Laboratory 5 - 14/11/16

Consider the following problem in the unknown $y(t, x)$:

$$\begin{cases} \dfrac{\partial y}{\partial t} + a\dfrac{\partial y}{\partial x} = 0, \quad -3 < x \leq 3,\ 0 < t \leq 2 \\ y(t, -3) = y_{inflow} \\ y(0, x) = y_0(x) = \begin{cases} \cos^4(\pi x), \quad \text{if } -0.5 \leq x \leq 0.5 \\ 0 \quad \text{otherwise,} \end{cases} \end{cases}$$

with $a > 0$ and $y_{inflow} = 0$, that has exact solution

$$y(x, t) = y_0(x - at),$$

obtained with the characteristic method.

Exercise 1

Implement the Matlab function for the backward Euler/centered scheme (BE/C). By fixing $a = 1$, the space step $h = 0.1$ and by choosing Δt such that the CFL number is equal to 0.5, solve the proposed problem with this scheme and compare the numerical solution with the exact one at different time frames. What happens for $CFL = 2$?

Exercise 2 (homework)

Define the error as $E := \max_n \max_j |y(t^n, x_j) - U_j^n|$, where x_j and t^n are the spatial and temporal nodes, respectively, and $y(t^n, x_j)$ and U_j^n are the exact solution and the numerical one at time t^n and at node x_j. Prove numerically the order of accuracy of the Upwind and Lax-Wendroff schemes by considering six successive halvings of h and Δt starting from $h = 0.1$ and setting the CFL number equal to 0.5.