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> # This program calculates the source term Q for the 2D Navier-
  Stokes equations -
> # Energy e
>
> restart;
> #with(CodeGeneration):
> #with(Student[VectorCalculus]):
> #SetCoordinates('cartesian'[x,y,z]):
> alias(rho = rho(x,y,z,t)): alias(p_an = p_an(x,y,z,t)):
> alias(u = u(x,y,z,t)): alias(u_an = u_an(x,y,z,t)):
> alias(v = v(x,y,z,t)): alias(v_an = v_an(x,y,z,t)):
> alias(w = w(x,y,z,t)): alias(w_an = w_an(x,y,z,t)):
> alias(p = p(x,y,z,t)): alias(p_an = p_an(x,y,z,t)):
> alias(t_xx = tau_xx(x,y,z,t)): alias(t_xy = tau_xy(x,y,z,t)): alias(t_xz = tau_xz(x,y,
  z,t)):
> alias(t_yy = tau_yy(x,y,z,t)): alias(t_yx = tau_yx(x,y,z,t)): alias(t_yz = tau_yz(x,y,
  z,t)):
> alias(t_zz = tau_zz(x,y,z,t)): alias(t_zx = tau_zx(x,y,z,t)): alias(t_zy = tau_zy(x,y,
  z,t)):
> alias(T = T(x,y,z,t)):
> alias(Q = Q(x,y,z,t)):

>
>
>
> #2D Navier-Stokes equation - ENERGY:
> #Diff(rho·e_t, t) + Diff(rho·u·e_t + p·u - u·tau_xx - v·tau_xy + q_x, x) + Diff(rho·v
  ·e_t + p·v - u·tau_xy - v·tau_yy + q_y, y);
> Diff(rho·e_t, t) + Diff(rho·u·e_t + p·u - u·tau_xx - v·tau_xy - w·tau_xz + q_x, x)
  + Diff(rho·v·e_t + p·v - u·tau_xy - v·tau_yy - w·tau_yz + q_y, y) + Diff(rho·w·e_t
  + p·w - u·tau_xz - v·tau_zy - w·tau_zz + q_z, z);

$$\frac{\partial}{\partial t} (\rho e_t) + \frac{\partial}{\partial x} (\rho u e_t + p u - u \tau_{xx} - v \tau_{xy} - w \tau_{xz} + q_x) + \frac{\partial}{\partial y} (\rho v e_t + p v - u \tau_{xy} - v \tau_{yy} - w \tau_{yz} + q_y) + \frac{\partial}{\partial z} (\rho w e_t + p w - u \tau_{xz} - v \tau_{zy} - w \tau_{zz} + q_z) \quad (1)$$

>
>
> #Auxiliary relations for energy:
> #p=rho·R·T;

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> T :=  $\frac{p}{\rho R}$  :
> e :=  $\frac{1}{\text{gamma} - 1} R \cdot T$ :
> e_t := e +  $\frac{(u^2 + v^2 + w^2)}{2}$  :
>  $\tau_{xx} := \frac{2}{3} \cdot \mu \left( 2 \left( \frac{\partial}{\partial x} u \right) - \left( \frac{\partial}{\partial y} v \right) - \left( \frac{\partial}{\partial z} w \right) \right)$  (2)

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>  $\tau_{xy} := \mu \left( \text{diff}(u, y) + \text{diff}(v, x) \right)$  :
>  $\tau_{xz} := \mu \left( \text{diff}(u, z) + \text{diff}(w, x) \right)$  :

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>  $\tau_{yy} := \frac{2}{3} \cdot \mu \left( 2 \left( \frac{\partial}{\partial y} v \right) - \left( \frac{\partial}{\partial x} u \right) - \left( \frac{\partial}{\partial z} w \right) \right)$  (3)

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>  $\tau_{yx} := \tau_{xy}$ :
>  $\tau_{yz} := \mu \left( \text{diff}(w, y) + \text{diff}(v, z) \right)$  :
>  $\tau_{zz} := \frac{2}{3} \cdot \mu \left( 2 \left( \text{diff}(w, z) - \text{diff}(u, x) - \text{diff}(v, y) \right) \right)$  :
>  $\tau_{zy} := \tau_{yz}$ :
>  $\tau_{zx} := \tau_{xz}$ :

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>  $q_x := -k \text{diff}(T, x)$  :
>  $q_y := -k \text{diff}(T, y)$  :
>  $q_z := -k \text{diff}(T, z)$  :
>  $\text{Diff}(\rho \cdot e_t, t) + \text{Diff}(\rho \cdot u \cdot e_t + p \cdot u - u \cdot \tau_{xx} - v \cdot \tau_{xy} - w \cdot \tau_{xz} + q_x, x)$ 
  +  $\text{Diff}(\rho \cdot v \cdot e_t + p \cdot v - u \cdot \tau_{xy} - v \cdot \tau_{yy} - w \cdot \tau_{yz} + q_y, y)$  +  $\text{Diff}(\rho \cdot w \cdot e_t$ 
  +  $p \cdot w - u \cdot \tau_{xz} - v \cdot \tau_{zy} - w \cdot \tau_{zz} + q_z, z)$  ;

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$$\frac{\partial}{\partial t} \left( \rho \left( \frac{p}{(\gamma-1)\rho} + \frac{1}{2} u^2 + \frac{1}{2} v^2 + \frac{1}{2} w^2 \right) \right) + \frac{\partial}{\partial x} \left( \rho u \left( \frac{p}{(\gamma-1)\rho} + \frac{1}{2} u^2 + \frac{1}{2} v^2 \right. \right. \quad (4)$$

$$\left. \left. + \frac{1}{2} w^2 \right) + p u - \frac{2}{3} u \mu \left( 2 \left( \frac{\partial}{\partial x} u \right) - \left( \frac{\partial}{\partial y} v \right) - \left( \frac{\partial}{\partial z} w \right) \right) - v \mu \left( \frac{\partial}{\partial y} u + \frac{\partial}{\partial x} v \right) \right.$$

$$\left. - w \mu \left( \frac{\partial}{\partial z} u + \frac{\partial}{\partial x} w \right) - k \left( \frac{\frac{\partial}{\partial x} p}{\rho R} - \frac{p \left( \frac{\partial}{\partial x} \rho \right)}{\rho^2 R} \right) \right) + \frac{\partial}{\partial y} \left( \rho v \left( \frac{p}{(\gamma-1)\rho} \right. \right.$$

$$\begin{aligned}
& + \frac{1}{2} u^2 + \frac{1}{2} v^2 + \frac{1}{2} w^2 \Big) + p v - u \mu \left( \frac{\partial}{\partial y} u + \frac{\partial}{\partial x} v \right) - \frac{2}{3} v \mu \left( 2 \left( \frac{\partial}{\partial y} v \right) \right. \\
& \left. - \left( \frac{\partial}{\partial x} u \right) - \left( \frac{\partial}{\partial z} w \right) \right) - w \mu \left( \frac{\partial}{\partial y} w + \frac{\partial}{\partial z} v \right) - k \left( \frac{\frac{\partial}{\partial y} p}{\rho R} - \frac{p \left( \frac{\partial}{\partial y} \rho \right)}{\rho^2 R} \right) \\
& + \frac{\partial}{\partial z} \left( \rho w \left( \frac{p}{(\gamma-1) \rho} + \frac{1}{2} u^2 + \frac{1}{2} v^2 + \frac{1}{2} w^2 \right) + p w - u \mu \left( \frac{\partial}{\partial z} u + \frac{\partial}{\partial x} w \right) \right. \\
& \left. - v \mu \left( \frac{\partial}{\partial y} w + \frac{\partial}{\partial z} v \right) - \frac{2}{3} w \mu \left( 2 \left( \frac{\partial}{\partial z} w \right) - \left( \frac{\partial}{\partial x} u \right) - \left( \frac{\partial}{\partial y} v \right) \right) - k \left( \frac{\frac{\partial}{\partial z} p}{\rho R} \right. \right. \\
& \left. \left. - \frac{p \left( \frac{\partial}{\partial z} \rho \right)}{\rho^2 R} \right) \right)
\end{aligned}$$

> # Energy equation, written as differential operator:

$$\begin{aligned}
> L1 := & \text{diff}(\text{rho} \cdot e_t, t) + \text{diff}(\text{rho} \cdot u \cdot e_t + p \cdot u - u \cdot \tau_{xx} - v \cdot \tau_{xy} - w \cdot \tau_{xz} + q_x, x) \\
& + \text{diff}(\text{rho} \cdot v \cdot e_t + p \cdot v - u \cdot \tau_{xy} - v \cdot \tau_{yy} - w \cdot \tau_{yz} + q_y, y) + \text{diff}(\text{rho} \cdot w \cdot e_t \\
& + p \cdot w - u \cdot \tau_{xz} - v \cdot \tau_{zy} - w \cdot \tau_{zz} + q_z, z);
\end{aligned}$$

$$\begin{aligned}
L1 := & -w \mu \left( \frac{\partial^2}{\partial y^2} w + \frac{\partial^2}{\partial z \partial y} v \right) - u \mu \left( \frac{\partial^2}{\partial z^2} u + \frac{\partial^2}{\partial z \partial x} w \right) - \frac{2}{3} \left( \frac{\partial}{\partial x} u \right) \mu \left( 2 \left( \frac{\partial}{\partial x} u \right) \right. \\
& \left. - \left( \frac{\partial}{\partial y} v \right) - \left( \frac{\partial}{\partial z} w \right) \right) - \left( \frac{\partial}{\partial x} v \right) \mu \left( \frac{\partial}{\partial y} u + \frac{\partial}{\partial x} v \right) + \rho \left( \frac{\partial}{\partial x} u \right) \left( \frac{p}{(\gamma-1) \rho} \right. \\
& \left. + \frac{1}{2} u^2 + \frac{1}{2} v^2 + \frac{1}{2} w^2 \right) + \left( \frac{\partial}{\partial x} \rho \right) u \left( \frac{p}{(\gamma-1) \rho} + \frac{1}{2} u^2 + \frac{1}{2} v^2 + \frac{1}{2} w^2 \right) \\
& + \rho u \left( \frac{\frac{\partial}{\partial x} p}{(\gamma-1) \rho} - \frac{p \left( \frac{\partial}{\partial x} \rho \right)}{(\gamma-1) \rho^2} + u \left( \frac{\partial}{\partial x} u \right) + v \left( \frac{\partial}{\partial x} v \right) + w \left( \frac{\partial}{\partial x} w \right) \right) \\
& - \left( \frac{\partial}{\partial x} w \right) \mu \left( \frac{\partial}{\partial z} u + \frac{\partial}{\partial x} w \right) + \left( \frac{\partial}{\partial y} \rho \right) v \left( \frac{p}{(\gamma-1) \rho} + \frac{1}{2} u^2 + \frac{1}{2} v^2 + \frac{1}{2} w^2 \right) \\
& + \rho \left( \frac{\partial}{\partial y} v \right) \left( \frac{p}{(\gamma-1) \rho} + \frac{1}{2} u^2 + \frac{1}{2} v^2 + \frac{1}{2} w^2 \right) + \rho v \left( \frac{\frac{\partial}{\partial y} p}{(\gamma-1) \rho} - \frac{p \left( \frac{\partial}{\partial y} \rho \right)}{(\gamma-1) \rho^2} \right. \\
& \left. + u \left( \frac{\partial}{\partial y} u \right) + v \left( \frac{\partial}{\partial y} v \right) + w \left( \frac{\partial}{\partial y} w \right) \right) - \left( \frac{\partial}{\partial y} u \right) \mu \left( \frac{\partial}{\partial y} u + \frac{\partial}{\partial x} v \right) \\
& - \frac{2}{3} \left( \frac{\partial}{\partial y} v \right) \mu \left( 2 \left( \frac{\partial}{\partial y} v \right) - \left( \frac{\partial}{\partial x} u \right) - \left( \frac{\partial}{\partial z} w \right) \right) - \left( \frac{\partial}{\partial y} w \right) \mu \left( \frac{\partial}{\partial y} w + \frac{\partial}{\partial z} v \right)
\end{aligned} \tag{5}$$

$$\begin{aligned}
& + \left( \frac{\partial}{\partial z} \rho \right) w \left( \frac{p}{(\gamma-1) \rho} + \frac{1}{2} u^2 + \frac{1}{2} v^2 + \frac{1}{2} w^2 \right) + \rho \left( \frac{\partial}{\partial z} w \right) \left( \frac{p}{(\gamma-1) \rho} \right. \\
& \left. + \frac{1}{2} u^2 + \frac{1}{2} v^2 + \frac{1}{2} w^2 \right) + \rho w \left( \frac{\frac{\partial}{\partial z} p}{(\gamma-1) \rho} - \frac{p \left( \frac{\partial}{\partial z} \rho \right)}{(\gamma-1) \rho^2} + u \left( \frac{\partial}{\partial z} u \right) \right. \\
& \left. + v \left( \frac{\partial}{\partial z} v \right) + w \left( \frac{\partial}{\partial z} w \right) \right) - \left( \frac{\partial}{\partial z} u \right) \mu \left( \frac{\partial}{\partial z} u + \frac{\partial}{\partial x} w \right) - \left( \frac{\partial}{\partial z} v \right) \mu \left( \frac{\partial}{\partial y} w \right. \\
& \left. + \frac{\partial}{\partial z} v \right) - \frac{2}{3} \left( \frac{\partial}{\partial z} w \right) \mu \left( 2 \left( \frac{\partial}{\partial z} w \right) - \left( \frac{\partial}{\partial x} u \right) - \left( \frac{\partial}{\partial y} v \right) \right) - \frac{2}{3} u \mu \left( 2 \left( \frac{\partial^2}{\partial x^2} u \right) \right. \\
& \left. - \left( \frac{\partial^2}{\partial y \partial x} v \right) - \left( \frac{\partial^2}{\partial z \partial x} w \right) \right) - v \mu \left( \frac{\partial^2}{\partial y \partial x} u + \frac{\partial^2}{\partial x^2} v \right) - w \mu \left( \frac{\partial^2}{\partial z \partial x} u + \frac{\partial^2}{\partial x^2} w \right) \\
& - u \mu \left( \frac{\partial^2}{\partial y^2} u + \frac{\partial^2}{\partial y \partial x} v \right) - \frac{2}{3} v \mu \left( 2 \left( \frac{\partial^2}{\partial y^2} v \right) - \left( \frac{\partial^2}{\partial y \partial x} u \right) - \left( \frac{\partial^2}{\partial z \partial y} w \right) \right) \\
& - v \mu \left( \frac{\partial^2}{\partial z \partial y} w + \frac{\partial^2}{\partial z^2} v \right) - \frac{2}{3} w \mu \left( 2 \left( \frac{\partial^2}{\partial z^2} w \right) - \left( \frac{\partial^2}{\partial z \partial x} u \right) - \left( \frac{\partial^2}{\partial z \partial y} v \right) \right) \\
& + \left( \frac{\partial}{\partial t} \rho \right) \left( \frac{p}{(\gamma-1) \rho} + \frac{1}{2} u^2 + \frac{1}{2} v^2 + \frac{1}{2} w^2 \right) + \rho \left( \frac{\frac{\partial}{\partial t} p}{(\gamma-1) \rho} - \frac{p \left( \frac{\partial}{\partial t} \rho \right)}{(\gamma-1) \rho^2} \right. \\
& \left. + u \left( \frac{\partial}{\partial t} u \right) + v \left( \frac{\partial}{\partial t} v \right) + w \left( \frac{\partial}{\partial t} w \right) \right) + \left( \frac{\partial}{\partial x} p \right) u + p \left( \frac{\partial}{\partial x} u \right) + \left( \frac{\partial}{\partial y} p \right) v \\
& + p \left( \frac{\partial}{\partial y} v \right) + \left( \frac{\partial}{\partial z} p \right) w + p \left( \frac{\partial}{\partial z} w \right) - k \left( \frac{\frac{\partial^2}{\partial x^2} p}{\rho R} - \frac{2 \left( \frac{\partial}{\partial x} p \right) \left( \frac{\partial}{\partial x} \rho \right)}{\rho^2 R} \right. \\
& \left. + \frac{2 p \left( \frac{\partial}{\partial x} \rho \right)^2}{\rho^3 R} - \frac{p \left( \frac{\partial^2}{\partial x^2} \rho \right)}{\rho^2 R} \right) - k \left( \frac{\frac{\partial^2}{\partial y^2} p}{\rho R} - \frac{2 \left( \frac{\partial}{\partial y} p \right) \left( \frac{\partial}{\partial y} \rho \right)}{\rho^2 R} \right. \\
& \left. + \frac{2 p \left( \frac{\partial}{\partial y} \rho \right)^2}{\rho^3 R} - \frac{p \left( \frac{\partial^2}{\partial y^2} \rho \right)}{\rho^2 R} \right) - k \left( \frac{\frac{\partial^2}{\partial z^2} p}{\rho R} - \frac{2 \left( \frac{\partial}{\partial z} p \right) \left( \frac{\partial}{\partial z} \rho \right)}{\rho^2 R} \right. \\
& \left. + \frac{2 p \left( \frac{\partial}{\partial z} \rho \right)^2}{\rho^3 R} - \frac{p \left( \frac{\partial^2}{\partial z^2} \rho \right)}{\rho^2 R} \right)
\end{aligned}$$

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> #L1:=factor(L1)
>
> #3D Analytical solution proposed by Roy, Smith & Ober, AIAA,

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2002:

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> rho_an := rho_0 + rho_x.sin( a_rhox.pi.x / L ) + rho_y.cos( a_rhoy.pi.y / L ) + rho_z
   .sin( a_rhoz.pi.z / L ) :
> u_an := u_0 + u_x.sin( a_ux.pi.x / L ) + u_y.cos( a_uy.pi.y / L ) + u_z
   .cos( a_uz.pi.z / L ) :
> v_an := v_0 + v_x.cos( a_vx.pi.x / L ) + v_y.sin( a_vy.pi.y / L ) + v_z
   .sin( a_vz.pi.z / L ) :
> w_an := w_0 + w_x.sin( a_wx.pi.x / L ) + w_y.sin( a_wy.pi.y / L ) + w_z
   .cos( a_wz.pi.z / L )
w_an := w_0 + w_x.sin( a_wx.pi.x / L ) + w_y.sin( a_wy.pi.y / L ) + w_z.cos( a_wz.pi.z / L )      (6)
> p_an := p_0 + p_x.cos( a_px.pi.x / L ) + p_y.sin( a_py.pi.y / L ) + p_z
   .cos( a_pz.pi.z / L ) :
```

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>
> # Applying operator L1 on u,v,rho and p, in order to obtain
  source term Q:
>
> Q := algsubs(u = u_an, L1) :
> Q := algsubs(v = v_an, Q) :
> Q := algsubs(w = w_an, Q) :
> Q := foldr( algsubs, Q, rho = rho_an, 1 / rho = 1 / rho_an ) :#foldr(algsubs, q_x, rho
  = rho_an, 1 / rho = 1 / rho_an);
> Q := algsubs(p = p_an, Q) :
> #Q:=simplify(Q, size):
  #not a good idea because it expandes the expression a lot!
> Q := collect(Q, [pi, a_px, a_py, a_pz, a_rhox, a_rhoy, a_rhoz, a_ux, a_uy, a_uz,
  a_vx, a_vy, a_vz, a_wx, a_wy, a_wz, gamma], distributed, factor) :
> Q_e := sort(Q, [a_px, a_py, a_pz, a_rhox, a_rhoy, a_rhoz, a_ux, a_uy, a_uz,
  a_vx, a_vy, a_vz, a_wx, a_wy, a_wz, pi, x, y, z], descending)
Q_e := (cos( a_px.pi.x / L ) p_x k a_px^2 pi^2) / (L^2 R (rho_0 + rho_x.sin( a_rhox.pi.x / L ))
```

$$\begin{aligned}
& + \text{rho\_y} \cos\left(\frac{a_{\text{rho}} y \pi}{L}\right) + \text{rho\_z} \sin\left(\frac{a_{\text{rho}} z \pi}{L}\right)\Big) \\
& - \left(2 \cos\left(\frac{a_{\text{rho}} x \pi}{L}\right) \text{rho\_x} \sin\left(\frac{a_{\text{px}} x \pi}{L}\right) p_x k a_{\text{px}} a_{\text{rho}} x \pi^2\right) \Bigg/ \left(L^2 R \left(rho_0 + rho_x \sin\left(\frac{a_{\text{rho}} x \pi}{L}\right) + rho_y \cos\left(\frac{a_{\text{rho}} y \pi}{L}\right)\right.\right. \\
& \left.\left. + rho_z \sin\left(\frac{a_{\text{rho}} z \pi}{L}\right)\right)\right)^2 \\
& + \left(\sin\left(\frac{a_{\text{py}} y \pi}{L}\right) p_y k a_{\text{py}}^2 \pi^2\right) \Bigg/ \left(L^2 R \left(rho_0 + rho_x \sin\left(\frac{a_{\text{rho}} x \pi}{L}\right) + rho_y \cos\left(\frac{a_{\text{rho}} y \pi}{L}\right)\right.\right. \\
& \left.\left. + rho_z \sin\left(\frac{a_{\text{rho}} z \pi}{L}\right)\right)\right) \\
& - \left(2 \sin\left(\frac{a_{\text{rho}} y \pi}{L}\right) \text{rho\_y} \cos\left(\frac{a_{\text{py}} y \pi}{L}\right) p_y k a_{\text{py}} a_{\text{rho}} y \pi^2\right) \Bigg/ \left(L^2 R \left(rho_0 + rho_x \sin\left(\frac{a_{\text{rho}} x \pi}{L}\right) + rho_y \cos\left(\frac{a_{\text{rho}} y \pi}{L}\right)\right.\right. \\
& \left.\left. + rho_z \sin\left(\frac{a_{\text{rho}} z \pi}{L}\right)\right)\right)^2 \\
& + \left(\cos\left(\frac{a_{\text{pz}} z \pi}{L}\right) p_z k a_{\text{pz}}^2 \pi^2\right) \Bigg/ \left(L^2 R \left(rho_0 + rho_x \sin\left(\frac{a_{\text{rho}} x \pi}{L}\right) + rho_y \cos\left(\frac{a_{\text{rho}} y \pi}{L}\right)\right.\right. \\
& \left.\left. + rho_z \sin\left(\frac{a_{\text{rho}} z \pi}{L}\right)\right)\right) \\
& - \left(2 \cos\left(\frac{a_{\text{rho}} z \pi}{L}\right) \text{rho\_z} \sin\left(\frac{a_{\text{pz}} z \pi}{L}\right) p_z k a_{\text{pz}} a_{\text{rho}} z \pi^2\right) \Bigg/ \left(L^2 R \left(rho_0 + rho_x \sin\left(\frac{a_{\text{rho}} x \pi}{L}\right) + rho_y \cos\left(\frac{a_{\text{rho}} y \pi}{L}\right)\right.\right. \\
& \left.\left. + rho_z \sin\left(\frac{a_{\text{rho}} z \pi}{L}\right)\right)\right)^2 \\
& - \left(k \text{rho\_x} \left(p_0 + p_x \cos\left(\frac{a_{\text{px}} x \pi}{L}\right) + p_y \sin\left(\frac{a_{\text{py}} y \pi}{L}\right) + p_z \cos\left(\frac{a_{\text{pz}} z \pi}{L}\right)\right) \left(\sin\left(\frac{a_{\text{rho}} x \pi}{L}\right) rho_0\right.\right. \\
& \left.\left. + \sin\left(\frac{a_{\text{rho}} z \pi}{L}\right) rho_z \sin\left(\frac{a_{\text{rho}} z \pi}{L}\right) + 2 rho_x \cos\left(\frac{a_{\text{rho}} x \pi}{L}\right)^2\right.\right. \\
& \left.\left. + rho_x \sin\left(\frac{a_{\text{rho}} x \pi}{L}\right)^2 + \sin\left(\frac{a_{\text{rho}} x \pi}{L}\right) rho_y \cos\left(\frac{a_{\text{rho}} y \pi}{L}\right)\right)\right. \\
& a_{\text{rho}} x \pi^2\Bigg) \Bigg/ \left(R \left(rho_0 + rho_x \sin\left(\frac{a_{\text{rho}} x \pi}{L}\right) + rho_y \cos\left(\frac{a_{\text{rho}} y \pi}{L}\right) + rho_z \sin\left(\frac{a_{\text{rho}} z \pi}{L}\right)\right)^3 L^2\right) \\
& - \left(k \text{rho\_y} \left(p_0 + p_x \cos\left(\frac{a_{\text{px}} x \pi}{L}\right) + p_y \sin\left(\frac{a_{\text{py}} y \pi}{L}\right) + p_z \cos\left(\frac{a_{\text{pz}} z \pi}{L}\right)\right) \left(\cos\left(\frac{a_{\text{rho}} y \pi}{L}\right) rho_0\right.\right. \\
& \left.\left. + rho_x \cos\left(\frac{a_{\text{rho}} x \pi}{L}\right)^2 + rho_x \sin\left(\frac{a_{\text{rho}} x \pi}{L}\right) \cos\left(\frac{a_{\text{rho}} y \pi}{L}\right)\right)\right)
\end{aligned}$$

$$\begin{aligned}
& \left. \frac{\partial}{\partial t} \left( a_{rho}^2 \pi^2 \right) \right/ \left( R \left( rho\_0 + rho\_x \sin\left(\frac{a\_rho x \pi x}{L}\right) \right. \right. \\
& \quad \left. \left. + rho\_y \cos\left(\frac{a\_rho y \pi y}{L}\right) + rho\_z \sin\left(\frac{a\_rho z \pi z}{L}\right) \right)^3 L^2 \right) \\
& \quad - \left( k rho\_z \left( p\_0 + p\_x \cos\left(\frac{a\_px \pi x}{L}\right) + p\_y \sin\left(\frac{a\_py \pi y}{L}\right) + p\_z \cos\left(\frac{a\_pz \pi z}{L}\right) \right) \right) \left( \sin\left(\frac{a\_rho x \pi x}{L}\right) \right. \\
& \quad \left. + rho\_z \sin\left(\frac{a\_rho z \pi z}{L}\right) \right)^2 + rho\_x \sin\left(\frac{a\_rho x \pi x}{L}\right) \sin\left(\frac{a\_rho z \pi z}{L}\right) \Big) \\
& \left. \frac{\partial}{\partial t} \left( a_{rho z}^2 \pi^2 \right) \right/ \left( R \left( rho\_0 + rho\_x \sin\left(\frac{a\_rho x \pi x}{L}\right) \right. \right. \\
& \quad \left. \left. + rho\_y \cos\left(\frac{a\_rho y \pi y}{L}\right) + rho\_z \sin\left(\frac{a\_rho z \pi z}{L}\right) \right)^3 L^2 \right) \\
& \quad + \frac{4}{3} \frac{1}{L^2} \left( u\_x \mu \left( -u\_x \cos\left(\frac{a\_ux \pi x}{L}\right)^2 + \sin\left(\frac{a\_ux \pi x}{L}\right) u\_0 \right. \right. \\
& \quad \left. \left. + u\_x \sin\left(\frac{a\_ux \pi x}{L}\right)^2 + \sin\left(\frac{a\_ux \pi x}{L}\right) u\_y \cos\left(\frac{a\_uy \pi y}{L}\right) \right. \right. \\
& \quad \left. \left. + \sin\left(\frac{a\_ux \pi x}{L}\right) u\_z \cos\left(\frac{a\_uz \pi z}{L}\right) \right) a\_ux^2 \pi^2 \right) \\
& \quad + \frac{4}{3} \frac{v\_y \cos\left(\frac{a\_vy \pi y}{L}\right) \mu u\_x \cos\left(\frac{a\_ux \pi x}{L}\right) a\_ux a\_vy \pi^2}{L^2} \\
& \quad - \frac{4}{3} \frac{w\_z \sin\left(\frac{a\_wz \pi z}{L}\right) \mu u\_x \cos\left(\frac{a\_ux \pi x}{L}\right) a\_ux a\_wz \pi^2}{L^2} \\
& \quad + \frac{1}{L^2} \left( u\_y \mu \left( -u\_y \sin\left(\frac{a\_uy \pi y}{L}\right)^2 + \cos\left(\frac{a\_uy \pi y}{L}\right) u\_0 \right. \right. \\
& \quad \left. \left. + \cos\left(\frac{a\_uy \pi y}{L}\right) u\_x \sin\left(\frac{a\_ux \pi x}{L}\right) + u\_y \cos\left(\frac{a\_uy \pi y}{L}\right)^2 \right. \right. \\
& \quad \left. \left. + \cos\left(\frac{a\_uy \pi y}{L}\right) u\_z \cos\left(\frac{a\_uz \pi z}{L}\right) \right) a\_uy^2 \pi^2 \right) \\
& \quad - \frac{2 v\_x \sin\left(\frac{a\_vx \pi x}{L}\right) \mu u\_y \sin\left(\frac{a\_uy \pi y}{L}\right) a\_uy a\_vx \pi^2}{L^2} + \frac{1}{L^2} \left( u\_z \mu \left( \right. \right. \\
& \quad \left. \left. - u\_z \sin\left(\frac{a\_uz \pi z}{L}\right)^2 + \cos\left(\frac{a\_uz \pi z}{L}\right) u\_0 \right) \right)
\end{aligned}$$

$$\begin{aligned}
& + \cos\left(\frac{a_{uz}\pi z}{L}\right) u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) + \cos\left(\frac{a_{uz}\pi z}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \\
& + u_z \cos\left(\frac{a_{uz}\pi z}{L}\right)^2 a_{uz}^2 \pi^2 \\
& + \frac{2 w_x \cos\left(\frac{a_{wx}\pi x}{L}\right) \mu u_z \sin\left(\frac{a_{uz}\pi z}{L}\right) a_{uz} a_{wx} \pi^2}{L^2} + \frac{1}{L^2} \left( \mu v_x \left( \right. \right. \\
& - v_x \sin\left(\frac{a_{vx}\pi x}{L}\right)^2 + \cos\left(\frac{a_{vx}\pi x}{L}\right) v_0 + v_x \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 \\
& + \cos\left(\frac{a_{vy}\pi y}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) + \cos\left(\frac{a_{vz}\pi z}{L}\right) v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) \left. \right) \\
& a_{vx}^2 \pi^2 \left. \right) + \frac{4}{3} \frac{1}{L^2} \left( \mu v_y \left( \sin\left(\frac{a_{vy}\pi y}{L}\right) v_0 \right. \right. \\
& + \sin\left(\frac{a_{vy}\pi y}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) + v_y \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 \\
& + \sin\left(\frac{a_{vy}\pi y}{L}\right) v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) - v_y \cos\left(\frac{a_{vy}\pi y}{L}\right)^2 \left. \right) a_{vy}^2 \pi^2 \\
& - \frac{4}{3} \frac{w_z \sin\left(\frac{a_{wz}\pi z}{L}\right) \mu v_y \cos\left(\frac{a_{vy}\pi y}{L}\right) a_{vy} a_{wz} \pi^2}{L^2} \\
& + \frac{1}{L^2} \left( v_z \mu \left( - v_z \cos\left(\frac{a_{vz}\pi z}{L}\right)^2 + \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 \right. \right. \\
& + \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) + \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \\
& + v_z \sin\left(\frac{a_{vz}\pi z}{L}\right)^2 \left. \right) a_{vz}^2 \pi^2 \left. \right) \\
& - \frac{2 w_y \cos\left(\frac{a_{wy}\pi y}{L}\right) \mu v_z \cos\left(\frac{a_{vz}\pi z}{L}\right) a_{vz} a_{wy} \pi^2}{L^2} + \frac{1}{L^2} \left( \mu w_x \left( \right. \right. \\
& - w_x \cos\left(\frac{a_{wx}\pi x}{L}\right)^2 + \sin\left(\frac{a_{wx}\pi x}{L}\right) w_0 + w_x \sin\left(\frac{a_{wx}\pi x}{L}\right)^2 \\
& + \sin\left(\frac{a_{wx}\pi x}{L}\right) w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) + \sin\left(\frac{a_{wx}\pi x}{L}\right) w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) \left. \right) \\
& a_{wx}^2 \pi^2 \left. \right) + \frac{1}{L^2} \left( \mu w_y \left( - w_y \cos\left(\frac{a_{wy}\pi y}{L}\right)^2 + \sin\left(\frac{a_{wy}\pi y}{L}\right) w_0 \right. \right)
\end{aligned}$$

$$\begin{aligned}
& + \sin\left(\frac{a_{wy}\pi y}{L}\right) w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) + w_y \sin\left(\frac{a_{wy}\pi y}{L}\right)^2 \\
& + \sin\left(\frac{a_{wy}\pi y}{L}\right) w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) \Big) a_{wy}^2 \pi^2 \Big) + \frac{4}{3} \frac{1}{L^2} \left( \mu w_z \left( \right. \right. \\
& - w_z \sin\left(\frac{a_{wz}\pi z}{L}\right)^2 + \cos\left(\frac{a_{wz}\pi z}{L}\right) w_0 \\
& + \cos\left(\frac{a_{wz}\pi z}{L}\right) w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) + \cos\left(\frac{a_{wz}\pi z}{L}\right) w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) \\
& \left. \left. + w_z \cos\left(\frac{a_{wz}\pi z}{L}\right)^2 \right) a_{wz}^2 \pi^2 \right) - \frac{1}{L(\gamma-1)} \left( \gamma \left( u_0 \right. \right. \\
& + u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) + u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) + u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \Big) \\
& p_x \sin\left(\frac{a_{px}\pi x}{L}\right) a_{px}\pi \Big) + \frac{1}{L(\gamma-1)} \left( \gamma \left( v_0 + v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \right. \right. \\
& + v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) + v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) \Big) p_y \cos\left(\frac{a_{py}\pi y}{L}\right) a_{py}\pi \Big) \\
& - \frac{1}{L(\gamma-1)} \left( \gamma \left( w_0 + w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) + w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) \right. \right. \\
& + w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) \Big) p_z \sin\left(\frac{a_{pz}\pi z}{L}\right) a_{pz}\pi \Big) + \frac{1}{2} \frac{1}{L} \left( \left( u_0 \right. \right. \\
& + u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) + u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) + u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \Big) \\
& \cos\left(\frac{a_{rhox}\pi x}{L}\right) rho_x \left( u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 + u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \right. \\
& \left. + u_z^2 \cos\left(\frac{a_{uz}\pi z}{L}\right)^2 + v_z^2 \sin\left(\frac{a_{vz}\pi z}{L}\right)^2 + v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 \right. \\
& \left. + v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 + w_z^2 \cos\left(\frac{a_{wz}\pi z}{L}\right)^2 + w_x^2 \sin\left(\frac{a_{wx}\pi x}{L}\right)^2 \right. \\
& \left. + w_y^2 \sin\left(\frac{a_{wy}\pi y}{L}\right)^2 + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \right. \\
& \left. + 2 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \right. \\
& \left. + 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \right. \\
& \left. + 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \right)
\end{aligned}$$

$$\begin{aligned}
& + 2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \\
& + 2 w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) \\
& + 2 w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) \\
& + 2 w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) + 2 u_0 u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \\
& + 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 + 2 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \\
& + 2 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) + 2 w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_0 \\
& + 2 w_0 w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) + 2 w_0 w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) \\
& + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) + 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) + u_0^2 + v_0^2 + w_0^2 \\
& + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \Big) a_{rhox} \pi \Big) - \frac{1}{2} \frac{1}{L} \Bigg( \Big( v_0 \\
& + v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) + v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) + v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) \Big) \\
& \sin\left(\frac{a_{rhoxy}\pi y}{L}\right) rho_y \Big( u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 + u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \\
& + u_z^2 \cos\left(\frac{a_{uz}\pi z}{L}\right)^2 + v_z^2 \sin\left(\frac{a_{vz}\pi z}{L}\right)^2 + v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 \\
& + v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 + w_z^2 \cos\left(\frac{a_{wz}\pi z}{L}\right)^2 + w_x^2 \sin\left(\frac{a_{wx}\pi x}{L}\right)^2 \\
& + w_y^2 \sin\left(\frac{a_{wy}\pi y}{L}\right)^2 + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \\
& + 2 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \\
& + 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \\
& + 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \\
& + 2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + 2 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_x \sin\left(\frac{a_w x \pi x}{L}\right) \\
& + 2 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) \\
& + 2 w_x \sin\left(\frac{a_w x \pi x}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) + 2 u_0 u_z \cos\left(\frac{a_u z \pi z}{L}\right) \\
& + 2 v_z \sin\left(\frac{a_v z \pi z}{L}\right) v_0 + 2 v_0 v_x \cos\left(\frac{a_v x \pi x}{L}\right) \\
& + 2 v_0 v_y \sin\left(\frac{a_v y \pi y}{L}\right) + 2 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_0 \\
& + 2 w_0 w_x \sin\left(\frac{a_w x \pi x}{L}\right) + 2 w_0 w_y \sin\left(\frac{a_w y \pi y}{L}\right) \\
& + 2 u_0 u_y \cos\left(\frac{a_u y \pi y}{L}\right) + 2 u_0 u_x \sin\left(\frac{a_u x \pi x}{L}\right) + u_0^2 + v_0^2 + w_0^2 \\
& + 2 u_x \sin\left(\frac{a_u x \pi x}{L}\right) u_y \cos\left(\frac{a_u y \pi y}{L}\right) \Big) a_r h o y \pi \Big) + \frac{1}{2} \frac{1}{L} \Big( \Big( w_0 \\
& + w_x \sin\left(\frac{a_w x \pi x}{L}\right) + w_y \sin\left(\frac{a_w y \pi y}{L}\right) + w_z \cos\left(\frac{a_w z \pi z}{L}\right) \Big) \\
& \cos\left(\frac{a_r h o z \pi z}{L}\right) r h o_z \Big( u_x^2 \sin\left(\frac{a_u x \pi x}{L}\right)^2 + u_y^2 \cos\left(\frac{a_u y \pi y}{L}\right)^2 \\
& + u_z^2 \cos\left(\frac{a_u z \pi z}{L}\right)^2 + v_z^2 \sin\left(\frac{a_v z \pi z}{L}\right)^2 + v_x^2 \cos\left(\frac{a_v x \pi x}{L}\right)^2 \\
& + v_y^2 \sin\left(\frac{a_v y \pi y}{L}\right)^2 + w_z^2 \cos\left(\frac{a_w z \pi z}{L}\right)^2 + w_x^2 \sin\left(\frac{a_w x \pi x}{L}\right)^2 \\
& + w_y^2 \sin\left(\frac{a_w y \pi y}{L}\right)^2 + 2 u_x \sin\left(\frac{a_u x \pi x}{L}\right) u_z \cos\left(\frac{a_u z \pi z}{L}\right) \\
& + 2 u_y \cos\left(\frac{a_u y \pi y}{L}\right) u_z \cos\left(\frac{a_u z \pi z}{L}\right) \\
& + 2 v_z \sin\left(\frac{a_v z \pi z}{L}\right) v_x \cos\left(\frac{a_v x \pi x}{L}\right) \\
& + 2 v_z \sin\left(\frac{a_v z \pi z}{L}\right) v_y \sin\left(\frac{a_v y \pi y}{L}\right) \\
& + 2 v_x \cos\left(\frac{a_v x \pi x}{L}\right) v_y \sin\left(\frac{a_v y \pi y}{L}\right) \\
& + 2 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_x \sin\left(\frac{a_w x \pi x}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + 2 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) \\
& + 2 w_x \sin\left(\frac{a_w x \pi x}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) + 2 u_0 u_z \cos\left(\frac{a_u z \pi z}{L}\right) \\
& + 2 v_z \sin\left(\frac{a_v z \pi z}{L}\right) v_0 + 2 v_0 v_x \cos\left(\frac{a_v x \pi x}{L}\right) \\
& + 2 v_0 v_y \sin\left(\frac{a_v y \pi y}{L}\right) + 2 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_0 \\
& + 2 w_0 w_x \sin\left(\frac{a_w x \pi x}{L}\right) + 2 w_0 w_y \sin\left(\frac{a_w y \pi y}{L}\right) \\
& + 2 u_0 u_y \cos\left(\frac{a_u y \pi y}{L}\right) + 2 u_0 u_x \sin\left(\frac{a_u x \pi x}{L}\right) + u_0^2 + v_0^2 + w_0^2 \\
& + 2 u_x \sin\left(\frac{a_u x \pi x}{L}\right) u_y \cos\left(\frac{a_u y \pi y}{L}\right) \Big) a_{rho z} \pi \\
& + \frac{1}{2} \frac{1}{L(\gamma-1)} \left( \left( 2 p_x \cos\left(\frac{a_p x \pi x}{L}\right) \gamma + 2 p_y \sin\left(\frac{a_p y \pi y}{L}\right) \gamma \right. \right. \\
& + 2 p_z \cos\left(\frac{a_p z \pi z}{L}\right) \gamma - 3 u_0^2 rho_0 - v_0^2 rho_0 - w_0^2 rho_0 \\
& - 3 u_0^2 rho_z \sin\left(\frac{a_rho z \pi z}{L}\right) - 3 u_0^2 rho_x \sin\left(\frac{a_rho x \pi x}{L}\right) \\
& - 3 u_0^2 rho_y \cos\left(\frac{a_rho y \pi y}{L}\right) - 3 u_x^2 \sin\left(\frac{a_u x \pi x}{L}\right)^2 rho_0 \\
& - 3 u_y^2 \cos\left(\frac{a_u y \pi y}{L}\right)^2 rho_0 - 3 u_z^2 \cos\left(\frac{a_u z \pi z}{L}\right)^2 rho_0 \\
& + 3 u_0^2 \gamma rho_0 - v_0^2 rho_z \sin\left(\frac{a_rho z \pi z}{L}\right) - v_0^2 rho_x \sin\left(\frac{a_rho x \pi x}{L}\right) \\
& - v_0^2 rho_y \cos\left(\frac{a_rho y \pi y}{L}\right) - w_0^2 rho_z \sin\left(\frac{a_rho z \pi z}{L}\right) \\
& - w_0^2 rho_x \sin\left(\frac{a_rho x \pi x}{L}\right) - w_0^2 rho_y \cos\left(\frac{a_rho y \pi y}{L}\right) + \gamma v_0^2 rho_0 \\
& + \gamma w_0^2 rho_0 - v_z^2 \sin\left(\frac{a_v z \pi z}{L}\right)^2 rho_0 - v_x^2 \cos\left(\frac{a_v x \pi x}{L}\right)^2 rho_0 \\
& - v_y^2 \sin\left(\frac{a_v y \pi y}{L}\right)^2 rho_0 - w_z^2 \cos\left(\frac{a_w z \pi z}{L}\right)^2 rho_0 \\
& - w_x^2 \sin\left(\frac{a_w x \pi x}{L}\right)^2 rho_0 - w_y^2 \sin\left(\frac{a_w y \pi y}{L}\right)^2 rho_0
\end{aligned}$$

$$\begin{aligned}
& -3 u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 rho_z \sin\left(\frac{a_rhoz\pi z}{L}\right) \\
& -3 u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -3 u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& -3 u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& -3 u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -3 u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& -3 u_z^2 \cos\left(\frac{a_{uz}\pi z}{L}\right)^2 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& -3 u_z^2 \cos\left(\frac{a_{uz}\pi z}{L}\right)^2 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -3 u_z^2 \cos\left(\frac{a_{uz}\pi z}{L}\right)^2 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& +3 u_0^2 \gamma rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) + 3 u_0^2 \gamma rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& +3 u_0^2 \gamma rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) - 6 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) rho_0 \\
& -6 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) rho_0 - 6 u_0 u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) rho_0 \\
& +3 u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 \gamma rho_0 + 3 u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma rho_0 \\
& +3 u_z^2 \cos\left(\frac{a_{uz}\pi z}{L}\right)^2 \gamma rho_0 + \gamma v_0^2 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& +\gamma v_0^2 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) + \gamma v_0^2 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& -2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 rho_0 - 2 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_0 \\
& -2 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_0 + \gamma w_0^2 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& +\gamma w_0^2 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) + \gamma w_0^2 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -2 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_0 \rho_0 - 2 w_0 w_x \sin\left(\frac{a_w x \pi x}{L}\right) \rho_0 \\
& - 2 w_0 w_y \sin\left(\frac{a_w y \pi y}{L}\right) \rho_0 + \gamma v_z^2 \sin^2\left(\frac{a_v z \pi z}{L}\right) \rho_0 \\
& + \gamma v_x^2 \cos^2\left(\frac{a_v x \pi x}{L}\right) \rho_0 + \gamma v_y^2 \sin^2\left(\frac{a_v y \pi y}{L}\right) \rho_0 \\
& + \gamma w_z^2 \cos^2\left(\frac{a_w z \pi z}{L}\right) \rho_0 + \gamma w_x^2 \sin^2\left(\frac{a_w x \pi x}{L}\right) \rho_0 \\
& + \gamma w_y^2 \sin^2\left(\frac{a_w y \pi y}{L}\right) \rho_0 \\
& - v_z^2 \sin^2\left(\frac{a_v z \pi z}{L}\right) \rho_0 \sin\left(\frac{a_r hoz \pi z}{L}\right) \\
& - v_z^2 \sin^2\left(\frac{a_v z \pi z}{L}\right) \rho_0 \sin\left(\frac{a_r rhox \pi x}{L}\right) \\
& - v_z^2 \sin^2\left(\frac{a_v z \pi z}{L}\right) \rho_0 \cos\left(\frac{a_r rho y \pi y}{L}\right) \\
& - v_x^2 \cos^2\left(\frac{a_v x \pi x}{L}\right) \rho_0 \sin\left(\frac{a_r hoz \pi z}{L}\right) \\
& - v_x^2 \cos^2\left(\frac{a_v x \pi x}{L}\right) \rho_0 \sin\left(\frac{a_r rhox \pi x}{L}\right) \\
& - v_x^2 \cos^2\left(\frac{a_v x \pi x}{L}\right) \rho_0 \cos\left(\frac{a_r rho y \pi y}{L}\right) \\
& - v_y^2 \sin^2\left(\frac{a_v y \pi y}{L}\right) \rho_0 \sin\left(\frac{a_r hoz \pi z}{L}\right) \\
& - v_y^2 \sin^2\left(\frac{a_v y \pi y}{L}\right) \rho_0 \sin\left(\frac{a_r rhox \pi x}{L}\right) \\
& - v_y^2 \sin^2\left(\frac{a_v y \pi y}{L}\right) \rho_0 \cos\left(\frac{a_r rho y \pi y}{L}\right) \\
& - w_z^2 \cos^2\left(\frac{a_w z \pi z}{L}\right) \rho_0 \sin\left(\frac{a_r hoz \pi z}{L}\right) \\
& - w_z^2 \cos^2\left(\frac{a_w z \pi z}{L}\right) \rho_0 \sin\left(\frac{a_r rhox \pi x}{L}\right) \\
& - w_z^2 \cos^2\left(\frac{a_w z \pi z}{L}\right) \rho_0 \cos\left(\frac{a_r rho y \pi y}{L}\right) \\
& - w_x^2 \sin^2\left(\frac{a_w x \pi x}{L}\right) \rho_0 \sin\left(\frac{a_r hoz \pi z}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -w_x^2 \sin\left(\frac{a_{wx}\pi x}{L}\right)^2 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -w_x^2 \sin\left(\frac{a_{wx}\pi x}{L}\right)^2 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& -w_y^2 \sin\left(\frac{a_{wy}\pi y}{L}\right)^2 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& -w_y^2 \sin\left(\frac{a_{wy}\pi y}{L}\right)^2 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -w_y^2 \sin\left(\frac{a_{wy}\pi y}{L}\right)^2 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 6 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 6 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 6 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 6 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 6 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 6 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 6 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 6 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 6 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + 2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 2 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 2 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 2 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 2 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 2 \gamma w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 2 \gamma w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 \gamma w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& - 6 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& - 6 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& - 6 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& - 6 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -6 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -6 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& -6 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& -6 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -6 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& +6 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& +6 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& +6 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& +6 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& +6 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& +6 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& +6 u_0 u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& +6 u_0 u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& +6 u_0 u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& +6 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_0 \\
& +6 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_0 \\
& +6 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_0 \\
& +2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_0
\end{aligned}$$

$$\begin{aligned}
& + 2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_0 \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_0 \\
& + 2 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_0 \\
& + 2 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_0 \\
& + 2 \gamma w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_0 \\
& - 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& - 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& - 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& - 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& - 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& - 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& - 2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 2 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_0 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 2 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_0 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_0 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 2 \gamma w_0 w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 2 \gamma w_0 w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 \gamma w_0 w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 2 \gamma w_0 w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 2 \gamma w_0 w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 \gamma w_0 w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& - 2 w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& - 2 w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& - 2 w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& - 2 w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -2 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_x \sin\left(\frac{a_r h o x \pi x}{L}\right) \\
& -2 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_y \cos\left(\frac{a_r h o y \pi y}{L}\right) \\
& -2 w_x \sin\left(\frac{a_w x \pi x}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_z \sin\left(\frac{a_r h o z \pi z}{L}\right) \\
& -2 w_x \sin\left(\frac{a_w x \pi x}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_x \sin\left(\frac{a_r h o x \pi x}{L}\right) \\
& -2 w_x \sin\left(\frac{a_w x \pi x}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_y \cos\left(\frac{a_r h o y \pi y}{L}\right) \\
& -6 u_x \sin\left(\frac{a_u x \pi x}{L}\right) u_y \cos\left(\frac{a_u y \pi y}{L}\right) rho_0 \\
& -6 u_x \sin\left(\frac{a_u x \pi x}{L}\right) u_z \cos\left(\frac{a_u z \pi z}{L}\right) rho_0 \\
& -6 u_y \cos\left(\frac{a_u y \pi y}{L}\right) u_z \cos\left(\frac{a_u z \pi z}{L}\right) rho_0 \\
& +6 u_0 u_x \sin\left(\frac{a_u x \pi x}{L}\right) \gamma rho_0 + 6 u_0 u_y \cos\left(\frac{a_u y \pi y}{L}\right) \gamma rho_0 \\
& +6 u_0 u_z \cos\left(\frac{a_u z \pi z}{L}\right) \gamma rho_0 \\
& -6 u_0 u_x \sin\left(\frac{a_u x \pi x}{L}\right) rho_z \sin\left(\frac{a_r h o z \pi z}{L}\right) \\
& -6 u_0 u_x \sin\left(\frac{a_u x \pi x}{L}\right) rho_x \sin\left(\frac{a_r h o x \pi x}{L}\right) \\
& -6 u_0 u_x \sin\left(\frac{a_u x \pi x}{L}\right) rho_y \cos\left(\frac{a_r h o y \pi y}{L}\right) \\
& -6 u_0 u_y \cos\left(\frac{a_u y \pi y}{L}\right) rho_z \sin\left(\frac{a_r h o z \pi z}{L}\right) \\
& -6 u_0 u_y \cos\left(\frac{a_u y \pi y}{L}\right) rho_x \sin\left(\frac{a_r h o x \pi x}{L}\right) \\
& -6 u_0 u_y \cos\left(\frac{a_u y \pi y}{L}\right) rho_y \cos\left(\frac{a_r h o y \pi y}{L}\right) \\
& -6 u_0 u_z \cos\left(\frac{a_u z \pi z}{L}\right) rho_z \sin\left(\frac{a_r h o z \pi z}{L}\right) \\
& -6 u_0 u_z \cos\left(\frac{a_u z \pi z}{L}\right) rho_x \sin\left(\frac{a_r h o x \pi x}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -6 u_0 u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) rho_y \cos\left(\frac{a_{rho y}\pi y}{L}\right) \\
& + 3 u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 \gamma rho_z \sin\left(\frac{a_{rho z}\pi z}{L}\right) \\
& + 3 u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 \gamma rho_x \sin\left(\frac{a_{rho x}\pi x}{L}\right) \\
& + 3 u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 \gamma rho_y \cos\left(\frac{a_{rho y}\pi y}{L}\right) \\
& + 3 u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma rho_z \sin\left(\frac{a_{rho z}\pi z}{L}\right) \\
& + 3 u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma rho_x \sin\left(\frac{a_{rho x}\pi x}{L}\right) \\
& + 3 u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma rho_y \cos\left(\frac{a_{rho y}\pi y}{L}\right) \\
& + 3 u_z^2 \cos\left(\frac{a_{uz}\pi z}{L}\right)^2 \gamma rho_z \sin\left(\frac{a_{rho z}\pi z}{L}\right) \\
& + 3 u_z^2 \cos\left(\frac{a_{uz}\pi z}{L}\right)^2 \gamma rho_x \sin\left(\frac{a_{rho x}\pi x}{L}\right) \\
& + 3 u_z^2 \cos\left(\frac{a_{uz}\pi z}{L}\right)^2 \gamma rho_y \cos\left(\frac{a_{rho y}\pi y}{L}\right) \\
& - 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 rho_z \sin\left(\frac{a_{rho z}\pi z}{L}\right) \\
& - 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 rho_x \sin\left(\frac{a_{rho x}\pi x}{L}\right) \\
& - 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 rho_y \cos\left(\frac{a_{rho y}\pi y}{L}\right) \\
& - 2 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_z \sin\left(\frac{a_{rho z}\pi z}{L}\right) \\
& - 2 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_x \sin\left(\frac{a_{rho x}\pi x}{L}\right) \\
& - 2 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_y \cos\left(\frac{a_{rho y}\pi y}{L}\right) \\
& - 2 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rho z}\pi z}{L}\right) \\
& - 2 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rho x}\pi x}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -2 v_0 v_y \sin\left(\frac{a_v y \pi y}{L}\right) rho_y \cos\left(\frac{a_r h o y \pi y}{L}\right) \\
& -2 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_0 rho_z \sin\left(\frac{a_r h o z \pi z}{L}\right) \\
& -2 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_0 rho_x \sin\left(\frac{a_r h o x \pi x}{L}\right) \\
& -2 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_0 rho_y \cos\left(\frac{a_r h o y \pi y}{L}\right) \\
& -2 w_0 w_x \sin\left(\frac{a_w x \pi x}{L}\right) rho_z \sin\left(\frac{a_r h o z \pi z}{L}\right) \\
& -2 w_0 w_x \sin\left(\frac{a_w x \pi x}{L}\right) rho_x \sin\left(\frac{a_r h o x \pi x}{L}\right) \\
& -2 w_0 w_x \sin\left(\frac{a_w x \pi x}{L}\right) rho_y \cos\left(\frac{a_r h o y \pi y}{L}\right) \\
& -2 w_0 w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_z \sin\left(\frac{a_r h o z \pi z}{L}\right) \\
& -2 w_0 w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_x \sin\left(\frac{a_r h o x \pi x}{L}\right) \\
& -2 w_0 w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_y \cos\left(\frac{a_r h o y \pi y}{L}\right) \\
& + \gamma v_z^2 \sin\left(\frac{a_v z \pi z}{L}\right)^2 rho_z \sin\left(\frac{a_r h o z \pi z}{L}\right) \\
& + \gamma v_z^2 \sin\left(\frac{a_v z \pi z}{L}\right)^2 rho_x \sin\left(\frac{a_r h o x \pi x}{L}\right) \\
& + \gamma v_z^2 \sin\left(\frac{a_v z \pi z}{L}\right)^2 rho_y \cos\left(\frac{a_r h o y \pi y}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_v x \pi x}{L}\right)^2 rho_z \sin\left(\frac{a_r h o z \pi z}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_v x \pi x}{L}\right)^2 rho_x \sin\left(\frac{a_r h o x \pi x}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_v x \pi x}{L}\right)^2 rho_y \cos\left(\frac{a_r h o y \pi y}{L}\right) \\
& + \gamma v_y^2 \sin\left(\frac{a_v y \pi y}{L}\right)^2 rho_z \sin\left(\frac{a_r h o z \pi z}{L}\right) \\
& + \gamma v_y^2 \sin\left(\frac{a_v y \pi y}{L}\right)^2 rho_x \sin\left(\frac{a_r h o x \pi x}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + \gamma v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 rho_y \cos\left(\frac{a_{rho}\pi y}{L}\right) \\
& - 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_0 \\
& - 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_0 \\
& - 2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_0 \\
& + 2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 rho_0 + 2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_0 \\
& + 2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_0 \\
& + \gamma w_z^2 \cos\left(\frac{a_{wz}\pi z}{L}\right)^2 rho_z \sin\left(\frac{a_{rho}\pi z}{L}\right) \\
& + \gamma w_z^2 \cos\left(\frac{a_{wz}\pi z}{L}\right)^2 rho_x \sin\left(\frac{a_{rho}\pi x}{L}\right) \\
& + \gamma w_z^2 \cos\left(\frac{a_{wz}\pi z}{L}\right)^2 rho_y \cos\left(\frac{a_{rho}\pi y}{L}\right) \\
& + \gamma w_x^2 \sin\left(\frac{a_{wx}\pi x}{L}\right)^2 rho_z \sin\left(\frac{a_{rho}\pi z}{L}\right) \\
& + \gamma w_x^2 \sin\left(\frac{a_{wx}\pi x}{L}\right)^2 rho_x \sin\left(\frac{a_{rho}\pi x}{L}\right) \\
& + \gamma w_x^2 \sin\left(\frac{a_{wx}\pi x}{L}\right)^2 rho_y \cos\left(\frac{a_{rho}\pi y}{L}\right) \\
& + \gamma w_y^2 \sin\left(\frac{a_{wy}\pi y}{L}\right)^2 rho_z \sin\left(\frac{a_{rho}\pi z}{L}\right) \\
& + \gamma w_y^2 \sin\left(\frac{a_{wy}\pi y}{L}\right)^2 rho_x \sin\left(\frac{a_{rho}\pi x}{L}\right) \\
& + \gamma w_y^2 \sin\left(\frac{a_{wy}\pi y}{L}\right)^2 rho_y \cos\left(\frac{a_{rho}\pi y}{L}\right) \\
& + 2 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_0 rho_0 + 2 \gamma w_0 w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_0 \\
& + 2 \gamma w_0 w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_0 \\
& - 2 w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_0
\end{aligned}$$

$$\begin{aligned}
& -2 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_0 \\
& -2 w_x \sin\left(\frac{a_w x \pi x}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_0 + 2 p_0 \gamma \\
& u_x \cos\left(\frac{a_u x \pi x}{L}\right) a_{ux} \pi \Big) - \left( \left( v_0 + v_x \cos\left(\frac{a_v x \pi x}{L}\right) \right. \right. \\
& \left. \left. + v_y \sin\left(\frac{a_v y \pi y}{L}\right) + v_z \sin\left(\frac{a_v z \pi z}{L}\right) \right) \left( -\sin\left(\frac{a_u y \pi y}{L}\right) u_y \right. \right. \\
& \left. \left. + rho_x \sin\left(\frac{a_rho x \pi x}{L}\right) + rho_y \cos\left(\frac{a_rho y \pi y}{L}\right) \right. \right. \\
& \left. \left. + rho_z \sin\left(\frac{a_rho z \pi z}{L}\right) \right)^2 \left( u_0 + u_x \sin\left(\frac{a_u x \pi x}{L}\right) + u_y \cos\left(\frac{a_u y \pi y}{L}\right) \right. \right. \\
& \left. \left. + u_z \cos\left(\frac{a_u z \pi z}{L}\right) \right) + \sin\left(\frac{a_u y \pi y}{L}\right) u_y \right. \right. \\
& \left. \left. + rho_x \sin\left(\frac{a_rho x \pi x}{L}\right) + rho_y \cos\left(\frac{a_rho y \pi y}{L}\right) \right. \right. \\
& \left. \left. + rho_z \sin\left(\frac{a_rho z \pi z}{L}\right) \right)^2 \left( u_0 + u_x \sin\left(\frac{a_u x \pi x}{L}\right) + u_y \cos\left(\frac{a_u y \pi y}{L}\right) \right. \right. \\
& \left. \left. + u_z \cos\left(\frac{a_u z \pi z}{L}\right) \right) \gamma \right) a_{uy} \pi \Big) \Big/ \left( (\gamma - 1) L \left( rho_0 \right. \right. \\
& \left. \left. + rho_x \sin\left(\frac{a_rho x \pi x}{L}\right) + rho_y \cos\left(\frac{a_rho y \pi y}{L}\right) \right. \right. \\
& \left. \left. + rho_z \sin\left(\frac{a_rho z \pi z}{L}\right) \right) \right) - \left( \left( w_0 + w_x \sin\left(\frac{a_w x \pi x}{L}\right) \right. \right. \\
& \left. \left. + w_y \sin\left(\frac{a_w y \pi y}{L}\right) + w_z \cos\left(\frac{a_w z \pi z}{L}\right) \right) \left( \right. \right. \\
& \left. \left. - \sin\left(\frac{a_u z \pi z}{L}\right) u_z \right. \left( rho_0 + rho_x \sin\left(\frac{a_rho x \pi x}{L}\right) \right. \right. \\
& \left. \left. + rho_y \cos\left(\frac{a_rho y \pi y}{L}\right) + rho_z \sin\left(\frac{a_rho z \pi z}{L}\right) \right)^2 \left( u_0 \right. \right. \\
& \left. \left. + u_x \sin\left(\frac{a_u x \pi x}{L}\right) + u_y \cos\left(\frac{a_u y \pi y}{L}\right) + u_z \cos\left(\frac{a_u z \pi z}{L}\right) \right) \right. \right. \\
& \left. \left. + \sin\left(\frac{a_u z \pi z}{L}\right) u_z \right. \left( rho_0 + rho_x \sin\left(\frac{a_rho x \pi x}{L}\right) \right. \right. \\
& \left. \left. + rho_y \cos\left(\frac{a_rho y \pi y}{L}\right) + rho_z \sin\left(\frac{a_rho z \pi z}{L}\right) \right)^2 \left( u_0 \right. \right)
\end{aligned}$$

$$\begin{aligned}
& + u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) + u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) + u_z \cos\left(\frac{a_{uz}\pi z}{L}\right)\right) \gamma \Big) a_{uz} \pi \Big) \\
& \Big/ \left( (\gamma - 1) L \left( rho_0 + rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) + rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \right. \right. \\
& \left. \left. + rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \right) \right) - \left( \left( u_0 + u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \right. \right. \\
& \left. \left. + u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) + u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \right) \right) \left( \right. \\
& \left. - \sin\left(\frac{a_{vx}\pi x}{L}\right) v_x \left( rho_0 + rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \right. \right. \\
& \left. \left. + rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) + rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \right)^2 \left( v_0 \right. \right. \\
& \left. \left. + v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) + v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) + v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) \right) \right) \\
& \left. + \sin\left(\frac{a_{vx}\pi x}{L}\right) v_x \left( rho_0 + rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \right. \right. \\
& \left. \left. + rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) + rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \right)^2 \left( v_0 \right. \right. \\
& \left. \left. + v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) + v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) + v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) \right) \right) \gamma \Big) a_{vx} \pi \Big) \\
& \Big/ \left( (\gamma - 1) L \left( rho_0 + rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) + rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \right. \right. \\
& \left. \left. + rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \right) \right) + \frac{1}{2} \frac{1}{L(\gamma - 1)} \left( \left( 2 p_x \cos\left(\frac{a_{px}\pi x}{L}\right) \gamma \right. \right. \\
& \left. \left. + 2 p_y \sin\left(\frac{a_{py}\pi y}{L}\right) \gamma + 2 p_z \cos\left(\frac{a_{pz}\pi z}{L}\right) \gamma - u_0^2 rho_0 - 3 v_0^2 rho_0 \right. \right. \\
& \left. \left. - w_0^2 rho_0 - u_0^2 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) - u_0^2 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \right. \right. \\
& \left. \left. - u_0^2 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) - u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 rho_0 \right. \right. \\
& \left. \left. - u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 rho_0 - u_z^2 \cos\left(\frac{a_{uz}\pi z}{L}\right)^2 rho_0 + u_0^2 \gamma rho_0 \right. \right. \\
& \left. \left. - 3 v_0^2 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) - 3 v_0^2 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \right. \right. \\
& \left. \left. - 3 v_0^2 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) - w_0^2 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \right. \right)
\end{aligned}$$

$$\begin{aligned}
& -w_{\_0}^2 \rho_{\_x} \sin\left(\frac{a_{\_rho} x \pi x}{L}\right) - w_{\_0}^2 \rho_{\_y} \cos\left(\frac{a_{\_rho} y \pi y}{L}\right) \\
& + 3 \gamma v_{\_0}^2 \rho_{\_0} + \gamma w_{\_0}^2 \rho_{\_0} - 3 v_{\_z}^2 \sin\left(\frac{a_{\_vz} \pi z}{L}\right)^2 \rho_{\_0} \\
& - 3 v_{\_x}^2 \cos\left(\frac{a_{\_vx} \pi x}{L}\right)^2 \rho_{\_0} - 3 v_{\_y}^2 \sin\left(\frac{a_{\_vy} \pi y}{L}\right)^2 \rho_{\_0} \\
& - w_{\_z}^2 \cos\left(\frac{a_{\_wz} \pi z}{L}\right)^2 \rho_{\_0} - w_{\_x}^2 \sin\left(\frac{a_{\_wx} \pi x}{L}\right)^2 \rho_{\_0} \\
& - w_{\_y}^2 \sin\left(\frac{a_{\_wy} \pi y}{L}\right)^2 \rho_{\_0} - u_{\_x}^2 \sin\left(\frac{a_{\_ux} \pi x}{L}\right)^2 \rho_{\_z} \sin\left(\frac{a_{\_rho} z \pi z}{L}\right) \\
& - u_{\_x}^2 \sin\left(\frac{a_{\_ux} \pi x}{L}\right)^2 \rho_{\_x} \sin\left(\frac{a_{\_rho} x \pi x}{L}\right) \\
& - u_{\_x}^2 \sin\left(\frac{a_{\_ux} \pi x}{L}\right)^2 \rho_{\_y} \cos\left(\frac{a_{\_rho} y \pi y}{L}\right) \\
& - u_{\_y}^2 \cos\left(\frac{a_{\_uy} \pi y}{L}\right)^2 \rho_{\_z} \sin\left(\frac{a_{\_rho} z \pi z}{L}\right) \\
& - u_{\_y}^2 \cos\left(\frac{a_{\_uy} \pi y}{L}\right)^2 \rho_{\_x} \sin\left(\frac{a_{\_rho} x \pi x}{L}\right) \\
& - u_{\_y}^2 \cos\left(\frac{a_{\_uy} \pi y}{L}\right)^2 \rho_{\_y} \cos\left(\frac{a_{\_rho} y \pi y}{L}\right) \\
& - u_{\_z}^2 \cos\left(\frac{a_{\_uz} \pi z}{L}\right)^2 \rho_{\_z} \sin\left(\frac{a_{\_rho} z \pi z}{L}\right) \\
& - u_{\_z}^2 \cos\left(\frac{a_{\_uz} \pi z}{L}\right)^2 \rho_{\_x} \sin\left(\frac{a_{\_rho} x \pi x}{L}\right) \\
& - u_{\_z}^2 \cos\left(\frac{a_{\_uz} \pi z}{L}\right)^2 \rho_{\_y} \cos\left(\frac{a_{\_rho} y \pi y}{L}\right) \\
& + u_{\_0}^2 \gamma \rho_{\_z} \sin\left(\frac{a_{\_rho} z \pi z}{L}\right) + u_{\_0}^2 \gamma \rho_{\_x} \sin\left(\frac{a_{\_rho} x \pi x}{L}\right) \\
& + u_{\_0}^2 \gamma \rho_{\_y} \cos\left(\frac{a_{\_rho} y \pi y}{L}\right) - 2 u_{\_0} u_{\_x} \sin\left(\frac{a_{\_ux} \pi x}{L}\right) \rho_{\_0} \\
& - 2 u_{\_0} u_{\_y} \cos\left(\frac{a_{\_uy} \pi y}{L}\right) \rho_{\_0} - 2 u_{\_0} u_{\_z} \cos\left(\frac{a_{\_uz} \pi z}{L}\right) \rho_{\_0} \\
& + u_{\_x}^2 \sin\left(\frac{a_{\_ux} \pi x}{L}\right)^2 \gamma \rho_{\_0} + u_{\_y}^2 \cos\left(\frac{a_{\_uy} \pi y}{L}\right)^2 \gamma \rho_{\_0} \\
& + u_{\_z}^2 \cos\left(\frac{a_{\_uz} \pi z}{L}\right)^2 \gamma \rho_{\_0} + 3 \gamma v_{\_0}^2 \rho_{\_z} \sin\left(\frac{a_{\_rho} z \pi z}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + 3 \gamma v_- \theta^2 rho_x \sin\left(\frac{a_{rho} \pi x}{L}\right) + 3 \gamma v_- \theta^2 rho_y \cos\left(\frac{a_{rho} \pi y}{L}\right) \\
& - 6 v_z \sin\left(\frac{a_{vz} \pi z}{L}\right) v_0 rho_0 - 6 v_0 v_x \cos\left(\frac{a_{vx} \pi x}{L}\right) rho_0 \\
& - 6 v_0 v_y \sin\left(\frac{a_{vy} \pi y}{L}\right) rho_0 + \gamma w_- \theta^2 rho_z \sin\left(\frac{a_{rho} \pi z}{L}\right) \\
& + \gamma w_- \theta^2 rho_x \sin\left(\frac{a_{rho} \pi x}{L}\right) + \gamma w_- \theta^2 rho_y \cos\left(\frac{a_{rho} \pi y}{L}\right) \\
& - 2 w_z \cos\left(\frac{a_{wz} \pi z}{L}\right) w_0 rho_0 - 2 w_0 w_x \sin\left(\frac{a_{wx} \pi x}{L}\right) rho_0 \\
& - 2 w_0 w_y \sin\left(\frac{a_{wy} \pi y}{L}\right) rho_0 + 3 \gamma v_z^2 \sin^2\left(\frac{a_{vz} \pi z}{L}\right) rho_0 \\
& + 3 \gamma v_x^2 \cos^2\left(\frac{a_{vx} \pi x}{L}\right) rho_0 + 3 \gamma v_y^2 \sin^2\left(\frac{a_{vy} \pi y}{L}\right) rho_0 \\
& + \gamma w_z^2 \cos^2\left(\frac{a_{wz} \pi z}{L}\right) rho_0 + \gamma w_x^2 \sin^2\left(\frac{a_{wx} \pi x}{L}\right) rho_0 \\
& + \gamma w_y^2 \sin^2\left(\frac{a_{wy} \pi y}{L}\right) rho_0 \\
& - 3 v_z^2 \sin^2\left(\frac{a_{vz} \pi z}{L}\right) rho_z \sin\left(\frac{a_{rho} \pi z}{L}\right) \\
& - 3 v_z^2 \sin^2\left(\frac{a_{vz} \pi z}{L}\right) rho_x \sin\left(\frac{a_{rho} \pi x}{L}\right) \\
& - 3 v_z^2 \sin^2\left(\frac{a_{vz} \pi z}{L}\right) rho_y \cos\left(\frac{a_{rho} \pi y}{L}\right) \\
& - 3 v_x^2 \cos^2\left(\frac{a_{vx} \pi x}{L}\right) rho_z \sin\left(\frac{a_{rho} \pi z}{L}\right) \\
& - 3 v_x^2 \cos^2\left(\frac{a_{vx} \pi x}{L}\right) rho_x \sin\left(\frac{a_{rho} \pi x}{L}\right) \\
& - 3 v_x^2 \cos^2\left(\frac{a_{vx} \pi x}{L}\right) rho_y \cos\left(\frac{a_{rho} \pi y}{L}\right) \\
& - 3 v_y^2 \sin^2\left(\frac{a_{vy} \pi y}{L}\right) rho_z \sin\left(\frac{a_{rho} \pi z}{L}\right) \\
& - 3 v_y^2 \sin^2\left(\frac{a_{vy} \pi y}{L}\right) rho_x \sin\left(\frac{a_{rho} \pi x}{L}\right) \\
& - 3 v_y^2 \sin^2\left(\frac{a_{vy} \pi y}{L}\right) rho_y \cos\left(\frac{a_{rho} \pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -w_z^2 \cos\left(\frac{a_wz\pi z}{L}\right)^2 rho_z \sin\left(\frac{a_rhoz\pi z}{L}\right) \\
& -w_z^2 \cos\left(\frac{a_wz\pi z}{L}\right)^2 rho_x \sin\left(\frac{a_rhox\pi x}{L}\right) \\
& -w_z^2 \cos\left(\frac{a_wz\pi z}{L}\right)^2 rho_y \cos\left(\frac{a_rho y\pi y}{L}\right) \\
& -w_x^2 \sin\left(\frac{a_wx\pi x}{L}\right)^2 rho_z \sin\left(\frac{a_rhoz\pi z}{L}\right) \\
& -w_x^2 \sin\left(\frac{a_wx\pi x}{L}\right)^2 rho_x \sin\left(\frac{a_rhox\pi x}{L}\right) \\
& -w_x^2 \sin\left(\frac{a_wx\pi x}{L}\right)^2 rho_y \cos\left(\frac{a_rho y\pi y}{L}\right) \\
& -w_y^2 \sin\left(\frac{a_wy\pi y}{L}\right)^2 rho_z \sin\left(\frac{a_rhoz\pi z}{L}\right) \\
& -w_y^2 \sin\left(\frac{a_wy\pi y}{L}\right)^2 rho_x \sin\left(\frac{a_rhox\pi x}{L}\right) \\
& -w_y^2 \sin\left(\frac{a_wy\pi y}{L}\right)^2 rho_y \cos\left(\frac{a_rho y\pi y}{L}\right) \\
& +2 u_x \sin\left(\frac{a_ux\pi x}{L}\right) u_y \cos\left(\frac{a_uy\pi y}{L}\right) \gamma rho_z \sin\left(\frac{a_rhoz\pi z}{L}\right) \\
& +2 u_x \sin\left(\frac{a_ux\pi x}{L}\right) u_y \cos\left(\frac{a_uy\pi y}{L}\right) \gamma rho_x \sin\left(\frac{a_rhox\pi x}{L}\right) \\
& +2 u_x \sin\left(\frac{a_ux\pi x}{L}\right) u_y \cos\left(\frac{a_uy\pi y}{L}\right) \gamma rho_y \cos\left(\frac{a_rho y\pi y}{L}\right) \\
& +2 u_x \sin\left(\frac{a_ux\pi x}{L}\right) u_z \cos\left(\frac{a_uz\pi z}{L}\right) \gamma rho_z \sin\left(\frac{a_rhoz\pi z}{L}\right) \\
& +2 u_x \sin\left(\frac{a_ux\pi x}{L}\right) u_z \cos\left(\frac{a_uz\pi z}{L}\right) \gamma rho_x \sin\left(\frac{a_rhox\pi x}{L}\right) \\
& +2 u_x \sin\left(\frac{a_ux\pi x}{L}\right) u_z \cos\left(\frac{a_uz\pi z}{L}\right) \gamma rho_y \cos\left(\frac{a_rho y\pi y}{L}\right) \\
& +2 u_y \cos\left(\frac{a_uy\pi y}{L}\right) u_z \cos\left(\frac{a_uz\pi z}{L}\right) \gamma rho_z \sin\left(\frac{a_rhoz\pi z}{L}\right) \\
& +2 u_y \cos\left(\frac{a_uy\pi y}{L}\right) u_z \cos\left(\frac{a_uz\pi z}{L}\right) \gamma rho_x \sin\left(\frac{a_rhox\pi x}{L}\right) \\
& +2 u_y \cos\left(\frac{a_uy\pi y}{L}\right) u_z \cos\left(\frac{a_uz\pi z}{L}\right) \gamma rho_y \cos\left(\frac{a_rho y\pi y}{L}\right)
\end{aligned}$$



$$\begin{aligned}
& -2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& -2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& -2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& -2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& -2 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& -2 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -2 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& +2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& +2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& +2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& +2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& +2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& +2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& +2 u_0 u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& +2 u_0 u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& +2 u_0 u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_0 \\
& + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_0 \\
& + 2 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_0 \\
& + 6 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_0 \\
& + 6 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_0 \\
& + 6 \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_0 \\
& + 2 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_0 \\
& + 2 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_0 \\
& + 2 \gamma w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_0 \\
& - 6 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& - 6 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& - 6 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& - 6 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& - 6 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& - 6 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& - 6 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& - 6 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& - 6 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + 6 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 \rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 6 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 \rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 6 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 \rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 6 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 6 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 6 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 6 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 6 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 6 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 2 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_0 \rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 2 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_0 \rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_0 \rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 2 \gamma w_0 w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) \rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 2 \gamma w_0 w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) \rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 \gamma w_0 w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) \rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 2 \gamma w_0 w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) \rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 2 \gamma w_0 w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) \rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 \gamma w_0 w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) \rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -2 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_x \sin\left(\frac{a_w x \pi x}{L}\right) rho_z \sin\left(\frac{a_r hoz \pi z}{L}\right) \\
& -2 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_x \sin\left(\frac{a_w x \pi x}{L}\right) rho_x \sin\left(\frac{a_r rho x \pi x}{L}\right) \\
& -2 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_x \sin\left(\frac{a_w x \pi x}{L}\right) rho_y \cos\left(\frac{a_r rho y \pi y}{L}\right) \\
& -2 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_z \sin\left(\frac{a_r hoz \pi z}{L}\right) \\
& -2 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_x \sin\left(\frac{a_r rho x \pi x}{L}\right) \\
& -2 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_y \cos\left(\frac{a_r rho y \pi y}{L}\right) \\
& -2 w_x \sin\left(\frac{a_w x \pi x}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_z \sin\left(\frac{a_r hoz \pi z}{L}\right) \\
& -2 w_x \sin\left(\frac{a_w x \pi x}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_x \sin\left(\frac{a_r rho x \pi x}{L}\right) \\
& -2 w_x \sin\left(\frac{a_w x \pi x}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_y \cos\left(\frac{a_r rho y \pi y}{L}\right) \\
& -2 u_x \sin\left(\frac{a_u x \pi x}{L}\right) u_y \cos\left(\frac{a_u y \pi y}{L}\right) rho_0 \\
& -2 u_x \sin\left(\frac{a_u x \pi x}{L}\right) u_z \cos\left(\frac{a_u z \pi z}{L}\right) rho_0 \\
& -2 u_y \cos\left(\frac{a_u y \pi y}{L}\right) u_z \cos\left(\frac{a_u z \pi z}{L}\right) rho_0 \\
& +2 u_0 u_x \sin\left(\frac{a_u x \pi x}{L}\right) \gamma rho_0 + 2 u_0 u_y \cos\left(\frac{a_u y \pi y}{L}\right) \gamma rho_0 \\
& +2 u_0 u_z \cos\left(\frac{a_u z \pi z}{L}\right) \gamma rho_0 \\
& -2 u_0 u_x \sin\left(\frac{a_u x \pi x}{L}\right) rho_z \sin\left(\frac{a_r hoz \pi z}{L}\right) \\
& -2 u_0 u_x \sin\left(\frac{a_u x \pi x}{L}\right) rho_x \sin\left(\frac{a_r rho x \pi x}{L}\right) \\
& -2 u_0 u_x \sin\left(\frac{a_u x \pi x}{L}\right) rho_y \cos\left(\frac{a_r rho y \pi y}{L}\right) \\
& -2 u_0 u_y \cos\left(\frac{a_u y \pi y}{L}\right) rho_z \sin\left(\frac{a_r hoz \pi z}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& -2 u_0 u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& -2 u_0 u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -2 u_0 u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 \gamma rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 \gamma rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 \gamma rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + u_z^2 \cos\left(\frac{a_{uz}\pi z}{L}\right)^2 \gamma rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + u_z^2 \cos\left(\frac{a_{uz}\pi z}{L}\right)^2 \gamma rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + u_z^2 \cos\left(\frac{a_{uz}\pi z}{L}\right)^2 \gamma rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& - 6 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& - 6 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& - 6 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& - 6 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -6 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -6 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& -6 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& -6 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -6 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& -2 w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_0 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& -2 w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_0 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -2 w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_0 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& -2 w_0 w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& -2 w_0 w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -2 w_0 w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& -2 w_0 w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& -2 w_0 w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -2 w_0 w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& +3 \gamma v_z^2 \sin\left(\frac{a_{vz}\pi z}{L}\right)^2 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& +3 \gamma v_z^2 \sin\left(\frac{a_{vz}\pi z}{L}\right)^2 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& +3 \gamma v_z^2 \sin\left(\frac{a_{vz}\pi z}{L}\right)^2 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& +3 \gamma v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + 3 \gamma v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 3 \gamma v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 3 \gamma v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 3 \gamma v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 3 \gamma v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& - 6 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_0 \\
& - 6 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_0 \\
& - 6 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_0 \\
& + 6 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 rho_0 + 6 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_0 \\
& + 6 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_0 \\
& + \gamma w_z^2 \cos\left(\frac{a_{wz}\pi z}{L}\right)^2 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + \gamma w_z^2 \cos\left(\frac{a_{wz}\pi z}{L}\right)^2 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + \gamma w_z^2 \cos\left(\frac{a_{wz}\pi z}{L}\right)^2 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + \gamma w_x^2 \sin\left(\frac{a_{wx}\pi x}{L}\right)^2 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + \gamma w_x^2 \sin\left(\frac{a_{wx}\pi x}{L}\right)^2 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + \gamma w_x^2 \sin\left(\frac{a_{wx}\pi x}{L}\right)^2 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + \gamma w_y^2 \sin\left(\frac{a_{wy}\pi y}{L}\right)^2 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + \gamma w_y^2 \sin\left(\frac{a_{wy}\pi y}{L}\right)^2 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + \gamma w_y^2 \sin\left(\frac{a_w y \pi y}{L}\right)^2 rho_y \cos\left(\frac{a_r h o y \pi y}{L}\right) \\
& + 2 \gamma w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_0 rho_0 + 2 \gamma w_0 w_x \sin\left(\frac{a_w x \pi x}{L}\right) rho_0 \\
& + 2 \gamma w_0 w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_0 \\
& - 2 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_x \sin\left(\frac{a_w x \pi x}{L}\right) rho_0 \\
& - 2 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_0 \\
& - 2 w_x \sin\left(\frac{a_w x \pi x}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_0 + 2 p_0 \gamma \\
& v_y \cos\left(\frac{a_v y \pi y}{L}\right) a_v y \pi - \left( \left( w_0 + w_x \sin\left(\frac{a_w x \pi x}{L}\right) \right. \right. \\
& \left. \left. + w_y \sin\left(\frac{a_w y \pi y}{L}\right) + w_z \cos\left(\frac{a_w z \pi z}{L}\right) \right) \right. \\
& \left( \cos\left(\frac{a_v z \pi z}{L}\right) v_z \left( rho_0 + rho_x \sin\left(\frac{a_r h o x \pi x}{L}\right) \right. \right. \\
& \left. \left. + rho_y \cos\left(\frac{a_r h o y \pi y}{L}\right) + rho_z \sin\left(\frac{a_r h o z \pi z}{L}\right) \right)^2 \left( v_0 \right. \right. \\
& \left. \left. + v_x \cos\left(\frac{a_v x \pi x}{L}\right) + v_y \sin\left(\frac{a_v y \pi y}{L}\right) + v_z \sin\left(\frac{a_v z \pi z}{L}\right) \right) \right. \\
& - \cos\left(\frac{a_v z \pi z}{L}\right) v_z \left( rho_0 + rho_x \sin\left(\frac{a_r h o x \pi x}{L}\right) \right. \\
& \left. + rho_y \cos\left(\frac{a_r h o y \pi y}{L}\right) + rho_z \sin\left(\frac{a_r h o z \pi z}{L}\right) \right)^2 \left( v_0 \right. \\
& \left. + v_x \cos\left(\frac{a_v x \pi x}{L}\right) + v_y \sin\left(\frac{a_v y \pi y}{L}\right) + v_z \sin\left(\frac{a_v z \pi z}{L}\right) \right) \gamma \right) a_v z \pi \Big) \\
& \Big/ \left( (\gamma - 1) L \left( rho_0 + rho_x \sin\left(\frac{a_r h o x \pi x}{L}\right) + rho_y \cos\left(\frac{a_r h o y \pi y}{L}\right) \right. \right. \\
& \left. \left. + rho_z \sin\left(\frac{a_r h o z \pi z}{L}\right) \right) \right) - \left( \left( u_0 + u_x \sin\left(\frac{a_u x \pi x}{L}\right) \right. \right. \\
& \left. \left. + u_y \cos\left(\frac{a_u y \pi y}{L}\right) + u_z \cos\left(\frac{a_u z \pi z}{L}\right) \right) \right. \\
& \left( \cos\left(\frac{a_w x \pi x}{L}\right) w_x \left( rho_0 + rho_x \sin\left(\frac{a_r h o x \pi x}{L}\right) \right. \right. \\
\end{aligned}$$

$$\begin{aligned}
& + rho\_y \cos\left(\frac{a\_rho y \pi y}{L}\right) + rho\_z \sin\left(\frac{a\_rho z \pi z}{L}\right)\right)^2 \left(w\_0 \right. \\
& + w\_x \sin\left(\frac{a\_wx \pi x}{L}\right) + w\_y \sin\left(\frac{a\_wy \pi y}{L}\right) + w\_z \cos\left(\frac{a\_wz \pi z}{L}\right) \\
& - \cos\left(\frac{a\_wx \pi x}{L}\right) w\_x \left(rho\_0 + rho\_x \sin\left(\frac{a\_rho x \pi x}{L}\right) \right. \\
& + rho\_y \cos\left(\frac{a\_rho y \pi y}{L}\right) + rho\_z \sin\left(\frac{a\_rho z \pi z}{L}\right)\right)^2 \left(w\_0 \right. \\
& + w\_x \sin\left(\frac{a\_wx \pi x}{L}\right) + w\_y \sin\left(\frac{a\_wy \pi y}{L}\right) + w\_z \cos\left(\frac{a\_wz \pi z}{L}\right) \\
& \left.\left.\right)\gamma\right) \\
& a\_wx \pi\Big) \Big/ \left((\gamma - 1) L \left(rho\_0 + rho\_x \sin\left(\frac{a\_rho x \pi x}{L}\right) \right. \right. \\
& + rho\_y \cos\left(\frac{a\_rho y \pi y}{L}\right) + rho\_z \sin\left(\frac{a\_rho z \pi z}{L}\right)\Big)\Big) - \left( \left(v\_0 \right. \right. \\
& + v\_x \cos\left(\frac{a\_vx \pi x}{L}\right) + v\_y \sin\left(\frac{a\_vy \pi y}{L}\right) + v\_z \sin\left(\frac{a\_vz \pi z}{L}\right) \\
& \left.\left.\right) \right. \\
& \left(\cos\left(\frac{a\_wy \pi y}{L}\right) w\_y \left(rho\_0 + rho\_x \sin\left(\frac{a\_rho x \pi x}{L}\right) \right. \right. \\
& + rho\_y \cos\left(\frac{a\_rho y \pi y}{L}\right) + rho\_z \sin\left(\frac{a\_rho z \pi z}{L}\right)\right)^2 \left(w\_0 \right. \\
& + w\_x \sin\left(\frac{a\_wx \pi x}{L}\right) + w\_y \sin\left(\frac{a\_wy \pi y}{L}\right) + w\_z \cos\left(\frac{a\_wz \pi z}{L}\right) \\
& - \cos\left(\frac{a\_wy \pi y}{L}\right) w\_y \left(rho\_0 + rho\_x \sin\left(\frac{a\_rho x \pi x}{L}\right) \right. \\
& + rho\_y \cos\left(\frac{a\_rho y \pi y}{L}\right) + rho\_z \sin\left(\frac{a\_rho z \pi z}{L}\right)\right)^2 \left(w\_0 \right. \\
& + w\_x \sin\left(\frac{a\_wx \pi x}{L}\right) + w\_y \sin\left(\frac{a\_wy \pi y}{L}\right) + w\_z \cos\left(\frac{a\_wz \pi z}{L}\right) \\
& \left.\left.\right)\gamma\right) \\
& a\_wy \pi\Big) \Big/ \left((\gamma - 1) L \left(rho\_0 + rho\_x \sin\left(\frac{a\_rho x \pi x}{L}\right) \right. \right. \\
& + rho\_y \cos\left(\frac{a\_rho y \pi y}{L}\right) + rho\_z \sin\left(\frac{a\_rho z \pi z}{L}\right)\Big)\Big) \\
& - \frac{1}{2} \frac{1}{L(\gamma - 1)} \left( \left(2 p\_x \cos\left(\frac{a\_px \pi x}{L}\right) \gamma + 2 p\_y \sin\left(\frac{a\_py \pi y}{L}\right) \gamma \right. \right. \\
& + 2 p\_z \cos\left(\frac{a\_pz \pi z}{L}\right) \gamma - u\_0^2 rho\_0 - v\_0^2 rho\_0 - 3 w\_0^2 rho\_0
\end{aligned}$$

$$\begin{aligned}
& - u_{\_0}^2 \rho_{\_z} \sin\left(\frac{a_{\_rhoz}\pi z}{L}\right) - u_{\_0}^2 \rho_{\_x} \sin\left(\frac{a_{\_rhox}\pi x}{L}\right) \\
& - u_{\_0}^2 \rho_{\_y} \cos\left(\frac{a_{\_rhoy}\pi y}{L}\right) - u_{\_x}^2 \sin\left(\frac{a_{\_ux}\pi x}{L}\right)^2 \rho_{\_0} \\
& - u_{\_y}^2 \cos\left(\frac{a_{\_uy}\pi y}{L}\right)^2 \rho_{\_0} - u_{\_z}^2 \cos\left(\frac{a_{\_uz}\pi z}{L}\right)^2 \rho_{\_0} + u_{\_0}^2 \gamma \rho_{\_0} \\
& - v_{\_0}^2 \rho_{\_z} \sin\left(\frac{a_{\_rhoz}\pi z}{L}\right) - v_{\_0}^2 \rho_{\_x} \sin\left(\frac{a_{\_rhox}\pi x}{L}\right) \\
& - v_{\_0}^2 \rho_{\_y} \cos\left(\frac{a_{\_rhoy}\pi y}{L}\right) - 3 w_{\_0}^2 \rho_{\_z} \sin\left(\frac{a_{\_rhoz}\pi z}{L}\right) \\
& - 3 w_{\_0}^2 \rho_{\_x} \sin\left(\frac{a_{\_rhox}\pi x}{L}\right) - 3 w_{\_0}^2 \rho_{\_y} \cos\left(\frac{a_{\_rhoy}\pi y}{L}\right) \\
& + \gamma v_{\_0}^2 \rho_{\_0} + 3 \gamma w_{\_0}^2 \rho_{\_0} - v_{\_z}^2 \sin\left(\frac{a_{\_vz}\pi z}{L}\right)^2 \rho_{\_0} \\
& - v_{\_x}^2 \cos\left(\frac{a_{\_vx}\pi x}{L}\right)^2 \rho_{\_0} - v_{\_y}^2 \sin\left(\frac{a_{\_vy}\pi y}{L}\right)^2 \rho_{\_0} \\
& - 3 w_{\_z}^2 \cos\left(\frac{a_{\_wz}\pi z}{L}\right)^2 \rho_{\_0} - 3 w_{\_x}^2 \sin\left(\frac{a_{\_wx}\pi x}{L}\right)^2 \rho_{\_0} \\
& - 3 w_{\_y}^2 \sin\left(\frac{a_{\_wy}\pi y}{L}\right)^2 \rho_{\_0} \\
& - u_{\_x}^2 \sin\left(\frac{a_{\_ux}\pi x}{L}\right)^2 \rho_{\_z} \sin\left(\frac{a_{\_rhoz}\pi z}{L}\right) \\
& - u_{\_x}^2 \sin\left(\frac{a_{\_ux}\pi x}{L}\right)^2 \rho_{\_x} \sin\left(\frac{a_{\_rhox}\pi x}{L}\right) \\
& - u_{\_x}^2 \sin\left(\frac{a_{\_ux}\pi x}{L}\right)^2 \rho_{\_y} \cos\left(\frac{a_{\_rhoy}\pi y}{L}\right) \\
& - u_{\_y}^2 \cos\left(\frac{a_{\_uy}\pi y}{L}\right)^2 \rho_{\_z} \sin\left(\frac{a_{\_rhoz}\pi z}{L}\right) \\
& - u_{\_y}^2 \cos\left(\frac{a_{\_uy}\pi y}{L}\right)^2 \rho_{\_x} \sin\left(\frac{a_{\_rhox}\pi x}{L}\right) \\
& - u_{\_y}^2 \cos\left(\frac{a_{\_uy}\pi y}{L}\right)^2 \rho_{\_y} \cos\left(\frac{a_{\_rhoy}\pi y}{L}\right) \\
& - u_{\_z}^2 \cos\left(\frac{a_{\_uz}\pi z}{L}\right)^2 \rho_{\_z} \sin\left(\frac{a_{\_rhoz}\pi z}{L}\right) \\
& - u_{\_z}^2 \cos\left(\frac{a_{\_uz}\pi z}{L}\right)^2 \rho_{\_x} \sin\left(\frac{a_{\_rhox}\pi x}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -u_z^2 \cos\left(\frac{a_{uz}\pi z}{L}\right)^2 rho_y \cos\left(\frac{a_{rho y}\pi y}{L}\right) \\
& + u_0^2 \gamma rho_z \sin\left(\frac{a_{rho z}\pi z}{L}\right) + u_0^2 \gamma rho_x \sin\left(\frac{a_{rho x}\pi x}{L}\right) \\
& + u_0^2 \gamma rho_y \cos\left(\frac{a_{rho y}\pi y}{L}\right) - 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) rho_0 \\
& - 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) rho_0 - 2 u_0 u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) rho_0 \\
& + u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 \gamma rho_0 + u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma rho_0 \\
& + u_z^2 \cos\left(\frac{a_{uz}\pi z}{L}\right)^2 \gamma rho_0 + \gamma v_0^2 rho_z \sin\left(\frac{a_{rho z}\pi z}{L}\right) \\
& + \gamma v_0^2 rho_x \sin\left(\frac{a_{rho x}\pi x}{L}\right) + \gamma v_0^2 rho_y \cos\left(\frac{a_{rho y}\pi y}{L}\right) \\
& - 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 rho_0 - 2 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_0 \\
& - 2 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_0 + 3 \gamma w_0^2 rho_z \sin\left(\frac{a_{rho z}\pi z}{L}\right) \\
& + 3 \gamma w_0^2 rho_x \sin\left(\frac{a_{rho x}\pi x}{L}\right) + 3 \gamma w_0^2 rho_y \cos\left(\frac{a_{rho y}\pi y}{L}\right) \\
& - 6 w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_0 rho_0 - 6 w_0 w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_0 \\
& - 6 w_0 w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_0 + \gamma v_z^2 \sin\left(\frac{a_{vz}\pi z}{L}\right)^2 rho_0 \\
& + \gamma v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 rho_0 + \gamma v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 rho_0 \\
& + 3 \gamma w_z^2 \cos\left(\frac{a_{wz}\pi z}{L}\right)^2 rho_0 + 3 \gamma w_x^2 \sin\left(\frac{a_{wx}\pi x}{L}\right)^2 rho_0 \\
& + 3 \gamma w_y^2 \sin\left(\frac{a_{wy}\pi y}{L}\right)^2 rho_0 \\
& - v_z^2 \sin\left(\frac{a_{vz}\pi z}{L}\right)^2 rho_z \sin\left(\frac{a_{rho z}\pi z}{L}\right) \\
& - v_z^2 \sin\left(\frac{a_{vz}\pi z}{L}\right)^2 rho_x \sin\left(\frac{a_{rho x}\pi x}{L}\right) \\
& - v_z^2 \sin\left(\frac{a_{vz}\pi z}{L}\right)^2 rho_y \cos\left(\frac{a_{rho y}\pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& -v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -v_x^2 \cos\left(\frac{a_{vx}\pi x}{L}\right)^2 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& -v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& -v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -v_y^2 \sin\left(\frac{a_{vy}\pi y}{L}\right)^2 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& -3 w_z^2 \cos\left(\frac{a_{wz}\pi z}{L}\right)^2 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& -3 w_z^2 \cos\left(\frac{a_{wz}\pi z}{L}\right)^2 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -3 w_z^2 \cos\left(\frac{a_{wz}\pi z}{L}\right)^2 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& -3 w_x^2 \sin\left(\frac{a_{wx}\pi x}{L}\right)^2 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& -3 w_x^2 \sin\left(\frac{a_{wx}\pi x}{L}\right)^2 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -3 w_x^2 \sin\left(\frac{a_{wx}\pi x}{L}\right)^2 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& -3 w_y^2 \sin\left(\frac{a_{wy}\pi y}{L}\right)^2 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& -3 w_y^2 \sin\left(\frac{a_{wy}\pi y}{L}\right)^2 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -3 w_y^2 \sin\left(\frac{a_{wy}\pi y}{L}\right)^2 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& +2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& +2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& +2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 2 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 2 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + 6 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 6 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 6 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + 6 \gamma w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_z \sin\left(\frac{a_rho z \pi z}{L}\right) \\
& + 6 \gamma w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_x \sin\left(\frac{a_rho x \pi x}{L}\right) \\
& + 6 \gamma w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_y \cos\left(\frac{a_rho y \pi y}{L}\right) \\
& + 6 \gamma w_x \sin\left(\frac{a_w x \pi x}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_z \sin\left(\frac{a_rho z \pi z}{L}\right) \\
& + 6 \gamma w_x \sin\left(\frac{a_w x \pi x}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_x \sin\left(\frac{a_rho x \pi x}{L}\right) \\
& + 6 \gamma w_x \sin\left(\frac{a_w x \pi x}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_y \cos\left(\frac{a_rho y \pi y}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_u x \pi x}{L}\right) u_y \cos\left(\frac{a_u y \pi y}{L}\right) rho_z \sin\left(\frac{a_rho z \pi z}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_u x \pi x}{L}\right) u_y \cos\left(\frac{a_u y \pi y}{L}\right) rho_x \sin\left(\frac{a_rho x \pi x}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_u x \pi x}{L}\right) u_y \cos\left(\frac{a_u y \pi y}{L}\right) rho_y \cos\left(\frac{a_rho y \pi y}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_u x \pi x}{L}\right) u_z \cos\left(\frac{a_u z \pi z}{L}\right) rho_z \sin\left(\frac{a_rho z \pi z}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_u x \pi x}{L}\right) u_z \cos\left(\frac{a_u z \pi z}{L}\right) rho_x \sin\left(\frac{a_rho x \pi x}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_u x \pi x}{L}\right) u_z \cos\left(\frac{a_u z \pi z}{L}\right) rho_y \cos\left(\frac{a_rho y \pi y}{L}\right) \\
& - 2 u_y \cos\left(\frac{a_u y \pi y}{L}\right) u_z \cos\left(\frac{a_u z \pi z}{L}\right) rho_z \sin\left(\frac{a_rho z \pi z}{L}\right) \\
& - 2 u_y \cos\left(\frac{a_u y \pi y}{L}\right) u_z \cos\left(\frac{a_u z \pi z}{L}\right) rho_x \sin\left(\frac{a_rho x \pi x}{L}\right) \\
& - 2 u_y \cos\left(\frac{a_u y \pi y}{L}\right) u_z \cos\left(\frac{a_u z \pi z}{L}\right) rho_y \cos\left(\frac{a_rho y \pi y}{L}\right) \\
& + 2 u_0 u_x \sin\left(\frac{a_u x \pi x}{L}\right) \gamma rho_z \sin\left(\frac{a_rho z \pi z}{L}\right) \\
& + 2 u_0 u_x \sin\left(\frac{a_u x \pi x}{L}\right) \gamma rho_x \sin\left(\frac{a_rho x \pi x}{L}\right) \\
& + 2 u_0 u_x \sin\left(\frac{a_u x \pi x}{L}\right) \gamma rho_y \cos\left(\frac{a_rho y \pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_y \cos\left(\frac{a_{rho}\pi y}{L}\right) \\
& + 2 u_0 u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + 2 u_0 u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + 2 u_0 u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_y \cos\left(\frac{a_{rho}\pi y}{L}\right) \\
& + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_0 \\
& + 2 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_0 \\
& + 2 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_0 \\
& + 2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_0 \\
& + 2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_0 \\
& + 2 \gamma v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_0 \\
& + 6 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_0 \\
& + 6 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_0 \\
& + 6 \gamma w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_0 \\
& - 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& - 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& - 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_y \cos\left(\frac{a_{rho}\pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& -2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rho}\pi y}{L}\right) \\
& -2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& -2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& -2 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rho}\pi y}{L}\right) \\
& +2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& +2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& +2 \gamma v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 rho_y \cos\left(\frac{a_{rho}\pi y}{L}\right) \\
& +2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& +2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& +2 \gamma v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_y \cos\left(\frac{a_{rho}\pi y}{L}\right) \\
& +2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& +2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& +2 \gamma v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rho}\pi y}{L}\right) \\
& +6 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_0 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& +6 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_0 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& +6 \gamma w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_0 rho_y \cos\left(\frac{a_{rho}\pi y}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + 6 \gamma w_0 w_x \sin\left(\frac{a_w x \pi x}{L}\right) rho_z \sin\left(\frac{a_r hoz \pi z}{L}\right) \\
& + 6 \gamma w_0 w_x \sin\left(\frac{a_w x \pi x}{L}\right) rho_x \sin\left(\frac{a_r rho x \pi x}{L}\right) \\
& + 6 \gamma w_0 w_x \sin\left(\frac{a_w x \pi x}{L}\right) rho_y \cos\left(\frac{a_r rho y \pi y}{L}\right) \\
& + 6 \gamma w_0 w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_z \sin\left(\frac{a_r hoz \pi z}{L}\right) \\
& + 6 \gamma w_0 w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_x \sin\left(\frac{a_r rho x \pi x}{L}\right) \\
& + 6 \gamma w_0 w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_y \cos\left(\frac{a_r rho y \pi y}{L}\right) \\
& - 6 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_x \sin\left(\frac{a_w x \pi x}{L}\right) rho_z \sin\left(\frac{a_r hoz \pi z}{L}\right) \\
& - 6 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_x \sin\left(\frac{a_w x \pi x}{L}\right) rho_x \sin\left(\frac{a_r rho x \pi x}{L}\right) \\
& - 6 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_x \sin\left(\frac{a_w x \pi x}{L}\right) rho_y \cos\left(\frac{a_r rho y \pi y}{L}\right) \\
& - 6 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_z \sin\left(\frac{a_r hoz \pi z}{L}\right) \\
& - 6 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_x \sin\left(\frac{a_r rho x \pi x}{L}\right) \\
& - 6 w_z \cos\left(\frac{a_w z \pi z}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_y \cos\left(\frac{a_r rho y \pi y}{L}\right) \\
& - 6 w_x \sin\left(\frac{a_w x \pi x}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_z \sin\left(\frac{a_r hoz \pi z}{L}\right) \\
& - 6 w_x \sin\left(\frac{a_w x \pi x}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_x \sin\left(\frac{a_r rho x \pi x}{L}\right) \\
& - 6 w_x \sin\left(\frac{a_w x \pi x}{L}\right) w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_y \cos\left(\frac{a_r rho y \pi y}{L}\right) \\
& - 2 u_x \sin\left(\frac{a_u x \pi x}{L}\right) u_y \cos\left(\frac{a_u y \pi y}{L}\right) rho_0 \\
& - 2 u_x \sin\left(\frac{a_u x \pi x}{L}\right) u_z \cos\left(\frac{a_u z \pi z}{L}\right) rho_0 \\
& - 2 u_y \cos\left(\frac{a_u y \pi y}{L}\right) u_z \cos\left(\frac{a_u z \pi z}{L}\right) rho_0
\end{aligned}$$

$$\begin{aligned}
& + 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \gamma rho_0 + 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) \gamma rho_0 \\
& + 2 u_0 u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) \gamma rho_0 \\
& - 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& - 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& - 2 u_0 u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& - 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& - 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& - 2 u_0 u_y \cos\left(\frac{a_{uy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& - 2 u_0 u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& - 2 u_0 u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& - 2 u_0 u_z \cos\left(\frac{a_{uz}\pi z}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 \gamma rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 \gamma rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + u_x^2 \sin\left(\frac{a_{ux}\pi x}{L}\right)^2 \gamma rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& + u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + u_y^2 \cos\left(\frac{a_{uy}\pi y}{L}\right)^2 \gamma rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& + u_z^2 \cos\left(\frac{a_{uz}\pi z}{L}\right)^2 \gamma rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& + u_z^2 \cos\left(\frac{a_{uz}\pi z}{L}\right)^2 \gamma rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& + u_z^2 \cos\left(\frac{a_{uz}\pi z}{L}\right)^2 \gamma rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& - 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& - 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& - 2 v_z \sin\left(\frac{a_{vz}\pi z}{L}\right) v_0 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& - 2 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& - 2 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& - 2 v_0 v_x \cos\left(\frac{a_{vx}\pi x}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& - 2 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& - 2 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& - 2 v_0 v_y \sin\left(\frac{a_{vy}\pi y}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& - 6 w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_0 rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& - 6 w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_0 rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& - 6 w_z \cos\left(\frac{a_{wz}\pi z}{L}\right) w_0 rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& - 6 w_0 w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right) \\
& - 6 w_0 w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_x \sin\left(\frac{a_{rhox}\pi x}{L}\right) \\
& - 6 w_0 w_x \sin\left(\frac{a_{wx}\pi x}{L}\right) rho_y \cos\left(\frac{a_{rhoy}\pi y}{L}\right) \\
& - 6 w_0 w_y \sin\left(\frac{a_{wy}\pi y}{L}\right) rho_z \sin\left(\frac{a_{rhoz}\pi z}{L}\right)
\end{aligned}$$

$$\begin{aligned}
& -6 w_0 w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_x \sin\left(\frac{a_r hox \pi x}{L}\right) \\
& -6 w_0 w_y \sin\left(\frac{a_w y \pi y}{L}\right) rho_y \cos\left(\frac{a_r hoy \pi y}{L}\right) \\
& + \gamma v_z^2 \sin\left(\frac{a_v z \pi z}{L}\right)^2 rho_z \sin\left(\frac{a_r hoz \pi z}{L}\right) \\
& + \gamma v_z^2 \sin\left(\frac{a_v z \pi z}{L}\right)^2 rho_x \sin\left(\frac{a_r hox \pi x}{L}\right) \\
& + \gamma v_z^2 \sin\left(\frac{a_v z \pi z}{L}\right)^2 rho_y \cos\left(\frac{a_r hoy \pi y}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_v x \pi x}{L}\right)^2 rho_z \sin\left(\frac{a_r hoz \pi z}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_v x \pi x}{L}\right)^2 rho_x \sin\left(\frac{a_r hox \pi x}{L}\right) \\
& + \gamma v_x^2 \cos\left(\frac{a_v x \pi x}{L}\right)^2 rho_y \cos\left(\frac{a_r hoy \pi y}{L}\right) \\
& + \gamma v_y^2 \sin\left(\frac{a_v y \pi y}{L}\right)^2 rho_z \sin\left(\frac{a_r hoz \pi z}{L}\right) \\
& + \gamma v_y^2 \sin\left(\frac{a_v y \pi y}{L}\right)^2 rho_x \sin\left(\frac{a_r hox \pi x}{L}\right) \\
& + \gamma v_y^2 \sin\left(\frac{a_v y \pi y}{L}\right)^2 rho_y \cos\left(\frac{a_r hoy \pi y}{L}\right) \\
& - 2 v_z \sin\left(\frac{a_v z \pi z}{L}\right) v_x \cos\left(\frac{a_v x \pi x}{L}\right) rho_0 \\
& - 2 v_z \sin\left(\frac{a_v z \pi z}{L}\right) v_y \sin\left(\frac{a_v y \pi y}{L}\right) rho_0 \\
& - 2 v_x \cos\left(\frac{a_v x \pi x}{L}\right) v_y \sin\left(\frac{a_v y \pi y}{L}\right) rho_0 \\
& + 2 \gamma v_z \sin\left(\frac{a_v z \pi z}{L}\right) v_0 rho_0 + 2 \gamma v_0 v_x \cos\left(\frac{a_v x \pi x}{L}\right) rho_0 \\
& + 2 \gamma v_0 v_y \sin\left(\frac{a_v y \pi y}{L}\right) rho_0 \\
& + 3 \gamma w_z^2 \cos\left(\frac{a_w z \pi z}{L}\right)^2 rho_z \sin\left(\frac{a_r hoz \pi z}{L}\right) \\
& + 3 \gamma w_z^2 \cos\left(\frac{a_w z \pi z}{L}\right)^2 rho_x \sin\left(\frac{a_r hox \pi x}{L}\right)
\end{aligned}$$

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+ 3 γ w_z² cos $\left(\frac{a_wz\pi z}{L}\right)^2 rho_y \cos\left(\frac{a_rho y \pi y}{L}\right)$ 
+ 3 γ w_x² sin $\left(\frac{a_wx\pi x}{L}\right)^2 rho_z \sin\left(\frac{a_rho z \pi z}{L}\right)$ 
+ 3 γ w_x² sin $\left(\frac{a_wx\pi x}{L}\right)^2 rho_x \sin\left(\frac{a_rho x \pi x}{L}\right)$ 
+ 3 γ w_x² sin $\left(\frac{a_wx\pi x}{L}\right)^2 rho_y \cos\left(\frac{a_rho y \pi y}{L}\right)$ 
+ 3 γ w_y² sin $\left(\frac{a_wy\pi y}{L}\right)^2 rho_z \sin\left(\frac{a_rho z \pi z}{L}\right)$ 
+ 3 γ w_y² sin $\left(\frac{a_wy\pi y}{L}\right)^2 rho_x \sin\left(\frac{a_rho x \pi x}{L}\right)$ 
+ 3 γ w_y² sin $\left(\frac{a_wy\pi y}{L}\right)^2 rho_y \cos\left(\frac{a_rho y \pi y}{L}\right)$ 
+ 6 γ w_z cos $\left(\frac{a_wz\pi z}{L}\right) w_0 rho_0 + 6 \gamma w_0 w_x \sin\left(\frac{a_wx\pi x}{L}\right) rho_0$ 
+ 6 γ w_0 w_y sin $\left(\frac{a_wy\pi y}{L}\right) rho_0$ 
- 6 w_z cos $\left(\frac{a_wz\pi z}{L}\right) w_x \sin\left(\frac{a_wx\pi x}{L}\right) rho_0$ 
- 6 w_z cos $\left(\frac{a_wz\pi z}{L}\right) w_y \sin\left(\frac{a_wy\pi y}{L}\right) rho_0$ 
- 6 w_x sin $\left(\frac{a_wx\pi x}{L}\right) w_y \sin\left(\frac{a_wy\pi y}{L}\right) rho_0 + 2 p_0 \gamma$ 
w_z sin $\left(\frac{a_wz\pi z}{L}\right) a_wz \pi$ 
> #-----
-----#
> #Q=T1+T2+T3+...+T14.
> #N:=simplify(Q_e, size)
>

```