.2))
qqline(residuals(linMod.b.2), col="red")
```



In the Q-Q Plot, we get that the tails behave a little bit better than in our original model. In conclusion, the new model appears to satisfy the constant variance assumption and the residuals appear to follow a normal distribution.

Part (c)

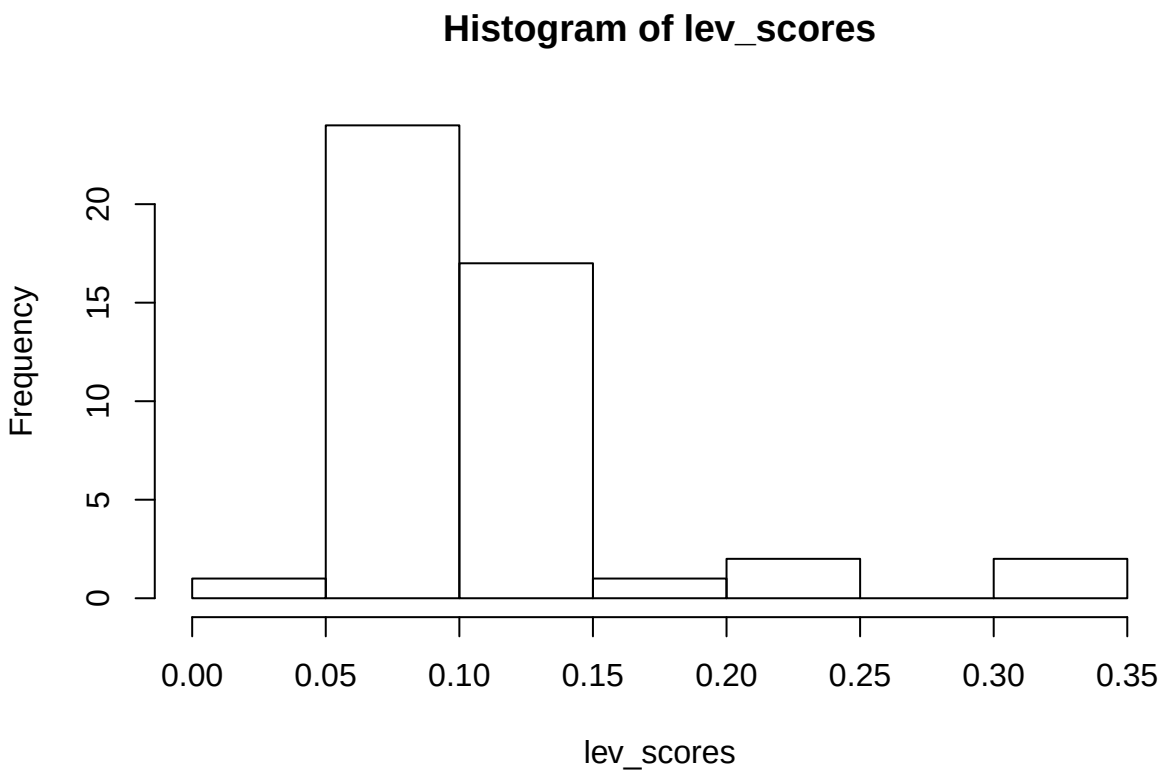
Next we'll compute the hat matrix H for the models we used in part(b).

NOTE: Since we used the same predictors, the hat matrix will be the same for the two models we ran in part(b).

```
intercept <- rep(1, length(Data.b$gamble)) # We create the 1 vector
predictors <- data.matrix(Data.b[,1:4]) # We obtain the columns for the predictors
X <- cbind(intercept,predictors) # We create the X matrix
H <- X %*% solve(t(X) %*% X) %*% t(X) # We create the hat matrix H
```

Remember that the leverage scores are the elements $h_{ii}, i \in \{1, ..., n\}$ in the diagonal of the hat matrix H. Next we obtain the leverage scores and plot a histogram of them.

```
lev_scores <- diag(H) # We obtain the leverage scores
hist(lev_scores)
```



Part (c.i)

We are concerned about having large leverage scores because they determine influential point in model fitting. In what follows we check if we have large leverage points in this data.

```
# Here we compute the mean leverage h_bar
p <- 5
n <- length(Data.b$gamble)
mean_lev <- p/n
outliers <- which(lev_scores > 2 * mean_lev)
outliers
```

```
## [1] 31 33 35 42
```

It seems that the observations 31,33,35 and 42 are influential points in regression our model.

Part (c.ii)

Here we will reestimate the model from part(b) after removing the points with large leverage scores.

NOTE: I first ran again the boxcox() function to obtain a new λ for the model with the deleted observations. However, the professor told me to just use the same value we used in part(b). So I'll stick to use $\lambda = 0.25$.

```
# Here we create a dataset dropping the large leverage points
Data.c <- data.frame(Data.b[-c(31,33,35,42),])
Y.tilde.c <- Y.tilde.b[-c(31,33,35,42)]
# We regress Y_tilde again and display our results
linMod.c.2 <- lm(Y.tilde.c ~ sex+status+income+verbal, data = Data.c)
summary(linMod.c.2)
```

```
##
## Call:
## lm(formula = Y.tilde.c ~ sex + status + income + verbal, data = Data.c)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.4314 -1.7963 -0.0286  1.7619  4.3426
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.20489    2.91021   0.758   0.4533
## sex         -2.20420    1.03376  -2.132   0.0395 *
## status       0.06429    0.03429   1.875   0.0685 .
## income       0.48531    0.18060   2.687   0.0106 *
## verbal      -0.63452    0.31691  -2.002   0.0524 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.693 on 38 degrees of freedom
## Multiple R-squared:  0.4122, Adjusted R-squared:  0.3504
## F-statistic: 6.663 on 4 and 38 DF,  p-value: 0.0003637
```

I plot again the summary of the model in part(b) once again just to make it easier to compare to the model in part(c).

```
summary(linMod.b.2)
```

```
##
## Call:
## lm(formula = Y.tilde.b ~ sex + status + income + verbal, data = Data.b)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.4536 -1.5945 -0.0122  1.3945  4.3834
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.09354    1.94384   1.077   0.2876
## sex         -2.21942    0.92814  -2.391   0.0213 *
## status       0.06365    0.03178   2.003   0.0516 .
## income       0.50381    0.11591   4.347 8.59e-05 ***
## verbal      -0.62329    0.24553  -2.539   0.0149 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.565 on 42 degrees of freedom
## Multiple R-squared:  0.5235, Adjusted R-squared:  0.4782
## F-statistic: 11.54 on 4 and 42 DF,  p-value: 2.078e-06
```

By comparing the parameter estimates and its standard error from the models in part(b) and part(c) we can notice that the most significante changes occurred in the predictors 'income' and 'verbal'. In one hand the estimated coefficients are really close but the Standard Error went up in both cases.

Moreover, the P-Values for both predictors went up -especially the P-Value for 'income'-. This means that those ppredictors weren't as important (but important nevertheless) as previously suggested by the results in part(b).

Part (d)

Here we compute the Cook's distance for every point for the models in part(b) and part(c).

```
dis.b <- cooks.distance(linMod.b.2) # Retrieve the cook's distance for all points
cut.b <- qf(.50, df1 = 5, df2 = 47-5)
inf.b <- which(dis.b > cut.b)
inf.b
```

```
## named integer(0)
```

```
dis.c <- cooks.distance(linMod.c.2) # Retrieve the cook's distance for all points
cut.c <- qf(.50, df1 = 5, df2 = 43-5)
inf.c <- which(dis.b > cut.b)
inf.c
```

```
## named integer(0)
```

Using the criteria we saw in class we find that there are no points that appear to be influential points. This means that the points with large leverage score didn't corrupt our model fit. Even with these outliers the values of Y were close to what we expected.