

Laboratory 7 - 05/12/16

Advection-diffusion problem

Consider the one-dimensional advection-diffusion equation:

$$\begin{cases} -\mu y'' + ay' = f(x), & x \in (\alpha, \beta), \\ y(\alpha) = y_\alpha, \\ y(\beta) = y_\beta, \end{cases} \quad (1)$$

with $\mu = 10^{-2}$ and $a = 1$. If $\alpha = 0$, $\beta = 1$, $y_\alpha = 0$, $y_\beta = 1$ and $f(x) = 0$, then the exact solution is given by

$$y(x) = \frac{e^{\frac{ax}{\mu}} - 1}{e^{\frac{a}{\mu}} - 1}.$$

- 1) Discretize problem (1) by using a second order centered finite difference scheme. Represent the exact solution and the numerical one obtained with a uniform grid with spatial step $h = 0.1$.
- 2) Repeat the previous exercise by using a value of the spatial step h such that the centered scheme is stable. Represent the exact solution and the numerical one.