Laboratory 9 - 19/12/16 - Solution

Exercise

1. For the implementation of the FE/C method, see file `FEheatEquation.m`.

2. By using the FE/C method to solve the proposed problem with $\alpha = 3/4$, we find the result plotted in Figure 1 (left).

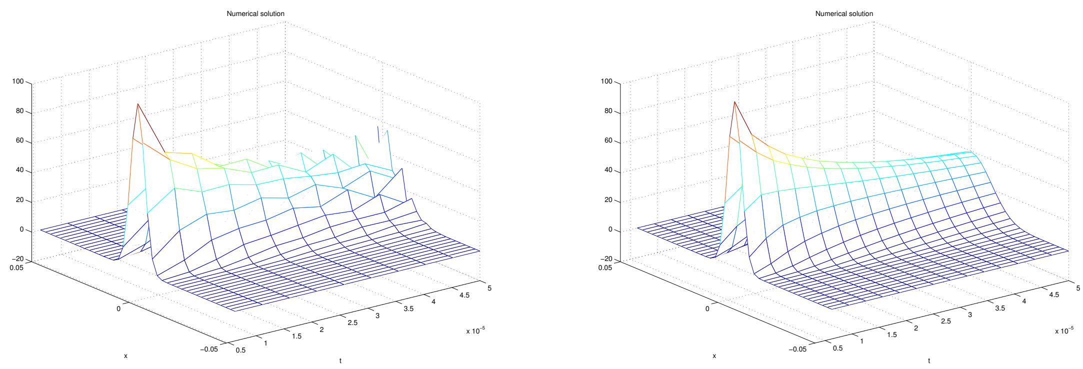

Figure 1: Numerical solution obtained with $\alpha = 3/4$ (left) and $\alpha = 1/2$ (right) by using the FE/C method.

As we can see from the result, for this particular choice of α, the FE/C method is unstable. This is due to the property of the FE/C method, that is conditionally stable. In particular, the FE/C method is stable under a condition like $\Delta t \leq Ch^2$, where C is a suitable constant.

By reducing the value of α, we expect to find a value such that the FE/C method is stable. Indeed, for $\alpha = 1/2$, the solution obtained with the FE/C method is stable, see Figure 1 (right). This is in accordance with the theoretical estimate $C = 1/(2\mu)$ ($\mu = 1$ in our case).

3. With the choice of $h = \overline{h} = 2L/M$ and $\Delta t = \alpha\overline{h}^2$, $\alpha = \frac{1}{2}$, the FE/C method produces a stable solution, indeed the condition $\Delta t \leq \frac{1}{2}h^2$ is fulfilled. By halving both the spatial

and time step, i.e., $h = \overline{h}/2$ and $\Delta t = \alpha \overline{h}^2/2$, this condition is no more satisfied and we obtain an unstable numerical solution (see Figure 2 (left)). In fact, the condition requires that Δt has to scale quadratically with respect to h in order to maintain it fulfilled (see Figure 2 (right)). For this reason, the FE/C method can be very expensive from the computational point of view.

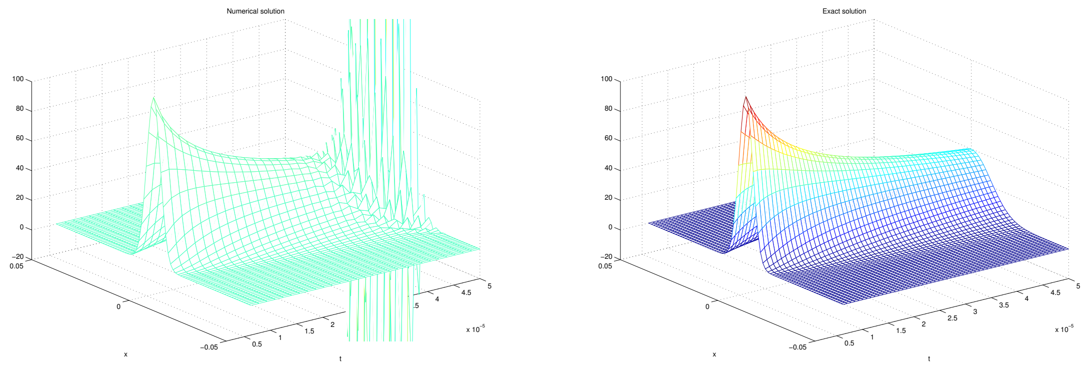

Figure 2: Numerical solution obtained with $\Delta t = \alpha \overline{h}^2/2, h = \overline{h}/2$ (left) and $\Delta t = \alpha \overline{h}^2/4, h = \overline{h}/2$ (right) by using the FE/C method.