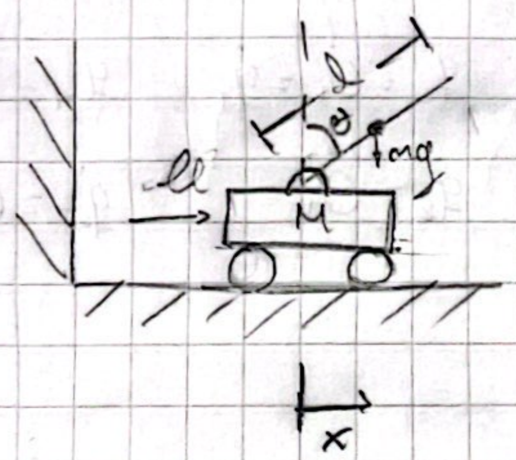
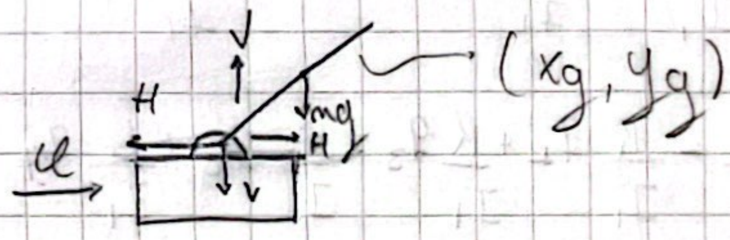
Sist mezclado traslaccional y rotaccional.





$$x_g = x + l\,\mathrm{Sen}\,\theta$$

$$y_g = l\,\mathrm{Cos}\,\theta$$

Rotational motion

$$I\ddot{\theta} = V l\,\mathrm{Sen}\,\theta - H l\,\mathrm{Cos}\,\theta \quad (1)$$

Horizontal motion

$$\frac{m\,d^2(x + l\,\mathrm{Sen}\,\theta)}{dt^2} = H$$

$$\rightarrow m\ddot{x} + \frac{m\,d^2 l \,Sen\theta}{dt^2} = H$$

$$= m\ddot{x} + ml \frac{d}{dt}\left(Cos\theta \cdot \dot{\theta}\right) =$$

$$= m\ddot{x} + ml\left(-Sen(\theta) \cdot \dot{\theta} \cdot \dot{\theta} + Cos\theta \cdot \ddot{\theta}\right) =$$

$$= m\ddot{x} - mlSen(\theta) \cdot \dot{\theta}^2 + mlCos\theta \cdot \ddot{\theta} = H \quad (2)$$

Vertical motion

$$\frac{m\,d^2(lCos\theta)}{dt^2} = V - mg \quad (3)$$

$$= ml\frac{d}{dt}\left(-Sen\theta \cdot \dot{\theta}\right)$$

$$= ml\left(-Cos\theta \cdot \dot{\theta} \cdot \dot{\theta} - Sen\theta \cdot \ddot{\theta}\right)$$

$$= -mlCos\theta \cdot \dot{\theta}^2 - mlSen\theta\, \ddot{\theta} = V - mg$$

Cart horizontal motion

$$M\ddot{x} = u - H \quad (4)$$

θ So small $\begin{cases} Sen\theta \approx \theta \\ Cos\theta \approx 1 \end{cases}$ $\quad \theta\dot{\theta}^2 = \varnothing$

(1) $\rightarrow$

$$I\ddot{\theta} = Vl\,\cancel{Sen}\theta - Hl\,\cancel{Cos\theta}$$

$$I\ddot{\theta} = Vl\theta - Hl \quad (5)$$

(2) $\rightarrow$

$$m\ddot{x} - ml\,\cancel{Sen\theta} \cdot \cancel{\dot{\theta}^2} + ml\,\cancel{Cos}\theta\, \ddot{\theta} = H$$

$$= m\ddot{x} + ml\ddot{\theta} = H$$

$$= m(\ddot{x} + l\ddot{\theta}) = H \quad (6)$$

(3) $\rightarrow$

$$0 = V - mg \quad (7)$$

Tomando (4) y (6)

$$M\ddot{x} = u - H \quad , \quad m(\ddot{x} + l\ddot{\theta}) = H$$

$$= M\ddot{x} = u - m(\ddot{x} + l\ddot{\theta})$$

$$= M\ddot{x} = u - m\ddot{x} - lm\ddot{\theta}$$

$$= M\ddot{x} + m\ddot{x} = u - ml\ddot{\theta}$$

$$= \ddot{x}(M+m) = u - ml\ddot{\theta}$$

$$= (M+m)\ddot{x} + ml\ddot{\theta} = u \quad (8)$$

Tomando (5), (6) ∧ (7)

$V = mg$

$$I\ddot{\theta} = Vl\theta - Hl \quad ; \quad H = m(\ddot{x} + l\ddot{\theta}) \; ; \; 0 = V - mg$$

$$\rightarrow I\ddot{\theta} = mgl\theta - m(\ddot{x} + l\ddot{\theta})l$$

$$= mgl\theta - ml\ddot{x} - ml\ddot{\theta}$$

$$I\ddot{\theta} = mgl\theta - l(m\ddot{x} - m\ddot{\theta}) \quad ó \quad (I + ml^2)\ddot{\theta} + ml\ddot{x} = mgl\theta$$

(9)

Tomando (8) y despejando $\ddot{x}$ para remplazar en (9)

$$\rightarrow \ddot{x} = \frac{u - ml\ddot{\theta}}{(M+m)} \quad ; \quad \rightarrow (I + ml^2)\ddot{\theta} + ml\left(\frac{u - ml\ddot{\theta}}{(M+m)}\right) = mgl\theta$$

$$= (I + ml^2)\ddot{\theta} + \frac{mlu}{M+m} - \frac{m^2l^2\ddot{\theta}}{M+m} = mgl\theta$$

$$= \left(I + ml^2 - \frac{m^2l^2}{M+m}\right)\ddot{\theta} + \frac{mlu}{M+m} = mgl\theta$$

$$= \ddot{\theta} = \frac{mgl\theta - \frac{mlu}{M+m}}{\left(I + ml^2 - \frac{m^2l^2}{M+m}\right)}$$

$$\frac{I(M+m) + ml^2(M+m) - m^2l^2}{M+m}$$

$$= \frac{IM + Im + mMl^2 + \cancel{m^2l^2} - \cancel{m^2l^2}}{M+m}$$

$$= \frac{I(M+m) + mMl^2}{M+m}$$

$$\rightarrow \ddot{\theta} = \frac{mgl\theta - \frac{mlu}{M+m}}{\frac{I(M+m) + mMl^2}{M+m}}$$

$$= \ddot{\theta} = \frac{(M+m)\,mgl\theta}{I(M+m) + mMl^2} - \frac{mlu}{I(M+m)+mMl^2} \qquad (10)$$

Tomando (8) y despejando $\ddot{\theta}$ para remplazar en (9)

$$\rightarrow \ddot{\theta} = \frac{u}{ml} - \frac{(M+m)\ddot{x}}{ml}$$

$$\rightarrow (I+ml^2)\left(\frac{u}{ml} - \frac{(M+m)\ddot{x}}{ml}\right) + ml\ddot{x} = mgl\theta$$

$$= \frac{Iu}{ml} - \frac{\ddot{x}I(M+m)}{ml} + \frac{\cancel{m}l^2u}{\cancel{m}l} - \frac{\ddot{x}\cancel{m}l^2(M+m)}{\cancel{ml}} + ml\ddot{x} = mgl\theta$$

$$= \frac{Iu}{ml} - \frac{\ddot{x}I(M+m)}{ml} + lu - \ddot{x}l(M+m) + ml\ddot{x} = mgl\theta$$

$$= u\left(\frac{I}{ml} + l\right) + \ddot{x}\left(ml - \frac{I(M+m)}{ml} - l(M+m)\right) = mgl\theta$$

$$= \ddot{x}\left(\frac{ml(ml) - I(M+m) - l(M+m)(ml)}{ml}\right) = mgl\theta - u\left(\frac{I + l(ml)}{ml}\right)$$

$$= \ddot{x}\left(\frac{\cancel{m^2l^2} - IM - Im - ml^2M - \cancel{m^2l^2}}{ml}\right) = mgl\theta - u\left(\frac{I+ml^2}{ml}\right)$$

$$= \ddot{x}\left(\frac{-I(M+m) - ml^2M}{ml}\right) = mgl\theta - u\left(\frac{I+ml^2}{ml}\right)$$

$$\ddot{x} = \frac{mgl\theta - u\left(\frac{I+ml^2}{ml}\right)}{\left(\frac{-I(M+m) - ml^2M}{\cancel{ml}}\right)}$$

$$= \ddot{x} = \frac{m^2l^2g\theta - uml\left(\frac{I+ml^2}{ml}\right)}{-I(M+m) - ml^2M}$$

$$= \ddot{x} = \frac{m^2l^2g\theta}{-I(M+m) - ml^2M} - \frac{u\left(I - ml^2\right)}{-I(M+m) - ml^2M} \qquad (11)$$

→ $q_1 = \Theta$ $\qquad q_3 = x$

$q_2 = \dot{q}_1 = \dot{\Theta}$ $\qquad q_4 = \dot{q}_3 = \dot{x}$

$\dot{q}_2 = \ddot{\Theta}$ $\qquad \dot{q}_4 = \ddot{x}$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{(M+m)\,mgl}{I(M+m)+mMl^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{m^2l^2g}{I(M+m)+ml^2M} & 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ \frac{ml}{I(M+m)+mMl^2} \\ 0 \\ \frac{(I-ml^2)}{-I(M+m)-mMl^2} \end{bmatrix} u$$

→ $\Theta = q_1 \qquad q_3 = x$