Pricing server's surplus capacity

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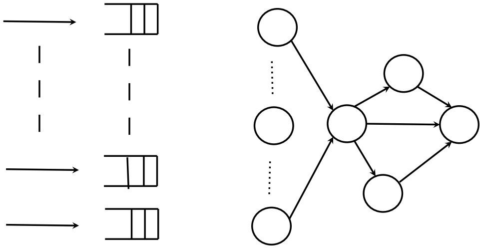
November 2, 2012

Outline

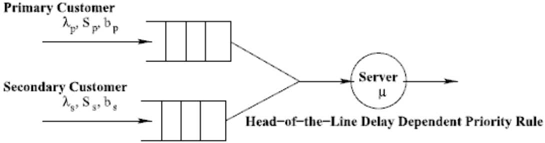
- Introduction
- Model by Sinha et. al. (2010) and some literature
- A proof of conjecture
- Current work
- Future work

Problem definition



- Multi class customers arriving over a network
- Introduce new class without affecting the service level of other classes.
- Scheduling of new class across classes
- Admission control of new class

Single node and two classes (Sinha et. al. (2010))



- There are two classes of customers, primary and secondary.
- Primary are the existing customers and their mean waiting time is promised below S_p.
- There is a surplus capacity to accommodate new customers.
- There is no pre-emption.
- The demand of new customers (secondary class) is sensitive to both unit admission price and mean waiting time.
- The problem is to quote the unit admission price and service level.

Implementation

- A delay dependent non pre-emptive priority is considered across classes (Klienrock (1964)).

$$q_p(t) = delay \times b_p$$

Notations:

λ_p	Arrival rate for primary class customers
λ_s	Arrival rate of secondary customers
S_p	Promised mean waiting time of primary class customers
S_s	Promised mean waiting time of secondary class customers
μ	Mean service rate of server
σ^2	Variance of service time
θ	Unit admission price charged to secondary customers
ψ	$= \frac{1+\sigma^2\mu^2}{2}$

Original Optimization problem P_0

$$\max_{\lambda_s,\theta,S_s,\beta} \theta\lambda_s \tag{1}$$

Subject to

$$W_p(\lambda_s, \beta) \leq S_p \tag{2}$$

$$S_s \geq W_s(\lambda_s, \beta) \tag{3}$$

$$\lambda_s \leq \mu - \lambda_p \tag{4}$$

$$\lambda_s \leq a - b\theta - cS_s \tag{5}$$

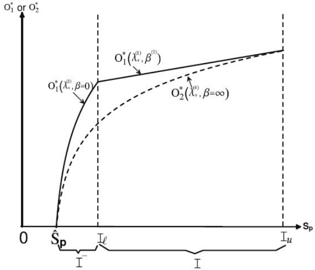
$$\lambda_s, \theta, S_s, \beta \geq 0 \tag{6}$$

- Constraint (2) and (3) are service level constraint for primary and secondary class customers respectively.
- Constraint (4) and (5) are system stability and demand constraint.
- Constraint (3) and (5) will be binding.

Optimization problem P_1 and P_2

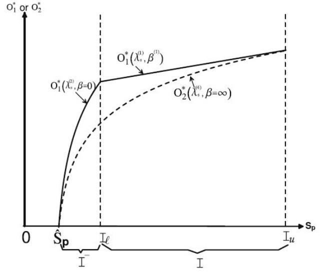
- One can reduce the four dimensional optimization problem (P_0) to two dimensional optimization problem in λ_s and β.
- Range of queue discipline management parameter, $0 \leq \beta \leq \infty$
- $\beta = \infty$ is also a valid decision.
- Optimization problem P_1 reduces to one dimensional convex optimization problem P_2 for $\beta = \infty$.
- To search for global optima, one needs to compare the objective of optimization problems P_1 and P_2

Search for global optima



An algorithm is proposed to find the global optima assuming that conjecture is true.

Search for global optima



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Conjecture

For $S_p \in I^-$, the optimal solution of the original problem (P_0) is given by the optimal solution of P_1.

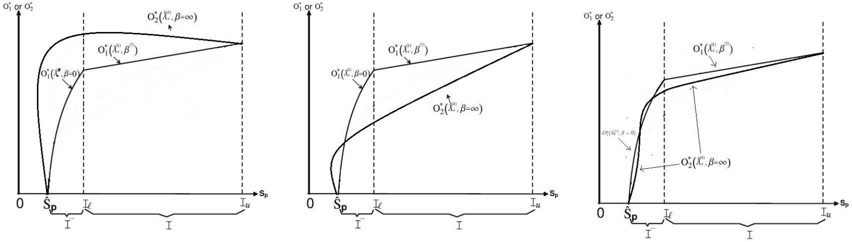
Theorem

Optimal solution for optimization problem P_2, i.e., O_2^* is increasing concave in interval $I^- \cup I$ while O_1^* is increasing concave in I^- and linearly increasing in I

- The fact that O_1^* is increasing concave in I^- and linearly increasing in I follows from Sinha et. al. (2010)
- To prove first part
  - Claim that $I^- \subset J^-$
  - Corollary that $\lambda_s^{(4)}$ is an increasing function of S_p
  - Some optimization and queueing based arguments

Corollary

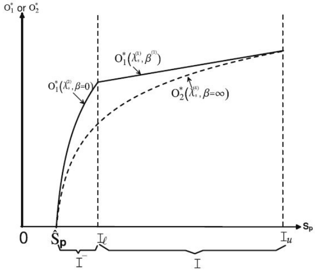
For $S_p \in I^- \cup I$, the optimal solution of P0 is given by optimal solution of P1.



- Contradiction from $O_2^*(\lambda_s^i, \infty) < O_1^*(\lambda_s^f, \beta^f)$ for $S_p \in I$
- Contradiction from infeasibility and $O_2^*(\lambda_s^i, \infty) < O_1^*(\lambda_s^f, \beta^f)$ at $\hat{S}_p + \epsilon$
- Contradiction from concavity of O_2^*

Proof of conjecture

The possible way for O_1^* and O_2^*



Conjecture. *For $S_p \in I^-$, the optimal solution of P0 is given by optimal solution of P1.*
Proof. Follows from above corollary.

A variation of model

- Delay dependent pre-emptive priority across classes
- Service time is exponential.
- Remaining settings are similar to Sinha et. al. (2010)
- Comparison of objectives
- Proposed an algorithm to find the global optimal operating parameters
- A comparative study of two priority policies
- A better algorithm by changing the priority policy
- Sensitivity analysis and numerical examples

Challenges in network

- Departure process of delay dependent priority
  - Departure process of M/M/1 queue (Bruke (1956))
  - Departure process of $\sum M_i/G_i/1$ queue (Stanford (1991))
- Stochastic approximation algorithms for constrained optimization via simulation
  - Scheduling parameter is not compact.
- Relative priority (Haviv et. al. (2007))
  - Relative priority and delay dependent priority are equivalent in two dimension.
  - Relative priority is complete in two dimension.

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Thank You