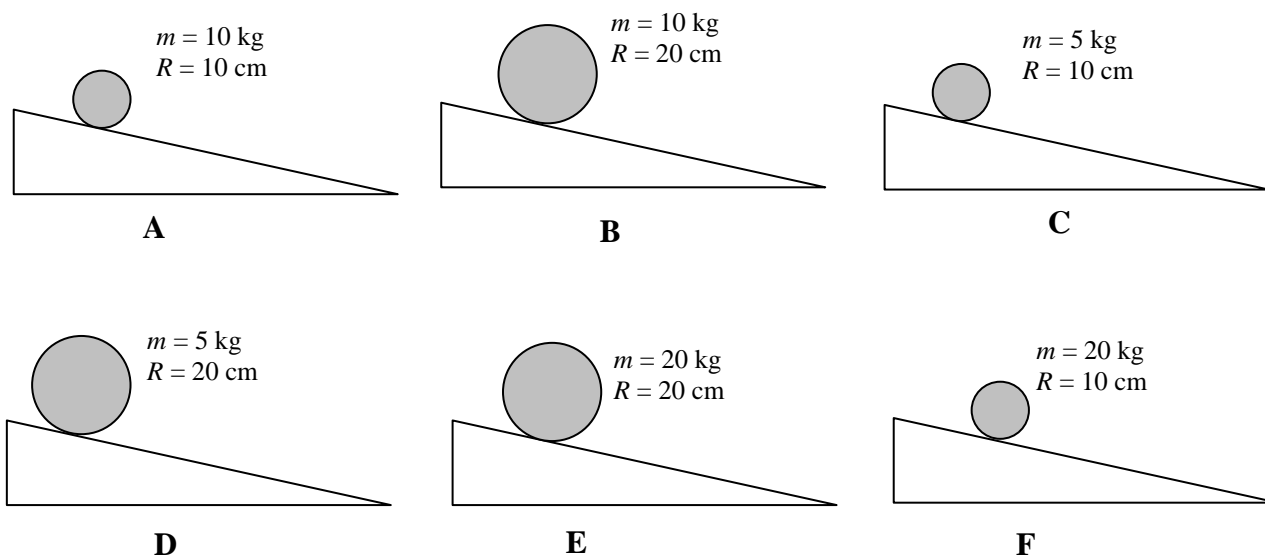# Cylinders Rolling Down Hills—Solution

Shown below are six cylinders of different materials that are rolled down the same hill. The diagrams show the masses (m) and radii (R) of the cylinders. The cylinders are all released from rest and roll without slipping the same distance down the incline.

Rank these situations from least to greatest in terms of the time it takes them to reach the bottom of the hill.

$m = 10$ kg
$R = 10$ cm

A

$m = 10$ kg
$R = 20$ cm

B

$m = 5$ kg
$R = 10$ cm

C

$m = 5$ kg
$R = 20$ cm

D

$m = 20$ kg
$R = 20$ cm

E

$m = 20$ kg
$R = 10$ cm

F

Least 1 _____ 2 _____ 3 _____ 4 _____ 5 _____ 6 _____ Greatest

Or, all of the cylinders reach the bottom at the same time. ___X___

Please carefully explain your reasoning.

$\tau = I\alpha$, and using the center of mass as the axis of rotation, the only force that creates a torque is the frictional force F_f. The moment of inertia of a cylinder is $I = \frac{1}{2}mR^2$ and $\alpha = \frac{a}{R}$. Putting this all together, we get:

$$\tau = I\alpha,$$

$$R \cdot F_f = \frac{1}{2}mR^2 \cdot \frac{a}{R},$$

$$F_f = \frac{1}{2}ma.$$

Using Newton's second law, $mg\sin\theta - F_f = ma$, so with a little bit of algebra we get $a = \frac{2}{3}g\sin\theta$. The acceleration does not depend on the mass or the radius, so all of the cylinders get to the bottom at the same time.

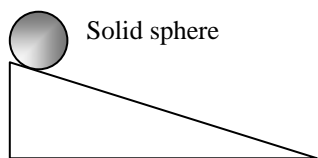How sure are you of your ranking?
Basically Guessed   Very Sure
                1   2   3   4   5   6   7   8   9   10
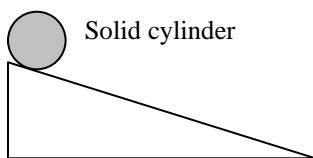
Various Objects Rolling Down a Hill: Speed at Bottom—Solution

Shown below are five objects of equal mass and radius. All five objects are released from rest and roll the same distance down the same hill without slipping.
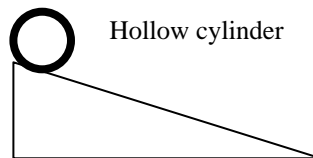
Rank the situations shown below from fastest to slowest in terms of their speed when they reach the bottom of the hill.
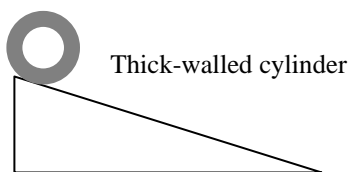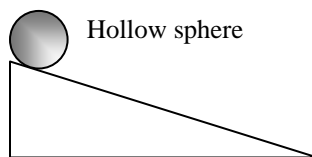
Solid sphere

Solid cylinder

Hollow cylinder

A

B

C

Thick-walled cylinder

Hollow sphere

D

E

Fastest 1 ____**A**_____ 2 _____**B**____ 3 _____**E**____ 4 ___**D**_____ 5 ____**C**_____ Slowest

Or, all of the objects are traveling at the same speed. _____

Please carefully explain your reasoning.

The object with the smallest moment of inertia will have the least rotational kinetic energy at the bottom and the most translational kinetic energy; therefore it will be moving the fastest.

How sure are you of your ranking?
Basically Guessed   Very Sure

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

Various Objects Rolling Down a Hill: Total Energy—Solution

Shown below are five objects of equal mass and radius. All five objects are released from rest and roll the same distance down the same hill without slipping.
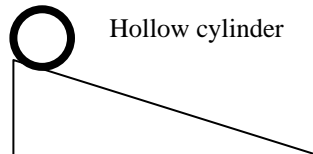
Rank the situations shown below from greatest to least in terms of their total mechanical energy when they reach the bottom of the hill.
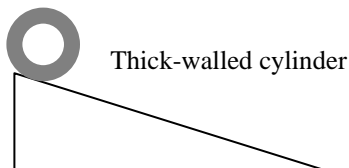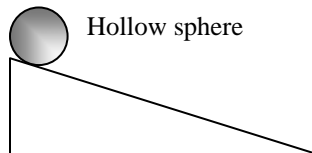
Solid sphere

A

Solid cylinder

B

Hollow cylinder

C

Thick-walled cylinder

D

Hollow sphere
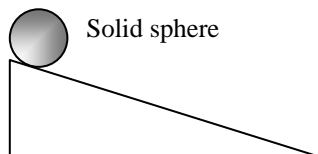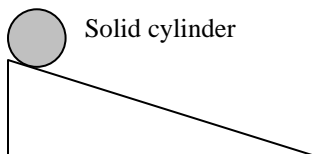
E

Greatest 1 _____ 2 _____ 3 _____ 4 _____ 5 _____ Least

Or, all of the objects have the same total energy. ___**X**_____

Please carefully explain your reasoning.

Energy is conserved, and since they all start from the same height, their initial potential energies will all be the same. This is converted to rotational and translational kinetic energies in different proportions, depending on the moment of inertia, but they will all have the same total energy at the bottom because they all started with the same potential energy.

How sure are you of your ranking?
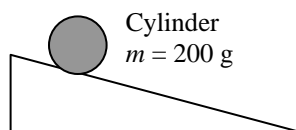Basically Guessed   Very Sure
            1   2   3   4   5   6   7   8   9   10
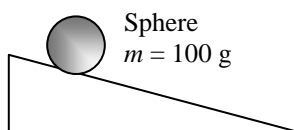
Cylinders and Spheres Rolling Down Hills I—Solution

Shown below are six objects. Some of them are cylinders, and some of them are spheres. They all have the same radius but different masses, as shown in the diagram. They are all released from rest from the same height on the hill and allowed to roll without slipping to the bottom of the hill.
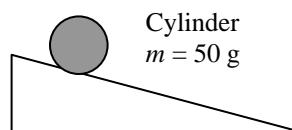
Rank these six situations from least to greatest in terms of the time it takes them to get to the bottom of the hill.
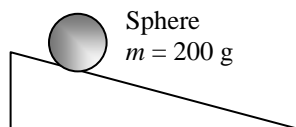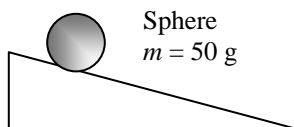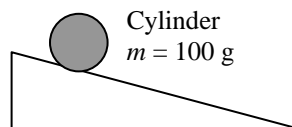
Cylinder
$m = 200$ g

Sphere
$m = 100$ g

Cylinder
$m = 50$ g

A

B

C

Sphere
$m = 200$ g

Sphere
$m = 50$ g

Cylinder
$m = 100$ g

D

E

F

Least 1 ___**BDE**_____ 2 ____**ACF**_____ 3 _____ 4 _____ 5 _____ 6 _____ Greatest

Or, all of the objects reach the bottom at the same time. _____

Please carefully explain your reasoning.

$\tau = I\alpha$, and using the center of mass as the axis of rotation, the only force that creates a torque is the frictional force F_f. The moment of inertia of a cylinder is $I = \frac{1}{2}mR^2$ and $\alpha = \frac{a}{R}$. Putting this all together, we get:

$$\tau = I\alpha,$$

$$R \cdot F_f = \frac{1}{2}mR^2 \cdot \frac{a}{R},$$

$$F_f = \frac{1}{2}ma.$$

Using Newton's second law, $mg\sin\theta - F_f = ma$, so with a little bit of algebra we get $a = \frac{2}{3}g\sin\theta$. The acceleration does not depend on the mass or the radius, so all of the cylinders get to the bottom at the same time. The moment of inertia of a sphere is $I = \frac{2}{5}mR^2$, so its corresponding acceleration is $a = \frac{5}{7}g\sin\theta$, which is slightly greater than that of the cylinders. All of the spheres get to the bottom at the same time, before all of the cylinders, independent of mass.

How sure are you of your ranking?
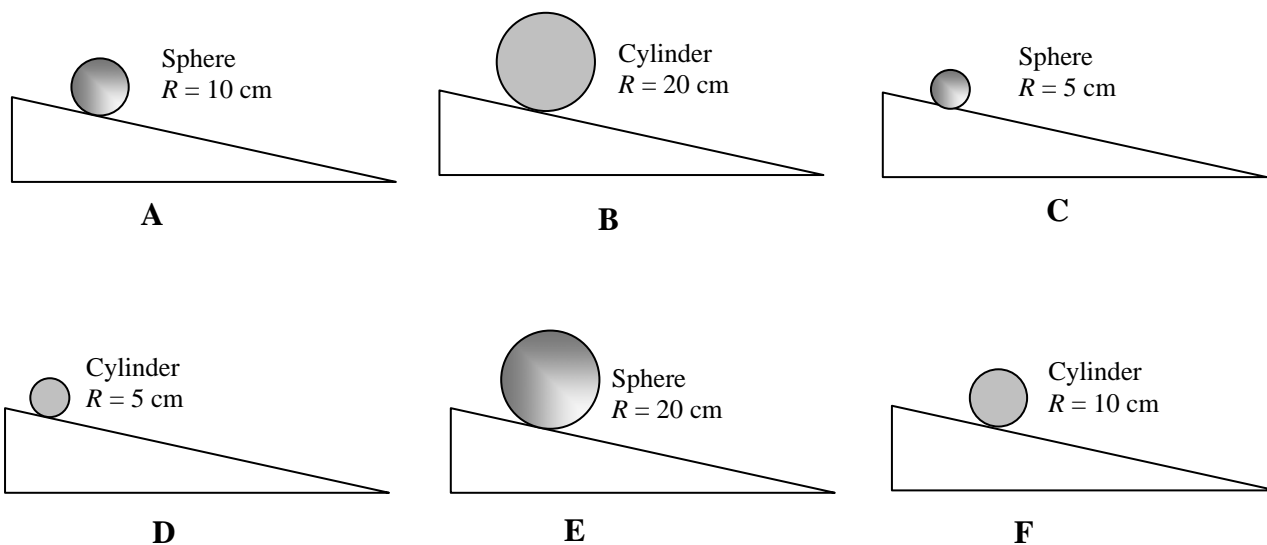Basically Guessed   Very Sure

1	2	3	4	5	6	7	8	9	10

Cylinders and Spheres Rolling Down Hills: Energy—Solution

Shown in the diagram below are six objects, either cylinders or spheres, all of the same mass but different radii. They are all released from rest from the same height on the hill and allowed to roll without slipping to the bottom of the hill.

Rank these six situations from greatest to least in terms of their total kinetic energy at the bottom of the hill.

Sphere
$R = 10$ cm

A

Cylinder
$R = 20$ cm

B

Sphere
$R = 5$ cm

C

Cylinder
$R = 5$ cm

D

Sphere
$R = 20$ cm

E

Cylinder
$R = 10$ cm

F

Greatest 1 _____ 2 _____ 3 _____ 4 _____ 5 _____ 6 _____ Least

Or, all of the objects have the same kinetic energy. _____**X**_____

Please carefully explain your reasoning.

Energy is conserved, and since they all start from the same height, their initial potential energies will all be the same. This is converted to rotational and translational kinetic energies in different proportions depending on the moment of inertia, but they will all have the same total energy at the bottom because they all started with the same potential energy.

How sure are you of your ranking?
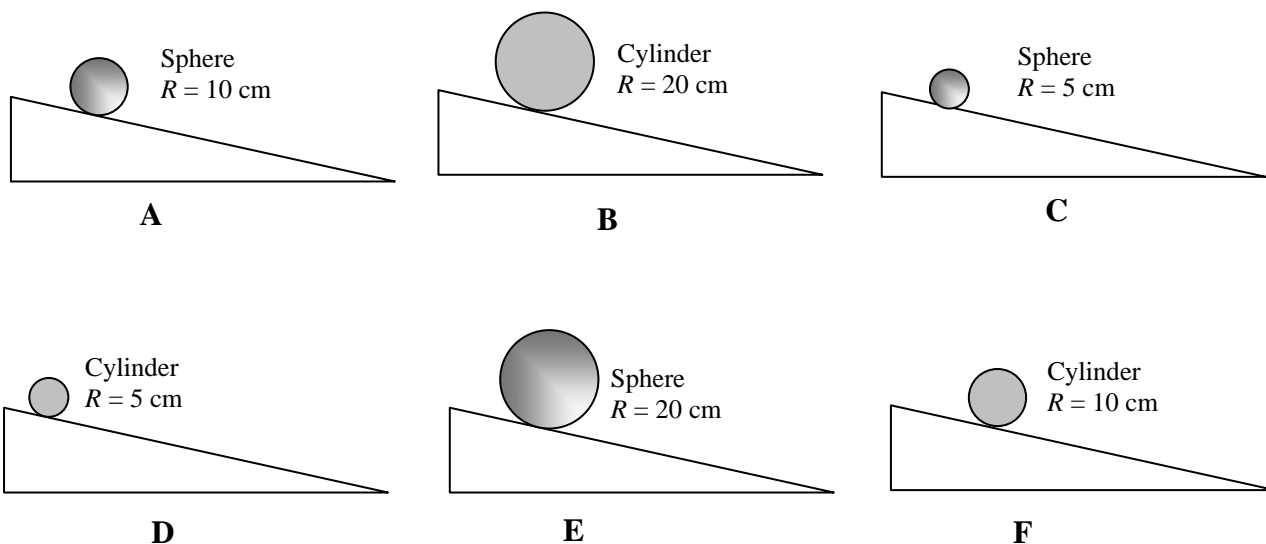Basically Guessed   Very Sure
          1   2   3   4   5   6   7   8   9   10

Cylinders and Spheres Rolling Down Hills II—Solution

Shown in the diagram below are six objects, either cylinders or spheres, all of the same mass but different radii. They are all released from rest from the same height on the hill and allowed to roll without slipping to the bottom of the hill.

Rank these six situations from least to greatest in terms of the time it takes them to roll down the hill.

Sphere
R = 10 cm

A

Cylinder
R = 20 cm

B

Sphere
R = 5 cm

C

Cylinder
R = 5 cm

D

Sphere
R = 20 cm

E

Cylinder
R = 10 cm

F

Least 1 ___**ACE**_____ 2 _____**BDF**____ 3 _____ 4 _____ 5 _____ 6 _____ Greatest

Or, all of the objects reach the bottom at the same time. _____

Please carefully explain your reasoning.

$\tau = I\alpha$, and using the center of mass as the axis of rotation, the only force that creates a torque is the frictional force F_f. The moment of inertia of a cylinder is $I = \dfrac{1}{2}mR^2$ and $\alpha = \dfrac{a}{R}$. Putting this all together, we get:

$$\tau = I\alpha, \qquad R \cdot F_f = \frac{1}{2}mR^2 \cdot \frac{a}{R}, \qquad F_f = \frac{1}{2}ma.$$

Using Newton's second law, $mg\sin\theta - F_f = ma$, so with a little bit of algebra we get $a = \dfrac{2}{3}g\sin\theta$. The acceleration does not depend on the mass or the radius, so all of the cylinders get to the bottom at the same time. The moment of inertia of a sphere is $I = \dfrac{2}{5}mR^2$, so its corresponding acceleration is $a = \dfrac{5}{7}g\sin\theta$, which is slightly greater than that of the cylinders. All of the spheres get to the bottom at the same time, before all of the cylinders, independent of radius.

How sure are you of your ranking?
Basically Guessed   Very Sure
                    1   2   3   4   5   6   7   8   9   10