STAT 2270 Homework 1 Fall 2020

Manuel Alejandro Garcia Acosta

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Exercise 4

Here we'll make use of the Hitters dataset found in the ISLR package in R. Note that ISLR here stands for the textbook "Introduction to Statistical Learning with applications in R" cited in the syllabus for this class. You may find the lab contained in Chapter 6 of ISLR very useful for the first few parts of this exercise.

Part (a)

Load in the Hitters dataset found in the ISLR package in R. Following the approach in ISLR, we'll use Salary as our response and treat the rest of the variables as predictors. Remove all rows of the data that contain missing values. We'll treat the remaining data as our working dataset for the remainder of this problem.

First I loaded the dataset and omitted the rows with missing values.

Note: I'm turning in the analysis performed after normalizing/scaling the numeric variables. This scaling included the 'Salary' variable. As we discussed, results were similar to those obtained by performing the analysis with the original data (without scaling the numeric variables).

```
# Loading the dataset
if(exists('Hitters')){
  rm(Hitters)
}else{
  fix(Hitters)
}
```

```
## Warning in rm(Hitters): object 'Hitters' not found
```

```
# Omitting the rows with missing values
Hitters <- na.omit(Hitters)
# Scaling all numeric variables including 'Salary'
ind <- sapply(Hitters, is.numeric)
Hitters[ind] <- lapply(Hitters[ind], scale)
rm(ind)
```

Part (b)

Construct a linear model and record the resulting MSE of the predictions on the (training) data. Don't worry about fixing up the linear model (e.g. removing terms that are not significant) – simply regress Salary on all remaining predictors and record the error.

NOTES:

- For computing the MSE I'm taking the mean of the residual sum of squares. I did this throughout this HW.
- As we discussed, the MSE I'll be reporting was the *training error*.

```r
# Fitting the linear model
fit <- lm(Salary ~ ., data = Hitters)
# I computed the MSE by dividing the residuals squared sum by n
# ISLR book just takes the mean of the residual sum of squares for the MSE
mse <- sum((Hitters$Salary - fit$fitted.values)^2)/(length(Hitters$Salary))
```

The MSE for this model is 0.4521583.

Part (c)

Now construct the best possible linear model you can find. You can do this however you like: removing terms that are not significant, using combinations of forward/backward selection, including interaction terms, including higher order polynomial terms etc., but don't use any of the regularization methods we discussed in class. Once you've found what you think is the best model, record the MSE.

Multicollinearity

I checked the data for collinearity and there seems to be some sort of multicollinearity in the dataset. Some of the covariates like 'CAtBat', 'CHits', 'CRuns' and 'CRBI' have VIF over 100. This was really high.

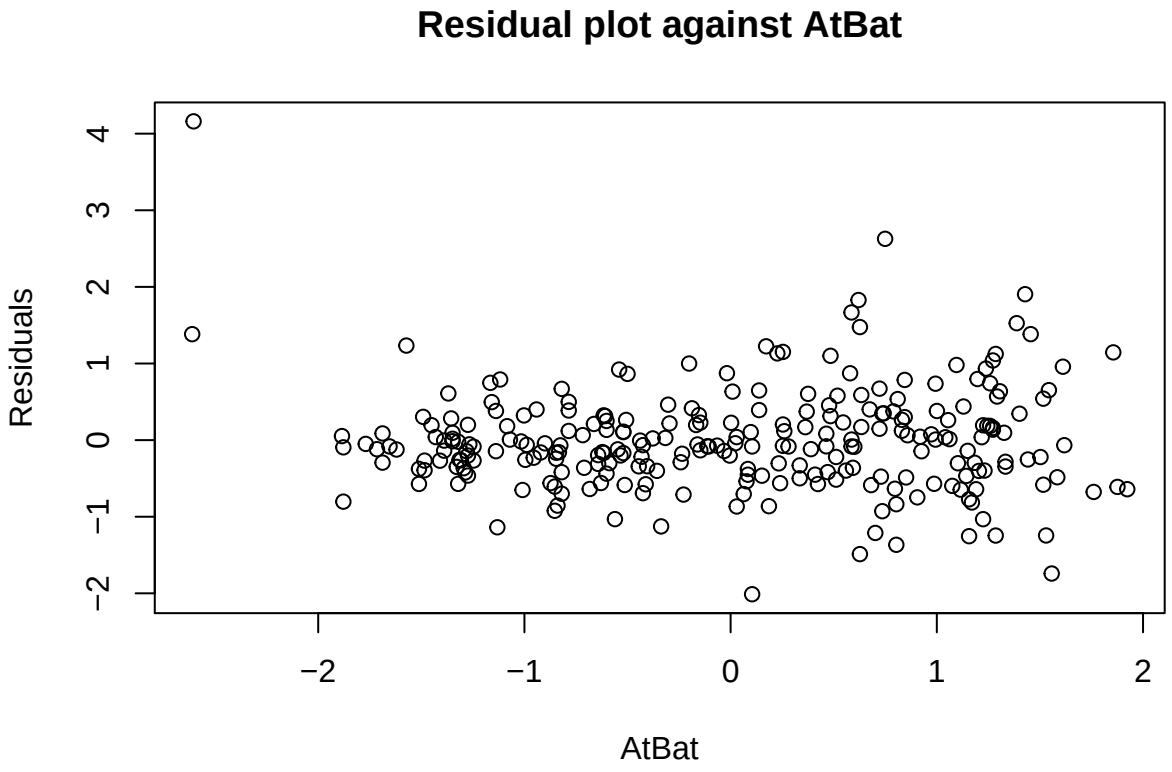
However, I decided not to discard them right away and started doing model selection with the saturated model (all features).

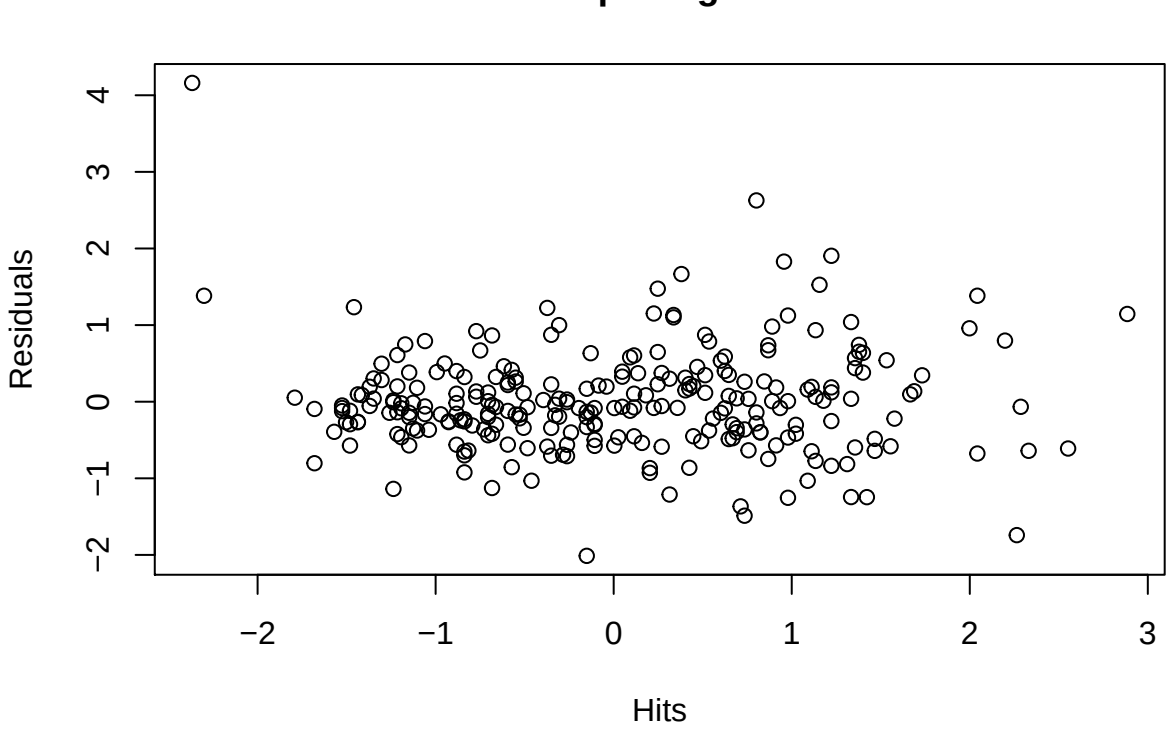
```r
# Computing the VIF for all the covariates
vif(lm(Salary ~ ., data = Hitters))
```

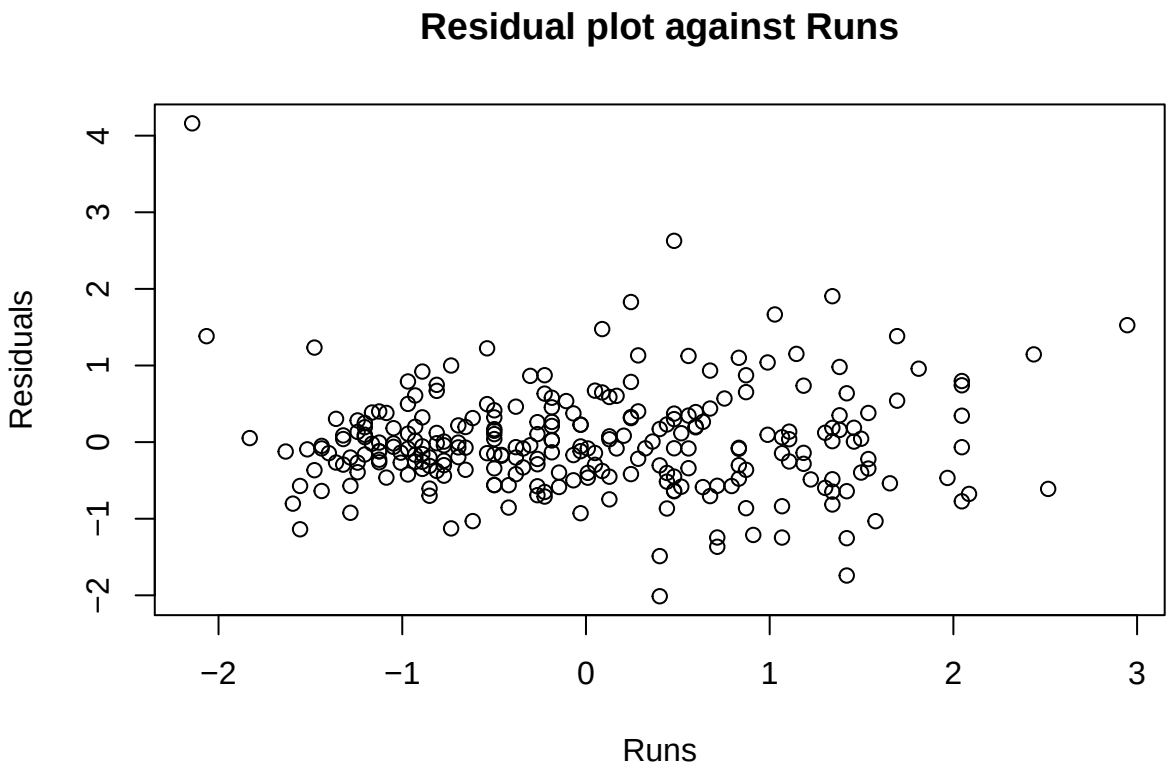
```
##      AtBat       Hits      HmRun       Runs        RBI      Walks
##  22.944366  30.281255   7.758668  15.246418  11.921715   4.148712
##      Years     CAtBat      CHits     CHmRun      CRuns       CRBI
##   9.313280 251.561160 502.954289  46.488462 162.520810 131.965858
##     CWalks     League   Division    PutOuts    Assists     Errors
##  19.744105   4.134115   1.075398   1.236317   2.709341   2.214543
##  NewLeague
##   4.099063
```

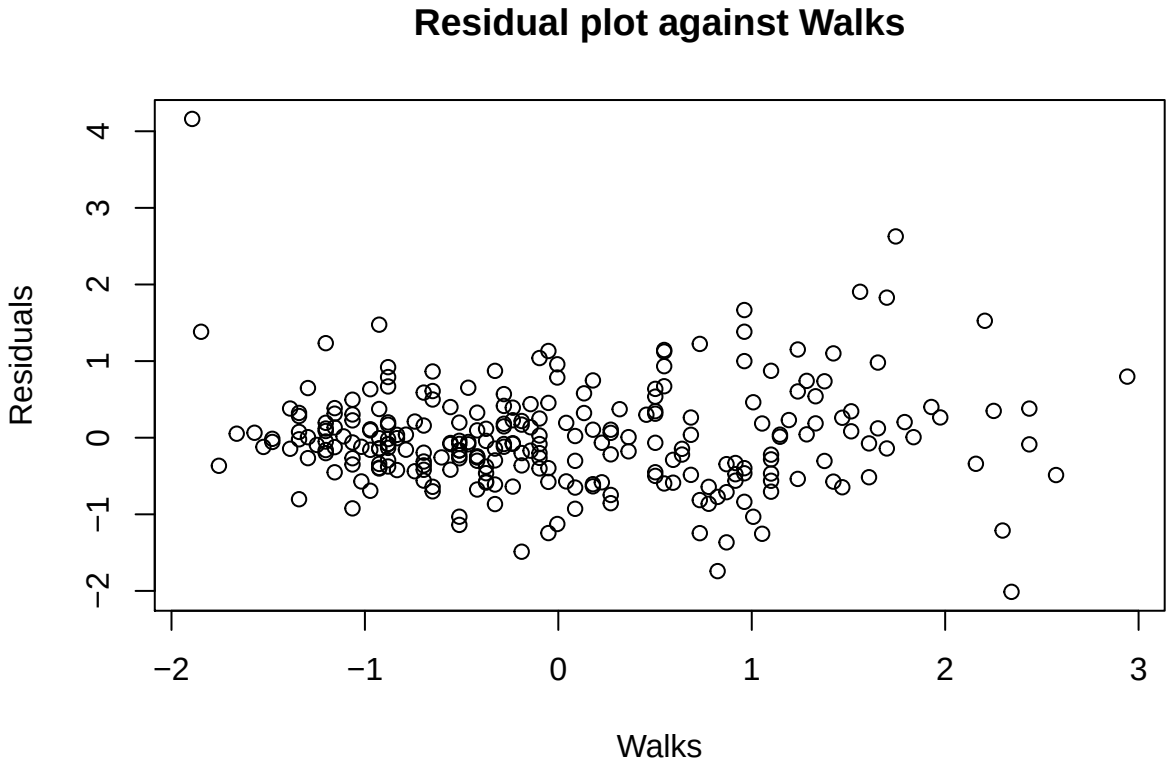
Adding covariates

After plotting the residuals of the model from part (b) vs the covariates, I tried adding to the model the squares of 'AtBat', 'Hits', 'Runs' and 'Walks'. While testing for the effects of the squares separately (a model with all original covariates plus one of these squares) all the p-values were below $\alpha = 0.05$.


Residual plot against AtBat
Residuals
AtBat
−2
−1
0
1
2
3
4


Residual plot against Hits
Residuals
Hits
−2
−1
0
1
2
3
4





After this I looked for interactions between variables, I did this in a single model.

```
# Fitting a model with all covariates and all possible interaction effects
fit2 <- lm(Salary ~ (.)^2, data = Hitters)
# summary(fit2)
```

While testing for the interaction terms, the interactions which seemed significant at $\alpha = 0.05$ where $AtBat * CHmRun$, $AtBat * CRuns$, $Hits * CHmRun$, $Walks * ChmRun$ and $CHmRun * CRBI$ and $PutOuts * Assists$.

Model selection

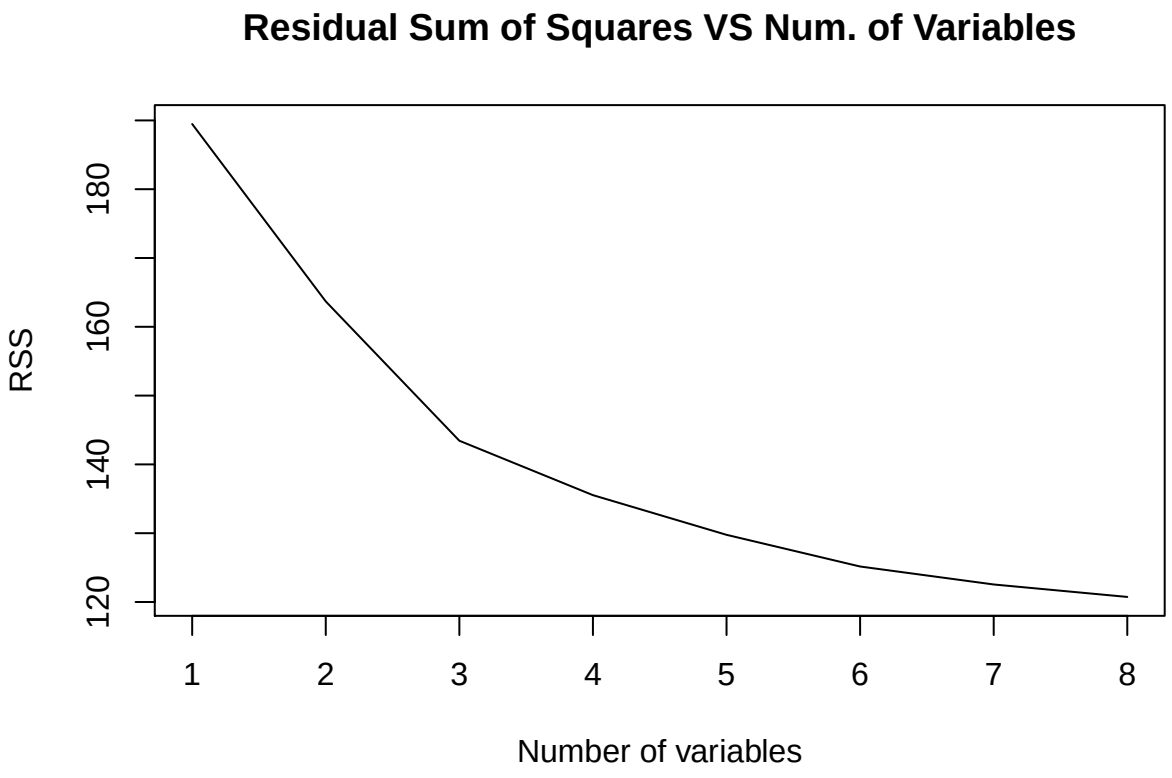
I procedeed to perform stepwise selection with $(\alpha_1 = 0.05, \alpha_2 = 0.1)$. I considered all original covariates plus the squared and interaction terms that appeared to be significant after my initial exploration.

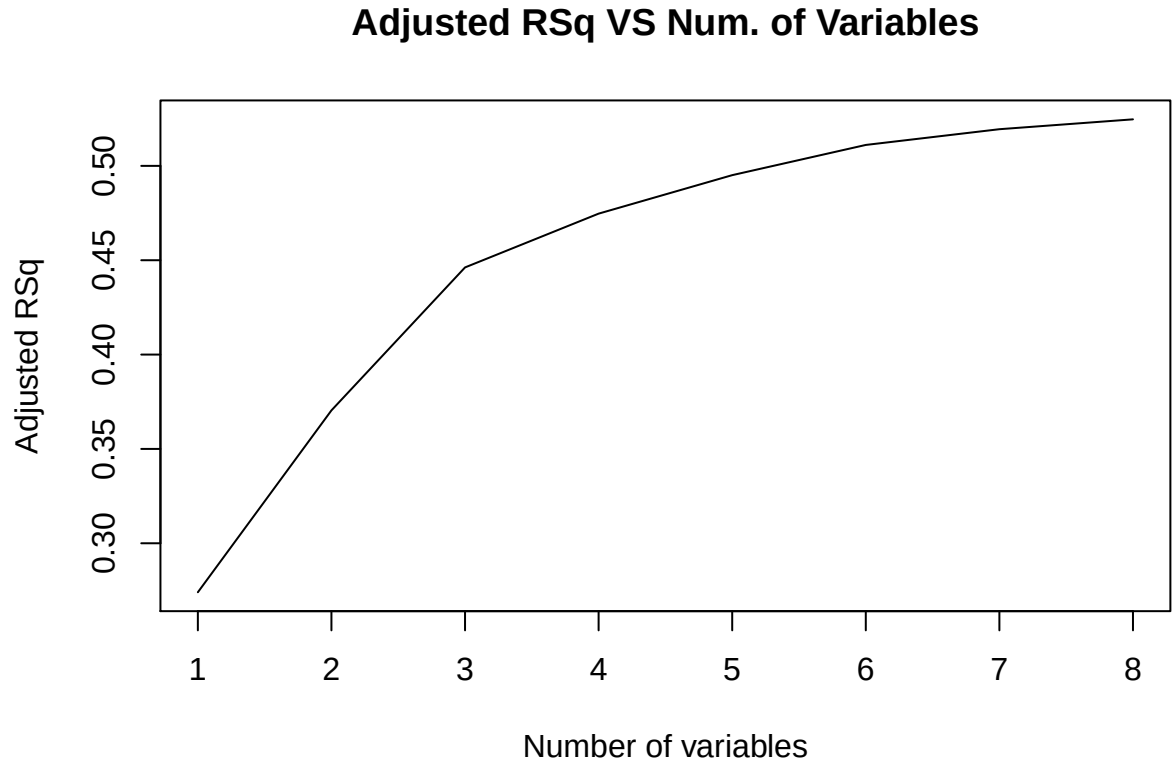
NOTE: I commented out the line for stepwise selection sice the output was way to long to include it on the pdf file version of the HW. You'll find the code on the Rmd file 'HW1(scaled).Rmd'

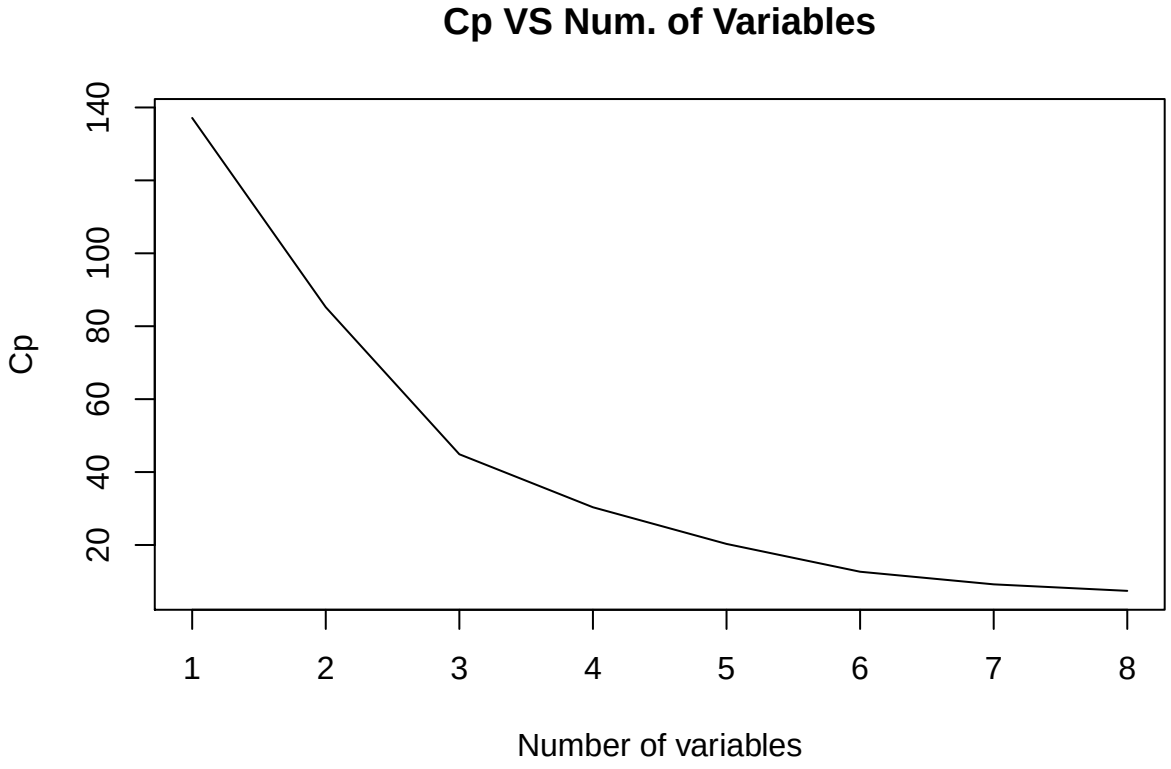
```
# Fitting the expanded model
fit3 <- lm(Salary ~ . + I(AtBat^2) + I(Hits^2)+ I(Runs^2)+ I(Walks^2) +
                  AtBat*CHmRun+ AtBat*CRuns + Hits*CHmRun + Walks*CHmRun +
                 CHmRun*CRBI + PutOuts*Assists, data = Hitters)
# Running stepwise selection for the proposed model
alpha.1 <- 0.05
alpha.2 <- 0.1
# step.wise <- ols_step_both_p(fit3, pent = alpha.1, prem = alpha.2)
```

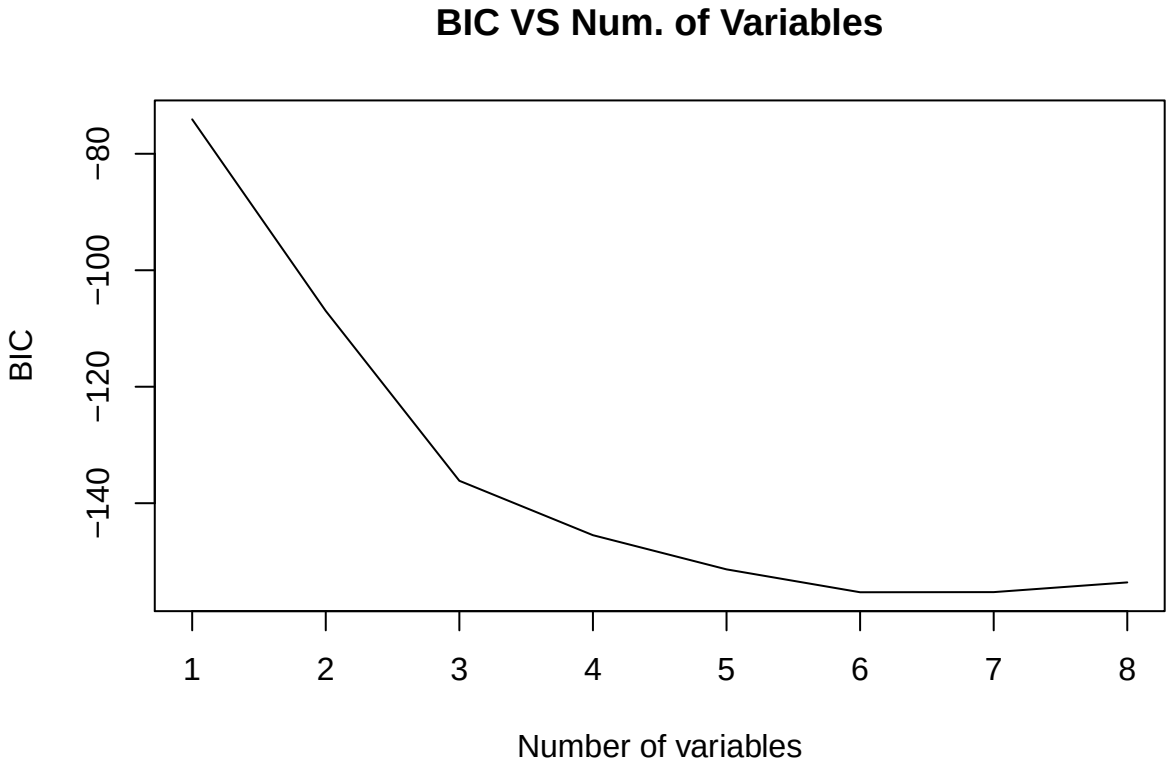
After running stepwise selection I got a model that includes 'Walks', 'CAtBat', 'CHmRun', 'Division', 'AtBat^2', 'Hits^2', 'Walks^2', 'PutOuts*Assists'. Next thing I wanted to do was look for the best submodel. I used RSS, Adjusted R^2, C_p and BIC to check which submodel was the 'best'. Next I plot each of these as a function of the number of features each 'best' subset has.

```
regfit <- regsubsets(Salary ~ Walks+CAtBat+CHmRun+Division+I(AtBat^2)+
                        I(Hits^2)+ I(Walks^2)+PutOuts*Assists, data = Hitters,
                        nvmax = 8)
reg.summary <- summary(regfit)
```




Adjusted RSq VS Num. of Variables
Adjusted RSq
0.30
0.35
0.40
0.45
0.50
1
2
3
4
5
6
7
8
Number of variables


Cp VS Num. of Variables
Cp
20
40
60
80
100
140
1
2
3
4
5
6
7
8
Number of variables



Results suggest that the best model for all criterias except BIC is the one with all 8 covariates ('Walks', 'CAtBat', 'CHmRun', 'Division', 'AtBatˆ2', 'Hitsˆ2', 'Walksˆ2', 'PutOuts*Assists'). This is exactly the same model I got after running stepwise selection. Using BIC the best model used 6 covariates, dropping 'AtBatˆ2' and 'Walksˆ2' from the full model. I'll pick the one with all 8 features to compute the MSE.

```
# Computing the MSE of the model
fit4 <- lm(Salary ~ Walks+CAtBat+CHmRun+Division+I(AtBat^2)+
             I(Hits^2)+I(Walks^2)+PutOuts*Assists, data = Hitters)
mse4 <- mean(fit4$residuals^2)
```

Finally, the MSE for the resulting model was 0.4583174, which this time around wasn't an improvement over the model from part (b). So sad : (.

Part (d)

Find the best model using ridge regression. Consider a range of tuning parameters and select the best model by cross validation. Record the value of the optimal tuning parameter as well as the values of the resulting coefficient estimates from that model.

NOTE: For solving this homework I selected the tunning parameters[1] while using 10-fold cross-validation.

For λ I created a vector with one thousand values ranging from $1 * 10^{10}$ to 0.01.

```
# Creating the design matrix and the response vector
# NOTE: I needed to erase the intercept because glmnet creates the intercept
#   automatically
x <- model.matrix(Salary~., data = Hitters, y=Salary)[,-1]
y <- Hitters$Salary
# Creating a set of possible values for lambda
```

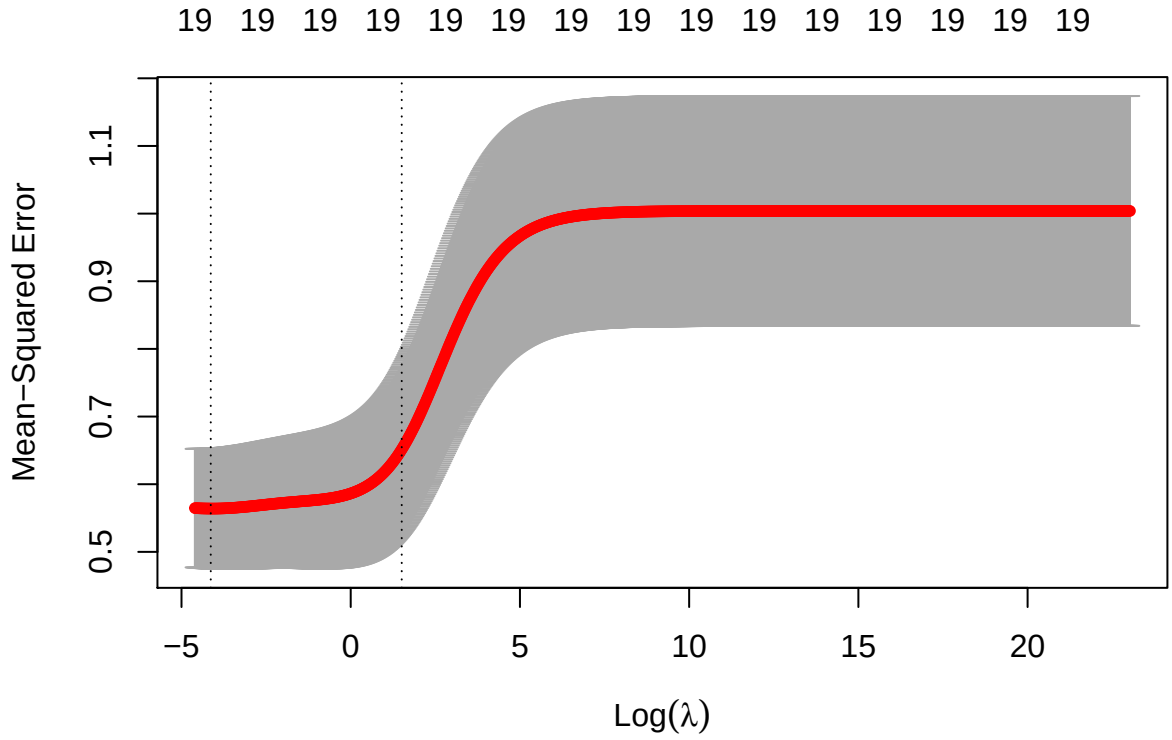
[1] λ for ridge regression, lasso and elastic net, α for elastic net

```
lambdas <- 10^seq(10,-2,length.out=1000)
# Fitting ridge regression for the vector of lambdas
fit.ridge <- glmnet(x,y,alpha = 0, lambda = lambdas)
```

NOTE: For reproducibility purposes I set a seed whenever I used k-fold cross-validation. This holds for all remaining parts of this exercise.

```
# Set a seed so results can be reproduced
set.seed(1989)
# Here I run 10-fold cross-validation
cv.fit.ridge <- cv.glmnet(x,y, alpha = 0, nfolds = 10, lambda = lambdas,
                          type.measure = 'mse')
```

I include a plot of how the MSE varies alongside $log(\lambda)$.



Next, I obtained the value for λ associated to the smallest cross-validation error as well as the coefficient estimates for that model. Notice that I got my results after running k-fold cross-validation only once.

```
# Obtain the lambda which gives smallest error
ridge.lambda <- cv.fit.ridge$lambda.min
```

The 'best' value for lambda was $\lambda = 0.0294082$. For this value, the ridge regression estimates are

```
# Run ridge regression with the 'best' lambda
best.ridge.fit <- glmnet(x,y, alpha = 0, lambda = ridge.lambda)
# Obtain the coefficients
coef.d <- as.matrix(coef(best.ridge.fit))
coef.d
```

```
##                       s0
## (Intercept)  0.1044910830
## AtBat       -0.4489162926
## Hits         0.4838906434
```

```
## HmRun       -0.0003714846
## Runs         0.0093836397
## RBI          0.0145780242
## Walks        0.2279359737
## Years       -0.1143765187
## CAtBat      -0.1729717323
## CHits        0.2495951091
## CHmRun       0.1232289847
## CRuns        0.4089154886
## CRBI         0.2573957294
## CWalks      -0.3032058674
## LeagueN      0.1330520778
## DivisionW   -0.2744835044
## PutOuts      0.1715134339
## Assists      0.0830109530
## Errors      -0.0560635510
## NewLeagueN  -0.0590067448
```

Finally, I computed the MSE for ridge regression using the best lambda. Here $MSE = 0.4711957$

```
# Computing the MSE of the model
ridge.pred <- as.numeric(predict(best.ridge.fit, s=ridge.lambda, newx=x))
mse.ridge <- mean((ridge.pred - y)^2)
```

Part (e)

Repeat part (d) with lasso.

For solving this part I selected the tunning parameter while using 10-fold cross-validation.

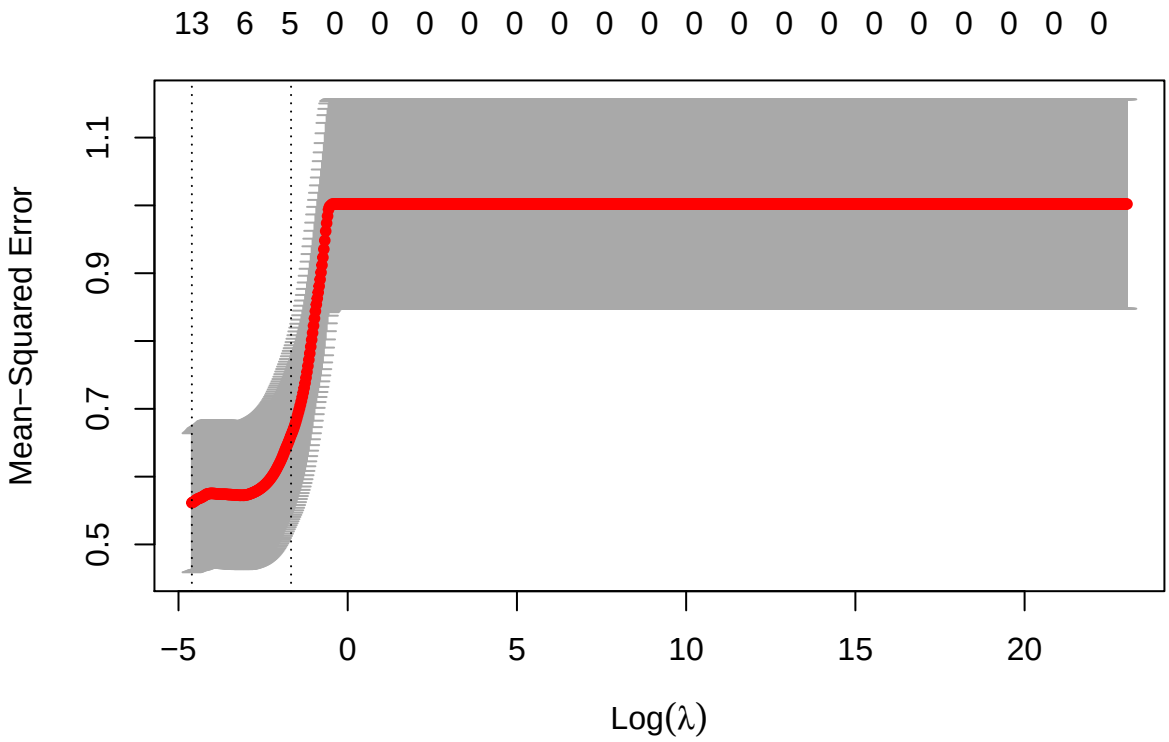
NOTE: I'm using the design matrix 'x', response vector 'y' and the vector of lambdas 'lambdas' created in part (d) for parts (e)-(f) also.

```
# Fitting lasso for vector of lambdas
fit.lasso <- glmnet(x,y,alpha = 1, lambda = lambdas)
```

I set the seed and run cross-validation next

```
# Set a seed so results can be reproduced
set.seed(1991)
# Here I run 10-fold cross-validation
cv.fit.lasso <- cv.glmnet(x,y, alpha = 1, nfolds = 10, lambda = lambdas,
                          type.measure = 'mse')
```

I include a plot of how the MSE varies alongside $log(\lambda)$.



Next, I obtained the value for λ associated to the smallest cross-validation error as well as the coefficient estimates for that model.

```
# Obtain the lambda which gives smallest error
lasso.lambda <- cv.fit.lasso$lambda.min
```

The 'best' value for lambda was $\lambda = 0.01$. For this value, the lasso estimates are

```
# Run lasso with the 'best' lambda
best.lasso.fit <- glmnet(x,y, alpha = 1, lambda = lasso.lambda)
# Obtain the coefficients
coef.e <- as.matrix(coef(best.lasso.fit))
coef.e
```

```
##                     s0
## (Intercept)  0.10324285
## AtBat       -0.34463433
## Hits         0.45813891
## HmRun        0.00000000
## Runs         0.00000000
## RBI          0.00000000
## Walks        0.18480554
## Years       -0.06879923
## CAtBat       0.00000000
## CHits        0.00000000
## CHmRun       0.06466104
## CRuns        0.37321427
## CRBI         0.27715798
## CWalks      -0.19318224
## LeagueN      0.06512583
## DivisionW   -0.26289904
## PutOuts      0.16107387
```

```
## Assists      0.02538619
## Errors      -0.01769622
## NewLeagueN   0.00000000
```

Finally, I computed the MSE for lasso using the best lambda. Here $MSE = 0.4750985$.

```
# Computing the MSE of the model
lasso.pred <- as.numeric(predict(best.lasso.fit, s=lasso.lambda, newx=x))
mse.lasso <- mean((lasso.pred - y)^2)
```

Part (f)

Repeat part (d) with elastic net.

NOTE: In order to solve this part of the problem I programmed a bunch of functions that allowed me to run elastic net through different values for α. I'm omitting the code on the pdf but they can be found in the Rmd file 'HW1(scaled).Rmd' of my HW.

Now, for elastic net I did basically three steps

1. Define a set of alphas and run k-fold cross-validation for each lambda
2. Choose best lambda for each alpha
3. Compare MSE for each of the lambdas obtained. Since each λ corresponds to a single α, this will give us the best pair of (α, λ).

The set of α's I used were $\{0, 0.01, 0.02, \ldots, 1\}$. Once again, I set a seed for reproducibility.

```
# Set a seed so results can be reproduced
set.seed(1995)
# Create vector of alphas
alphas <- seq(from = 0, to = 1 , by = 0.01)
# Run k-fold cross-validation for the set of alphas defined above
out <- cv.alpha.vect(alphas = alphas, feat = x, res = y, l = lambdas)
# Get a vector of the MSE associated to the 'best' lambda for each alpha
elastic.mse <- cv.mse.vect(out, featu = x, resp = y)
# Select the lowest MSE over elastic.mse
index <- which.min(elastic.mse)
# Gets the minimum MSE over all pairs (alpha,lambda)
mse.elastic <- elastic.mse[index]
```

The best pair (α, λ) was (0, 0.01) with $MSE_{\alpha,\lambda} = 0.459424$. This intuitively tells us that lasso performs a little bit better than ridge regression while using the original covariates since the 'best' alpha was close to 1. The elastic net estimates are

```
# Get best pair (alpha, lambda)
elastic.alpha <- alphas[index]
elastic.lambda <- get.lambda(out[[index]])
# Get coefficient estimates
best.elastic.fit <- glmnet(x,y, alpha = elastic.alpha, lambda = elastic.lambda)
coef.f <- as.matrix(coef(best.elastic.fit))
coef.f
```

```
##                      s0
## (Intercept)  0.103585498
## AtBat       -0.501385053
## Hits         0.539513150
## HmRun        0.010386175
```

```
## Runs        -0.010381587
## RBI          0.005137531
## Walks        0.243671407
## Years       -0.110240346
## CAtBat      -0.249177748
## CHits        0.260499762
## CHmRun       0.117605909
## CRuns        0.484681834
## CRBI         0.279534371
## CWalks      -0.336891060
## LeagueN      0.134818178
## DivisionW   -0.272915060
## PutOuts      0.172680010
## Assists      0.089869591
## Errors      -0.055339259
## NewLeagueN  -0.060572313
```

Part (g)

Compare the models in parts (b)-(f). Does any one model (or a few models) stand out as being substantially better than the others?

By comparing MSE's of the different models. I can state that between the models which only use the original features, OLS (0.4521583) and elastic net (0.459424) performed better than ridge regression (0.4711957) and lasso (0.4750985). The model proposed in part (c) outperformed all regularization methods (0.4583174) but not OLS using MSE as **selection criteria**.

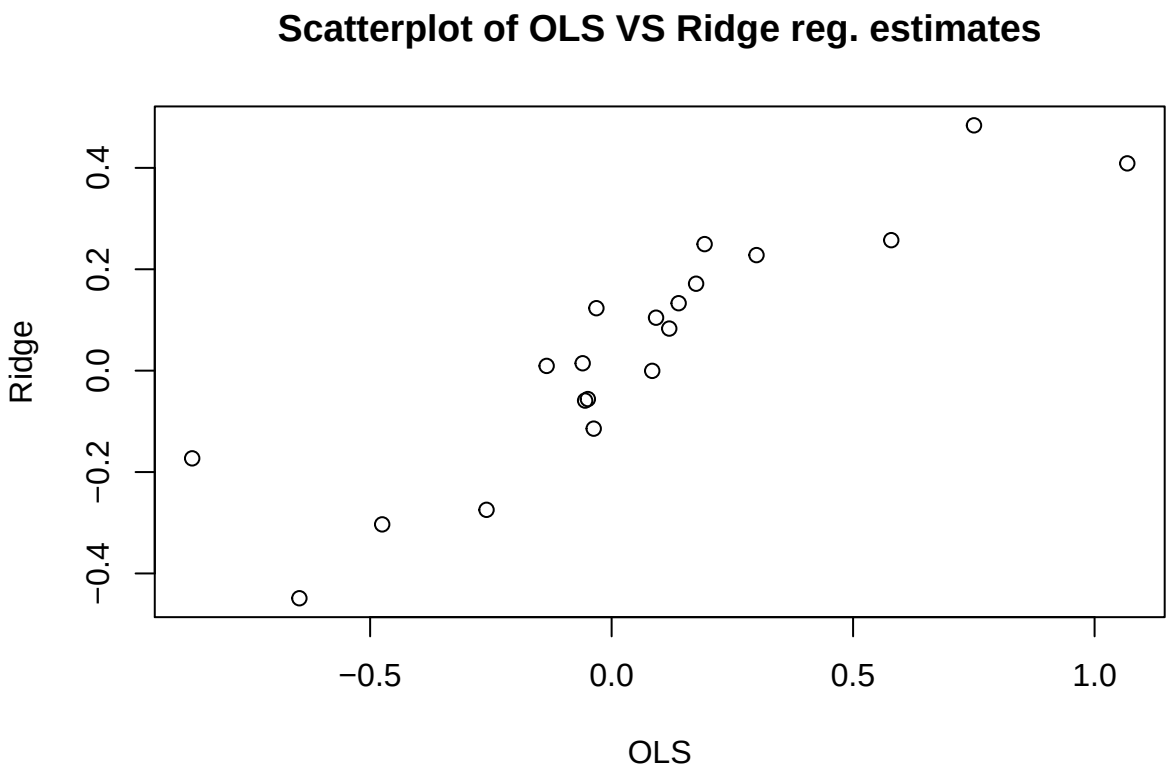
Part (h)

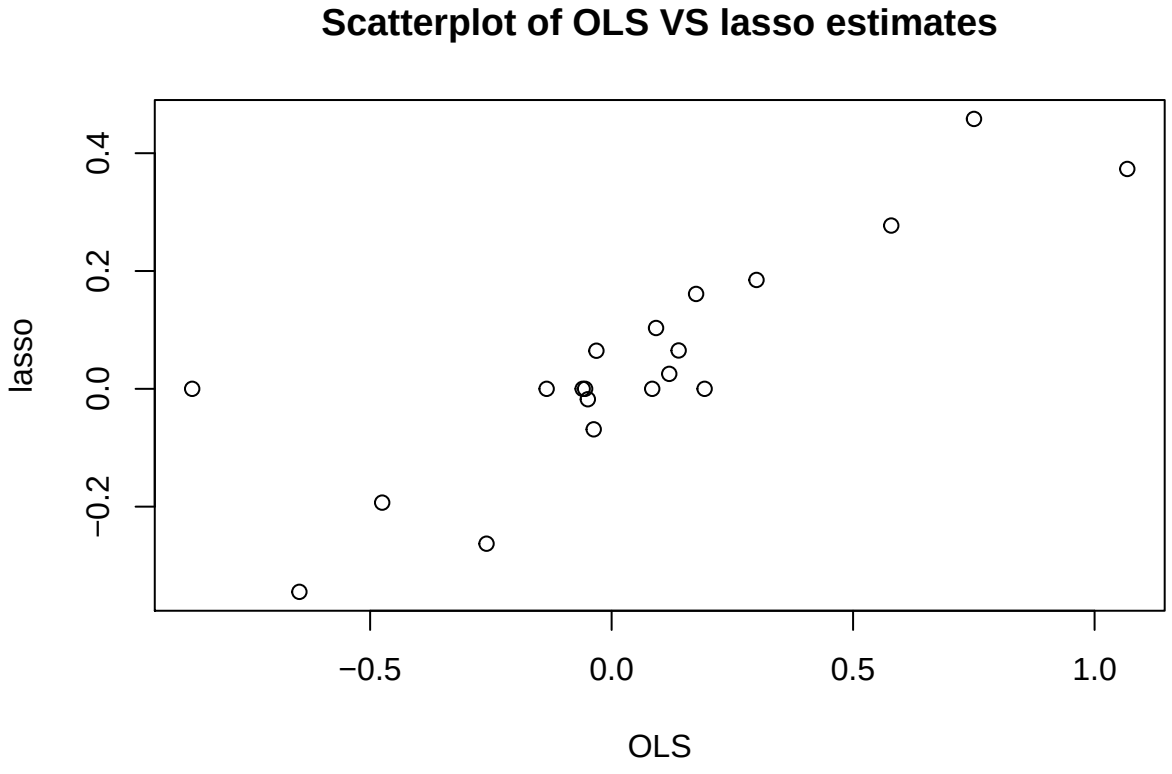
Make a scatterplot of coefficient estimates from part (b) vs those found in part (d). Repeat this to compare the results in (b) with those from (e) and also from (f). Are those coefficients with the smallest estimates those most penalized in parts (d)-(f)?

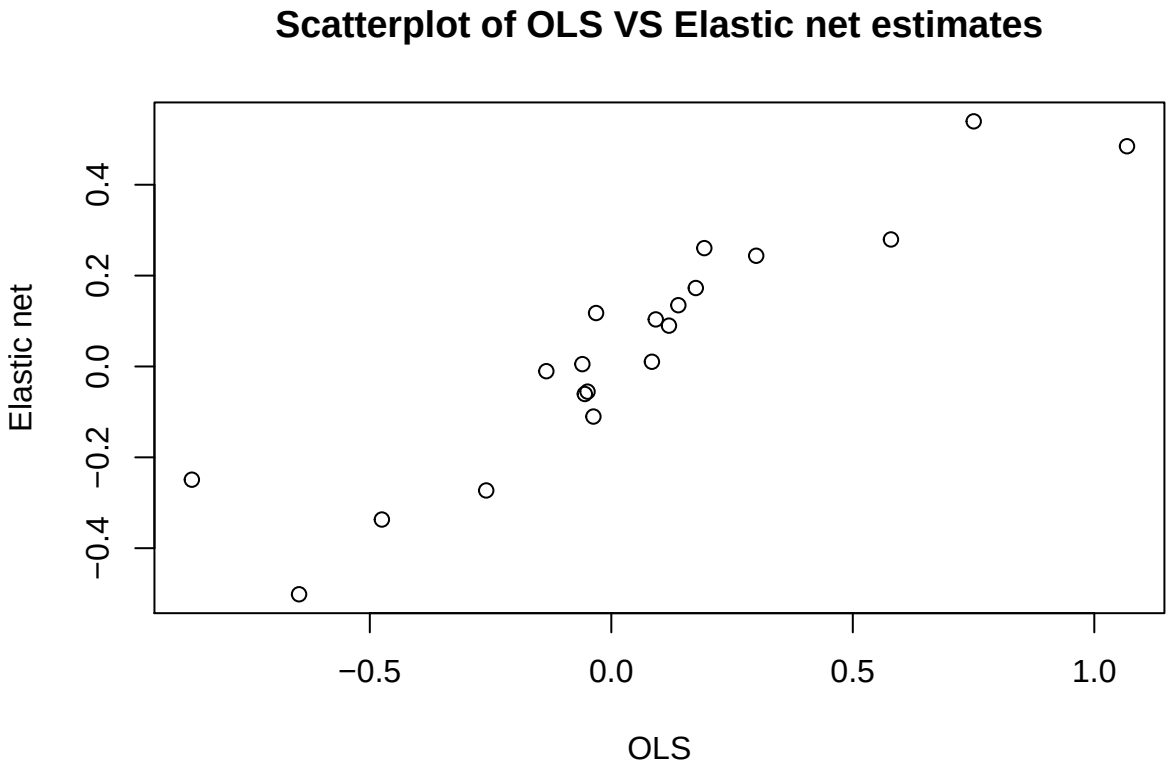
Here I will compute the coefficients for OLS and regularization methods after scaling the numeric variables.

While plotting the coefficients estimates from OLS against those obtained by using ridge regression, lasso and elastic net we can tell that OLS estimates which where close to 0 (either negative or positive) were the most penalized regarless of the regularization method used.

In all three plots, visible clusters around $(0, 0)$ appear.


Scatterplot of OLS VS Ridge reg. estimates
Ridge
OLS
−0.4
−0.2
0.0
0.2
0.4
−0.5
0.0
0.5
1.0


Scatterplot of OLS VS lasso estimates
lasso
OLS
−0.2
0.0
0.2
0.4
−0.5
0.0
0.5
1.0



Part (i)

Repeat parts (d)-(f) with a dataset that includes interactions and higher order polynomial terms. Do these models appear to be better than the others you've fit thus far?

For this exercise I'll be adding the extra variables that I proposed on part (c).

NOTE: Here some comments about what I did for this part.

- I created a new design matrix 'x.ext'. It will be used for all regularization methods.
- Since I'm still trying to predict 'Salary' I'll keep up using the response vector 'y' created in part (d).
- I'll use the same set of vectors for the hyper-parameters that I used to tune λ ('lambdas') and α ('alphas') in parts (d)-(f).

Ridge regression

I did pretty much the same as in part (d) after adding the higher order polynomial and interaction terms.

```
# Create the new design matrix
# NOTE: Needed to erase the intercept because glmnet creates the intercept
#   automatically
x.ext <- model.matrix(Salary ~ . + I(AtBat^2) + I(Hits^2)+ I(Runs^2)+ I(Walks^2)+
                  AtBat*CHmRun+ AtBat*CRuns + Hits*CHmRun + Walks*CHmRun+
                 CHmRun*CRBI + PutOuts*Assists, data = Hitters, y=Salary)[,-1]
```

After setting the seed I used k-fold cross-validation to select λ.

```
# Set a seed so results can be reproduced
set.seed(1990)
```

```
# Here I run 10-fold cross-validation
cv.fit.ext.ridge <- cv.glmnet(x.ext,y, alpha = 0, nfolds = 10, lambda = lambdas,
                              type.measure = 'mse')
```

```
# Obtain the lambda which gives smallest error
ext.ridge.lambda <- cv.fit.ext.ridge$lambda.min
```

The 'best' value for lambda was $\lambda = 0.02110203$. For this value, the ridge regression estimates are

```
# Run ridge regression with the 'best' lambda
best.ext.ridge.fit <- glmnet(x.ext,y, alpha = 0, lambda = ext.ridge.lambda)
# Obtain the coefficients
coef.ext.d <- as.matrix(coef(best.ext.ridge.fit))
coef.ext.d
```

```
##                             s0
## (Intercept)     -0.070499610
## AtBat           -0.137799284
## Hits             0.342048666
## HmRun           -0.107711776
## Runs            -0.005014960
## RBI             -0.053801033
## Walks            0.126911104
## Years           -0.113469482
## CAtBat          -0.136970481
## CHits            0.067587271
## CHmRun           0.566556520
## CRuns            0.300544557
## CRBI             0.304031212
## CWalks          -0.131248686
## LeagueN          0.151420882
## DivisionW       -0.207720387
## PutOuts          0.072332920
## Assists         -0.003224363
## Errors          -0.043576005
## NewLeagueN      -0.022816853
## I(AtBat^2)       0.044984872
## I(Hits^2)        0.102863586
## I(Runs^2)       -0.034205632
## I(Walks^2)       0.128600354
## AtBat:CHmRun    -0.185685390
## AtBat:CRuns      0.061420493
## Hits:CHmRun      0.343845804
## Walks:CHmRun     0.014520894
## CHmRun:CRBI     -0.203398031
## PutOuts:Assists -0.394998988
```

Finally, the MSE for this model was 0.3148338.

```
# Computing the MSE of the model
ext.ridge.pred <- as.numeric(predict(best.ext.ridge.fit, s=ext.ridge.lambda,
                                     newx=x.ext))
mse.ext.ridge <- mean((ext.ridge.pred - y)^2)
```

Lasso

I did pretty much the same as in part (e) after adding the higher order polynomial and interaction terms.

NOTE: As mentioned, I'll keep using the same design matrix as above.

I set the seed and used k-fold cross-validation to select λ.

```
# Set a seed so results can be reproduced
set.seed(2017)
# Here I run 10-fold cross-validation
cv.fit.ext.lasso <- cv.glmnet(x.ext,y, alpha = 1, nfolds = 10, lambda = lambdas,
                              type.measure = 'mse')
```

```
# Obtain the lambda which gives smallest error
ext.lasso.lambda <- cv.fit.ext.lasso$lambda.min
```

The 'best' value for lambda was $\lambda = 0.01$. For this value, the lasso estimates are

```
# Run lasso with the 'best' lambda
best.ext.lasso.fit <- glmnet(x.ext,y, alpha = 1, lambda = ext.lasso.lambda)
# Obtain the coefficients
coef.ext.e <- as.matrix(coef(best.ext.lasso.fit))
coef.ext.e
```

```
##                           s0
## (Intercept)     -0.10055869
## AtBat            0.00000000
## Hits             0.20844358
## HmRun           -0.08567701
## Runs             0.00000000
## RBI              0.00000000
## Walks            0.08171496
## Years            0.00000000
## CAtBat           0.00000000
## CHits            0.00000000
## CHmRun           0.45219060
## CRuns            0.12785674
## CRBI             0.19233666
## CWalks           0.00000000
## LeagueN          0.10652381
## DivisionW       -0.19064574
## PutOuts          0.06728482
## Assists          0.00000000
## Errors          -0.01745085
## NewLeagueN       0.00000000
## I(AtBat^2)       0.00000000
## I(Hits^2)        0.12092918
## I(Runs^2)        0.00000000
## I(Walks^2)       0.11343796
## AtBat:CHmRun     0.00000000
## AtBat:CRuns      0.02108065
## Hits:CHmRun      0.22668101
## Walks:CHmRun     0.00000000
## CHmRun:CRBI     -0.16015669
## PutOuts:Assists -0.33067231
```

Finally, the MSE for this model was 0.3270579.

```r
# Computing the MSE of the model
ext.lasso.pred <- as.numeric(predict(best.ext.lasso.fit, s=ext.lasso.lambda,
                                     newx=x.ext))
mse.ext.lasso <- mean((ext.lasso.pred - y)^2)
```

Elastic net

NOTE: I'll be using the functions I defined for part (f). Also, followed the same steps as in part (f).

```r
# Set a seed so results can be reproduced
set.seed(2018)
# Run k-fold cross-validation for the set of alphas
out.ext <- cv.alpha.vect(alphas = alphas, feat = x.ext, res = y, l = lambdas)
# Get a vector of the MSE associated to the 'best' lambda for each alpha
ext.elastic.mse <- cv.mse.vect(out.ext, featu = x.ext, resp = y)
# Select the lowest MSE over elastic.mse
index.ext <- which.min(ext.elastic.mse)
# Gets the minimum MSE over all pairs (alpha,lambda)
mse.ext.elastic <- ext.elastic.mse[index.ext]
```

The best pair (α, λ) was (0.02, 0.01056876) with $MSE_{\alpha,\lambda} = 0.3091055$. This time elastic net came closer to ridge regression. The elastic net estimates are

```r
# Get best pair (alpha, lambda)
elastic.alpha <- alphas[index.ext]
elastic.lambda <- get.lambda(out[[index.ext]])
# Get coefficient estimates
best.elastic.fit <- glmnet(x,y, alpha = elastic.alpha, lambda = elastic.lambda)
coef.f <- as.matrix(coef(best.elastic.fit))
coef.f
```

```
##                      s0
## (Intercept)  0.078165873
## AtBat        0.000000000
## Hits         0.113280175
## HmRun        0.000000000
## Runs         0.060200212
## RBI          0.043927601
## Walks        0.094121669
## Years        0.000000000
## CAtBat       0.050945075
## CHits        0.100880948
## CHmRun       0.085086665
## CRuns        0.102835182
## CRBI         0.107026343
## CWalks       0.000000000
## LeagueN      0.062306592
## DivisionW   -0.214671980
## PutOuts      0.125899524
## Assists      0.007182533
## Errors      -0.020555784
## NewLeagueN   0.003954126
```

Using the MSE as **performance criteria**, we can say that these models appear to be 'better'. All new models performed better than any of the models in parts (b)-(f). This means also that the models improved

the first iteration of each regularization method; elastic net (0.3091055) performed better than ridge regression (0.3148338) and lasso (0.3270579). The 'best' model overall judging by the MSE was elastic net with the added features.

Ex4 part (j)

Manuel Alejandro Garcia Acosta

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Part (j)

Now add some noise to the response. That is, replace **Salary** by **Salary** $+ \epsilon$ where $\epsilon \sim N(0, \sigma^2)$ for some chosen value of σ. Again, try a standard linear model, ridge, lasso and elastic net where each model is optimally tuned. Repeat this for several increasing values of σ^2. On one plot, plot the error of each model against the amount of additional noise. Does one model or a certain subset of models tend to perform better or worse for large or small values of σ? Is there some intuition for this? Discuss.

For part (j) I decided to not scale 'Salary'. I just scaled the features and then I added the noise to 'Salary'. Noise was added as $\epsilon \sim N(0, \sigma^2)$ to 'Salary' for several values of σ. I made two plots of σ vs MSE, one for small values of σ and the other one for big values.

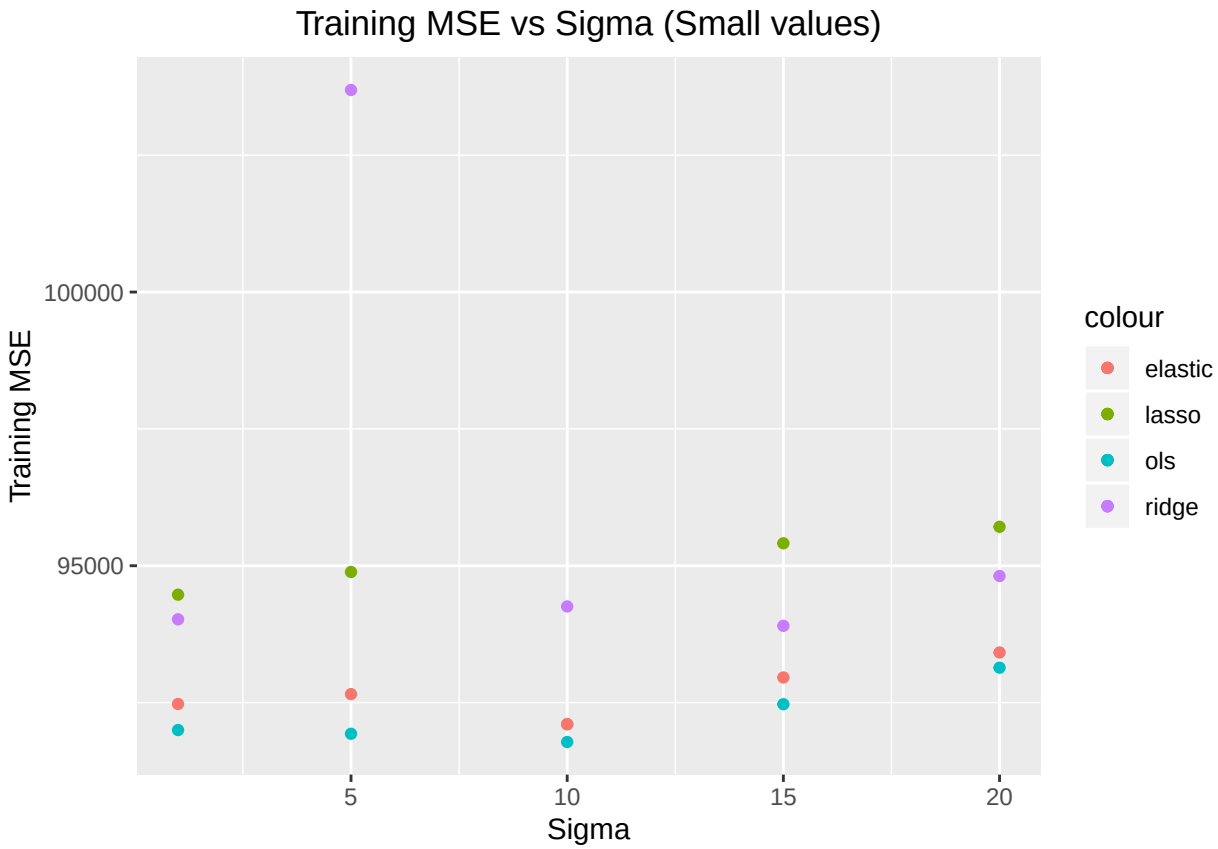
Interestingly, the results were similar to what I got in part (g). OLS and elastic net performed better than ridge regression and lasso for both small and large values of σ. Moreover, for large values of σ all regularization methods start performing way worse than OLS, even elastic net.

After our discussions now I get this happened because I used *training error* to compare the models. Using *test error* OLS should perform better for smaller values of σ and the regularization methods should perfom better than OLS for large values of σ.

Notes:

- Here I'm using quite a bit of code here, but I'm esentially using the same functions as in parts (b)-(i). I'm omitting the code here but you can find it on the Rmd file 'part_j.Rmd' I'm attaching.
- For this part I ran all the models using all the original features, i.e., the models were the same as parts (b) and (d)-(f) with additional noise.
- I continued using seeds for reproducibility.

This is the plot for small values of σ, selected values were 1, 5, 10, 15, 20.



This is the plot for large values of σ, selected values were 25, 30, 35, 40, 45.


Training MSE vs Sigma (Large values)
105000
100000
95000
Training MSE
25
30
35
40
45
Sigma
colour
elastic
lasso
ols
ridge