TA 1: Production Function Estimation

Manuel V. Montesinos

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Before we start...

- **Format of TA sessions**: guideline for each problem set (with presentation/code).
- **Grading of the course**: 4 problem sets (TA) + research proposal.
- **Office hours**: upon request, at Office B3-118.
- **Contact**: manuel.montesinos@barcelonagse.eu.
- **PS answers** must be sent by email in a zipped file named `surname1_ps1`.
    - Your code should be easy to follow (comment it). I will check if it goes through.

Firm-Level Estimation

A simple Cobb-Douglas production framework

Two inputs used in production, **capital** k_{it} and **labor** l_{it}, with **total factor productivity** ζ_{it}:

$$y_{it} = \zeta_{it} k_{it}^{\alpha} l_{it}^{\beta}$$

Taking **logs** allows us to estimate α and β by **OLS**:

$$\ln y_{it} = \alpha \ln k_{it} + \beta \ln l_{it} + \nu_{it} + \epsilon_{it},$$

with $\nu_{it} = \ln \zeta_{it}$, and **measurement error** ϵ_{it} unobserved by the econometrician.

Potential biases

Simultaneity bias: the firm knows ν_{it} when choosing k_{it} and l_{it}.

Other problems:

- **Attenuation bias** because of measurement error in inputs.
- **Selection bias** because of endogenous exit of firms: only the most productive survive.

Solutions:

- Instrumental variables (e.g. input prices).
- Fixed effects.
- Dynamic panel data.
- Control function approaches: Olley and Pakes (1996) and Levinsohn and Petrin (2003).

Olley and Pakes (1996)

Control function approach: look for observables that can control for unobserved total factor productivity ν_{it}.

Modifications of demands:

$$i_{it} = F_K(k_{it}, l_{it-1}, \nu_{it}, \boldsymbol{r_{it}}) \quad \text{and} \quad l_{it} = F_L(k_{it}, l_{it-1}, \nu_{it}, \boldsymbol{r_{it}}),$$

where i_{it} is investment at time t, and $\boldsymbol{r_{it}} = (r_{it}, w_{it})'$ are factor prices.

Time-to-build: i_{it} is not productive until $t+1$ where $k_{t+1} = (1-\delta)k_{it} + i_{it}$.

Olley and Pakes (1996): method

First stage: estimate β in

$$\ln y_{it} = \beta \ln l_{it} + \phi_t(l_{it-1}, k_{it}, i_{it}) + \epsilon_{it},$$

where

$$\phi_t(l_{it-1}, k_{it}, i_{it}) \equiv \alpha \ln k_{it} + F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r_{it}}),$$

which can be estimated by kernel regression or polynomial series approximations, such as

$$F_K^{-1}(l_{it-1}, k_{it}, i_{it}, \boldsymbol{r_{it}}) = \varphi_0 l_{it-1} + \varphi_1 l_{it-1}^2 + \varphi_2 l_{it-1}^3 + \varphi_3 k_{it} + \varphi_4 k_{it}^2 + \varphi_5 k_{it}^3 + \varphi_6 i_{it}.$$

Second stage: estimate α using

$$\hat{\phi}_{it} = \alpha \ln k_{it} + h\left(\hat{\phi}_{it-1} - \alpha \ln k_{it-1}\right) + \xi_{it},$$

with

$$\hat{\phi}_{it} \equiv \ln y_{it} - \hat{\beta} \ln l_{it}.$$

Olley and Pakes (1996): method

$$\hat{\phi}_{it} = \alpha \ln k_{it} + h\left(\hat{\phi}_{it-1} - \alpha \ln k_{it-1}\right) + \xi_{it},$$

The argument of $h(\cdot)$ depends on α (unknown) $\rightarrow$ **Recursive semiparametric method**:

1. Set an initial guess $\alpha^{(0)}$.
2. Compute: $\hat{\phi}_{it-1} - \alpha^{(0)} \ln k_{it-1}$.
3. Update α as the coefficient of $\ln k_{it}$ in the semi-parametric regression above.
4. Iterate until convergence.

Aggregate Production Functions

Nested CES

Convenient way to estimate elasticities of substitution across inputs (or allow for imperfect substitutability across them). In

$$Y = A\left[\alpha K^{\rho} + (1-\alpha)L^{\rho}\right]^{1/\rho}$$

- ρ is the **substitution parameter**.
- $\sigma = 1/(1-\rho)$ is the **elasticity of substitution**, which measures the per cent change in the capital-labor ratio in response to a 1 per cent change in their prices in a competitive market.

Two main advantages:

- **Log-linear** relation between relative prices and relative inputs.
- The elasticity of substitution between two inputs **inside one nest** can be estimated without information on the inputs or parameters in the **nests that lie above**.

Example: Borjas (2003)

Research question: what is the labor market impact of immigration?

Previous studies exploit the geographic clustering of immigrants and compare native employment opportunities across regions.

- The estimated impact of immigration on native wages is around **zero** (inconsistent with the textbook model).
- **Problem**: they ignore that economic conditions tend to equalize across cities and regions.

Example: Borjas (2003)

Paying attention to **skill groups** (education and work experience) can help: similarly educated workers who have different levels of experience are not perfect substitutes.

This paper exploits variation in **supply shifts across education-experience groups** to estimate the effect of immigration on native wages.

- Immigration is not balanced evenly across all experience cells within education groups, and this supply imbalance changes over time.

Results: immigration has indeed harmed the employment opportunities of competing native workers.

Nested CES

Consider the production function

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha},$$

with

$$L_t \equiv \left[\sum_i \theta_{it} L_{it}^{\rho}\right]^{\frac{1}{\rho}}, \quad L_{it} \equiv \left[\sum_j \gamma_{ij} L_{ijt}^{\eta}\right]^{\frac{1}{\eta}}, \quad \text{and} \quad L_{ijt} \equiv \left[\lambda L_{ijNt}^{\phi} + (1-\lambda) L_{ijMt}^{\phi}\right]^{\frac{1}{\phi}}.$$

- Assume $\alpha = 0.3$.
- i as index for education groups.
- j for experience groups.
- M for immigrants and N for natives respectively.

Nested CES: estimation

Sequential estimation based on **first order conditions**:

- **Lowest level**: estimate λ and ϕ using the relative wages and labor of natives and immigrants:

$$\ln \frac{w_{ijMt}}{w_{ijNt}} = \ln\left(\frac{1-\lambda}{\lambda}\right) + (\phi - 1)\ln \frac{L_{ijMt}}{L_{ijNt}}.$$

- Construct **next level aggregates** L_{ijt} and **estimate** γ_{ij} and η:

$$\ln w_{ijt} = \kappa_t + \pi_{it} + \ln \gamma_{ij} + (\eta - 1)\ln L_{ijt}.$$

Impose $\sum_j \gamma_{ij} = 1$ for every education group i to get γ_{ij}. The education-experience fixed effects $\ln \hat{\gamma}_{ij}$ can be normalized using the logistic function:

$$\gamma_{ij} = \frac{\exp(\ln \hat{\gamma}_{ij})}{\sum_j \exp(\ln \hat{\gamma}_{ij})}.$$

Nested CES: estimation

- Construct L_{it} and estimate θ_{it} and ρ:

$$\ln w_{it} = \kappa_t + \ln \theta_{it} + (\rho - 1) \ln L_{it}$$

  The education-time fixed effects $\ln \hat{\theta}_{it}$ also have to be normalized for $\sum_i \theta_{it} = 1$.

- With the estimated θ_{it} and ρ, L_t can be constructed.

References

Levinsohn, James and Amil Petrin, "Estimating Production Functions Using Inputs to Control for Unobservables," *Review of Economic Studies*, April 2003, *70* (2), 317–341.

Olley, G. Steven and Ariel Pakes, "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica*, November 1996, *64* (6), 1263.