

Egalitarian and Congestion Aware Truthful Airport Slot Allocation Mechanism

Aasheesh Dixit^{*1}, Garima Shakya^{*2}, Suresh Kumar Jakhar¹, and Swaprava Nath²

¹Indian Institute of Management Lucknow, {fpm18021, skj}@iiml.ac.in

²Indian Institute of Technology Kanpur, {garima, swaprava}@cse.iitk.ac.in, *: Equal contribution

October 21, 2020

Abstract

We propose a mechanism to allocate slots fairly at congested airports. This mechanism: (a) ensures that the slots are allocated according to the *true* valuations of airlines, (b) provides fair opportunities to the flights connecting remote cities to large airports, and (c) controls the number of flights in each slot to minimize congestion. The mechanism draws inspiration from economic theory. It allocates the slots based on an *affine maximizer* allocation rule and charges payments to the airlines such that they are incentivized to reveal their true valuations. The allocation also optimizes the occupancy of every slot to keep them as uncongested as possible. The formulation solves an optimal integral solution in strongly polynomial time. We conduct experiments on the data collected from two major airports in India. We also compare our results with existing allocations and also with the allocations based on the International Air Transport Association (IATA) guidelines. The computational results show that the *social utility* generated using our mechanism is 20-30% higher than IATA and current allocations.

Keywords: airport slot allocation, congestion cost, social welfare, mechanism design, strongly polynomial algorithm.

1 Introduction

In the last two decades or so, an overwhelming increase in demand for air transportation coupled with political, physical, and institutional constraints for capacity expansion resulted in the congestion and delays at the world's major commercial airports. As demand for airport slots¹ exceeds its available capacity, it results in more holding time for the permission to land or take-off, which leads to congestion. Airport congestion is veritably imposing a tremendous cost² on the world economy which includes additional aircraft operating costs, passenger delay costs, etc. The other externalities are environmental and noise pollution around the congested airport, while aircraft wait in queue with engines fired up. The contemporary research on airport slot allocations is primarily focused on demand-side solutions to mitigate congestion as it has the potential to restore the demand-capacity balance over a medium to short time horizon with

¹The daily runway scheduling period of an airport is divided into time intervals of a fixed length (e.g., 15 minutes) called slots (Androutsopoulos et al., 2020). The slot allocation to an airline movement means the permission to use the airport for landing or taking-off within a particular time interval (slot).

²A study commissioned by the Federal Aviation Administration (FAA) estimates the total cost of delay to-be around \$31.2 billion for calendar year 2007 (Churchill et al., 2010)

fairly low investments (Fairbrother and Zografos, 2020). The demand management strategies to manage slots at congested airport ranges from various administrative tools to market-based mechanisms.

The administrative approaches for slot allocation include grandfather rights, first-come-first-serve, lotteries, new entrant rule, etc. (Ball et al., 2006). These approaches essentially are government or institutional interventions through rules and regulations to allocate scarce slots at congested airports. In the grandfathering rights, the historical precedence of the allocated slots is maintained (Sieg, 2010). Whereas, in the first-come-first-serve rule, the slots are assigned based on their time of arrival and airlines queue up for runway and gate access (Fan and Odoni, 2002). In the new entrant rule, the preference is given to new entrants for the vacant/newly available slots. For example, at Frankfurt airport a considerable capacity of slots at a newly constructed runway were allocated to new entrants (IATA, 2017). The use of administrative instruments may also be warranted to achieve certain other social goals. For example, a varied set of administrative instruments have been used by governments across the world to promote remote connectivity. Indeed, air transportation has emerged as a ‘lifeline service’ to connect remote cities where land transportation is not a real option (Fageda et al., 2018). The link between air connectivity and economic growth is also well established (Brueckner, 2003; Alderighi et al., 2017). The study by Bilotkach et al. (2015) shows a strong correlation between the regional growth and connectivity to various new destinations. Fageda et al. (2018) observe that an appropriate mechanism can help governments to effectively promote regional air transport which otherwise is excluded under normal market conditions. An administrative approach may also respond to a country’s varied political goals, for example, UK governments’ objective to connect Heathrow airport to the rest of the UK (Burghouwt, 2017).

The extant literature also points out several issues with administrative approaches, with one of the biggest criticisms being that they are economically inefficient. The runway resource at congested airports in peak periods is a valuable and scarce resource and grand-fathering or lotteries are inefficient and random ways to allocate them (Zografos and Madas, 2003; Zografos et al., 2013). Therefore, the slots should be allocated to the airlines which value them the most (Cao and Kanafani, 2000). Another criticism for grandfathering rule is that it creates entry barriers to new airlines (Vaze and Barnhart, 2012) and encourages legacy carriers to overschedule flights to avoid losing the allocated slots (Harsha, 2009). This also prevents the effective competitive pressure on these incumbents from the new entrants especially the low-cost carriers. For example, British Airways holds 51% of the take-off and landing capacity at London Heathrow (the busiest airport in Europe) with an estimated worth of 742 million pounds (Horton, 2020). This airport is operating at its 100% capacity, and this makes it incredibly difficult for the new entrant to obtain a slot. Moreover, when there are no monetary payments involved in slot allocation, the airlines may overdemand the slots to minimize deviation from their preferred schedule (Vaze and Barnhart, 2012) or to gain market power through slot hoarding (Sheng et al., 2019). Historically it has been established that government interventions lead to inefficiencies and airlines may exploit the loopholes in the regulations (Burghouwt, 2017).

Researchers have shown that market-based mechanisms, result in the efficient allocation of scarce airport resources (Czerny and Zhang, 2014; Madas and Zografos, 2010). The market mechanisms would essentially reveal the true economic value of the resources at the congested airport. They provide a flexible and transparent approach for balancing supply-demand mismatch through pricing. These mechanisms also provide equal opportunities to the legacy carriers as well as the new entrant and promote healthy competition. Various market-based mechanisms such as congestion pricing, auctions, and secondary trading of slots have been proposed. Congestion pricing favours charging airlines a fee based on several proposed rules such as:

- Pricing based on the marginal cost of delays (Vickrey, 1969; Carlin and Park, 1970).

- Differential pricing across carriers by considering small and large airlines (Brueckner, 2009).
- Pricing based on contribution to the infeasibility of the ideal solution (Castelli et al., 2012).
- Pricing based on types of passengers when airlines price discriminates (Czerny and Zhang, 2014).
- Pricing based on desired amount of delay that airlines are ready to buy (Mehta and Vazirani, 2020).

Congestion pricing results in considerable welfare gains, which can minimize total delay and helps in attaining ideal slot allocations (Daniel and Harback, 2009; Czerny, 2010). Daniel (2011) reported that congestion pricing could help in saving of \$72 million to \$105 million annually at Canadian airports by reducing delays and associated costs. A drawback of congestion pricing is that the fee has to be iteratively varied not only in time but also among different airports depending on the degree of congestion set up by the airport administration. The other market-based approach widely discussed in the extant literature is auctioning, where airline bids for slots. An auction-based mechanism maximizes the social welfare as it allocates slots purely based on valuation maximization (Harsha, 2009). Ball et al. (2020) proposed a quantity-contingent based auction mechanism to allocate the slots at the congested airports. Basso and Zhang (2010) compared congestion pricing with slot auctions and conclude that there is no clear winner for airport profit maximization.

The market-based mechanisms despite being regarded as the most efficient way to allocate slots at congested airports, also have certain shortcomings. They are considered to be detrimental for flights from remote cities due to their low valuations as compared to flights from big cities. The low profitability and inconsistent load factor of movements from remote cities may limit their ability to win slots at an auction or pay pure market-based congestion prices. In fact, any mechanism solely based on the transfer of money will be unfavourable to remote communities and will lead to an exclusion of air-service to these cities (Green, 2007; Harsha, 2009; Sheard, 2014). The use of administrative instruments may be warranted to achieve social objectives by ensuring connectivity to peripheral regions and support the population of remote regions. The need to combine market-based and administrative instruments leads to a hybrid mechanism, which allocates the slots based on efficiency goal (valuation maximization) but at the same time also ensures slot opportunities for flights connecting to remote cities. This paper addresses this dual goal of valuation-based slot allocation with remote city connectivity (social objective) at the congested airport.

The key lever for congestion minimization is to limit the number of allocated movements (landings/takeoffs) in a particular slot. The most important decision here is to determine the number of movements to assign, based on the trade-off between the cost of delay and resource utilization. In our proposed mechanism, we determine the number of movements in each slot based on the trade-off between an increase in valuations due to additionally allocated movement and the resultant increase in congestion cost. It considers a proper balance between flight delays and the extent of the services offered (the number of flights scheduled). As the number of movements scheduled in each slot decreases, it would result in a decrease in congestion level and resultant delays. Based on a case study of LaGuardia Airport, Li et al. (2010) found that airport congestion can be minimized by limiting the number of allocated movements in a slot. Similarly, Swaroop et al. (2012) found that more than two-thirds of the total system-delays can be reduced by capping the slot allocation. We propose that if the number of movements in a slot are allocated up to a certain limit then it may not result in significant congestion and delays. However, if we allocate movements in a slot beyond a certain limit then it will start adding to congestion and delays. Therefore, we have imposed a penalty in the form of congestion cost for

every additionally allocated movement beyond a limit. The determination of allocation limit depends on various factors such as weather conditions in a particular season, aircraft mix and, the skill of the air traffic controllers and pilots, and other factors. Historical airport delay data can be used to determine these allocation limits.

Our Contributions

The goal of this paper is to devise a slot allocation mechanism that can integrate various (administrative and market-based) instruments into an overall slot allocation strategy that satisfies multiple criteria such as congestion mitigation, weighted efficiency³, and truthfulness (considering that the true valuation of the airlines are their private information). It is worth noting that there is a dearth of models in the extant literature that can simultaneously consider the efficiency and social welfare goals in slot allocation. In this paper, we propose a mechanism with the following distinctive features:

- It allocate slots based on the reported valuations of the airlines.
- It incorporates the remote city opportunity factor (social progress index and population of the remote city with adjustable weights) which caters to social obligations for slot allocation.
- It incorporates congestion cost and controls the number of movements allocated in each slot by considering the cost-benefit trade-offs. Controlling the number of allocations can be viewed as a mechanism for managing flight arrival and departure delays. The decrease in the number of allocations in each time window would result in a decrease in congestion level and resultant delays.
- Our mechanism provides a fine balance between three competing goals: slot allocation on basis of valuations, remote city connectivity, and congestion mitigation, yet ensures computational tractability. Usually, these objectives counter are conflicting in nature.

In particular, we show the following results.

- Our mechanism is truthful in dominant strategies (Theorem 1).
- It also incentivizes voluntary participation of the airlines (Theorem 2).
- The slot allocation problem is generally an integer program which is known to belong to a computationally intractable class (NP-complete). However, we show that our formulation is solvable in strongly polynomial time (Theorem 5).
- The experiments (Section 5) show that the social utility generated using our mechanism is 20 – 30% higher than IATA and Current allocations. The individual utility of movements is higher than IATA and Current allocation, while the payment made by movement is based on the type of connection provided by the flight movement.

To the best of our knowledge, this is the first mechanism that simultaneously considers multiple aspects for slot allocation, which we call Egalitarian and Congestion Aware Truthful Slot allocation (ECATS) mechanism. The remainder of the paper is organized as follows: The model and the desirable properties are formalized in Section 2. The mechanism and its theoretical properties are presented in Sections 3 and 4 respectively. We present the experimental results in Section 5. We conclude the paper in Section 6.

³Efficiency usually implies maximizing the sum of allocated valuations, which can be biased towards flights from metro cities. The weighted efficiency essentially means sum of weighted valuations where larger weights are assigned to the flights from remote cities to provide a fair opportunity in the slot allocation.

2 The Model

We divide the availability of the airport in disjoint time intervals or *slots*⁴. Let the set of time slots be $S = \{1, 2, \dots, n\}$. Each slot $j \in S$ has a capacity C_j , which is the maximum number of flights or movements that can be accommodated, $C_j \in \mathbb{Z}_+, \forall j \in S$. Let $M = \{1, 2, \dots, m\}$ be the set of movements. Every movement has a potentially different valuation for the different slots in S . The valuation of movement $i \in M$ for slot $j \in S$ is denoted by $v_{ij} \in \mathbb{R}_{\geq 0}$. For every movement $i \in M$, v_i be the vector of i 's valuations for every slot in S and $V = [v_{ij}], i \in M, j \in S$. We represent the set of all feasible allocations of the flights to the slots as $A = [x_{ij}], i \in M, j \in S$, where, $x_{ij} = 1$ if movement i is assigned slot j and is equal to 0 otherwise. We assume that each movement can be assigned to at most one slot.

We will use the shorthand $v_i(A)$ to denote the valuation of movement i in the allocation A , i.e., it will be equal to v_{ik} , if i is assigned slot k in A , and zero if i is unallocated in A . We assume that the valuation vector v_i is a private information of agent⁵ i , for all $i \in N$.

The planner decides the allocation A and charge payments $p = (p_i, i \in N)$ to each of the movements. We assume that every agent wants a more valued slot to be assigned to her and wants to pay less. Therefore, the net payoff of an agent is assumed to follow a standard *quasi-linear form* (Shoham and Leyton-Brown, 2008).

$$u_i((A, d), v_i) = v_i(A) - d_i. \quad (1)$$

Denote the set of all allocations by \mathcal{A} and $p_i \in \mathbb{R}, \forall i \in N$. The planner does not know the true valuations of the agents. Therefore he needs the agents to report their valuations and decide the allocation and the payments. This leaves the opportunity for an agent to misrepresent her true valuation, e.g., an agent can report a higher valuation to get a prioritized scheduling. To distinguish, we use v_{ij} for the true valuation and v'_{ij} for reported valuations. We will use the shorthand $v = (v_i)_{i \in N}$ to denote the true valuation profile represented as an $n \times m$ real matrix, and v' to denote the reported valuation profile. The notation v_{-i} denotes the valuation profile of the agents except i . The decision problem of the planner is therefore formally captured by the following function.

DEFINITION 1 (Airline Scheduling Function (ASF)). *An airline scheduling function (ASF) is a mapping $f : \mathbb{R}^{m \times n} \rightarrow \mathcal{A} \times \mathbb{R}^n$ that maps the reported valuations to an allocation and payment for every agent. Hence, $f(v') = (A(v'), p(v'))$, where A and p are the allocation and payment functions respectively.*⁶

We assume that when the number of movements in a slot exceeds a pre-defined threshold, congestion and delays start kicking in. We propose a division of the slot capacity into two parts: congestion-free and congestion prone. If the number of allocated movements in a time slot j exceeds a defined threshold, $(1 - \lambda)C_j$, we say that the slot j is congested. The determination of the threshold, λ , depends on various factors such as airport infrastructure and local weather conditions. Historical data can be used to determine this allocation limit. Therefore, the following term captures a measure of congestion level in the slot corresponding to the allocation

⁴We find that the term ‘windows’ and ‘slots’ are used interchangeably in the literature. Following the definition of slot provided by Androutsopoulos et al. (2020), Mehta and Vazirani (2020) and Ribeiro et al. (2018), we refer to each disjoint time interval as a slot with defined capacity.

⁵We will use the terms agent, player, and movement interchangeably in this paper.

⁶We overload the notation A and d to denote both functions and values of those functions, since their use will be clear from the context.

A and is equal to the number of allocated flights in the congestion prone capacity.

$$e_j(A) = \left(\sum_{i \in M} x_{ij} - C_j(1 - \lambda) \right)^+ \quad \forall j \in S \quad (2)$$

We have considered the above expression for congestion as it models the congestion in real settings. However, our results can be easily extended for any e_j that is linear in x .

2.1 Remote City Opportunity Factor

Inadequate air connectivity to metro cities is considered a major obstacle for the local economic development of remote cities. It is shown that poor air connectivity services inhibit local employment growth by limiting the city's attractiveness for new businesses and reducing the viability of existing businesses (Brueckner, 2003). In the existing literature, empirical studies from European Union, United States, and China have established a positive impact of metro airport connectivity on remote cities' regional growth (Brafman and Tennenholtz, 2003; Yao and Yang, 2008). A study based on airports in Canada shows that an increase of 1126 additional air-travel passengers can create one person-year of employment (Benell and Prentice, 1993). Yao and Yang (2008) showed that a 10% increase in population density could lead to a 1.7% increase in air passenger volume in China. Therefore, the remote cities' adequate population is also an important criterion to make air connectivity from remote cities economically viable. By considering the above two factors (economic progress and population), we propose the following remote city opportunity factor (RCOF):

$$\rho_i = \alpha_i \frac{\gamma_{\max} - \gamma_i + \delta}{\sum_{i \in N} \{\gamma_{\max} - \gamma_i\} + \delta} + (1 - \alpha_i) \frac{\omega_i - \omega_{\min} + \delta}{\sum_{i \in N} \{\omega_i - \omega_{\min}\} + \delta} \quad (3)$$

where γ is the social progress index (SPI)⁷ and ω is the population of the city. γ_{\max} takes the maximum value of SPI and ω_{\min} is the minimum population among all the cities in consideration. Here α_i indicates the relative weight assigned to SPI and population to calculate ρ_i . A small factor δ with $\lim_{\delta \rightarrow 0}$, is added to avoid a division-by-zero error. The value of ρ_i ranges between 0 to 1.

The SPI is an innovative way to measure the development of a region. It is a widely used index and is measured by the thirty-five indicators related to basic human needs, foundations of well-being, and opportunities for the city's progress. The framework is closely coherent with all sustainable development goals (SDGs) parameters. This simple but rigorous framework makes it an invaluable proxy measure of SDG performance. It captures a wide range of measures involving social and environmental factors, thus proving to be a monitoring and guiding mechanism for national policy decisions and assisting businesses in planning corporate social responsibility activities. The SPI values for various countries and their cities are available at <https://www.socialprogress.org>. For India, the social progress performance data of 562 districts is collected and maintained by the Institute for Competitiveness, India. As can be seen from the above equation, the formulation is designed to give higher priority to the cities with a lower value of SPI and a high population to benefit from air connectivity.

2.2 Desirable Properties

In this section, we formally define a few desirable properties that an ASF should satisfy. Since the mechanism can only access the movements' reported values, for a truly efficient slot alloca-

⁷<https://www.socialprogress.org/>

tion, it is needed that the reported valuations must be the true values. The following property ensures that every movement is incentivized to reveal the values truthfully.

DEFINITION 2 (Dominant Strategy Truthfulness). *An ASF $f = (A(\cdot), p(\cdot))$ is truthful in dominant strategies if for every $v_i, v'_i \in \mathbb{R}_{\geq 0}^n, i \in M$*

$$v_i(A(v_i, v_{-i})) - p_i(v_i, v_{-i}) \geq v_i(A(v'_i, v_{-i})) - p_i(v'_i, v_{-i}).$$

The inequality above shows that if the true value of agent i is v_i , the allocation and payment resulting from reporting it ‘truthfully’ maximizes her payoff *irrespective of the reports of the other agents*.

The next property ensures that it is always weakly beneficial for every rational agent to participate in such a mechanism.

DEFINITION 3 (Individual Rationality). *An ASF $f = (A(\cdot), p(\cdot))$ is individually rational if for every v , and $i \in M$*

$$v_i(A(v_i, v_{-i})) - p_i(v_i, v_{-i}) \geq 0.$$

For a large airport, the number of movements and slots and capacities of slots are large, this leads to an exponential increase in the size of \mathcal{A} . For such a setting, the allocations and payments are desired to be computed in time polynomial in number of movements and slots. We consider mechanisms that are *strongly polynomial* as defined below.

DEFINITION 4 (Strongly Polynomial). *The algorithm runs in strongly polynomial time if (Grötschel et al., 1993)*

1. *the number of arithmetic operations (addition, subtraction, multiplication, division, and comparison) in the arithmetic model of computation is bounded by a polynomial in the number of integers in the input instance; and*
2. *the space used by the algorithm is bounded by a polynomial in the size of the input.*

In the following section, we introduce the proposed mechanism.

3 The Proposed Mechanism

We propose the ASF ECATS (Egalitarian and Congestion Aware Truthful Slot allocation) in the quasi-linear (QL) environment. The allocation function uses the airlines’ reported valuations for slots and the RCOF to find a socially egalitarian allocation where each movement is weighted with their RCOFs. It also puts an additive penalty for congestion and maximizes this affine sum.

The optimal allocation is therefore computed by the following integer linear program.

$$\begin{aligned} & \underset{x}{\operatorname{argmax}} \quad \sum_{j \in S} \sum_{i \in M} (\rho_i v_{ij} x_{ij}) - e_j(x) g \\ & \text{subject to} \quad \sum_{i \in M} x_{ij} \leq C_j, \quad \forall j \in S \\ & \quad \sum_{j \in S} x_{ij} \leq 1, \quad \forall i \in M \\ & \quad x_{ij} \in \{0, 1\}, \quad \forall i \in M, \quad \forall j \in S. \end{aligned} \tag{4}$$

The first term of the objective function multiplies each movement's valuation with their RCOFs (ρ_i s) to equalize the opportunities to the flights to and from every city. To cater to the congestion problem, the second term of the objective function subtracts the total congestion cost. The first constraint in the above optimization problem is the capacity constraint of each slot. The second constraint ensures that none of the movements are assigned to more than one slot.

Allocation The allocation function $\mathcal{A}(\cdot)$ of ECATS computes the LP relaxation of IP 4 as follows.

$$\begin{aligned}
& \underset{x}{\operatorname{argmax}} && \sum_{j \in S} \sum_{i \in M} (\rho_i v_{ij} x_{ij}) - e_j(x) g \\
& \text{subject to} && \sum_{i \in M} x_{ij} \leq C_j, \quad \forall j \in S \\
& && \sum_{j \in S} x_{ij} \leq 1, \quad \forall i \in M \\
& && x_{ij} \geq 0, \quad \forall i \in M, \quad \forall j \in S
\end{aligned} \tag{5}$$

In Section 4, we prove that the solution of the above LP will always be integral and therefore coincides with the solution of IP 4.

Payment The payment function p in ECATS is given by,

$$p_i(v_i, v_{-i}) = \begin{cases} \frac{1}{\rho_i} \left(h_i(v_{-i}) - \left(\sum_{k \in M \setminus \{i\}} \rho_k v_k (A(v_i, v_{-i})) - g \sum_{j \in S} e_j(A(v_i, v_{-i})) \right) \right) & \rho_i > 0 \\ 0 & \rho_i = 0 \end{cases} \tag{6}$$

where,

$$h_i(v_{-i}) = \sum_{k \in M \setminus \{i\}} \rho_k v_k (A(v_{-i})) - g \sum_{j \in S} e_j(A(v_{-i}))$$

The payment function for an airline i is proportional to the difference between the value of optimal objective function when i is absent and present respectively. The payment is inspired by the idea of marginal contributions in the affine maximizers (Roberts, 1979).

4 Theoretical results

In this section, we present the theoretical guarantees for the properties of ECATS. For better readability, some of the proofs are deferred to the appendix.

Our first result shows that under ECATS, none of the players can get better utility by misreporting her true information.

THEOREM 1. *ECATS is dominant strategy truthful.*

The above result implies that, irrespective of the reported valuations of the other movements, a given movement's utility is maximized when it reports its valuations truthfully. Our next result shows that the movements are incentivized to voluntarily participate in ECATS.

THEOREM 2. *ECATS is individually rational for every movement.*

The proof shows that every movement gets non-negative utility from participating in ECATS. The next few results show that even if the allocation problem of IP 4 falls in a computationally intractable class, its special structure can be used to find a tractable solution.

THEOREM 3. *The allocation of ECATS given by LP 5 has an integral optimal solution and is polynomially solvable.*

Proof. Using the definition of e_j in Equation (2), LP 5 can be written as follows

$$\begin{aligned}
& \max_{x,w} \quad \sum_{j \in S} \sum_{i \in M} (\rho_i v_{ij} x_{ij}) - w_j g \\
& \text{subject to} \quad \sum_{i \in M} x_{ij} \leq C_j, \quad \forall j \in S \\
& \quad \sum_{j \in S} x_{ij} \leq 1, \quad \forall i \in M \\
& \quad \sum_{i \in M} x_{ij} - w_j \leq T_j, \quad \forall j \in S \\
& \quad x_{ij} \geq 0, \quad w_j \geq 0, \quad \forall i \in M, \quad \forall j \in S
\end{aligned} \tag{7}$$

where, $T_j = C_j(1 - \lambda)$.

CLAIM 1. *The coefficient matrix of optimization problem in Equation (7) is totally unimodular (TU).*

Proof. First we linearize the variables x and w of the optimization problem into a single vector \bar{x} as $(x_{11}, x_{12}, \dots, x_{1n}, \dots, x_{m1}, x_{m2}, \dots, x_{mn}, w_1, w_2, \dots, w_n)^\top$. The constraints in the Equation (7) can be written as $Z\bar{x} \leq b$, where, $Z_{(m+2n) \times (mn+n)}$ is the coefficient matrix and $b_{(m+2n) \times 1}$ is the bound vector as shown below.

$$\begin{bmatrix}
I_{n \times n} & I_{n \times n} & I_{n \times n} & \dots & I_{n \times n} & 0 \\
\mathbb{1}_{1 \times n} & 0 & \dots & 0 & 0 & 0 \\
0 & \mathbb{1}_{1 \times n} & \dots & 0 & 0 & 0 \\
0 & 0 & \ddots & 0 & 0 & 0 \\
0 & 0 & 0 & \mathbb{1}_{1 \times n} & 0 & 0 \\
0 & 0 & 0 & 0 & \mathbb{1}_{1 \times n} & 0 \\
I_{n \times n} & I_{n \times n} & I_{n \times n} & \dots & I_{n \times n} & -I_{n \times n}
\end{bmatrix}
\begin{bmatrix}
x_{11} \\
x_{12} \\
\vdots \\
x_{1n} \\
\vdots \\
x_{m1} \\
x_{m2} \\
\vdots \\
x_{mn} \\
w_1 \\
w_2 \\
\vdots \\
w_n
\end{bmatrix}
\leq
\begin{bmatrix}
C_1 \\
C_2 \\
\vdots \\
C_n \\
1 \\
1 \\
\vdots \\
1 \\
T_1 \\
T_2 \\
\vdots \\
T_n
\end{bmatrix}$$

The first n rows of Z has m ($n \times n$) identity matrices followed by $(0)_{n \times n}$. Each of the next m rows has exactly one $(1 \times n)$ vector of all 1s staggered as shown above. The last n rows are similar to the first n rows with the last n columns being a negative identity matrix of dimension $n \times n$.

We use the Ghouila-Houri (GH) characterization (Ghouila-Houri, 1962; de Werra, 1981) to prove that Z is TU, which says that a matrix $Z_{p \times q}$ is TU if and only if any subset R of rows, $R \subseteq \{1, 2, \dots, p\}$, can be partitioned into two subsets R_1 and R_2 , such that, $\sum_{i \in R_1} z_{ij} - \sum_{i \in R_2} z_{ij} \in \{1, 0, -1\}$, where $j = 1, 2, \dots, q$. In our case, each column of the coefficient matrix Z has at most three 1's or one -1 . Note that, the rows can be easily partitioned into *three* classes: class 1 consists of first n rows, class 2 consists of next m rows, class 3 consists of last n rows. There are three exhaustive cases for the subset R of rows of Z .

1. R consists of rows from all three classes of rows.
2. R consists of rows from any two classes of rows.
3. R consists of rows from exactly one class of rows.

For case 1, we find the two disjoint subsets R_1 and R_2 of R in an iterative manner. Begin with the partition where R_1 consists of rows from the first two classes of rows of Z in R , and R_2 consists of rows from last class rows of Z in R . There can be a situation where for a certain column j , the sum $\sum_{i \in R_1} z_{ij} = 2$ and the $\sum_{i \in R_2} z_{ij} = 0$. This can only happen when for column j and rows in R_1 , the sum of z_{ij} s where i 's are from the rows from class 1 is 1, for the i 's in class 2 there exists a row that has n 1s intersecting with j , and the sum of z_{ij} s where i 's are from the rows in R_2 has no 1s in j . In such a case, we move that row in class 2 from R_1 to R_2 . Repeat this procedure for every such column until no such situation exists. This procedure is guaranteed to converge to a partition of R such that GH conditions are met.

Cases 2 and 3 are straightforward. In case 2, the partition R_1 and R_2 are the intersections of R with the respective classes of rows. For case 3, any partition of R satisfies GH conditions. Hence proved. \square

The optimization problem in Equation (7) has a TU coefficient matrix, which implies LP 5 yields an optimal solution in integers and is solvable in polynomial time. \square

Our next result shows that finding the integral solution can be solved in strongly polynomial time by reducing Equation (7) to the b -matching problem.

DEFINITION 5 (*b -Matching Problem (Tamir and Mitchell, 1998)*). Consider a graph $G = (V, E)$, where V is the set of nodes and E is the set of edges. Each edge $e_{u,v} \in E$ between any two nodes $u, v \in V$, has a cost $c_{u,v}$. Let $b = (b_1, b_2, \dots, b_{|V|})$. A b -matching problem for G is to find the non-negative integer edge weights $w_{u,v}$ which maximises the total cost, $\sum_{u,v \in V} c_{u,v} w_{u,v}$ where the sum of weight on edges incident to a node u is no more than b_u , $\forall u \in V$.

THEOREM 4. The allocation of ECATS, given by LP 5, is implementable in strongly polynomial time.

Proof. Consider an edge weighted bipartite graph, $G = (P, Q, E)$, where $P = M \cup \{t_j | j \in S\}$ and $Q = S$. The edges in E are in two disjoint partitions, E_1 and E_2 . The first partition is $E_1 = \{(e_{i,j}) | i \in M \text{ and } j \in Q\}$. Set the cost $c_{i,j} = \rho_i v_{ij}, \forall e_{i,j} \in E_1$, where ρ_i and v_{ij} are the RCOF and valuation of movement i for slot j respectively. The other partition is $E_2 = \{e_{t_j,j} | t_j \in P \setminus M \text{ and } j \in Q\}$ having the cost $c_{t_j,j} = g$, where g is per unit congestion cost. Define b as, $b_i = 1$ for every $i \in M$, $b_j = C_j$ for every $j \in Q$ and for every $t_j \in P \setminus M$, $b_{t_j} = \lambda C_j$ ⁸. Figure 1 shows the edge-weighted bipartite graph $G = (P, Q, E)$. The red and blue colored edges denote the subsets E_1 and E_2 respectively. To make the notations simpler, we represent $e_{i,j}$ as (i, j) .

The objective function of the b -matching problem over G (with y being the optimization variables) is

$$\sum_{j \in Q} \sum_{i \in M} \rho_i v_{ij} y_{i,j} + \sum_{j \in Q} g y_{t_j,j},$$

with the constraints, $\sum_{v \in Q} y_{u,v} \leq b_u$ and $\sum_{u \in P} y_{u,v} \leq b_v$, $\forall u \in P, v \in Q$. By the definition of b -matching problem, the optimal solution y^* of b -matching of G has,

$$y_{t_j,j}^* = \begin{cases} C_j - \sum_{i \in M} y_{i,j}^* & \sum_{i \in M} y_{i,j}^* - C_j(1 - \lambda) > 0 \\ \lambda C_j & \sum_{i \in M} y_{i,j}^* - C_j(1 - \lambda) \leq 0 \end{cases} \quad (8)$$

⁸This is the size of congestion prone capacity of slot j .

We divide the set of slots S in two disjoint subsets S_1, S_2 with respect to \bar{x} such that, $S_1 = \{j | \sum_{i \in M} \bar{x}_{i,j} - C_j(1 - \lambda) > 0\}$ and $S_2 = \{j | \sum_{i \in M} \bar{x}_{i,j} - C_j(1 - \lambda) \leq 0\}$. Similarly, divide S in two disjoint subsets S_3, S_4 with respect to x^* such that, $S_3 = \{j | \sum_{i \in M} x_{i,j}^* - C_j(1 - \lambda) > 0\}$ and $S_4 = \{j | \sum_{i \in M} x_{i,j}^* - C_j(1 - \lambda) \leq 0\}$.

$$\begin{aligned} \sum_{j \in S} \sum_{i \in M} \rho_i v_{ij} \bar{x}_{i,j} + g \sum_{j \in S_1} \left(C_j - \lambda C_j - \sum_{i \in M} \bar{x}_{i,j} \right) \\ > \sum_{j \in S} \sum_{i \in M} \rho_i v_{ij} x_{i,j}^* + g \sum_{j \in S_3} \left(C_j - \lambda C_j - \sum_{i \in M} x_{i,j}^* \right) \end{aligned}$$

Adding $g \sum_{j \in S} \lambda C_j$ on both sides of the above inequality,

$$\begin{aligned} \sum_{j \in S} \sum_{i \in M} \rho_i v_{ij} \bar{x}_{i,j} + g \sum_{j \in S_1} \left(C_j - \sum_{i \in M} \bar{x}_{i,j} \right) + g \sum_{j \in S_2} \lambda C_j \\ > \sum_{j \in S} \sum_{i \in M} \rho_i v_{ij} x_{i,j}^* + g \sum_{j \in S_3} \left(C_j - \sum_{i \in M} x_{i,j}^* \right) + g \sum_{j \in S_4} \lambda C_j \end{aligned}$$

Let \bar{y} is a solution of b -matching for G corresponding to the solution \bar{x} of LP in Equation (5), then the expression at right side in above inequality is the value of objective function for b -matching problem, where $\forall j \in S_1$ the case 1 of Equation (8) is true and $\forall j \in S_2$ case 2 of Equation (8) is true. The above inequality implies that the value of the objective function of the b -matching problem for solution \bar{y} is more than that for y^* . Therefore, the above inequality contradicts with y^* being the optimal solution of the b -matching for graph G , which implies that x^* is the optimal solution for the LP in Equation (5).

To prove the other direction of Lemma 1, we construct y^* from the optimal solution x^* for the LP in Equation (5) as, $y_{i,j}^* = x_{i,j}^*$, for $e_{i,j} \in E_1$ and, $y_{t_j,j}^*$ using Equation (8) for $e_{t_j,j} \in E_2$. The proof follows by similar argument in the reverse order. \square

Lemma 1 shows that for every instance of the LP in Equation (5), there exists an instance of b -matching problem such that the optimal solution of that instance of b -matching problem gives the optimal solution for LP in Equation (5). Therefore, LP in Equation (5) is solvable in strongly polynomial time as there exist a combinatorial strongly polynomial algorithm to solve the b -matching problem (Anstee, 1987). \square

As the computation of payment for an airline $i \in M$ requires computation of a socially optimal allocation $\forall j \in M \setminus \{i\}$ using the LP in Equation (5), we get Corollary 1.

COROLLARY 1. *The computation of payments for all the airlines is implementable in strongly polynomial time.*

Following the Theorem 4 and Corollary 1, we get Theorem 5.

THEOREM 5. *There is a combinatorial strongly polynomial time algorithm for computing the allocation and payments $f = (A, p)$ in ECATS.*

5 Experimental Results

In this section, we investigate the performance of ECATS in real-world scenarios. While ECATS satisfies several desirable properties of a slot allocation mechanism, its performance with varying congestion cost, use of RCOF and its relevance are not theoretically captured. This is why an experimental study is called for.

For the experiment, we obtain data from two airports in India – Indira Gandhi International Airport (DEL) and Chennai International Airport (MAA). In the year 2018, DEL handled around 70 million passengers and was the 12th busiest airport in the world and the 6th busiest in Asia. It is designated as level 3 airport and has three near-parallel runways. The MAA has a handling capacity of 22.5 million passengers and is the 49th busiest airport in Asia. The DEL is a coordinated airport⁹ with high congestion while MAA is a non-coordinated¹⁰ airport with comparatively low congestion. The purpose of choosing these two airports is to evaluate the performance of ECATS in terms of the allocation and payments under different demand/congestion profiles.

We divide the slot capacity of the airport into two parts by setting a threshold of $(1 - \lambda)$ fraction of the slot capacity. If the number of movements is less than or equal to this threshold, we consider the slot to be congestion-free and consider it congestion prone otherwise. For the experiments, we assume $\lambda = 0.2$. We also compare ECATS with (a) the *Current allocation*, i.e., the Current movement to slot allocation, and (b) the *IATA guidelines*, which uses the optimization approach proposed by Ribeiro et al. (2018) and is based on the IATA guidelines. The model of Ribeiro et al. (2018) minimizes the number of slot requests rejected or displaced. It optimizes the allocation based on the slot availability and airline requests, while accounting for various priorities and requirements included in the IATA guidelines.

5.1 Summary of the data

We collect the data of flight schedule with landing and take-off slot details, city of arrival/departure, flight number and service provider. We obtain the flight movement data between January 21 to 25, 2020, for which an average of 867 movements occurred per day for DEL and an average of 220 movements happened per day for MAA airport.¹¹ We assign the valuation to each movement based on the revenue generated by the movement. Revenue is obtained by multiplying the ticket prices with aircraft capacity and an average load factor of that origin-destination pair. The ticket price data were collected from a booking website¹² for a time period of those five consecutive days for all the movements. The average load factor is defined as the ratio of the number of passengers carried and the available seats. For the data of aircraft capacity and the load factor of different destinations, we refer to the annual financial results of two public airlines: Indigo and Spicejet, which together have the largest fleet size and operates on the maximum number of routes with almost 65% market share in India. The valuation of the movements for other time slots was calculated by generating a random value from the histogram of the valuations (which acts as an empirical distribution) for each slot. The method helps in capturing the ticket fare variation across different time slots. Since the prices are all in Indian Rupees (INR), the unit of the valuations is INR as well.

Next, we calculate the remote city opportunity factor (RCOF) for which we require two types of data:

1. Social progress index of the origin/destination cities.
2. The population of the cities.

⁹Coordinated Airports are the ones where landing airlines have to acquire landing rights and ensure its operation during a specific time period. The slots are administered by the airport operator or by a government aviation regulator. Landing/takeoff demand at these airports exceeds its capacity.

¹⁰In non-coordinated airport, the principles governing slot allocation are less stringent and airlines periodically submit proposed schedules to the administrating authority.

¹¹We chose these dates to obtain the normal air movement patterns before COVID-19 and also because the data were most detailed during this period.

¹²www.ixigo.com

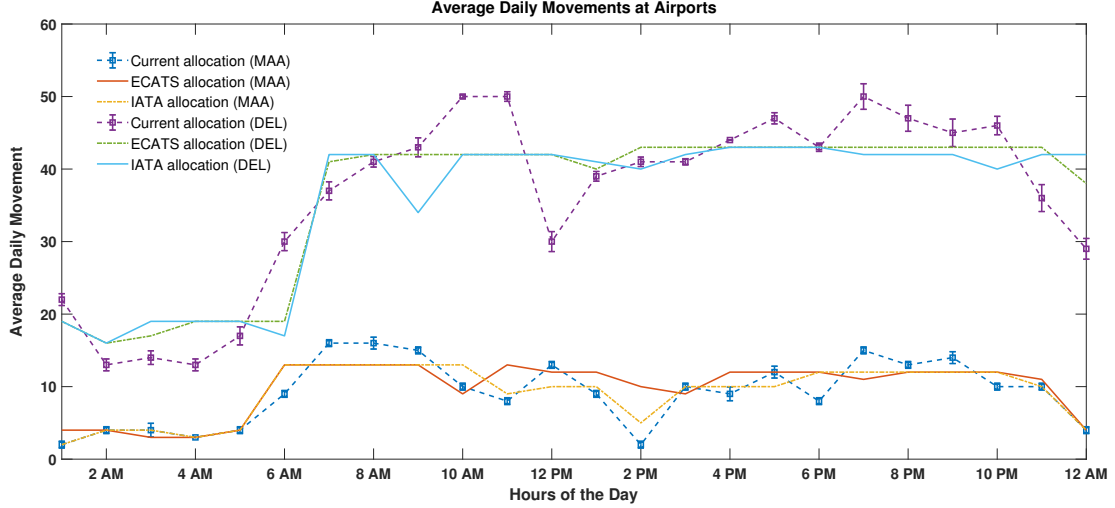


Figure 2: Average daily movements in the two airports.

The social progress index data is provided by the Institute for Competitiveness, India¹³. The population data is obtained from the Office of the Registrar General and Census Commissioner, India¹⁴. The capacity of both the airports for a one-hour time slot is assumed to be the *average* of the maximum number of movements handled in those slots in the past.

On plotting the average daily movements of the two airports (see Figure 2), we observe that flight movements are almost four times higher in DEL than MAA. The flight schedule has a higher variation under the Current allocation with few time slots having a large number of allocated movements. ECATS provides a comparatively uniform flight movement with fewer fluctuations. A stable air traffic movement minimizes losses in operational quality and improves resource allocation in the airport. Airports witness situation when terminals are ‘clogged’ because of mis-allocation of airport resources. In other words, avoiding traffic fluctuations and having a stable traffic movements will aid in optimal utilization of airport resources. Incorporation of congestion cost plays an important role in evenness of slot allocations, which is discussed in the next subsection.

5.2 Effect of congestion cost on the individual and social utility

We study the effect of congestion cost at the individual and social levels. We define the individual utility as the sum of utilities (i.e., the difference of valuation and payment for each movement) divided by the total number of allotted movements. The social utility is the value of objective function in the optimization problem given by Equation (4). Note that since the valuations are in INR, the unit of the utilities is also INR. We find that the individual utility decreases with congestion cost, but the utility is still significantly higher than the other two mechanisms (Figures 3 and 4). In addition, the utility of each movement is positive, which signifies that the airlines stand to gain by participating in the proposed mechanism.

We also observe that the individual utility is comparable for the case of different type of connections (metro, capital and remote cities). The result suggest that airlines will gain similar utility by operating in trunk routes or by providing connection to remote cities. The individual utility for ECATS decreases with increase in congestion cost but is unchanged for the other two

¹³<https://competitiveness.in>

¹⁴<https://censusindia.gov.in>

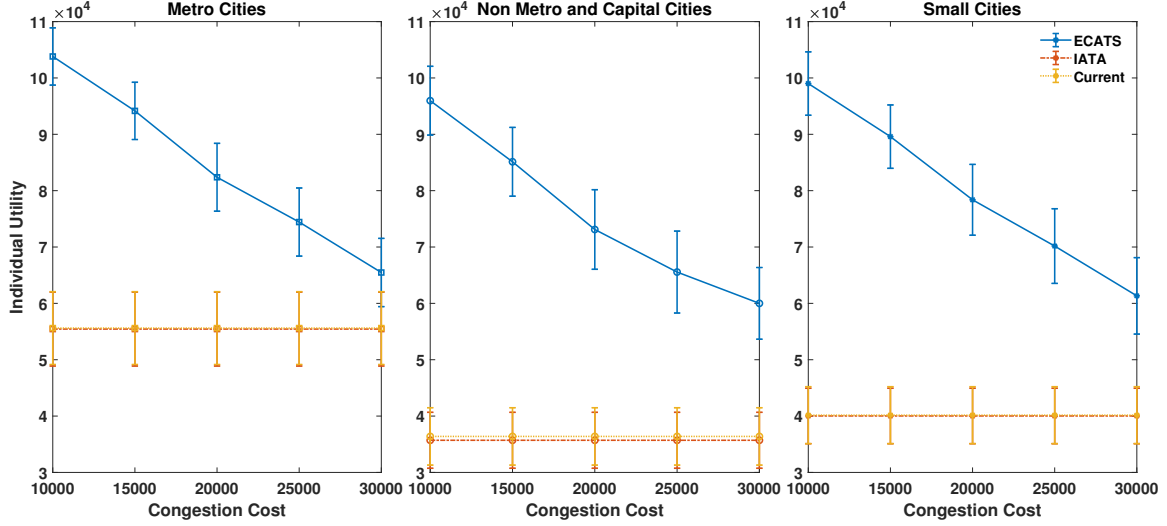


Figure 3: Individual utility of the movements at Delhi (DEL) airport.

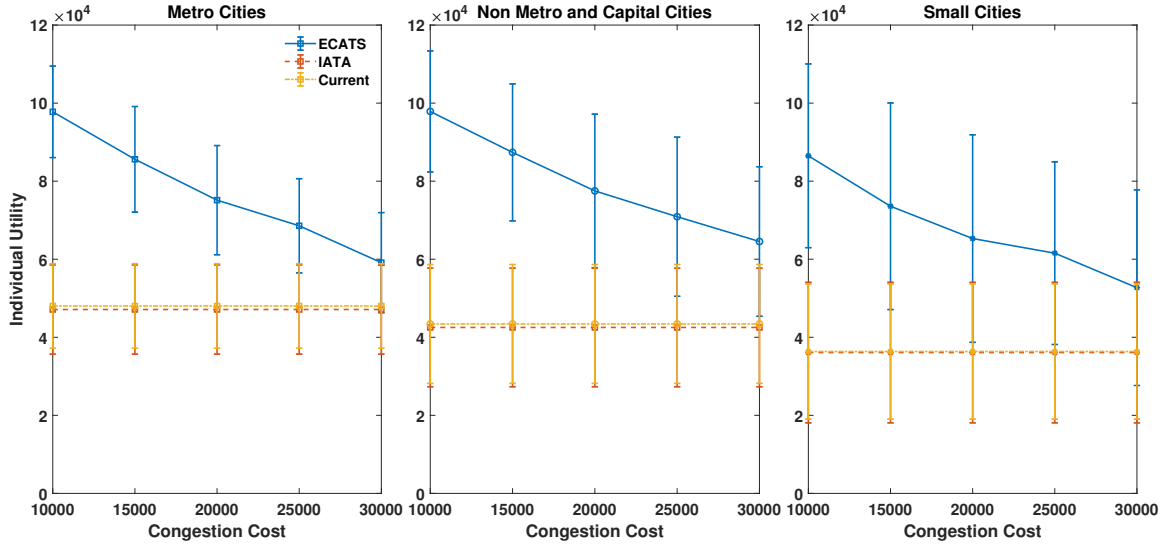


Figure 4: Individual utility of the movements at Chennai (MAA) airport.

mechanisms. This is because there was no direct consideration of valuation maximization and congestion cost for slot allocations in the IATA and Current allocations.

Figures 5 and 6 show that the social utility decreases with increase in congestion cost. As the congestion cost increases, ECATS considers the trade-off between allocating slots above the threshold capacity (and hence increasing congestion) and rejecting the slot request. Only movements with high weighted valuation are allocated these slots, and the movements with low values are rejected. We see that the social utility of ECATS is higher than IATA based approach as the latter mechanism is focused on minimizing the displacement of the requested slot, and does not take congestion cost into account at the time of allocation. These experiments demonstrate that a value-sensitive allocation like ECATS can improve both the individual and social utility by a significant amount.

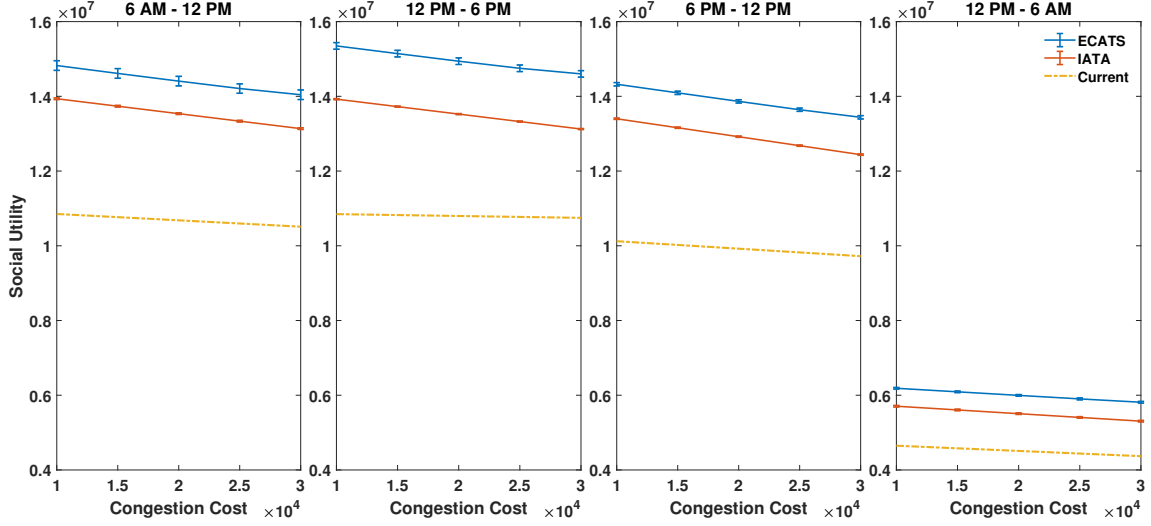


Figure 5: Social utility versus congestion cost: Delhi airport.

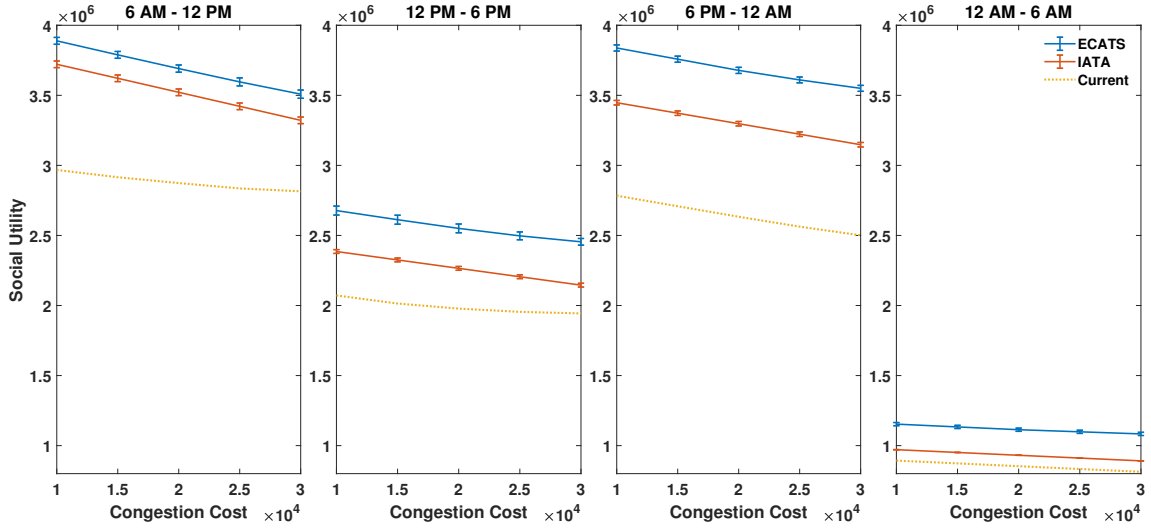


Figure 6: Social utility versus congestion cost: Chennai airport.

5.3 Effect of the congestion cost on the payments

One of the main features of ECATS is the payment function that makes the mechanism truthful. In this section, we study the effect of congestion cost on resulting average payments of the movements that are allocated a slot. The payment for each movement is calculated by equation 6. Figures 7 and 8 show that the trend of the average payments is upward. The social utility is also plotted in these figures for a reference of the level of the payments. Figure 7 and 8 helps in depicting the increasing payment with congestion cost along with decreasing social utility, while Figures 5 and 6 exhibit comparison of social utility across different mechanisms.

5.4 Remote city opportunity factor (RCOF) and its impact

The flights to metro cities and financial hubs have a high valuation since the ticket prices and load factor is generally high, which gives them an advantage if the allocation is done solely

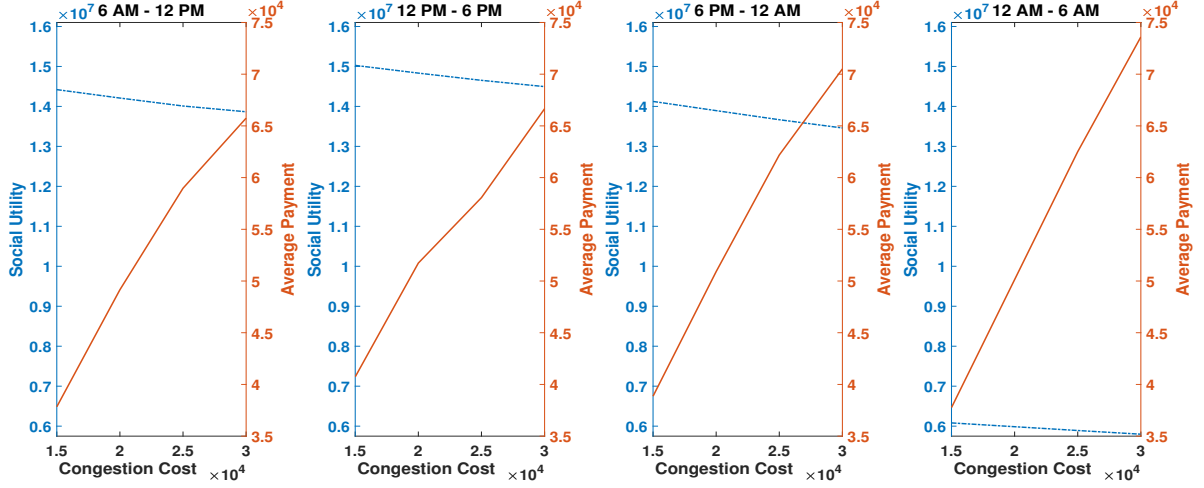


Figure 7: Social utility and average payment versus congestion cost: Delhi airport.

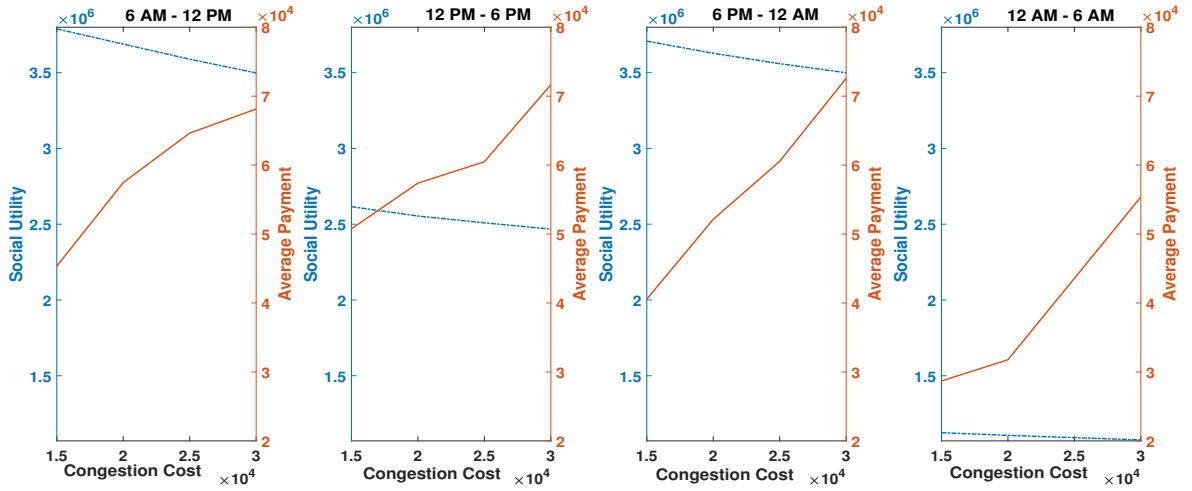


Figure 8: Social utility and average payment versus congestion cost: Chennai airport.

based on valuation. The ticket fare and demand on remote routes is comparatively less due to their low purchasing power or due to fare cap imposed by the government. Hence, to provide equal opportunities to remote cities, we used the remote city opportunity factor (RCOF) given by Equation (3) in the slot allocation process. It assigns a higher weight to cities with a low SPI and a considerable amount of population to have a benefit of air connectivity. Since the RCOF is multiplied with the valuations of the movements in the objective function of Equation (4), the chances of allocation of the remote cities in larger airports increase and at the same time, the factor $1/\rho_i$ in the payment function leads to a lower payment for such flights.

We observe that the individual utilities of the flights to and from remote cities are comparable to the metro cities despite their low valuations (see Figures 3 and 4), both for the case of Delhi and Chennai airport. This happens because the average slot payment for metro cities is higher than non-metro state capitals, which in turn is higher than in remote cities. Hence, ECATS is more egalitarian for slot allocations to flights from different cities.

From the regulator's point of view, ECATS provide a mechanism that offers opportunities to flights from remote cities and provide an incentive to airlines to operate in public service offering routes, which otherwise are neglected without regulatory intervention. On the other

hand, between two cities with the same value of ρ_i the city with high valuation will win the slot. Moreover, our mechanism is not totally unfavourable to metro cities as in our RCOF calculation, we have also incorporated the population of the cities. The metro cities with a higher population will have more slots. The policymaker can adjust the relative weight of SPI and population to decide how much importance they want to give for remote connectivity. Moreover, when a city makes progress on the economic front, its SPI value improves, leading to similar values of ρ_i for most cities. This reduces the difference given by $\gamma_{max} - \gamma_i$ ($SPI_{max} - SPI_i$), thus reducing the preferential treatment of the city. In the long run, with the development of cities, the social progress gap improves and administrative approach may not be required. Therefore, the ECATS will evolve over time and moves in the direction of purely market-based mechanism.

As shown in Figure 7 and Figure 8, an increase in congestion cost lead to more average payment by airlines. Higher payments essentially means more internalization of the congestion cost by the airlines, which earlier they imposed on society. This has positive benefits to society in terms of lower environmental and noise pollution along with less waiting time for the passengers. As shown, social utility decreases with an increase in congestion cost. The reason for the same is that we have only considered airline utilities in social utility calculations. Since airlines pay more by internalizing the cost of congestion, their utilities go down. However, with reduced congestion, airlines will be saving in terms of more utilization of aircrafts, crew members and less fuel burn.

The social utility generated using ECATS is 20-30% higher than IATA and Current allocation as shown in Table 1. It is interesting to note that IATA allocation performs better then existing allocation by 10-15%, while in turn our mechanism outperforms IATA. The properties of ECATS help us in considering congestion cost and RCOF in the objective function and incentivize a truthful value revelation by the airlines. The payment mechanism also takes into account congestion cost and RCOF, thereby driving airline to operate in less congested slots and connect to remote cities. To the best of our knowledge, this is the first study to consider all the three goals and provide a truthful mechanism for airport slot allocation.

6 Conclusions

The present study has examined the slot allocation process from the multi-objective viewpoint. We start with consideration of two major problems of the airline industry, namely congestion and regional connectivity. We combined the features of market-based and administrative instruments in a single dominant strategy incentive compatible mechanism with the goal of truthfulness and social connectivity. The proposed mechanism provides a flexible and transparent approach for slot allocation and the analytical results offer meaningful policy implications. The mechanism is solvable in polynomial time. The significant contributions of this study are as follows.

First, the proposed mechanism provides a fine balance between three competing goals: efficiency, remote city connection and congestion mitigation. A new model is proposed for optimizing the allocation of the slot on the basis of valuation of the movements and the congestion cost. The valuation of different movements is based on ticket prices and load factor. The model is efficient as it allocates slots to movements based on valuations. It also considers regional connectivity by incorporating remote city opportunity factor. The weights of population and social progress index of the cities can be flexibly changed based on the varying goals of the policy makers. In addition, the relationship between airport capacity and congestion is examined using variable capacity. It captures the phenomena of congestion when capacity used exceed its limits. The threshold limit of airport capacity can be adjusted for different airports as it depends on factors such as types of aircraft and technology used at airports. The threshold limit

Table 1: Percent improvement in social utility of ECATS vis-a-vis Current and IATA based allocations.

		Chennai (MAA) Airport		Delhi (DEL) Airport	
		% Improvement w.r.t.			
Time Interval	Congestion Cost	Current allocation	IATA allocation	Current allocation	IATA allocation
6AM - 12PM	15000	29.9%	4.6%	35.1%	6.2%
	20000	28.4%	4.8%	33.5%	6.4%
	25000	26.6%	5.1%	31.7%	6.4%
	30000	24.3%	5.6%	30.4%	6.9%
12PM - 6PM	15000	29.9%	12.3%	43.2%	10.2%
	20000	29.1%	12.6%	42.4%	10.3%
	25000	28.4%	13.2%	41.1%	10.5%
	30000	27.0%	13.4%	39.6%	10.8%
6PM - 12AM	15000	36.9%	11.4%	34.2%	7.1%
	20000	37.8%	11.5%	33.3%	7.3%
	25000	38.9%	12.0%	31.9%	7.6%
	30000	39.9%	12.8%	29.9%	8.0%
12AM - 6AM	15000	28.7%	19.2%	32.1%	8.4%
	20000	29.7%	19.6%	30.5%	8.7%
	25000	31.0%	20.6%	28.5%	8.9%
	30000	32.4%	21.6%	26.5%	9.2%

could serve as an upper limit for the non-congested capacity of the airport. Our mechanism brings in flexibility where:

- Congestion and its associated cost can be tuned based on the historical congestion data of the airport.
- The threshold limit of capacity can be adjusted for different airports.
- The mechanism can be tweaked according to the objective of airport/priority of the policy maker, as it provides adjustable weights of social progress index and population of the remote city, which caters to social obligations for slot allocation.

The second contribution of this study comes from the fact that proposed mechanism is strategy-proof and individually rational (the agents' valuations for the chosen allocations are nonnegative). Besides, our mechanism is incentive compatible in dominant strategies. None of the players can get better utility by misreporting their true valuation for the slots. The formulation is polynomial solvable as LP relaxation yields an optimal solution in integers in polynomial time.

The third contribution of this study is its payment rule. The payment rule is designed in such a way that it captures the externality imposed by a movement on the others. It is calculated as the difference between the social welfare generated without existence of the movement and the sum of weighted valuations of all the other airlines if it participates in the allocation. The resulting payment captures two properties:

- The contribution of particular movement to congestion and
- The type of connectivity provided by it (Metro city, non-metro capital city or small city).

The mechanism is designed in such a way that it assigns low payment for remote city connections and provides an incentive for airlines to operate on such routes. Moreover, the high payment value for trunk routes discourages airlines to increase frequency only on such routes. The mechanism also considers the congestion externality imposed by a movement and charge fee based on the contribution of movement. Hence, the payment mechanism emulates properties of Pigouvian tax, which is used to correct inefficient market by imposing a fee equal to the marginal cost of the negative externality. The proposed payment also drives our mechanism to be incentive-compatible and charge a higher cost to movements with negative externalities.

Limitations and Future Research Directions

Like any other research, our study also has several limitations. The future studies can extend the body of literature on slot allocation by incorporating these limitations into their models. Firstly, our mechanism is not budget balanced. Future studies may try to achieve together both efficiency and budget balance criteria. We have considered a single airport for slot allocation. Future studies may consider multiple airports for slot allocations simultaneously. Congestion in a single slot has a cascading effect on subsequent movements. Future studies may consider rolling capacity constraint for different slots. In our remote city connectivity factor, we have considered the social progress index and population of the cities. Future studies may also incorporate the availability of other modes of transportation such as rail, road and water to these cities.

References

- Marco Alderighi, Alberto A Gaggero, and Claudio A Piga. The hidden side of dynamic pricing: Evidence from the airline market. *Available at SSRN 3085585*, 2017.
- Konstantinos N Androutsopoulos, Eleftherios G Manousakis, and Michael A Madas. Modeling and solving a bi-objective airport slot scheduling problem. *European Journal of Operational Research*, 284(1):135–151, 2020.
- Richard P Anstee. A polynomial algorithm for b-matchings: an alternative approach. *Information Processing Letters*, 24(3):153–157, 1987.
- Michael Ball, George Donohue, and Karla Hoffman. Auctions for the safe, efficient, and equitable allocation of airspace system resources. *Combinatorial auctions*, 1, 2006.
- Michael O Ball, Alexander S Estes, Mark Hansen, and Yulin Liu. Quantity-contingent auctions and allocation of airport slots. *Transportation Science*, 54(4):858–881, 2020.
- Leonardo J Basso and Anming Zhang. Pricing vs. slot policies when airport profits matter. *Transportation Research Part B: Methodological*, 44(3):381–391, 2010.
- Dave W Benell and Barry E Prentice. A regression model for predicting the economic impacts of canadian airports. *Logistics and Transportation Review*, 29(2):139, 1993.
- Volodymyr Bilotkach, Alberto A Gaggero, and Claudio A Piga. Airline pricing under different market conditions: Evidence from european low-cost carriers. *Tourism Management*, 47: 152–163, 2015.
- R.I. Brafman and M. Tennenholtz. R-max-a general polynomial time algorithm for near-optimal reinforcement learning. *The Journal of Machine Learning Research*, 3:213–231, 2003.

- Jan K Brueckner. Airline traffic and urban economic development. *Urban Studies*, 40(8):1455–1469, 2003.
- Jan K Brueckner. Price vs. quantity-based approaches to airport congestion management. *Journal of Public Economics*, 93(5-6):681–690, 2009.
- Guillaume Burghouwt. Influencing air connectivity outcomes. International Transport Forum Discussion Paper, 2017.
- Jia-Ming Cao and Adib Kanafani. The value of runway time slots for airlines. *European Journal of Operational Research*, 126(3):491–500, 2000.
- Alan Carlin and Rolla Edward Park. Marginal cost pricing of airport runway capacity. *The American Economic Review*, pages 310–319, 1970.
- Lorenzo Castelli, Paola Pellegrini, and Raffaele Pesenti. Airport slot allocation in europe: economic efficiency and fairness. *International journal of revenue management*, 6(1-2):28–44, 2012.
- Andrew M Churchill, David J Lovell, and Michael O Ball. Flight delay propagation impact on strategic air traffic flow management. *Transportation Research Record*, 2177(1):105–113, 2010.
- Achim I Czerny. Airport congestion management under uncertainty. *Transportation Research Part B: Methodological*, 44(3):371–380, 2010.
- Achim I Czerny and Anming Zhang. Airport congestion pricing when airlines price discriminate. *Transportation Research Part B: Methodological*, 65:77–89, 2014.
- Joseph I Daniel. Congestion pricing of canadian airports. *Canadian Journal of Economics/Revue canadienne d’économique*, 44(1):290–324, 2011.
- Joseph I Daniel and Katherine Thomas Harback. Pricing the major us hub airports. *Journal of Urban Economics*, 66(1):33–56, 2009.
- Dominique de Werra. On some characterisations of totally unimodular matrices. *Mathematical Programming*, 20(1):14–21, 1981.
- Xavier Fageda, Ancor Suárez-Alemán, Tomas Serebrisky, and Reinaldo Fioravanti. Air connectivity in remote regions: A comprehensive review of existing transport policies worldwide. *Journal of Air Transport Management*, 66:65–75, 2018.
- Jamie Fairbrother and Konstantinos G Zografos. Optimal scheduling of slots with season segmentation. *European Journal of Operational Research*, 2020.
- Terence P Fan and Amedeo R Odoni. A practical perspective on airport demand management. *Air Traffic Control Quarterly*, 10(3):285–306, 2002.
- Alain Ghouila-Houri. Caractérisation des matrices totalement unimodulaires. *Comptes Rendus Hebdomadaires des Séances de l’Académie des Sciences (Paris)*, 254:1192–1194, 1962.
- Richard K Green. Airports and economic development. *Real estate economics*, 35(1):91–112, 2007.

- Martin Grötschel, László Lovász, and Alexander Schrijver. Complexity, oracles, and numerical computation. In *Geometric Algorithms and Combinatorial Optimization*, pages 21–45. Springer, 1993.
- Pavithra Harsha. *Mitigating airport congestion: market mechanisms and airline response models*. PhD thesis, Massachusetts Institute of Technology, 2009.
- Will Horton. British airways cuts threaten crown jewel of slots at london heathrow and gatwick. *The Forbes*, May 2020. URL <https://www.forbes.com/sites/willhorton1/2020/05/04/british-airways-cuts-threaten-crown-jewel-of-slots-at-london-heathrow-and-gatwick/#359a052b7cdf>. Accessed on May 4th 2020.
- IATA. Worldwide slot guidelines, eighth ed. IATA, 2017. URL <https://www.iata.org/policy/slots/Documents/wsg-8-english.pdf>.
- Zhi-Chun Li, William HK Lam, SC Wong, and Xiaowen Fu. Optimal route allocation in a liberalizing airline market. *Transportation Research Part B: Methodological*, 44(7):886–902, 2010.
- Michael A Madas and Konstantinos G Zografos. Airport slot allocation: a time for change? *Transport Policy*, 17(4):274–285, 2010.
- Ruta Mehta and Vijay V Vazirani. An incentive compatible, efficient market for air traffic flow management. *Theoretical Computer Science*, 818:41–50, 2020.
- Nuno Antunes Ribeiro, Alexandre Jacquillat, António Pais Antunes, Amedeo R Odoni, and João P Pita. An optimization approach for airport slot allocation under iata guidelines. *Transportation Research Part B: Methodological*, 112:132–156, 2018.
- Kevin Roberts. *The Characterization of Implementable Choice Rules*, chapter Aggregation and Revelation of Preferences, pages 321–348. North Holland Publishing, 1979.
- Nicholas Sheard. Airports and urban sectoral employment. *Journal of Urban Economics*, 80:133–152, 2014.
- Dian Sheng, Zhi-Chun Li, and Xiaowen Fu. Modeling the effects of airline slot hoarding behavior under the grandfather rights with use-it-or-lose-it rule. *Transportation Research Part E: Logistics and Transportation Review*, 122:48–61, 2019.
- Y. Shoham and K. Leyton-Brown. *Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations*. Cambridge University Press, 2008.
- Gernot Sieg. Grandfather rights in the market for airport slots. *Transportation Research Part B: Methodological*, 44(1):29–37, 2010.
- Prem Swaroop, Bo Zou, Michael O Ball, and Mark Hansen. Do more us airports need slot controls? a welfare based approach to determine slot levels. *Transportation Research Part B: Methodological*, 46(9):1239–1259, 2012.
- Arie Tamir and Joseph SB Mitchell. A maximumb-matching problem arising from median location models with applications to the roommates problem. *Mathematical Programming*, 80(2):171–194, 1998.
- Vikrant Vaze and Cynthia Barnhart. Modeling airline frequency competition for airport congestion mitigation. *Transportation Science*, 46(4):512–535, 2012.

William S Vickrey. Congestion theory and transport investment. *The American Economic Review*, 59(2):251–260, 1969.

Shujie Yao and Xiuyun Yang. Airport development and regional economic growth in china. *Available at SSRN 1101574*, 2008.

Konstantinos G Zografos and Michael A Madas. Critical assessment of airport demand management strategies in europe and the united states: Comparative perspective. *Transportation research record*, 1850(1):41–48, 2003.

Konstantinos G Zografos, Michael A Madas, and Yiannis Salouras. A decision support system for total airport operations management and planning. *Journal of Advanced Transportation*, 47(2):170–189, 2013.

Appendix

Proof of Theorem 1. In terms of utilities, the theorem says,

$$u_i(f(v_i, v_{-i}), V) \geq u_i(f(v'_i, v_{-i}), V), \quad \forall v_i, v'_i \in \mathbb{R}^{|n|} \quad \forall i \in M \quad (9)$$

Let us assume for the contradiction that, there exist a movement i for which the corresponding airline has true valuations for the slots as, v_i , but misreports it as v'_i (the corresponding value function is v'_i), and gets better utility,

$$u_i(f(v'_i, v_{-i}), V) > u_i(f(v_i, v_{-i}), V) \quad (10)$$

Suppose $\mathcal{A}(v'_i, v_{-i}) = x'$ and $\mathcal{A}(v_i, v_{-i}) = x^*$. By definition of the utility function,

$$\begin{aligned} u_i(x^*, V) &= v_i(x^*) - p_i(v_i, v_{-i}) \\ &= v_i(x^*) - \frac{1}{\rho_i} \left(h_i(v_{-i}) - \left(\sum_{k \in M \setminus \{i\}} \rho_k v_k(x^*) - g \sum_{j \in S} e_j(x^*) \right) \right) \\ &= v_i(x^*) + \frac{1}{\rho_i} \left(\sum_{k \in M \setminus \{i\}} \rho_k v_k(x^*) - g \sum_{j \in S} e_j(x^*) \right) - \frac{1}{\rho_i} \left(h_i(v_{-i}) \right) \\ &= \frac{1}{\rho_i} \left(\sum_{k \in M} \rho_k v_k(x^*) - g \sum_{j \in S} e_j(x^*) \right) - \frac{1}{\rho_i} \left(h_i(v_{-i}) \right) \end{aligned} \quad (11)$$

and,

$$\begin{aligned} u_i(x', (v_i, v_{-i})) &= v_i(x') - p_i(v'_i, v_{-i}) \\ &= \frac{1}{\rho_i} \left(\sum_{k \in M} \rho_k v_k(x') - g \sum_{j \in S} e_j(x') \right) - \frac{1}{\rho_i} \left(h_i(v_{-i}) \right) \end{aligned} \quad (12)$$

From inequality 10,

$$\begin{aligned} &\frac{1}{\rho_i} \left(\sum_{k \in M} \rho_k v_k(x') - g \sum_{j \in S} e_j(x') \right) - \frac{1}{\rho_i} \left(h_i(v_{-i}) \right) \\ &> \frac{1}{\rho_i} \left(\sum_{k \in M} \rho_k v_k(x^*) - g \sum_{j \in S} e_j(x^*) \right) - \frac{1}{\rho_i} \left(h_i(v_{-i}) \right) \end{aligned}$$

$$\sum_{k \in M} \rho_k v_k(x') - g \sum_{j \in S} e_j(x') > \sum_{k \in M} \rho_k v_k(x^*) - g \sum_{j \in S} e_j(x^*)$$

The above inequality leads to the contradiction with the fact that x^* is the socially optimal allocation. And therefore, the best strategy for airlines is to report the valuations of their movements truthfully. \square

Proof of Theorem 2. Consider the allocation given by \mathcal{A} as x^* . The utility of the airline i is, $u_i((x^*, p(V)), V) = v_i(x^*) - p_i(V) \forall i \in M$. $v_i(\mathcal{A}(V)) \geq 0$, as it is the valuation by the allocation of slots. If $\rho_i = 0$ then $u_i((x^*, p(V)), V) \geq 0$. For the case when $\rho_i > 0$, the proof is as follows: by expanding the expression for p in the definition of utility, we get

$$\begin{aligned} u_i((x^*, p(V)), V) &= v_i(x^*) - \frac{1}{\rho_i} \left(h_i(v_{-i}) - \left(\sum_{k \in M \setminus \{i\}} \rho_k v_k(x^*) - g \sum_{j \in S} e_j(x^*) \right) \right) \\ &= v_i(x^*) - \frac{1}{\rho_i} \left(h_i(v_{-i}) \right) + \frac{1}{\rho_i} \left(\sum_{k \in M \setminus \{i\}} \rho_k v_k(x^*) - g \sum_{j \in S} e_j(x^*) \right) \\ &= \frac{1}{\rho_i} \left(\sum_{k \in M} \rho_k v_k(x^*) - g \sum_{j \in S} e_j(x^*) \right) \\ &\quad - \frac{1}{\rho_i} \left(\sum_{k \in M \setminus \{i\}} \rho_k v_k(\mathcal{A}(v_{-i})) - g \sum_{j \in S} e_j(\mathcal{A}(v_{-i})) \right) \end{aligned}$$

By adding and subtracting $\frac{1}{\rho_i} (v_i(\mathcal{A}(v_{-i})))$,

$$\begin{aligned} &= \frac{1}{\rho_i} \left(\sum_{k \in M} \rho_k v_k(x^*) - g \sum_{j \in S} e_j(x^*) \right) \\ &\quad - \frac{1}{\rho_i} \left(\sum_{k \in M} \rho_k v_k(\mathcal{A}(v_{-i})) - g \sum_{j \in S} e_j(\mathcal{A}(v_{-i})) \right) + \frac{1}{\rho_i} (v_i(\mathcal{A}(v_{-i}))) \\ &= \underbrace{\left(\sum_{k \in M} \rho_k v_k(x^*) - g \sum_{j \in S} e_j(x^*) \right) - \left(\sum_{k \in M} \rho_k v_k(\mathcal{A}(v_{-i})) - g \sum_{j \in S} e_j(\mathcal{A}(v_{-i})) \right)}_{\geq 0} \\ &\quad + \underbrace{v_i(\mathcal{A}(v_{-i}))}_{\geq 0} \end{aligned}$$

The difference of first two terms is non-negative by the definition of x^* , the socially optimal allocation, and the third term is non-negative as it is the valuation by the allocation of slots, which proves that ECATS is individually rational. \square