TA 2: Applications of Discrete and Dynamic Choice Models

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Human Capital Accumulation: Heckman, Lochner, and Taber (1998)

Literature on inequality: partial equilibrium.

This paper:

- Develops and estimates an **overlapping generations**, **general equilibrium** model of labor earnings, skill formation and physical capital accumulation with heterogeneous human capital.
- Applies new methods for estimating the demand for unobserved **human capital** and the substitution between **skills** and **capital** in aggregate technology.
- Quantifies the mechanisms behind **increasing wage inequality** in the US.

Model

- Life-cycle maximization problem for **consumption** and **investment on human capital**.

- Discrete choice problem to determine the **education** decision at the beginning of the working life.

- **Aggregate production function** with skill-biased technical change.

Model

Life-cycle problem

The agent chooses **consumption** c and **human capital investment** g to maximize life-time utility:

$$V(h_a, b_a, e, i_t, r_{et}) \equiv \max_{c,g} \left\{ \frac{c^{1-\gamma}}{1-\gamma} + \beta V(h_{a+1}, b_{a+1}, e, i_{t+1}, r_{et+1}) \right\},$$

$$\text{s.t.} \quad b_{a+1} \leq b_a(1+i_t) + r_{et} h_a(1-g) - c.$$

On-the-job human capital accumulates as:

$$h_{a+t}(\omega, e) = \omega g^{\eta_e} h_a(\omega, e)^{\psi_e} + (1-\delta) h_a(\omega, e),$$

where η_e is the human capital investment productivity and ψ_e the productivity of the human capital endowment.

Model

Earnings

Earnings are determined by aggregate **skill prices**, individual **human capital endowments**, and individual **human capital investment decisions**:

$$w(a, t, h_a(\omega, e)) = r_{et} h_a(\omega, e)(1 - g).$$

For $a \geq a^*$, individuals no longer invest on human capital. Assume that $a \in \{1, 2, 3\}$, and $g = 0$ for $a > 1$.

Model

Education decision

The **education choice** e is made at the beginning, comparing lifetime discounted utility plus some non-pecuniary benefits with the cost of education:

$$\max_e [V^E(\omega, e, t) - \pi_e + \varepsilon_e],$$

Model

Aggregate output and skill units

Aggregate output Y_t is determined by the following nested CES technology:

$$Y_t = \left\{ \alpha K_t^{\phi} + (1-\alpha)\left[\theta_t L_{S_t}^{\rho} + (1-\theta_t) L_{U_t}^{\rho}\right]^{\frac{\phi}{\rho}} \right\}^{\frac{1}{\phi}}.$$

Skill-biased technical change, determined by the evolution of θ_t, is given by a time trend:

$$\ln\left(\frac{\theta_t}{1-\theta_t}\right) = \ln\left(\frac{\theta_0}{1-\theta_0}\right) + \varphi t.$$

The **aggregate stock of skills** is described by:

$$\frac{WageBill_{et}}{r_{et}(1-\delta)^t} = \frac{L_{et}}{(1-\delta)^t}.$$

Estimation

Estimation

Data: wages and education groups $e \in \{U, S\}$ of individuals at age a and calendar time t.

- At each time t, we observe individuals in different periods of their life a.

How to proceed:

1. Estimate skill prices r_{et}.
2. Estimate production function parameters θ_0, φ and ρ.
3. Estimate human capital function parameters η_e and ψ_e.
4. Estimate education decision parameters (not in the problem set).

Estimation

Step 1: Estimate skill prices

At old ages, $(a > 1)$, individuals no longer invest in human capital (i.e., $g = 0$). Therefore

$$w(a^* + 1, t + 1, h_{a^*+1}) \equiv r_{et+1} h_{a^*+1} = r_{et+1} h_{a^*}(1 - \delta),$$

which implies

$$\frac{w(a^* + l, t + l, h_{a^*+l})}{w(a^*, t, h_{a^*})} = \frac{r_{et+l}(1 - \delta)^l}{r_{et}}.$$

Assume $\delta = 0$ and $r_{et} = 1$ for $t = 1, ..., 4$ to recover r_{et} from the equation above.

The aggregate stocks of skill units L_{et} can be recovered from the estimated skill prices:

$$\frac{WageBill_{et}}{r_{et}(1 - \delta)^t} = \frac{L_{et}}{(1 - \delta)^t}.$$

Estimation

Step 2: Production function parameters

As in Chapter 1, the relative demands of the two labor inputs give an expression for the relative skill prices:

$$\ln\left(\frac{\partial Y_t/\partial L_{S_t}}{\partial Y_t/\partial L_{U_t}}\right) = \ln\left(\frac{r_{St}}{r_{Ut}}\right) = \ln\left(\frac{\theta_t}{1-\theta_t}\right) + (\rho-1)\ln\frac{L_{St}}{L_{Ut}}.$$

If we assume that

$$\ln\left(\frac{\theta_t}{1-\theta_t}\right) = \ln\left(\frac{\theta_0}{1-\theta_0}\right) + \varphi t,$$

we get

$$\ln\left(\frac{r_{St}}{r_{Ut}}\right) = \ln\left(\frac{\theta_0}{1-\theta_0}\right) + \varphi t + (\rho-1)\ln\left(\frac{L_{St}}{L_{Ut}}\right),$$

whose coefficients can be recovered from a linear regression.

Estimation

Step 3: Estimate human capital function parameters

Parameters to estimate: ψ_e, η_e.

1. Fix the values of β, γ (values in the literature), solve the **life-cyle problem** by backwards induction, and write h_1 as a function of g.

2. Find the optimal investment decision in each t for each education e as the **fixed-point** of $g = f(h_a, h_{a+1}, \eta_e, \psi_e)$.

3. Estimate η_e and ψ_e by **NLS**, minimizing the distance between actual and simulated wages:

$$\sum_i \sum_a \left[w_{i,a} - \widehat{w}_a\left(\eta_e, \psi_e, r_{et}, h_a(e)\right)\right]^2 .$$

Estimation

Step 3.1: Solve the life-cycle problem

- Start by the last period ($a = 3$). Remember that $g = 0$ and assume that all income is consumed ($c_3 =$ assets $+$ wage) to get V_3^*.

- Go to $a = 2$. Substitute the continuation value by V_3^*. Use the human capital production function to write h_2 in terms of h_3 in the budget constraint (again, $g = 0$).

- In V_3^*, replace b_3 by the budget constraint obtained in the previous step, so that the agent's problem in $a = 2$ is only in terms of c_2.

- Take FOC with respect to c_2 and solve it. You are supposed to obtain something like $c_2^* = Ab_2 + Bh_3$, with A and B gathering variables and parameters. Use it to write V_2^*.

Estimation

Step 3.1: Solve the life-cycle problem

- Go to $a = 1$. Substitute the continuation value by V_2^*.

- Take FOC for c_1 and g.

- After solving for c_1^*, you will need to check that $c_1^* > 0$.

- g will only appear in the continuation value. With the information of the previous step, you will realize that

$$\frac{\partial h_1}{\partial g}(1-g) - h_1 = 0$$

  for the FOC for g to hold. From this, you can obtain $h_1 = \frac{1}{1-g}$.

Estimation

Step 3.2: Fixed-point of g

The fixed-point algorithm will allow you to get the optimal g for each $e \in \{U, S\}$ and t:

- Use the human capital production function

$$h_{a+t}(\omega, e) = \omega g^{\eta_e} h_a(\omega, e)^{\psi_e} + (1-\delta) h_a(\omega, e)$$

  to write $g = f(h_a, h_{a+1}, \eta_e, \psi_e)$.

- Since h depends on g, we can implement a fixed-point algorithm for g for given values of η_e and ψ_e:

  - Make an initial guess of g.
  - Compute $h_a = \frac{1}{1-g}$, and recover h_{a+1} from the earnings equation.
  - With h_a and h_{a+1}, compute $g_{\text{new}} = f(h_a, h_{a+1}, \eta_e, \psi_e)$.
  - Iterate until convergence.

Estimation

Step 3.2: Fixed-point of g

The algorithm might fail to converge. If that is the case, add and **intermediate step**:

$$g'_{\text{new}} = \frac{mg - g_{\text{new}}}{m-1},$$

where $m < 0$.

Estimation

Step 3.3: Estimate η_e and ψ_e

The human capital parameters η_e and ψ_e can be estimated by NLS, minimizing

$$\sum_i \sum_a \left[w_{i,a} - \widehat{w}_a\left(\eta_e, \psi_e, r_{et}, h_a(e)\right)\right]^2.$$

The procedure is the following:

- Make an initial guess for η_e and ψ_e.
- Solve the fixed-point problem for g.
- Use g^* to get h_a and h_{a+1}, and simulate wages $\widehat{w}$ with the earnings equation.
- Evaluate the difference between actual and simulated wages.
- Update your guess of η_e and ψ_e and iterate until convergence.