Laboratory 8 - 12/12/16

Advection-diffusion problem

Exercise 1

Consider the one-dimensional advection-diffusion equation:

$$\begin{cases} -\mu y'' + a y' = f(x), & x \in (\alpha, \beta), \\ y(\alpha) = y_\alpha, \\ y(\beta) = y_\beta, \end{cases} \tag{1}$$

with $\mu = 10^{-2}$ and $a = 1$. If $\alpha = 0$, $\beta = 1$, $y_\alpha = 0$, $y_\beta = 1$ and $f(x) = 0$, then the exact solution is given by

$$y(x) = \frac{e^{\frac{ax}{\mu}} - 1}{e^{\frac{a}{\mu}} - 1}.$$

a) Discretize problem (1) by using the Upwind scheme for the approximation of the first derivative. Represent the exact solution and the numerical one obtained with a uniform grid with spatial step $h = 0.1$.

b) Implement a centered scheme with a generic stabilization for the resolution of problem (1).

Exercise 2

Estimate the order of accuracy of the centered and Upwind schemes by computing the error in the infinity norm $\|e_h\| := max_i |y(x_i) - U_i|$, with x_i the generic node and U_i the corresponding numerical solution, for $h = 0.002$, 0.001, 0.0005 and 0.00025.