Laboratory 3 - 24/10/16

Consider the following problem in the unknown $y(t, x)$:

$$\begin{cases} \dfrac{\partial y}{\partial t} + a\dfrac{\partial y}{\partial x} = 0, \quad -3 < x \leq 3,\ 0 < t \leq 4.5 \\ y(t, -3) = 0 \\ y(0, x) = y_0(x) = \begin{cases} \cos^2(\pi x), & \text{if } -0.5 \leq x \leq 0.5 \\ 0 & \text{otherwise,} \end{cases} \end{cases} \tag{1}$$

with $a > 0$, that has exact solution

$$y(x, t) = y_0(x - at),$$

obtained with the characteristic method.

Exercise 1

Implement the Matlab function for the forward Euler/centered method (FE/C).

Exercise 2

Solve problem (1) with the FE/C method by choosing $a = 0.5$, $h = 0.1$ and $\Delta t = 0.1$ and compare the numerical solution with the exact one at different time steps. What happens by decreasing h and/or Δt?