where $\phi = \rho, u, v, w$ or p, and $f_s(\cdot)$ functions denote either sine or cosine function. Note that in this case, ϕ_x, ϕ_y and ϕ_z are constants and the subscripts do not denote differentiation.

Although **?** provide the constants used in the manufactured solutions for the 2D supersonic and subsonic cases for Euler and Navier-Stokes equations, only the source term for the 2D mass conservation equation (3.20) is presented.

Source terms for mass conservation (Q_ρ), momentum (Q_u, Q_v and Q_w) and total energy (Q_{e_t}) equations are obtained by symbolic manipulations of compressible steady Euler equations above using Maple 13 (**?**) and are presented in the following sections for the one, two and three-dimensional cases.

3.2.2.1 1D Steady Euler

The manufactured analytical solutions (3.52) for each one of the variables in one-dimensional case of Euler equations are:

$$\begin{aligned}
\rho\left(x\right) &= \rho_0 + \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
u\left(x\right) &= u_0 + u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \\
p\left(x\right) &= p_0 + p_x \cos\left(\frac{a_{px}\pi x}{L}\right)
\end{aligned} \tag{3.26}$$

The MMS applied to Euler equations consists in modifying the 1D Euler equations (3.20) – (3.22) by adding a source term to the right-hand side of each equation:

$$\begin{aligned}
&\frac{\partial(\rho u)}{\partial x} = Q_\rho \\
&\frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(p)}{\partial x} = Q_u \\
&\frac{\partial(\rho u e_t)}{\partial x} + \frac{\partial(p u)}{\partial x} = Q_{e_t}
\end{aligned} \tag{3.27}$$

so the modified set of equations (3.27) conveniently has the analytical solution given in Equation (3.53).

Source terms Q_ρ, Q_u and Q_{e_t} are obtained by symbolic manipulations of equations above using Maple and are presented in the following sections. The following auxiliary variables have been included in order to improve readability and computational efficiency:

$$\begin{aligned}
\mathtt{Rho}_1 &= \rho_0 + \rho_x \sin\left(\frac{a_{\rho x}\pi x}{L}\right) \\
U_1 &= u_0 + u_x \sin\left(\frac{a_{ux}\pi x}{L}\right) \\
\mathtt{P}_1 &= p_0 + p_x \cos\left(\frac{a_{px}\pi x}{L}\right)
\end{aligned}$$

where the subscripts refer to the 1D case.

The mass conservation equation written as an operator is:

$$\mathcal{L} = \frac{\partial(\rho u)}{\partial x}$$