Laboratory 9 - 19/12/16

Consider the following problem:

$$\begin{cases} \dfrac{\partial y}{\partial t} - \dfrac{\partial^2 y}{\partial x^2} = 0, \quad -L < x < L,\, t_0 < t \leq T, \\ y(-L,t) = y(L,t) = 0, \quad t_0 < t \leq T, \\ y(x,t_0) = \Gamma(x,t_0), \quad -L < x < L, \end{cases} \tag{1}$$

where

$$\Gamma(x,t) = \frac{1}{\sqrt{4\pi t}} e^{-\frac{x^2}{4t}}$$

is the fundamental solution of the heat equation. The solution of the proposed problem is essentially equivalent to Γ provided that the size of the domain is large enough to guarantee that the boundary conditions do not interfere with the region where $y >> 0$.

Exercise

Let $L = 0.05$ and $T = 5 \cdot 10^{-5}$. Consider a uniform discretization of the interval $(-L, L)$, with $M = 40$, $h = 2L/M$ and $\Delta t = \alpha\, h^2$, and with $t_0 = h^2$. The approximation of the proposed problem through the forward Euler method in combination with a centered scheme for the space discretization (FE/C method) requires the computation of the sequence of vectors $\boldsymbol{U}^{n+1} \in \mathbb{R}^{M-1}$, such that

$$\boldsymbol{U}^{n+1} = (I - \Delta t A)\boldsymbol{U}^n,$$

where $A \in \mathbb{R}^{(M-1)\times(M-1)}$ represents the discretization of the second order spatial derivative.

1. Implement the FE/C method in Matlab.
2. Use such method to approximate the solution of the proposed problem with $\alpha = 3/4$ and $\alpha = 1/2$. Compare the numerical solutions with the exact one and interpret the results.
3. By fixing $\alpha = 1/2$, compute the numerical solution with the FE/C method by halving both h and Δt and interpret the result.