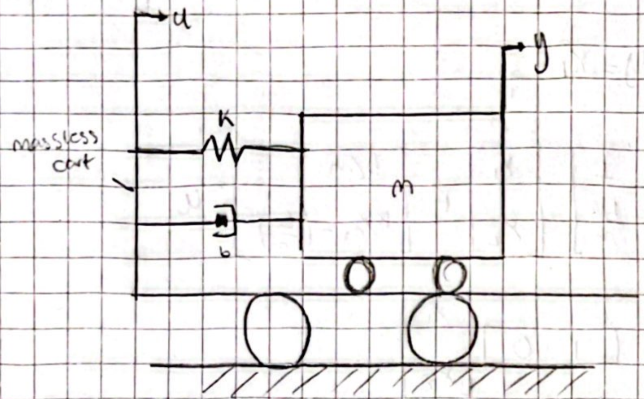
Punto #1 Parcial DS   Manuel Guo Cardona   20211005061

1) Ejemplo 3-3 Ogata.



$y > u$

Considering a $t < 0$ $u(t)$ = displacement → input (cart)
$y(t)$ = displacement of the mass → output

$\rightarrow \; m.a = \Sigma F \rightarrow m\frac{d^2y}{dt^2} = -b\left(\frac{dy}{dt} - \frac{du}{dt}\right) - k(y-u)$

$\rightarrow \; \mathcal{L} = (ms^2+bs+k)\, Y(s) = (bs+k)\, U(s)$

$\rightarrow \; G(s) = \frac{Y(s)}{U(s)} = \frac{(bs+k)}{(ms^2+bs+k)}$

→ Getting the state-space model

$$\ddot{y} + \frac{b}{m}\dot{y} + \frac{k}{m}y = \frac{b}{m}\dot{u} + \frac{k}{m}u$$

Standar form →
$$\ddot{y} + a_1\dot{y} + a_2 y = b_0\ddot{u} + b_1\dot{u} + b_2 u$$

where $a_1 = \frac{b}{m}$, $a_2 = \frac{k}{m}$ $b_0 = 0$ $b_1 = \frac{b}{m}$ $b_2 = \frac{k}{m}$

$\beta_0 = b_0 = 0$ $\beta_1 = b_1 - a_1\beta_0 = \frac{b}{m}$

$\beta_2 = b_2 - a_1\beta_1 - a_2\beta_0 = \frac{k}{m} - \left(\frac{b}{m}\right)^2$

$\rightarrow \; x_1 = y - \beta_0 u = y$

$x_2 = \dot{x}_1 - \beta_1 u = \dot{x}_1 - \frac{b}{m}u$

$$\dot{x}_1 = x_2 + \beta_1 u = x_2 + \frac{b}{m} u$$

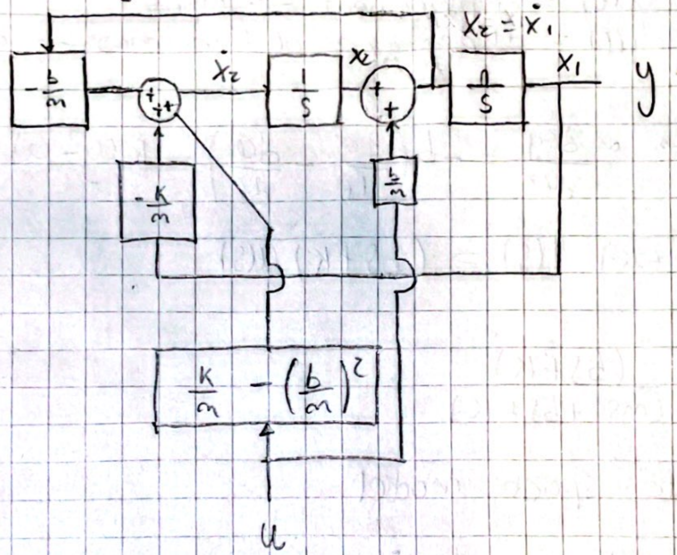
$$\dot{x}_2 = -a_2 x_1 - a_1 x_2 + \beta_2 u = -\frac{K}{m} x_1 - \frac{b}{m} x_2 + \left[\frac{K}{m} - \left(\frac{b}{m}\right)^2\right] u$$

$$y = x_1$$

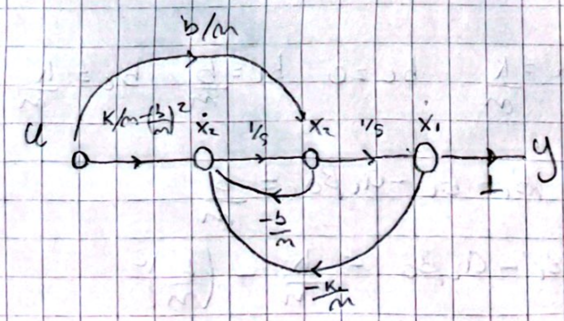
$$\rightarrow \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} b/m \\ K/m - \left(\frac{b}{m}\right)^2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

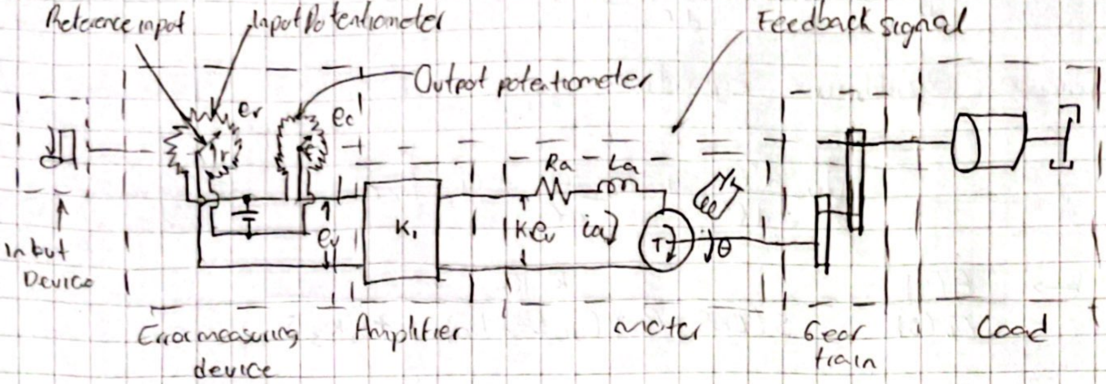
· Block diagram



· Signal flow diagram



2) Ejemplo A-3-9.



$$e = r - c$$

↳ error signal

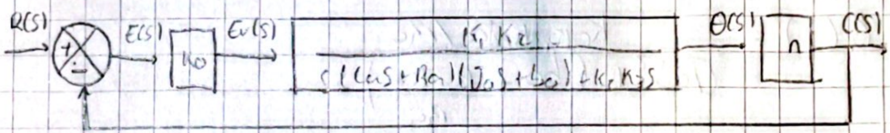
- $e_r - e_c = e_v$ → error voltage

$$e_r = K_0 r \qquad e_c = K_0 c$$

$T = K_2 i_a$ → Motor torque constant

↳ Torque

- Induced voltage $e_b = K_3 \frac{d\theta}{dt}$



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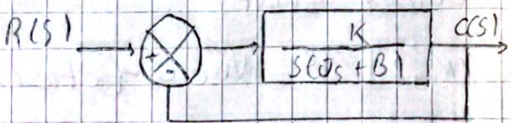


Block Diagram for the system

- Armadure voltage $e_a = K_1 e_v$ → output

$$\rightarrow L_a \frac{di_a}{dt} + R_a i_a + e_b = e_a$$

Differential equation for the armadure

$$= L_a \frac{di_a}{dt} + R_a i_a + K_3 \frac{d\theta}{dt} = K_1 e_v$$

→ Torque equilibrium equation

$$J_o \frac{d^2\theta}{dt^2} + b_o \frac{d\theta}{dt} = T = K_2 i_a$$

$$\rightarrow \frac{\theta(S)}{E_v(S)} = \frac{K_1 K_2}{S(L_a S + R_a)(J_o S + b_o) + K_2 K_3 S}$$

$$C(S) = n\theta(S)$$

$$\rightarrow E_v(S) = K_0 [R(S) - C(S)] = K_0 E(S)$$

$$\rightarrow G(S) = \frac{C(S)}{\cancel{\theta(S)}} \cdot \frac{\cancel{\theta(S)}}{\cancel{E_v(S)}} \cdot \frac{\cancel{E_v(S)}}{E(S)} = \frac{C(S)}{E(S)}$$

$$= \frac{K_0 K_1 K_2 n}{S[(L_a S + R_a)(J_o S + b_o) + K_2 K_3]}$$

→ L_a is small so →

$$G(S) = \frac{K_0 K_1 K_2 n / R_a}{J_o S^2 + \left(b_o + \frac{K_2 K_3}{R_a}\right) S}$$

$J = J_o / n^2$ = moment of inertia referred to the output shaft

$B = [b_o + (K_2 K_3 / R_a)] / n^2$ = Viscous-friction coefficient

$K = K_0 K_1 K_2 / n R_a$

$$\rightarrow G(S) = \frac{K_m}{S(T_m S + 1)}$$

Where $K_m = \frac{K}{B}$, $T_m = \frac{J}{B} = \frac{R_a J_o}{R_a b_o + K_2 K_3}$

· Now another way to find $G(S)$

if $E(S) = (R(S) - C(S))$ so

$$\frac{C(S)}{R(S) - C(S)} = \frac{K}{JS^2 + BS}$$

$$C(S) = R\left(\frac{K}{JS^2+BS}\right) - C(S)\left(\frac{K}{JS^2+BS}\right)$$

$$C(S)\left(1 + \frac{K}{JS^2+BS}\right) = R\left(\frac{K}{JS^2+BS}\right)$$

$$C(S) = \frac{R\left(\frac{K}{JS^2+BS}\right)}{\frac{K}{JS^2+BS} + 1} \rightarrow H(S) = \frac{C(S)}{R(S)}$$

$$H(S) = \frac{\frac{K}{JS^2+BS}}{\frac{K}{JS^2+BS} + 1} \div \frac{\cancel{1}\frac{K}{JS^2+BS}}{\cancel{1}\frac{K}{JS^2+BS}}$$

$$H(S) = \frac{1}{1 + \frac{JS^2+BS}{K}}$$

$$H(S) = \frac{K}{JS^2+BS+K}$$ Finally the complete Transfer function Including the potentiometers

· For consider the state variables

$$C(S)(JS^2+BS+K) = K(R(S))$$

$$J\ddot{C} + B\dot{C} + KC = KR$$

$$\rightarrow J\ddot{C} = KR - B\dot{C} + KC$$

$$\ddot{C} = \frac{K}{J}R - \frac{B}{J}\dot{C} + \frac{K}{J}C$$

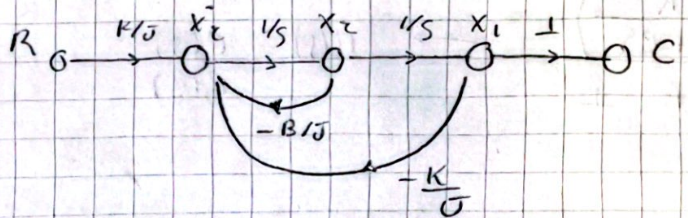
$\rightarrow \quad x_1 = C \qquad x_2 = \dot{x}_1 = \dot{C} \qquad \dot{x}_2 = \ddot{x}_1 = \ddot{C}$

$$\dot{x}_2 = -\frac{B}{J} x_2 - \frac{K}{J} x_1 + \frac{K}{J} R$$

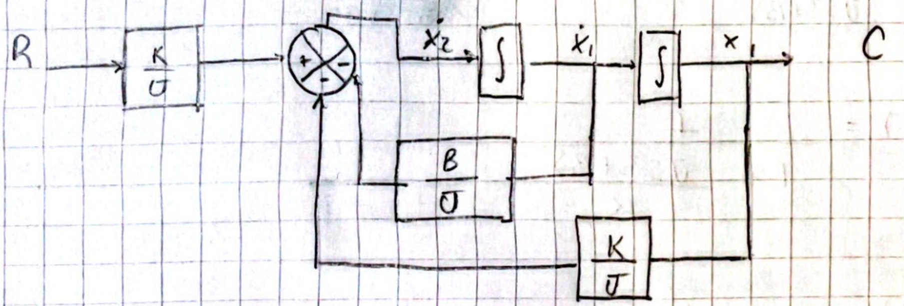
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -K/J & -B/J \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ K/J \end{bmatrix} R$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

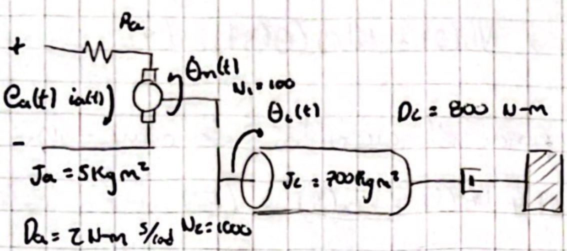
- Signal flow diagram

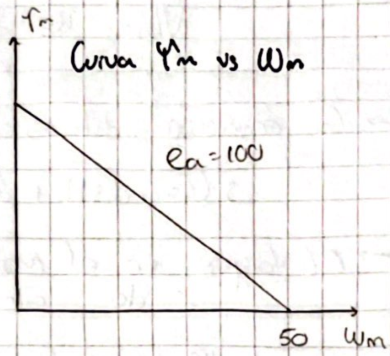


- Block diagram



3) Ejercicio 2-23 Norman Nise





→ Considerando la inercia total en el motor:

$$J_m = J_a + J_L\left(\frac{N_1}{N_2}\right)^2$$

• Si J_a → Inercia del eje del motor y J_L es la inercia de la carga

$$J_m = (5\,Kgm^2) + (700\,Kgm^2)\left(\frac{100}{1000}\right)^2 = 12\,Kgm^2 \quad (1)$$

→ Considerando el factor de amortiguamiento del sistema

$$D_m = D_a + D_L\left(\frac{N_1}{N_2}\right)^2$$

• D_a = Amortiguamiento eje motor
• D_L = Amortiguamiento carga

$$D_m = (2\,N \cdot m\ s/rad) + (800\ N \cdot m\ s/rad)\left(\frac{100}{1000}\right)^2$$
$$= 10\ N \cdot m\ s/rad \quad (2)$$

→ Considerando la curva Torque-Velocidad:

$$T = 500;\quad W_{\text{sin-carga}} = 50;\quad e_a = 100\,V$$

→ Al considerar las constantes eléctricas del sistema

$$\frac{K_T}{R_a} = \frac{T_{max}}{e_a} = \frac{500\ N/m}{100\ V} = 5\ N/mV \quad (3)$$

$$K_b = \frac{e_a}{W_{\text{sin-carga}}} = \frac{100\,V}{50\ rad/s} = 2\ V\ s/rad$$

→ Para un flujo constante, el voltaje inducido (V_b) es directamente proporcional a la velocidad angular.

$$V_b = K_b \frac{d\theta}{dt} \longrightarrow V_b(s) = K_b s(\theta(s)) \quad (4)$$

→ La ecuación diferencial para el circuito de armadura

$$s(L_a i_a(s)) + R_a i_a(s) + V_b(s) = E_a(s) \quad (5)$$

→ El torque en el motor es proporcional a la corriente de armadura.

$$T_m(s) = K_t\, i_a(s) \quad (6)$$

→ La ecuación para el equilibrio del torque

$$(J_m s^2 + D_m s)\,\theta_m(s) = T_m(s) \quad (7)$$

→ Remplazando (7) en (5) y considerando la pequeña

$$\left(\frac{R_a}{K_t}(J_m s + D_m) + K_b\right) s\theta_m(s) = E_a(s)$$

→ Despejando $\theta_m(s)/E_a(s)$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t / R_a J_m}{s\left(s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right)} \quad (8)$$

Esta corresponde a la ecuación de transferencia.

→ Remplazando los valores conocidos

$$\frac{\theta_m(s)}{E_a(s)} = \frac{5/12}{s\left(s + \frac{1}{12}(10 + (5)(2))\right)} = \frac{0,417}{s(s+1,667)}$$

→ Considerando el radio de los engranajes $N_1/N_2 = 1/10$

$$\frac{\theta_c(s)}{E_a(s)} = \frac{0,417(N_1/N_2)}{s(s+1,667)} = \frac{0,0417}{s(s+1,667)}$$

$$= \frac{\theta_c(s)}{E_a(s)} = \frac{0,0417}{s^2+1,667s}$$

$$\theta_c(s)(s^2 + 1,667s) = E_a(s)(0,0417)$$

→ Realizando transformada inversa de Laplace

$$\ddot{\theta} + 1{,}667\dot{\theta} = 0{,}047 E_a \quad (10)$$

$$X_1 = \theta \,,\; X_2 = \dot{X}_1 = \dot{\theta} \,,\; \dot{X}_2 = \ddot{\theta}$$

→ Remplazando en (10)

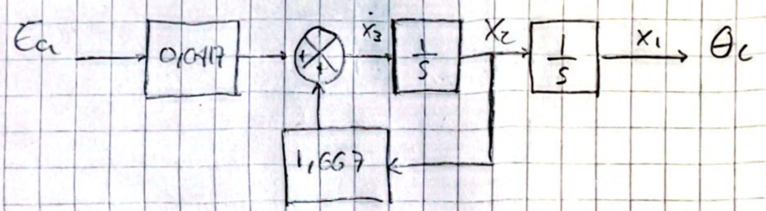
$$\rightarrow \dot{X}_2 + 1{,}667 X_2 = 0{,}047 E_a$$

$$= \dot{X}_2 = 0{,}047 E_a - 1{,}667 X_2$$

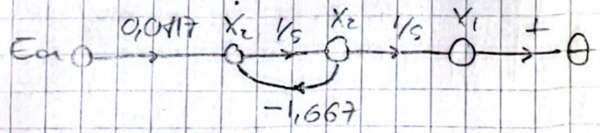
→ La matriz de variables de estado:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -1{,}667 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0{,}0417 \end{bmatrix} E_a$$

$$\theta_c = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$



→ El diagrama de flujo de Señal



4) Comparación

En los ejercicios 2-23 de Norman Nise y A 3-a de Ogata se tiene un análisis similar respecto a las consideraciones mecanicas y electricas del sistema. Sin embargo, en el ejercicio 2-23 de Norman Nise se tiene un sistema únicamente compuesto por un motor DC que a diferencia del servomotor no posee el sistema de engranajes que realimentan el estado del potenciometro de control el cual modifica la realimentación del Sistema y cambia la posición de la carga según sea necesario.

De esta forma, la ausencia de la parte de realimentación del sistema en el ejercicio de Norman Nise se refleja directamente con la ecuación de transferencia final de cada sistema que a su vez modifica los diagramas de bloque y flujo de señal junto con la matriz de variables de estado.

Finalmente, la diferencia entre ambos ejercicios se concluye en el uso de realimentación en el ejercicio A-3-9 de Ogata en el que se debe partir de la ecuación de transferencia obtenida para la parte de motor DC y considerar la realimentación del sistema.

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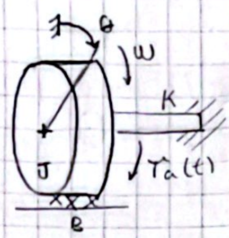
1.



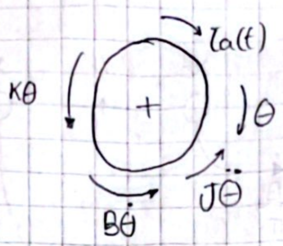


Diagrama de cuerpo libre

$$\longrightarrow \quad J\ddot{\theta} + B\dot{\theta} + K\theta = \tau_a(t)$$

Asignando Variables de estado

$$X_1 = \theta \;, \quad X_2 = \dot{X}_1 = \dot{\theta} \;, \quad \dot{X}_2 = \ddot{X}_1 = \ddot{\theta}$$

Remplazando las variables

$$\longrightarrow \quad J\dot{X}_2 + BX_2 + KX_1 = \tau_a(t)$$

$$= \quad \dot{X}_2 = \frac{\tau_a}{J} - \frac{B}{J}X_2 - \frac{K}{J}X_1$$

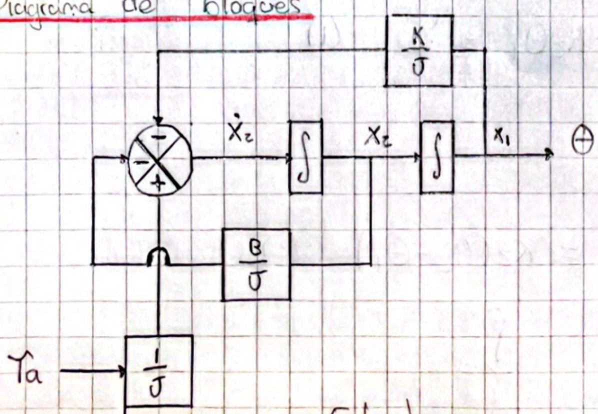
$$\longrightarrow \quad \theta = X_1$$

• Espacio de estados

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -K/J & -B/J \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/J \end{bmatrix} \tau_a$$

$$\theta = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

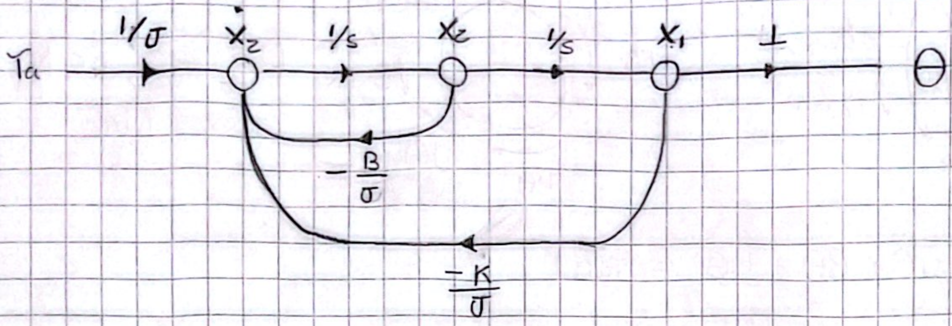
• Diagrama de bloques



Salida

Entrada

- Diagrama flujo señal



- Funcion de transferencia

→ Aplicando laplace $\mathcal{L}$

$$\rightarrow \quad JS^2\theta(S) + BS\theta(S) + K\theta(S) = Ta(S)$$

$$= \quad \theta(S)\,(JS^2 + BS + K) = Ta(S)$$

$$= \quad \frac{\theta(S)}{Ta(S)} = \frac{1}{JS^2 + BS + K} = G(S)$$

2.

$\theta_2 > \theta_1$

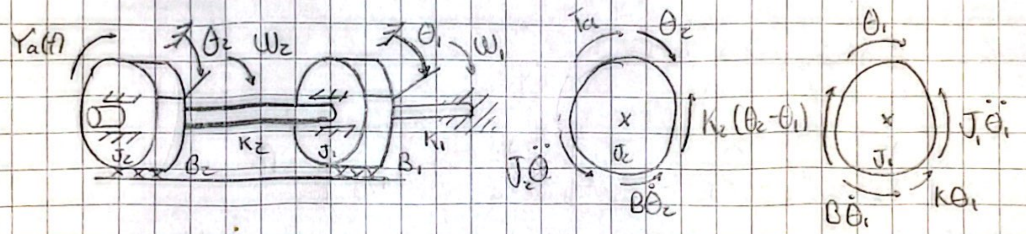


Diagrama cuerpo libre

→ Para θ_2

$$J_2\ddot{\theta}_2 + B_2\dot{\theta}_2 + K_2\theta_2 - K_2\theta_1 = Ta \quad (1)$$

→ Para θ_1

$$J_1\ddot{\theta}_1 + B_1\dot{\theta}_1 + K_1\theta_1 = K_2(\theta_2 - \theta_1) \quad (2)$$

· Función de transferencia

→ Para (1)

$$= J_2 S^2 \Theta_2(S) + B_2 S \Theta_2(S) + K_2 \Theta_2(S) - K_2 \Theta_1(S) = T_a(S)$$

$$= \Theta_2(S) \left(J_2 S^2 + B_2 S + K_2 \right) - K_2 \Theta_1(S) = T_a(S) \quad (3)$$

→ Para (2)

$$= J_1 S^2 \Theta_1(S) + B_1 S \Theta_1(S) + K_1 \Theta_1(S) + K_2 \Theta_1(S) = K_2 \Theta_2(S)$$

$$= \Theta_1(S) \left(J_1 S^2 + B_1 S + K_1 + K_2 \right) = K_2 \Theta_2(S) \quad (4)$$

→ Despejando $\Theta_1(S)$ de (4) y despejando en (3)

$$= \Theta_1(S) = \frac{K_2 \Theta_2(S)}{J_1 S^2 + B_1 S + K_1 + K_2}$$

→

$$\Theta_2(S) \left(J_2 S^2 + B_2 S + K_2 \right) - \frac{K_2^2 \Theta_2(S)}{J_1 S^2 + B_1 S + K_1 + K_2} = T_a(S)$$

$$= \Theta_2(S) \left(J_2 S^2 + B_2 S + K_2 - \frac{K_2^2}{J_1 S^2 + B_1 S + K_1 + K_2} \right) = T_a(S)$$

$$= \frac{\Theta_2(S)}{T_a(S)} \left(\frac{J_2 J_1 S^4 + J_2 B_1 S^3 + J_2 K_1 S^2 + J_2 K_2 S^2 + J_1 B_2 S^3 + B_2 B_1 S^2 + B_2 K_1 S + B_2 K_2 S + J_1 K_2 S^2 + B_1 K_2 S + K_1 K_2 + \cancel{K_2^2} - \cancel{K_2^2}}{J_1 S^2 + B_1 S + K_1 + K_2} \right)$$

$$= 1$$

$$= \frac{\Theta_2(S)}{T_a(S)} \left(\frac{J_2 J_1 S^4 + S^3 (J_2 B_1 + J_1 B_2) + S^2 (J_2 K_1 + J_2 K_2 + B_2 B_1 + J_1 K_2) + S (B_2 K_1 + B_2 K_2 + B_1 K_2) + K_1 K_2}{J_1 S^2 + B_1 S + K_1 + K_2} \right)$$

$$= 1$$

$$= \frac{\Theta_2(S)}{T_a(S)} = \frac{J_1 S^2 + B_1 S + K_1 + K_2}{J_2 J_1 S^4 + S^3 (J_2 B_1 + J_1 B_2) + S^2 (J_2 K_1 + J_2 K_2 + B_2 B_1 + J_1 K_2) + S (B_2 K_1 + B_2 K_2 + B_1 K_2) + K_1 K_2}$$

$$\rightarrow \Theta_2(S) \left(J_2 J_1 S^4 + S^3 (J_2 B_1 + J_1 B_2) + S^2 (J_2 (K_1 + K_2) + B_2 B_1 + J_1 K_2) + S (B_2 (K_1 + K_2) + B_1 K_2) + K_1 K_2 \right)$$

$$= T_a(S) \left(J_1 S^2 + B_1 S + K_1 + K_2 \right)$$

→

$$\xrightarrow{T_a(s)} \boxed{\dfrac{1}{J_2 J_1 S^4 + S^3(J_2 B_1 + J_1 B_2) + S^2(J_2(K_1+K_2) + B_2 B_1 + J_1 K_2) + S(B_2(K_1+K_2) + B_1 K_2) + K_1 K_2}} \xrightarrow{X_1(s)} \boxed{J_1 S^2 + B_1 S + K_1 + K_2} \longrightarrow \theta_2(s)$$

→ $\mathcal{L}^{-1}$

- $= J_2 J_1 \ddddot{X} + (J_2 B_1 + J_1 B_2)\dddot{X} + (J_2(K_1+K_2) + B_2 B_1 + J_1 K_2)\ddot{X} + (B_2(K_1+K_2) + B_1 K_2)\dot{X} + K_1 K_2 X = T_a$
- $= J_1 \ddot{X} + B_1 \dot{X} + (K_1 + K_2) X = \theta_2$

$$\begin{aligned}
X_1 &= X_1 \\
X_2 &= \dot{X}_1 = \dot{X} \\
X_3 &= \dot{X}_2 = \ddot{X} \\
X_4 &= \dot{X}_3 = \dddot{X} \\
\dot{X}_4 &= \ddddot{X}
\end{aligned}$$

Variables de estado

- $= J_2 J_1 \dot{X}_4 + (J_2 B_1 + J_1 B_2) X_4 + (J_2(K_1+K_2) + B_2 B_1 + J_1 K_2) X_3 + (B_2(K_1+K_2) + B_1 K_2) X_2 + K_1 K_2 X_1 = T_a$

  $= \dot{X}_4 = -\dfrac{K_1 K_2}{J_2 J_1} X_1 - \dfrac{(B_2(K_1+K_2) + B_1 K_2)}{J_2 J_1} X_2 - \dfrac{(J_2(K_1+K_2) + B_2 B_1 + J_1 K_2)}{J_2 J_1} X_3 - \dfrac{(J_2 B_1 + J_1 B_2)}{J_2 J_1} X_4 + \dfrac{T_a}{J_2 J_1}$

- $= \theta_2 = (K_1 + K_2) X_1 + B_1 X_2 + J_1 X_3$

→ Espacio de Estados

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \\ \dot{X}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{K_1 K_2}{J_2 J_1} & -\dfrac{(B_2(K_1+K_2) + B_1 K_2)}{J_2 J_1} & -\dfrac{(J_2(K_1+K_2) + B_2 B_1 + J_1 K_2)}{J_2 J_1} & -\dfrac{(J_2 B_1 + J_1 B_2)}{J_2 J_1} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{J_2 J_1} \end{bmatrix} T_a$$

$$\theta_2 = \begin{bmatrix} (K_1+K_2) & B_1 & J_1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}$$

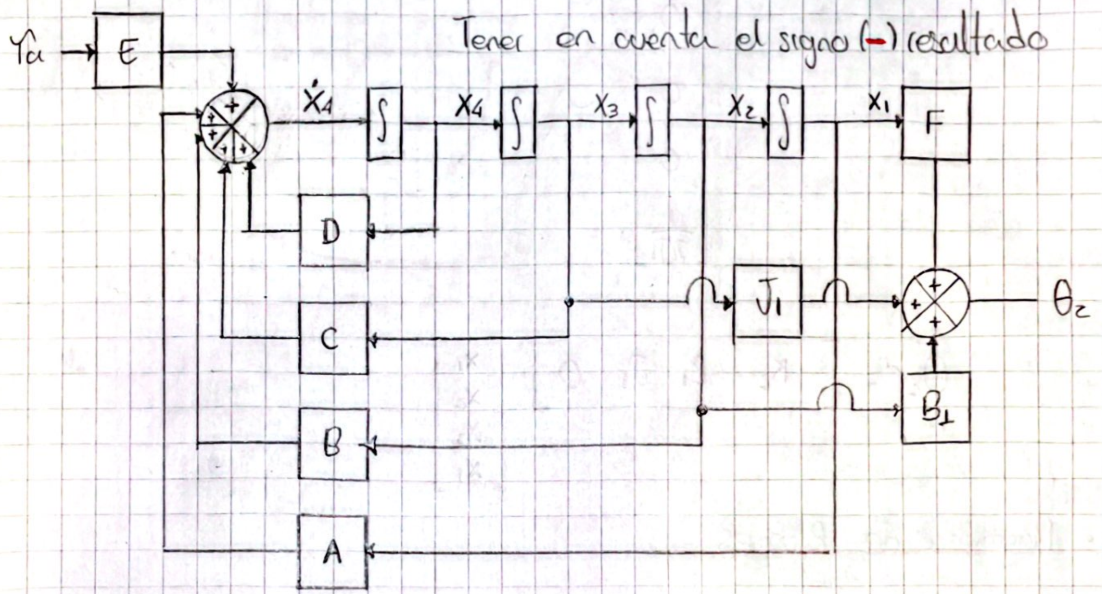
- Diagrama de Bloques

Para el diagrama de bloques y el diagrama de flujo de señal se asignaron las siguientes variables.

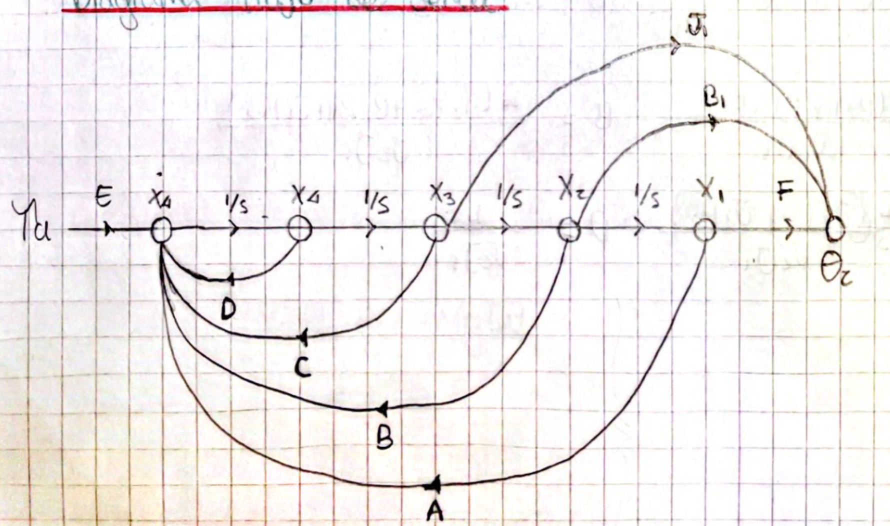
$$A = \frac{-K_1K_2}{J_1J_2} \quad B = \frac{-(B_2(K_1+K_2)+B_1K_2)}{J_2J_1} \quad C = \frac{-(J_2(K_1+K_2)+B_2B_1+J_1K_2)}{J_2J_1}$$

$$D = \frac{-(J_2B_1+J_1B_2)}{J_2J_1} \quad E = \frac{1}{J_2J_1} \quad F = (K_1+K_2)$$

Tener en cuenta el signo (-) resultado



- Diagrama Flujo de Señal



3. Considerando $K_1 = 0$ para punto 2.

- Función de transferencia

$$\frac{\theta_2(s)}{Ya(s)} = \frac{J_1 S^2 + B_1 S + K_2}{J_2 J_1 S^4 + S^3(J_2 B_1 + J_1 B_2) + S^2(J_2 K_2 + B_2 B_1 + J_1 K_2) + S(B_2 K_2 + B_1 K_2)}$$

- Espacio de estados

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-(B_2 K_2 + B_1 K_2)}{J_2 J_1} & \frac{-(J_2 K_2 + B_2 B_1 + J_1 K_2)}{J_2 J_1} & \frac{-(J_2 B_1 + J_1 B_2)}{J_2 J_1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_2 J_1} \end{bmatrix} \hat{Ya}$$

$$\theta_2 = \begin{bmatrix} K_2 & B_1 & J_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

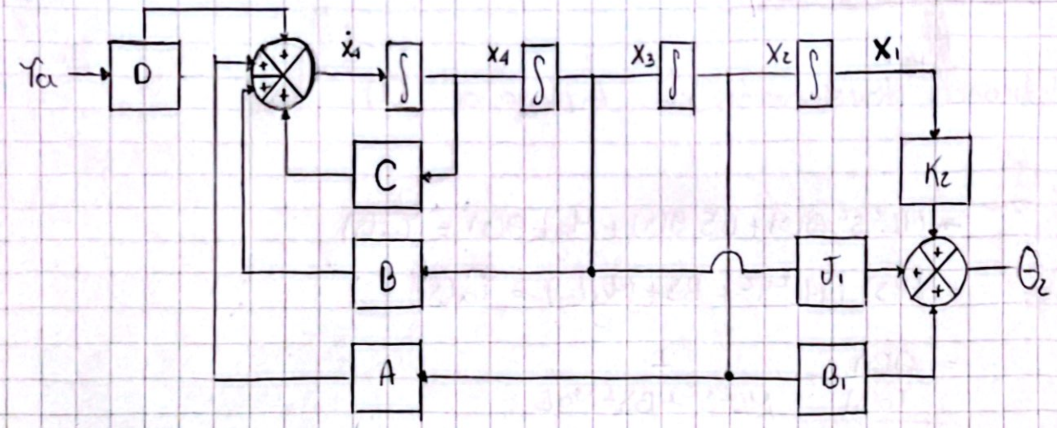
- Diagrama de Bloques

Al igual que el punto 2. Se asignaron variables.

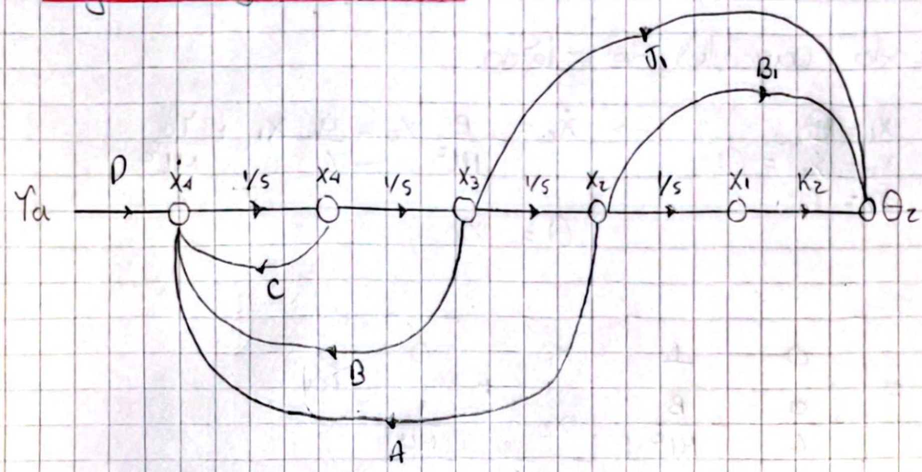
$$A = \frac{-(B_2 K_2 + B_1 K_2)}{J_2 J_1} \qquad B = \frac{-(J_2 K_2 + B_2 B_1 + J_1 K_2)}{J_2 J_1}$$

$$C = \frac{-(J_2 B_1 + J_1 B_2)}{J_2 J_1} \qquad D = \frac{1}{J_2 J_1}$$

1.



• Diagrama Flujo de señal



4.

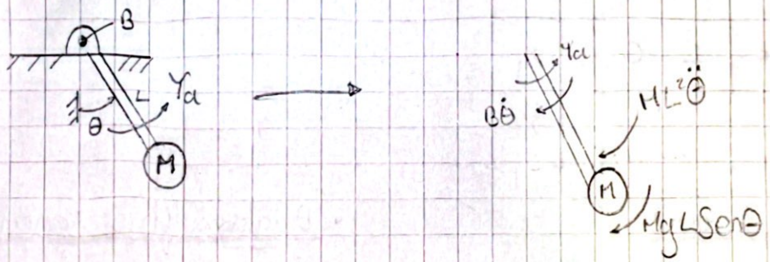


Diagrama Cuerpo Libre.

→ El momento de inercia corresponde a $J = ML^2$

$$\rightarrow ML^2\ddot{\theta} + B\dot{\theta} + MgL\,Sen\theta = Ya(t) \quad (1)$$

→ Para pequeños valores de $\theta \rightarrow Sen\theta \simeq \theta$

$$\rightarrow ML^2\ddot{\theta} + B\dot{\theta} + MgL\theta = Ya(t)$$

$$= \ddot{\theta} = -\frac{MgL}{ML^2}\theta - \frac{B}{ML^2}\dot{\theta} + \frac{Ya}{ML^2}$$

$$= \ddot{\theta} = -\frac{B}{ML^2}\dot{\theta} - \frac{g}{L}\theta + \frac{Ya}{ML^2} \quad (2)$$

- **Función transferencia**

Aplicando transformada de Laplace a (1)

$\to \mathcal{L}$

$$\to ML^2 S^2\, \theta(S) + BS\, \theta(S) + MgL\, \theta(S) = \hat{Y_a}(S)$$

$$= \theta(S)\, (ML^2 S^2 + BS + MgL) = \hat{Y_a}(S)$$

$$= \frac{\theta(S)}{\hat{Y_a}(S)} = \frac{1}{ML^2 S^2 + BS + MgL}$$

- **Espacio de estados**

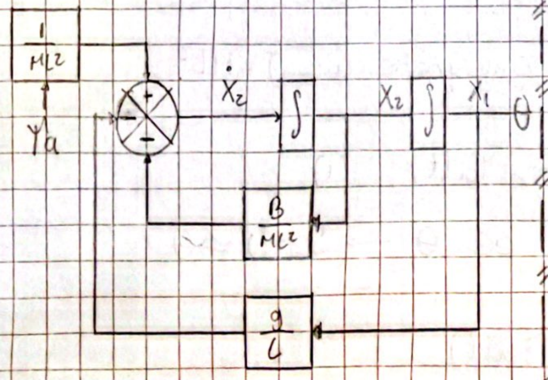
Asignando variables de estado.

$$x_1 = \theta \qquad \to \quad \dot{x}_2 = -\frac{B}{ML^2} x_2 - \frac{g}{L} x_1 + \frac{Y_a}{ML^2}$$
$$x_2 = \dot{x}_1 = \dot{\theta}$$
$$\dot{x}_2 = \ddot{\theta} \qquad\qquad \theta = x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{L} & -\frac{B}{ML^2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{ML^2} \end{bmatrix} Y_a$$

$$\theta = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

- **Diagrama de bloques**



- **Diagrama flujo señal**