Laboratory 4 - 07/11/16

Consider the following problem in the unknown $y(t, x)$:

$$\begin{cases} \dfrac{\partial y}{\partial t} + a\dfrac{\partial y}{\partial x} = 0, \quad -3 < x \leq 3, \ 0 < t \leq 2 \\ y(t, -3) = 0 \\ y(0, x) = y_0(x) = \begin{cases} \cos^4(\pi x), \quad \text{if } -0.5 \leq x \leq 0.5 \\ 0 \quad \text{otherwise,} \end{cases} \end{cases} \tag{1}$$

with $a > 0$, that has exact solution

$$y(x, t) = y_0(x - at),$$

obtained with the characteristic method.

Exercise 1

Implement the Upwind (UW) method for the solution of problem (1).

Exercise 2

The UW and the Lax-Wendroff (LW) methods can be interpreted as centered schemes with the addition of numerical viscosity. Write a Matlab function that implements both methods by taking the proper numerical viscosity as input parameter.

Exercise 3

Solve problem (1) with the UW and LW schemes implemented in the previous exercises with $a = 1$, $h = 0.1$, $\Delta t = 0.05$ and compare the numerical solutions with the exact one at different time steps. What happens by decreasing h and/or increasing Δt?