Methodology for Portfolio Monte Carlo Simulation and Performance Assessment

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1 Introduction

This document describes the methodology for the **Portfolio Monte Carlo Simulation and Performance Assessment** project. The primary objectives are to evaluate the historical performance of a long-only ETF portfolio from 2015 to 2023 and to assess the accuracy of a Monte Carlo simulation in forecasting the portfolio's performance. The ETFs were selected as top-performing by asset class based on 2015 data.

2 Methodology

2.1 Data Collection and Portfolio Construction

The portfolio comprises selected ETFs representing various asset classes, weighted according to a discretionary allocation strategy. Ticker symbols and weights are stored in '"ETFsList.csv"'. Two portfolio structures are analyzed:

- **Discretionary Weighting**: Each ETF is weighted based on conventional knowledge and market insights.
- **Equal-Weighting**: Each ETF receives an equal weight within the portfolio.

Daily price data for each ETF is retrieved using Yahoo Finance, with returns calculated as the percentage change in closing prices:

$$\text{Return}_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{1}$$

where P_t is the closing price on day t.

2.2 Monte Carlo Simulation

The Monte Carlo simulation forecasts portfolio performance over the period 2015-2023, using pre-2015 returns to calculate expected returns and volatility. Monte Carlo simulation is a method for estimating the future performance of a portfolio by generating multiple potential outcomes based on historical returns and volatility patterns. Each simulation run reflects a possible path the portfolio might take under stochastic influences, allowing us to estimate both expected performance and potential deviations.

2.2.1 Mean and Covariance Calculation

Using historical returns, we calculate the mean return vector, μ, and covariance matrix, Σ, for the portfolio:

$$\mu = E[\text{Returns}] \tag{2}$$

$$\Sigma = E[(\text{Returns} - \mu)(\text{Returns} - \mu)^T] \tag{3}$$

2.2.2 Daily Returns Simulation

Each Monte Carlo iteration models daily returns for each asset in the portfolio. Returns are drawn from a multivariate normal distribution, incorporating the calculated mean and covariance, to reflect both the expected return and asset correlations. The formula for daily simulated returns is:

$$\text{Returns}_t = \mu + L \cdot Z_t \tag{4}$$

where Z_t is a vector of independent standard normal variables, and L is the Cholesky decomposition of Σ, ensuring realistic correlations among assets.

The portfolio value is simulated over T days for each of the N simulations:

$$\text{Portfolio Value}_t = \text{Portfolio Value}_{t-1} \times (1 + \sum_i w_i \cdot \text{Returns}_{i,t}) \tag{5}$$

where w_i is the weight of asset i in the portfolio.

2.2.3 Simulation Boundaries

The simulation generates a forecast range (mean, upper, and lower bounds) for the portfolio's value. The boundaries are set at ± 1 standard deviation around the mean, representing the range within which we expect the actual portfolio performance to fall approximately 68% of the time. These boundaries are important as they provide a range of expected outcomes, allowing investors to gauge potential downside and upside risks. If actual performance consistently falls outside this range, it may indicate that assumptions used in the simulation require reassessment.

2.3 Comparing Equal vs. Discretionary Weighting

Comparing equal and discretionary weighting strategies is essential to understand how portfolio composition choices impact performance. An equal-weighted strategy treats all assets equally, which can reduce concentration risk and provide a baseline for diversification benefits. The discretionary strategy, informed by market insights and risk tolerance, may enhance performance by focusing more weight on certain assets. Analyzing both approaches helps highlight the benefits and potential trade-offs of each allocation strategy, offering insights into whether active (discretionary) allocation justifies any additional risk.

2.4 Risk Metrics Calculation

Several risk metrics are calculated to evaluate the portfolio's performance and downside risk:

2.4.1 Value at Risk (VaR)

VaR at a confidence level α represents the maximum expected portfolio loss:

$$\text{VaR}_\alpha = \text{Percentile}_\alpha(\text{Portfolio Returns}) \tag{6}$$

2.4.2 Conditional Value at Risk (CVaR)

CVaR measures the average loss if the portfolio return is less than or equal to the VaR threshold:

$$\text{CVaR}_\alpha = E[\text{Returns}|\text{Returns} \leq \text{VaR}_\alpha] \tag{7}$$

2.4.3 Drawdown

Drawdown measures the peak-to-trough decline in portfolio value:

$$\text{Drawdown}_t = \frac{\text{Portfolio Value}_t - \max(\text{Portfolio Value}_{0,\ldots,t})}{\max(\text{Portfolio Value}_{0,\ldots,t})} \tag{8}$$

3 Assumptions

- The portfolio is assumed to be long-only, with no leverage or short positions.
- The returns are assumed to follow a normal distribution, justifying the Monte Carlo simulation approach.
- The ETFs selected are expected to represent their asset classes consistently over the forecast period.

4 Conclusion

The **Portfolio Monte Carlo Simulation and Performance Assessment** project provides insights into the risk-adjusted performance of a multi-asset ETF portfolio, comparing it to the SP 500 and evaluating the simulation's forecasting accuracy. Results show that the portfolio offers superior drawdown protection and risk-adjusted returns relative to the SP 500, with actual performance falling within the Monte Carlo simulation's expected bounds.