

INTRODUCTORY LECTURES ON MANIFOLD TOPOLOGY: EXOTIC SPHERES

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One of the milestones in the study of high-dimensional manifolds is Milnor's discovery of exotic spheres (that is, smooth manifolds that are homeomorphic but not diffeomorphic to the standard sphere) [Mil56], and their classification in homotopy theoretical terms due to Kervaire and Milnor [KM63].

The goal of these lectures is to explain Milnor's construction of an exotic 7-sphere and Kervaire–Milnor's approach to the classification of these spheres in every dimension. Along the way, we will touch on several central techniques in manifold topology such as characteristic classes, bordism theory, and surgery.

This online mini-course is aimed at graduate students with interest in topology, geometry, and related areas. It consists of six one-hour lectures, one every Monday, Wednesday, and Friday at 4pm (London time), starting on September 7, 2020.

Prerequisites. Familiarity with the following topics is desirable:

- (1) **Algebraic topology.** CW-complexes, (co)homology and the cup-product, homotopy groups, fibrations and the long exact sequence in homotopy groups, the theorems of Seifert–van Kampen, Hurewicz, and Whitehead. This material is covered in most introductory textbooks to algebraic topology, for instance [Hat02] or [Bre93].
- (2) **Differential topology.** Vector bundles and general fibre bundles, smooth manifolds, the tubular neighbourhood theorem, and Whitney's embedding theorem. These topics appear for example in [Bre93] or [Hir94].

REFERENCES

- [Bre93] G.E. Bredon. *Topology and geometry*, volume 139 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1993.
- [Hat02] A. Hatcher. *Algebraic topology*. Cambridge University Press, Cambridge, 2002.
- [Hir94] M.W. Hirsch. *Differential topology*, volume 33 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1994. Corrected reprint of the 1976 original.
- [KM63] M.A. Kervaire and J.W. Milnor. Groups of homotopy spheres. I. *Ann. of Math. (2)*, 77:504–537, 1963.
- [Mil56] J. Milnor. On manifolds homeomorphic to the 7-sphere. *Ann. of Math. (2)*, 64:399–405, 1956.

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