Control and Simulation for a Two Link System.

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1 Introduction

Let us start with the position of M_1 and M_2 We know that these positions can be split to they're x and y components so for M1 we get

$$x_1 = l_1 \cos\theta_1$$

$$y_1 = l_1 \sin\theta_1$$

For M_2, we can form its the x and y component of both l_1 and l_2

$$x_2 = l_1 \cos\theta_1 + l_2 \cos\theta_2$$

$$y_2 = l_1 \sin\theta_1 + l_2 \sin\theta_2$$

Take the derivative for the velocity with respect to time for M_1.

$$\dot{x_1}\,(t) = -l_1\dot{\theta_1}\sin\theta_1$$

$$\dot{y_1}\,(t) = l_1\dot{\theta_1}\cos\theta_1$$

Note: $\dot{\theta}_i = \frac{d\theta}{dt}$ and $\dot{x_i}(t) = \frac{dx_i(t)}{dt}$ where $i = 1, 2$

Take the derivative for the velocity with respect to time for M_2.

$$\dot{x_2}\,(t) = -l_1\dot{\theta_1}\sin\theta_1 - l_2\dot{\theta_2}\sin\theta_2$$

$$\dot{y_2}\,(t) = l_1\dot{\theta_1}\cos\theta_1 + l_2\dot{\theta_2}\cos\theta_2$$

Notice how incorporating two different angles changing over time complicates the creation of state space equations. This complexity arises because each angle influences the system in a unique way, leading to interdependent dynamics between the components.

The Lagrangian method is essential as it allows for a unified approach to model these interactions within a single framework.

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = 0$$

$$\mathcal{L} = KE - PE$$

By capturing both kinetic and potential energies of the system, the Lagrangian formulation seamlessly integrates the effects of all varying parameters, such as the angles θ_1 and θ_2, into a comprehensive model.

This is particularly crucial for systems like a double pendulum, where the position and velocity of the second mass, M_2, are affected by the motion of both links.