# Control Theory

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October 2024

## 1 Introduction

Time-invariant system: A and B are constants, not dependent of time. Time-invariant means that the rules don't change over time

$$\dot{x}(t) = Ax(t) + Bu(t)$$

Because we are dealing with Time Invariant Process, we will have the dynamic equations:

$$\dot{x} = Ax + Bu$$

Where $A$ is the state matrix and $B$ is the input matrix, defined as follows:

$$A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & \dots & b_{1m} \\ \vdots & \ddots & \vdots \\ b_{n1} & \dots & b_{nm} \end{bmatrix}$$

Where

$$\dot{x}_1 = \frac{dx_1}{dt} = a_{11}(t)x_1 + ... + a_{1k}(t)x_k + b_{11}(t)u_1 + ... + b_{11}(t)u_1$$

$$\dot{x}_2 = \frac{dx_2}{dt} = a_{21}(t)x_1 + ... + a_{2k}(t)x_k + b_{21}(t)u_1 + ... + b_{21}(t)u_1$$

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$$\dot{x}_n = \frac{dx_n}{dt} = a_{n1}(t)x_1 + ... + a_{nk}(t)x_k + b_{n1}(t)u_1 + ... + b_{n1}(t)u_1$$

Note that A(t) is always a square where B(t) is not always.

Sometimes knowing the state of the system is unimportant and we need to analyze the output where $y(t)$ is the output vector

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \cdot \\ \cdot \\ \cdot \\ y_m(t) \end{bmatrix}$$

Output vector is assumed to be a linear combination of the state and the input

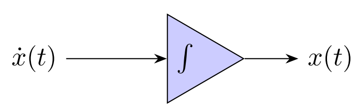
$$y(t) = C(t)x(t) + D(t)u(t)$$

where $C(t)$ and $D(t)$ are constants if the system is time-invariant.

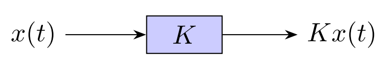
The output vector is sometimes called the 'observation' vector because it is a vector where the outputs can be measured through suitable sensors. The observation vector contains observation equations.

# 2 Block Diagrams

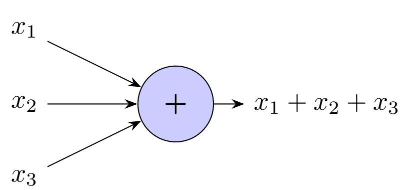
Integrator




Gain




Summer




# 3 Lagrangian Equations

Provide a brief overview or derivation of Lagrangian mechanics as applied to control systems.

## 4 Frequency Domain Analysis

Analyzing systems in the frequency domain helps understand how systems respond to sinusoidal inputs at different frequencies. This approach is crucial for understanding the behavior of systems like filters or oscillators.