

Descriptive Statistics



Descriptive Statistics

- Frequency Distribution
- Plots on Distributions
- Central Tendency – Mean Median Mode
- Skewness, Kurtosis
- Quiz check
- Measure of Dispersion – Variability and Spread
- Range, IQR, Percentile, Boxplot
- Variance, Standard Deviation z Score
- Scatterplot and Correlation
- Quiz Check



Frequency Distributions

- A frequency distribution shows us a summarized grouping of data divided into mutually exclusive classes and the number of occurrences(Frequency) in a class.
- It is a way of showing unorganized data notably to show results of an election, income of people for a certain region, sales of a product within a certain period, student loan amounts of graduates, etc.
- Some of the graphs that can be used with frequency distributions are [histograms](#), [bar charts](#), [pie charts](#), etc.
- Frequency distributions are used for both qualitative and quantitative data.



Qualitative - Bar Graph

- A **bar chart** or **bar graph** is a chart or graph that presents **categorical data** with **rectangular** bars with **heights** or **lengths** proportional to the values that they represent. The bars can be plotted vertically or horizontally. A vertical bar chart is sometimes called a **column chart**.



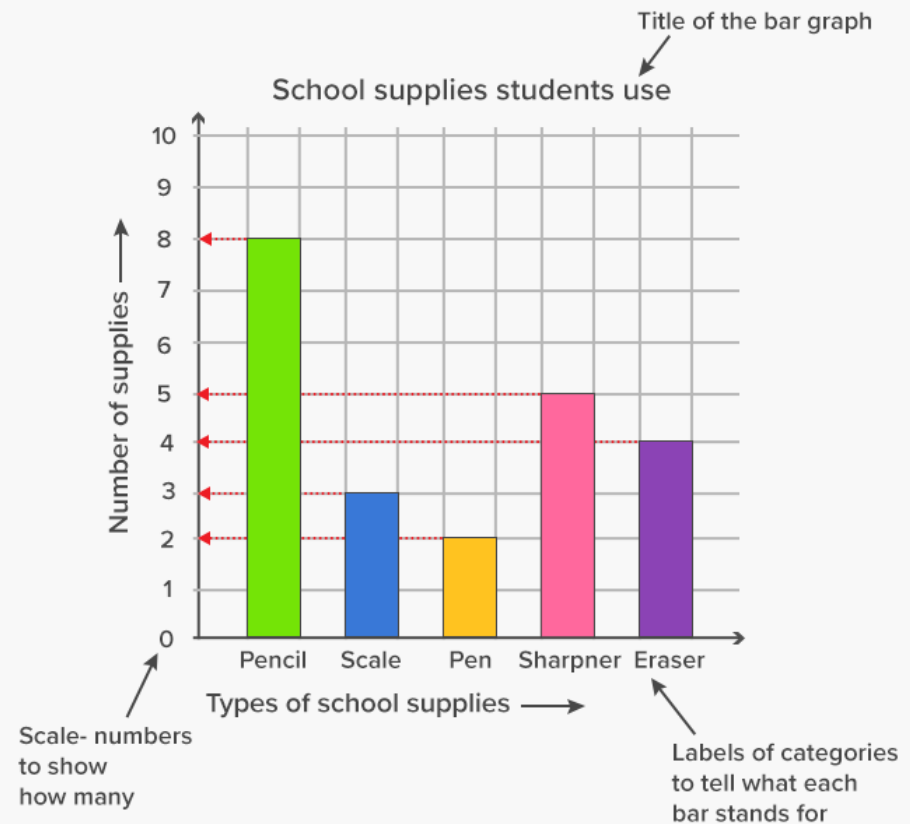
Graphical Representation of Bar

- A bar graph shows comparisons among **discrete categories**. One axis (X – axis) of the chart shows the specific categories(Class) being compared, and the other axis (Y – axis) represents a measured value(Frequency).

Class (School Supplies)	Frequency(No.of Suppliers)
Pencil	8
Scale	3
Pen	2
Sharpener	5
Eraser	4

Note: Frequency in Pandas are `Value_Counts()`

Source: Wikipedia and image from splashlearn.com



Qualitative - Pie Chart

- Pie charts work by splitting your data **into distinct groups or categories**. The chart consists of a circle split into wedge-shaped slices, and each slice represents a group.
- The size of each slice is proportional to how many are in each group compared with the others.
- The number in a particular group is called the **frequency**



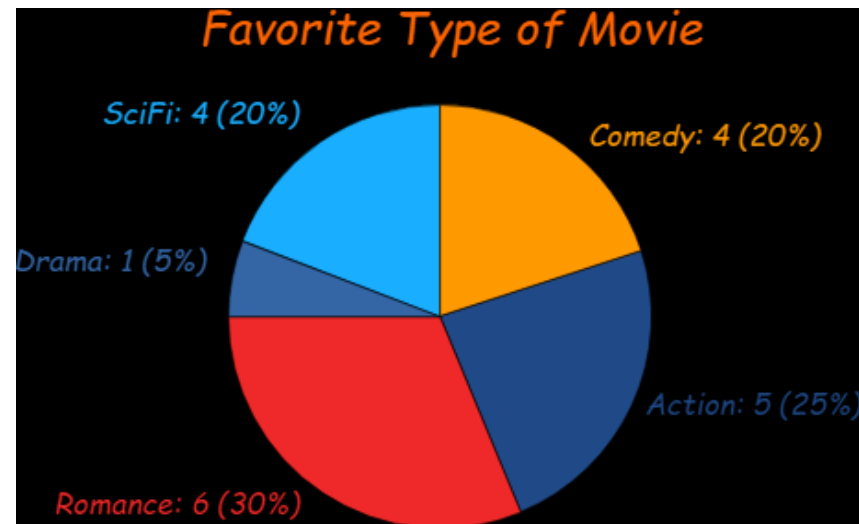
Pie Chart Calculation & Representation

Comedy	Action	Romance	Drama	SciFi
4	5	6	1	4

Next, divide each value by the total and multiply by 100 to get a percent:

Comedy	Action	Romance	Drama	SciFi	TOTAL
4	5	6	1	4	20
$\frac{4}{20}$ = 20%	$\frac{5}{20}$ = 25%	$\frac{6}{20}$ = 30%	$\frac{1}{20}$ = 5%	$\frac{4}{20}$ = 20%	100%

- The larger the slice, the greater the relative popularity of that group.
- Here Action is relative popularity from others



Quantitative - Histogram

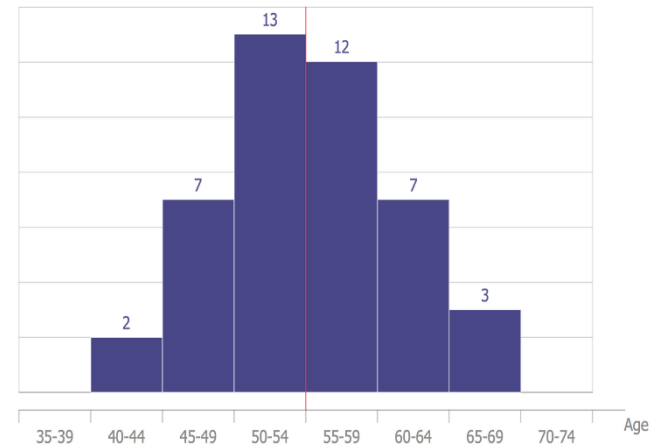
- Histogram is a graphical representation of the **Frequency Distribution** of data in which the X-axis represents the classes and the Y-axis represents the frequencies in bars.
- To construct a histogram from a **continuous variable** you first need to split the **data into intervals**, called **bins**.
- Lets consider ages of 44 People working in a company below list.
- [41,44, 45,46,46,47,48,49,49, 50,50,50,50,51,52,52,53,53,54,55,54,53, 55,55,55,56,56,57,57,57,58,58,59,59,59, 60,61,61,61,62,63,64, 66,68,69]



Visual Representation - Histogram

- Considering **bin size** equal to 5 you will get distribution as mention below. Also notice the histogram you will get from the distribution shown in the table.

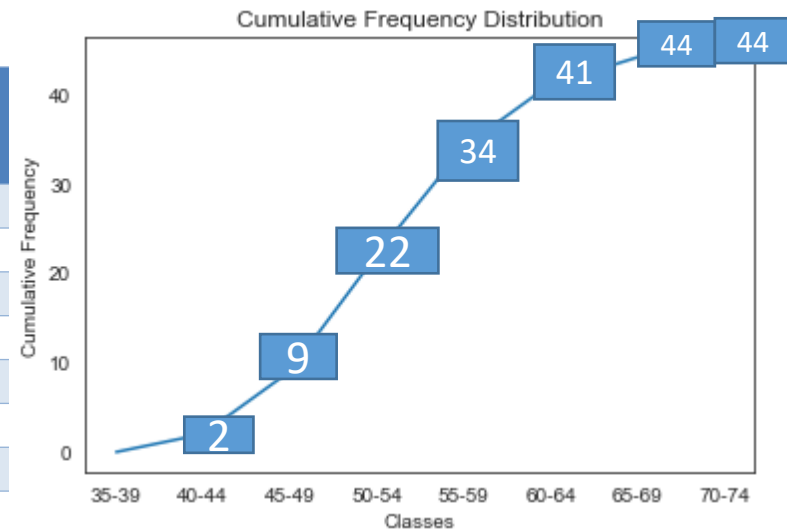
Class	Frequency	Scores included in Bin
35-39	0	
40-44	2	41,44
45-49	7	45,46,46,47,48,49,49
50-54	13	50,50,50,50,51,52,52,53,53,54,55,54,53
55-59	12	55,55,55,56,56,57,57,57,58,58,59,59,59
60-64	7	60,61,61,61,62,63,64
65-69	3	66,68,69
70-74	0	



Cumulative Frequency

A type of frequency distribution that shows how **many observations are above or below the lower boundaries of the classes.**

Class	Frequency	Relative Frequency	Cumulative Frequency	Cumulative Relative Frequency
35-39	0	0	0	0
40-44	2	0.045454545	2	0.04545455
45-49	7	0.159090909	9	0.20454545
50-54	13	0.295454545	22	0.5
55-59	12	0.272727273	34	0.77272727
60-64	7	0.159090909	41	0.93181818
65-69	3	0.068181818	44	1
70-74	0	0	44	1
Total	44	1		



Describing Data through Statistics

Descriptive Statistics



The Central Tendencies

Virat want to join a health club in a activity that has others in the same age group as him. He is 28 years old. Mean ages for **YOGA**, **GYM** and **SWIMMING** classes are

22 years



30 years



17 years



The Central Tendencies

Yoga class composition

Age (years)	19	22	23
Frequency, f	1	3	2



$$\text{Mean, } \mu = \frac{\sum x}{n} =$$

$$\frac{19 * 1 + 22 * 3 + 23 * 2}{1 + 3 + 2} \approx 22$$



The Central Tendencies

Power workout class composition

Age (years)	20	22	23	90
Frequency, f	4	8	5	1



$$\text{Mean, } \mu = \frac{\sum x}{n} =$$

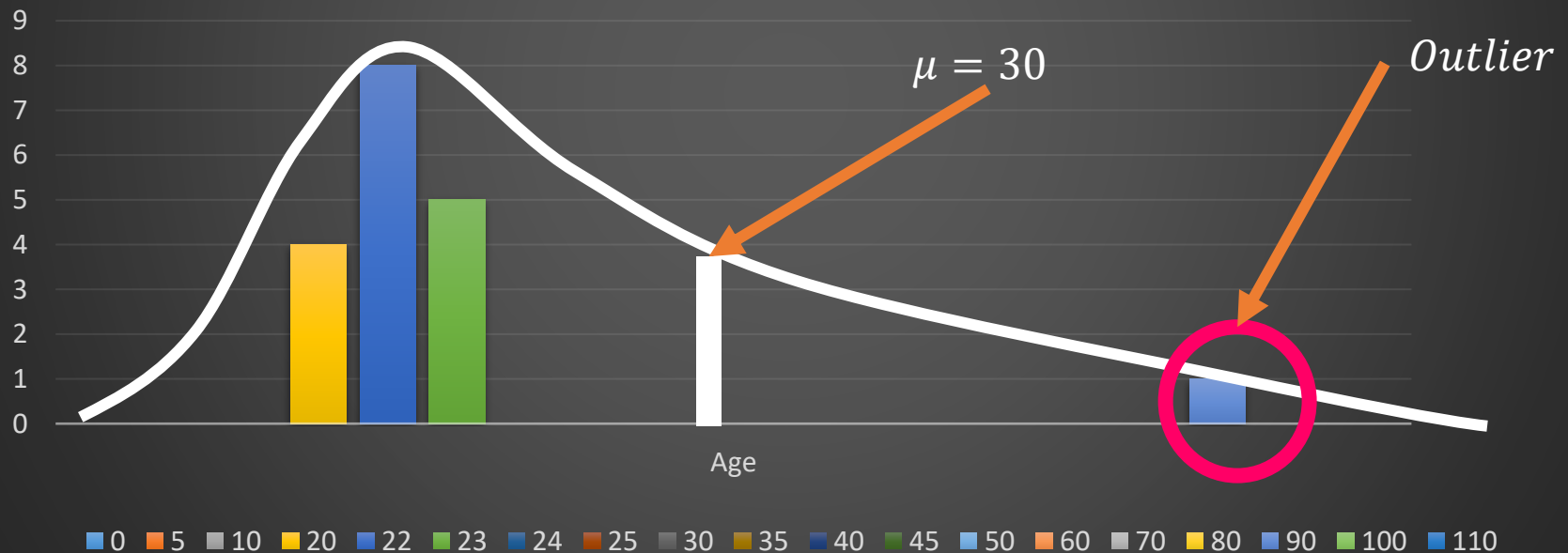
$$\frac{20 * 4 + 22 * 8 + 23 * 5 + 90 * 1}{4 + 8 + 5 + 1} = 30$$



The Central Tendencies

Power Workout Class Composition

Age (years)	20	22	23	90
Frequency, f	4	8	5	1



Disadvantage of Mean

- Finding mean is not a good approach as the '**Mean is often affected by Outliers**' or in simple words if there are some observations larger or smaller than majority of the other observations then the mean **tends to deviate towards these values**.
- To generalize it if the distribution of datasets is **skewed(troubled by outliers)**, we do not choose mean. Here we will have **to go for Median**.



The Central Tendencies – Median the mid-point

Age (years)	20	22	23	90
Frequency, f	4	8	5	1

- Data has outlier

How to find the median in three steps:

- 1. Line your numbers up in order, from smallest to largest.
- 2. If you have an odd number of values, the median is the one in the middle. If you have n numbers, the middle number is at position $(n + 1) / 2$.
- 3. If you have an even number of values, get the median by adding the two middle ones together and dividing by 2. You can find the midpoint by calculating $(n + 1) / 2$. The two middle numbers are on either side of this point.

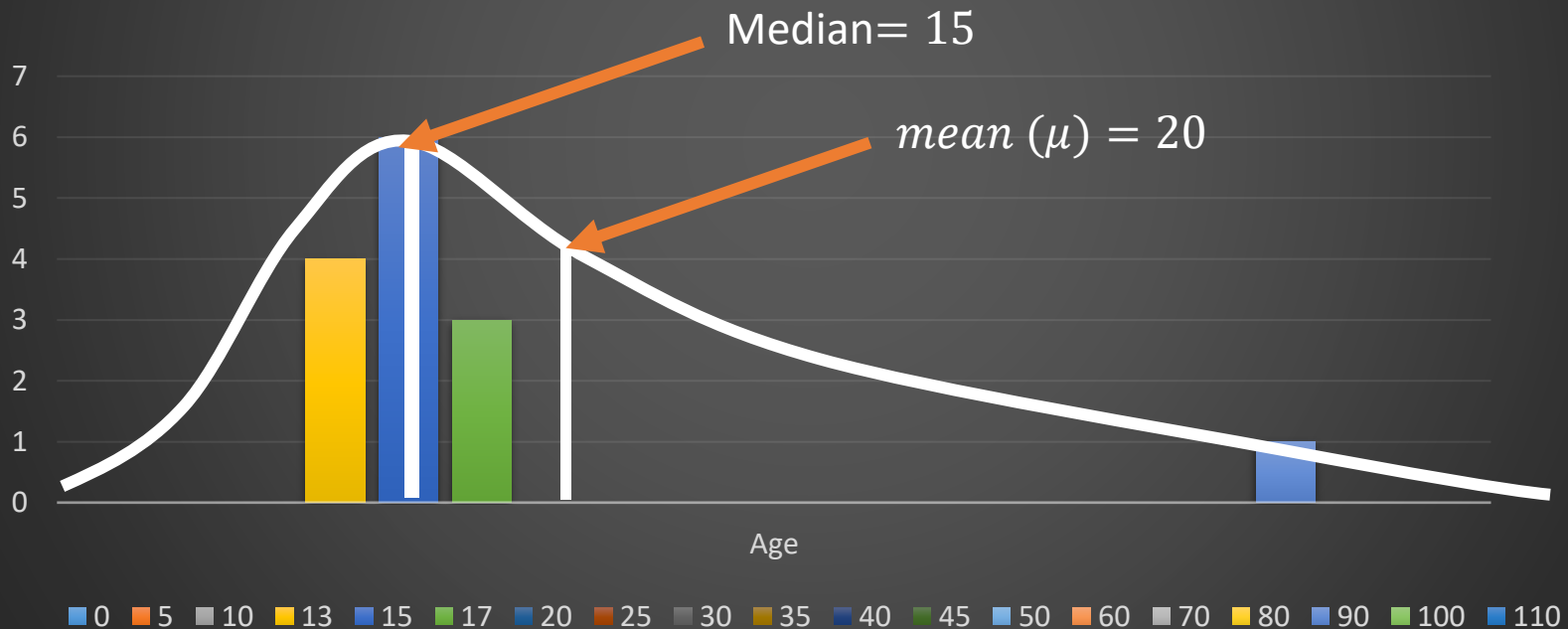
20, 20, 20, 20, 22, 22, 22, 22, 22, 22, 22, 23, 23, 23, 23, 23, 90



The Central Tendencies

Power Workout Class Composition

Age (years)	20	15	17	90
Frequency, f	4	6	3	1



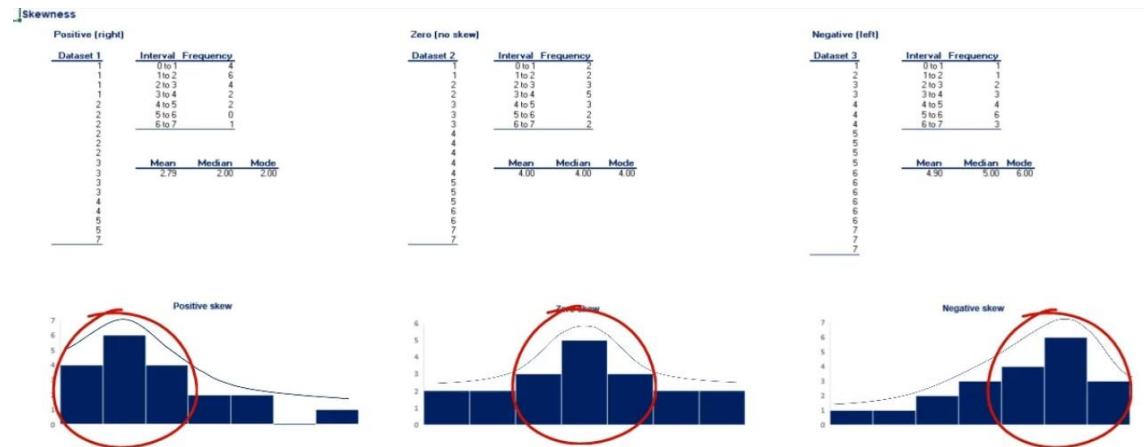
Skewness

- Skewness basically gives the shape of normal distribution of values.
- Skewness is asymmetry in a statistical distribution, in which the curve appears distorted or skewed either to the left or to the right.
- Skewness can be quantified to define the extent to which a distribution differs from a normal distribution.



Skewness

- Skewness tells us a lot about where the data is situated.

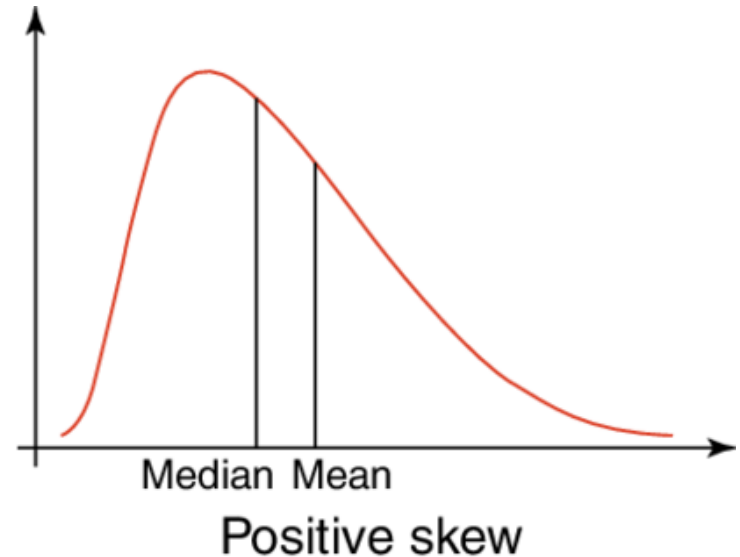


- In fact, the mean, median and mode should be used together to get a good understanding of the dataset.
- Measures of asymmetry like skewness are the link between central tendency measures and probability theory.
- This ultimately allows us to get a more complete understanding of the data we are working with.



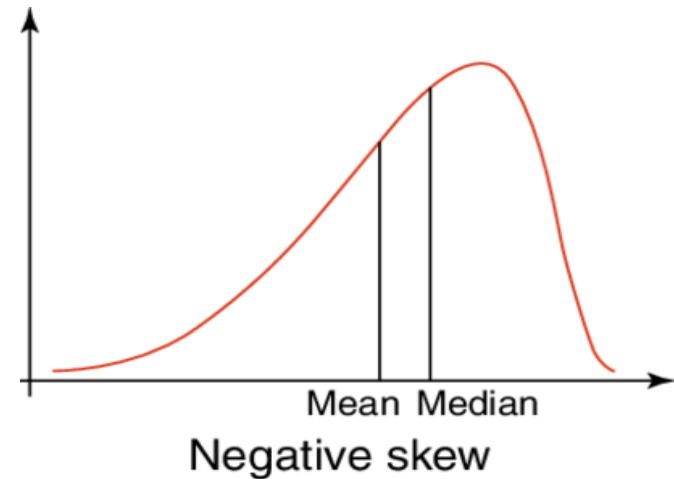
Positive Skewness

A positively skewed distribution means that the extreme data results are larger. This skews the data in that it brings the mean (average) up. The mean will be larger than the median in a Positively skewed distribution.



Negative Skewness

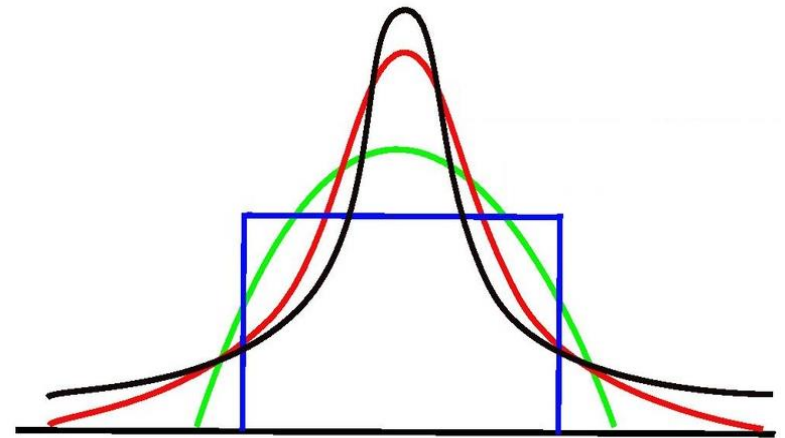
A negatively skewed distribution means the opposite: that the extreme data results are smaller. This means that the mean is brought down, and the median is larger than the mean in a negatively skewed distribution.



Kurtosis

The exact interpretation of the measure of Kurtosis used to be disputed, but is now settled. It's about existence of outliers.

Kurtosis is a measure of whether the data are heavy-tailed (profusion of outliers) or light-tailed (lack of outliers) relative to a normal distribution.



The Central Tendencies

Virat is disturbed and wants some relaxation. He joins the swimming class where mean age is 17 years. He didn't understand why they were asking where his kid was...

Age (Years)	1	2	3	30	31	32	33
Frequency, f	3	4	3	1	3	2	4

$\mu \approx 17 \text{ Years}$

Median ?



What happens to Median if another kid or adult is added ?



The Central Tendencies

Age (Years)	1	2	3	30	31	32	33
Frequency f	3	4	3	1	3	2	4

What is the mode – the most frequently occurring data point ?

Mode is 2 and 4



The Central Tendencies

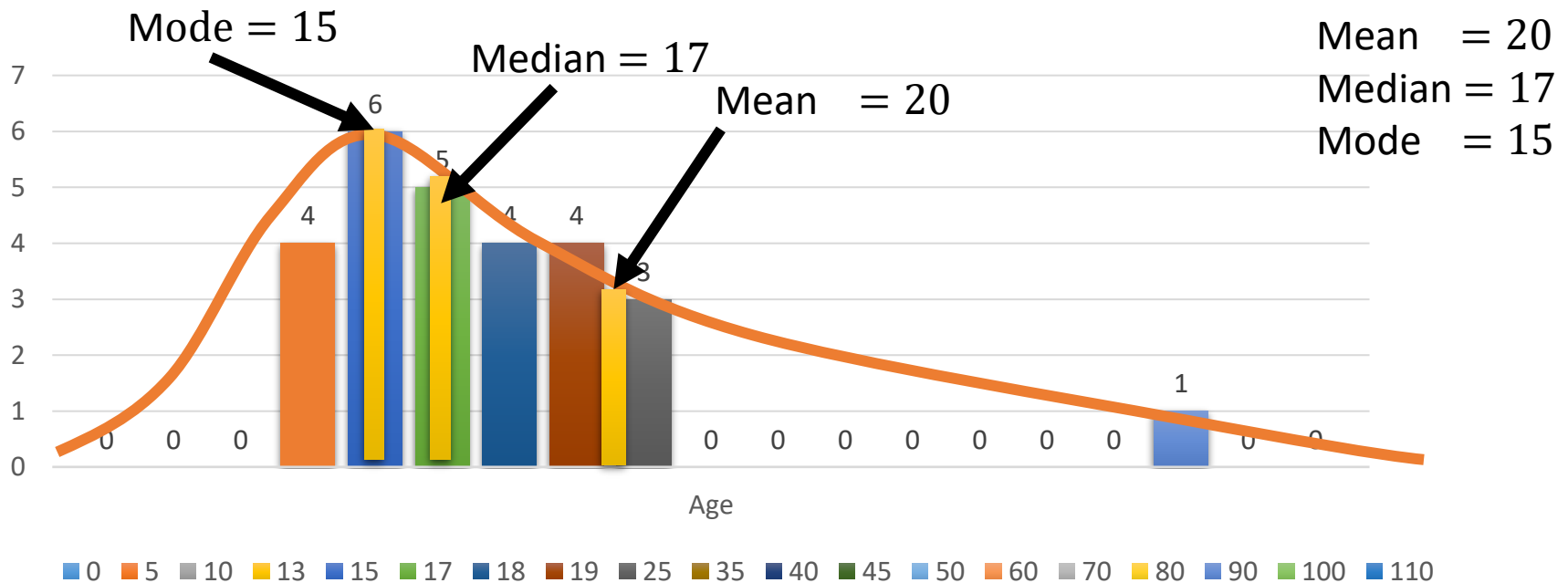
- Mean and Median need not be in the dataset but Mode has to be in it.
- **Mode** is also the only central-tendency statistic that works with **categorical data**.



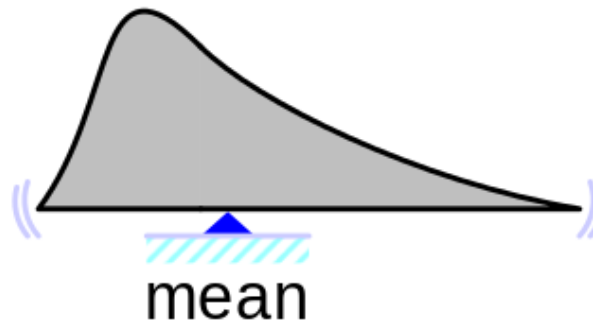
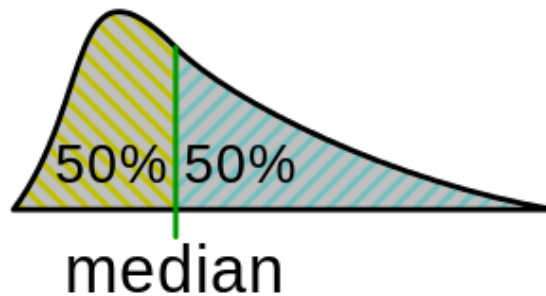
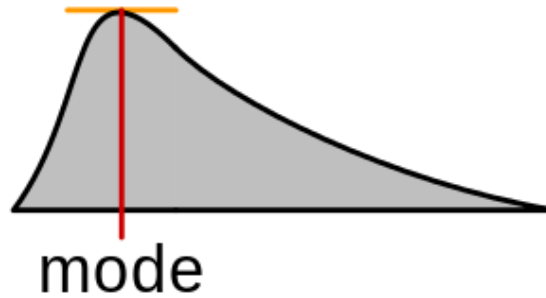
The Central Tendencies

Power Workout Class Composition

Age (years)	13	15	17	18	19	25	90
Frequency, f	4	6	5	4	4	3	1



The Central Tendencies



The Central Tendencies

The management of Good Heart Inc. wants to give all its employees a raise. They are unable to decide if they should give a straight Rs. 2000 to everyone or to increase salaries by 10% across the board. The mean salary is Rs. 50,000, the median is Rs. 20,000 and the mode is Rs. 10,000.

How do these central tendencies change in both cases?





QUIZ on Central Tendency

- Mean Simple Calculation
- Median
- Skewness (Mean median and Mode Position)



Measuring Variability and Spread

IQR, Range, Variance, Standard Deviation



Why Measure Of Spread?

- It is quite often, the average only gives part of the picture.
- Averages give us a way of determining where the centre of a set of data is, but they don't tell us how the data varies.



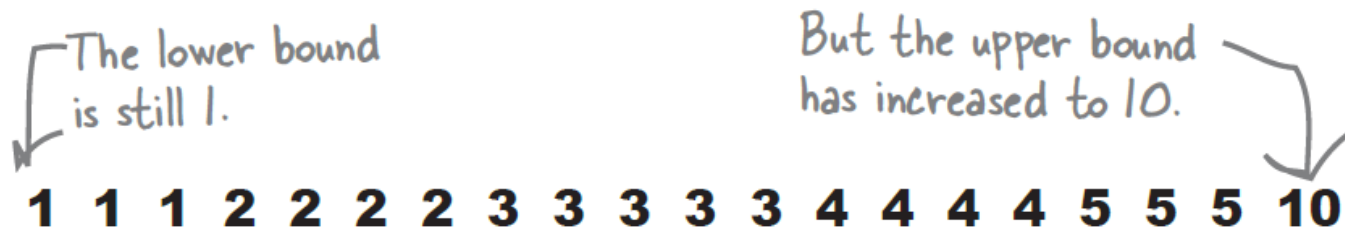
Different Measure of Spread

- Range
- IQR
- Variance
- Standard Deviation



Range

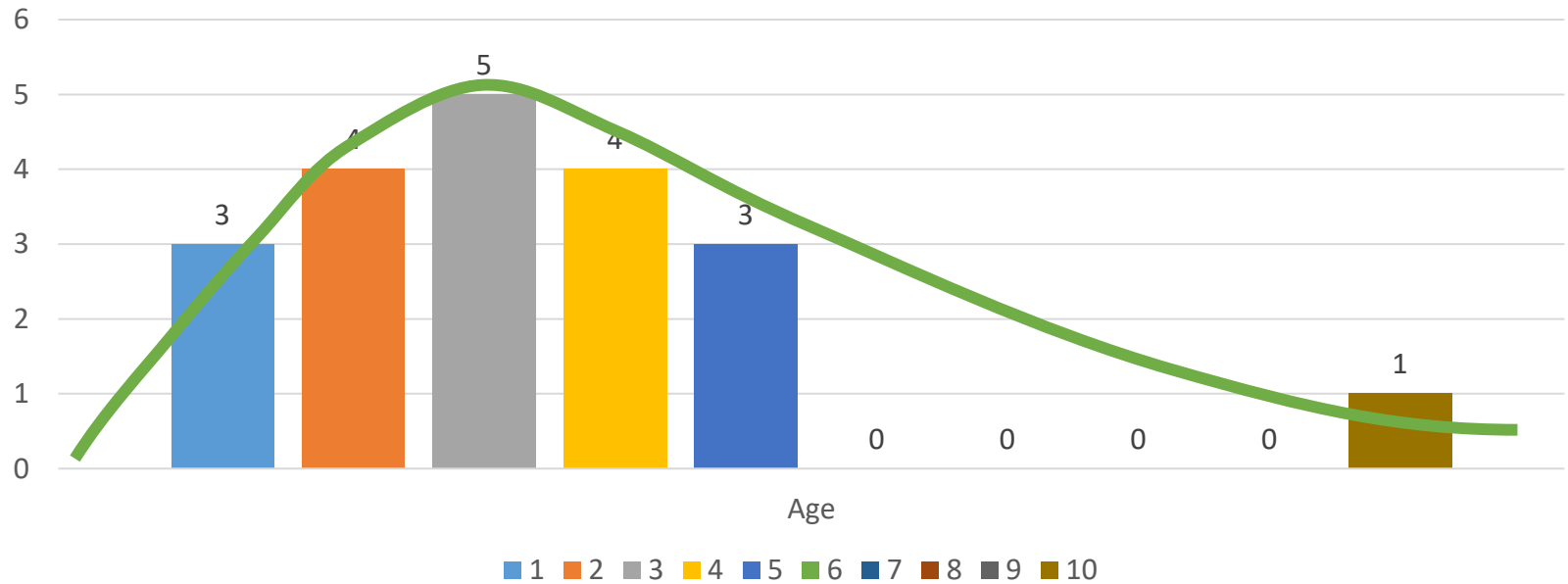
The range is a way of measuring how spread out a set of values are. It's given by Upper bound - Lower bound where the upper bound is the highest value, and the lower bound the lowest.



$$\begin{aligned}\text{Range} &= \text{upper bound} - \text{lower bound} \\ &= 10 - 1 \\ &= 9 \\ \text{so, the range is } 9\end{aligned}$$

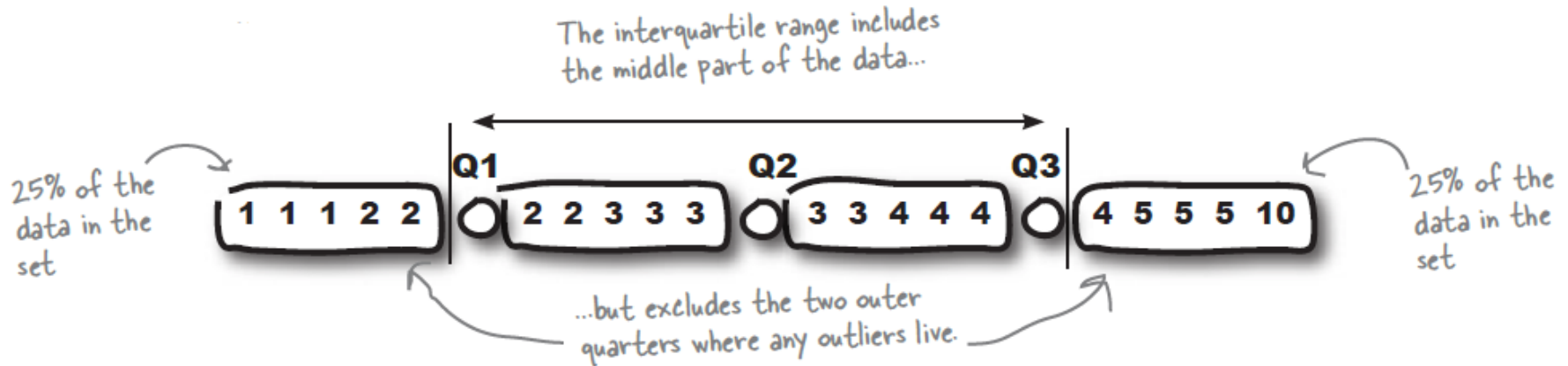
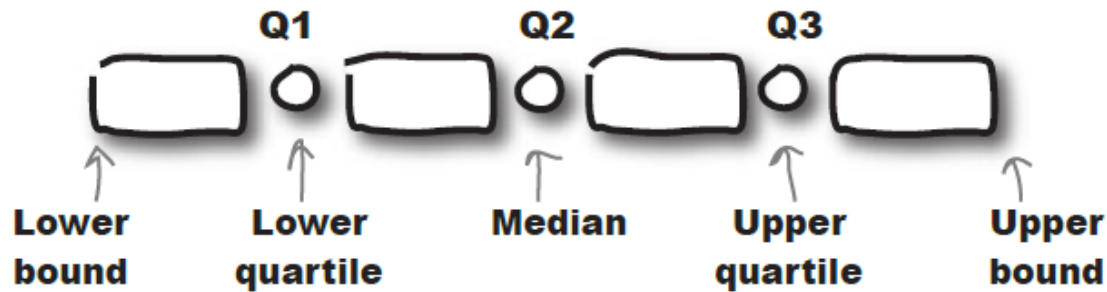


Kids Age



Quartiles will rescue the problem

Quartiles of a set of data is a very similar process to finding the median.



Quartiles

Quartiles : division of the data set into 4 regions If we have n data-points then the Quartile boundaries are given by

Lower quartile (25th percentile, Q1) = $\left(\frac{1*(n-1)}{4} + 1\right)^{th}$

Middle quartile = Median = $\left(\frac{2*(n-1)}{4} + 1\right)^{th} = \frac{(n+1)}{2}^{th}$

Upper quartile (75th percentile, Q3) = $\left(\frac{3*(n-1)}{4} + 1\right)^{th}$



IQR and Percentiles

Interquartile range:

$$\text{IQR} = Q3 - Q1 \text{ (central 50\% of data)}$$

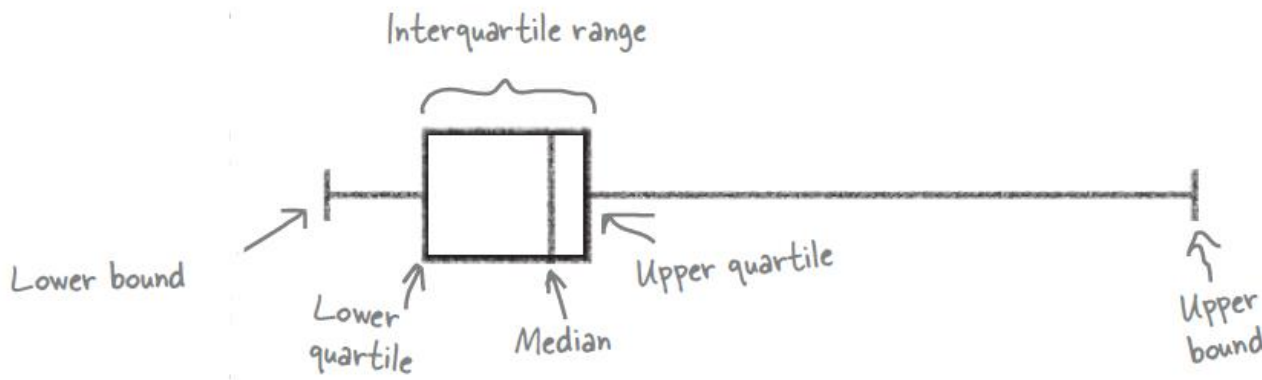
Percentile: divide the dataset into 100 regions

$$pth \text{ Percentile} = \left(\frac{p*(n-1)}{100} + 1 \right) th$$



Box Plot In Descriptive Stats

- In [descriptive statistics](#), a **box plot** or **boxplot** is a method for graphically depicting groups of numerical data through their [quartiles](#). Box plots may also have lines extending from the boxes (*whiskers*) indicating variability outside the upper and lower quartiles, hence the terms **box-and-whisker plot** and **box-and-whisker diagram**.
- [Outliers](#) may be plotted as individual points.



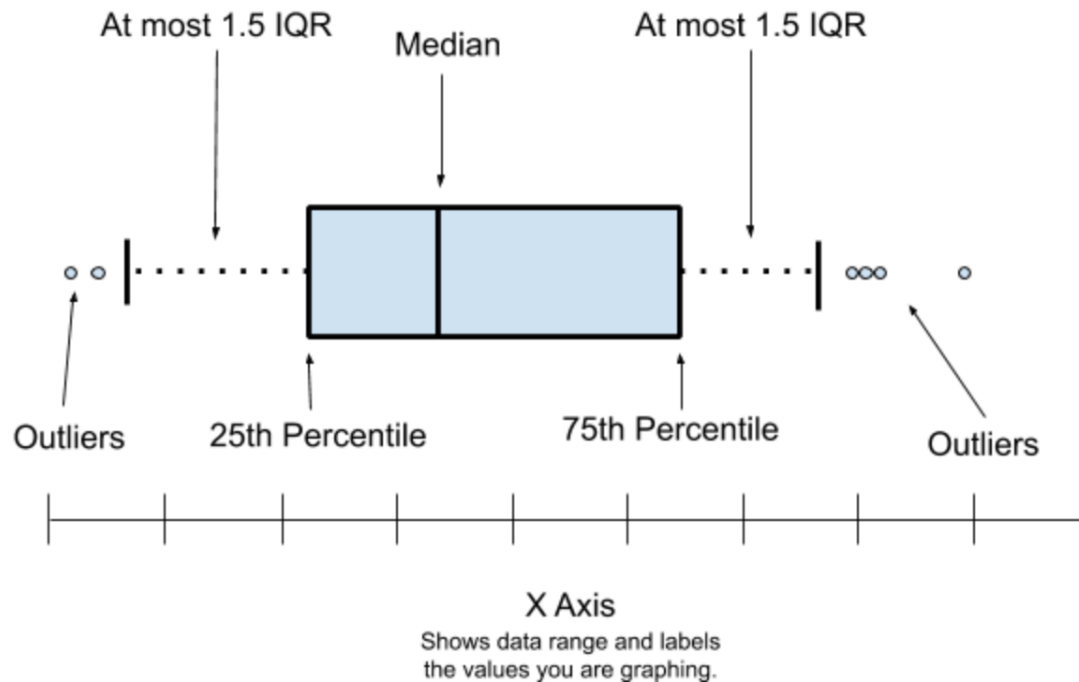
Box – Whisker Plot

- The Box and Whisker plot allows you to visualize the spread in the data easily
- **Steps:**
 - Compute the **Q1, Median and Q3** for the data. Compute **$IQR = Q3 - Q1$**
 - The Box of the plot is drawn from Q3 to Q1 (50% of data is contained within the box)
 - The Whiskers are a maximum of $1.5 * IQR$ from the **top and the bottom** of the box.
 - If there are no data points at $1.5 * IQR$, then pick an actual data point within the range of the Whiskers
 - Points lying outside the $1.5 * IQR$ from the box ends are considered as **Outliers**.



Boxplot Visual Representation

- Lets Understand the IQR and Identifying the Outliers



Advantage of IQR:

- The main advantage of the IQR is that it is not affected by outliers because it doesn't take into account observations below Q1 or above Q3.
- It might still be useful to look for possible outliers in your study.
- As a rule of thumb, observations can be qualified as outliers when they lie more than 1.5 IQR below the first quartile or 1.5 IQR above the third quartile.

$$\text{Outliers} = Q1 - 1.5 * IQR \quad \text{OR} \\ = Q3 + 1.5 * IQR$$

• Source: publiclab.org

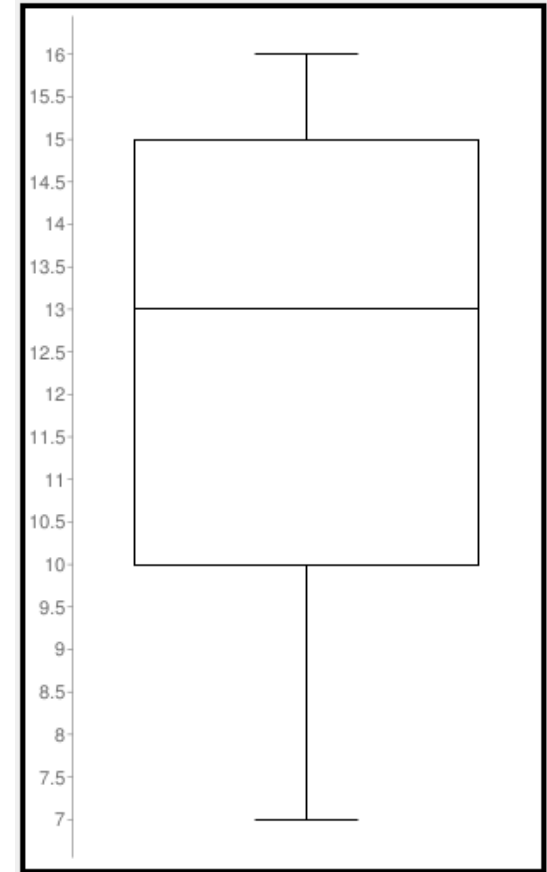


Interpreting Box-whisker plot

Age of kids in a party

Which of the following statements are true?

- All of the students are less than 17 years old
- At least 75% of the students are 10 years old or older
- There is only one 16 year old at the party
- The youngest kid is 7 years old
- Exactly half the kids are older than 13 in a party



Variance

The variance is a way of measuring spread, and it's the average of the distance of values from the mean squared.

$$\sigma^2 = \frac{\sum (x - \mu)^2}{n}$$

This is a method of measuring spread



Standard deviation

- Standard deviation is a way of saying how far typical values are from the mean.
- The smaller the standard deviation, the closer values are to the mean.
- The smallest value the standard deviation can take is 0.

$$\sigma = \sqrt{\text{Variance}}$$
$$\sigma = \sqrt{\frac{\sum (x - \mu)^2}{n}}$$

This is a method of measuring spread



Z - Score

- How far is any given data point from the mean ?
(Distance)
 - Z – score can help us answer
- How many standard deviation away (above and below) from the mean is a data point ?
- Units for Z- score is “standard deviation”
- Z – score is measure of distance from mean.

$$Z = \frac{x - \mu}{\sigma}$$



Value	Mean	value- Mean	Z	Outlier ?
1	3.26	-2.26	-1.1037	Not outlier
1	3.26	-2.26	-1.1037	Not outlier
1	3.26	-2.26	-1.1037	Not outlier
2	3.26	-1.26	-0.6160	Not outlier
2	3.26	-1.26	-0.6160	Not outlier
2	3.26	-1.26	-0.6160	Not outlier
2	3.26	-1.26	-0.6160	Not outlier
3	3.26	-0.26	-0.6160	Not outlier
3	3.26	-0.26	-0.1283	Not outlier
3	3.26	-0.26	-0.1283	Not outlier
3	3.26	-0.26	-0.1283	Not outlier
3	3.26	-0.26	-0.1283	Not outlier
4	3.26	1.74	0.3593	Not outlier
4	3.26	1.74	0.3593	Not outlier
4	3.26	1.74	0.3593	Not outlier
4	3.26	1.74	0.3593	Not outlier
5	3.26	2.74	0.8470	Not outlier
5	3.26	2.74	0.8470	Not outlier
10	3.26	7.74	3.28	outlier

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Value	Mean	value- Mean	Z	Outlier ?
1	3.26	-2.26	-1.1037	Not outlier
1	3.26	-2.26	-1.1037	Not outlier
1	3.26	-2.26	-1.1037	Not outlier
2	3.26	-1.26	-0.6160	Not outlier
2	3.26	-1.26	-0.6160	Not outlier
2	3.26	-1.26	-0.6160	Not outlier
2	3.26	-1.26	-0.6160	Not outlier
3	3.26	-0.26	-0.6160	Not outlier
3	3.26	-0.26	-0.1283	Not outlier
3	3.26	-0.26	-0.1283	Not outlier
3	3.26	-0.26	-0.1283	Not outlier
3	3.26	-0.26	-0.1283	Not outlier
4	3.26	1.74	0.3593	Not outlier
4	3.26	1.74	0.3593	Not outlier
4	3.26	1.74	0.3593	Not outlier
4	3.26	1.74	0.3593	Not outlier
5	3.26	2.74	0.8470	Not outlier
5	3.26	2.74	0.8470	Not outlier
10	3.26	7.74	3.28	outlier

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Action Check

Basketball coach is in a dilemma choosing between 3 players all having the **same average scores**.

Points Scored per game	7	8	9	10	11	12	13
Frequency, f	1	1	2	2	2	1	1

Points Scored per game	7	9	10	11	13
Frequency, f	1	2	4	2	1

Points Scored per game	3	6	7	10	11	13	30
Frequency, f	2	1	2	3	1	1	1



Measuring Variability and Spread

Exclude outliers scientifically – Quartiles

Points Scored per game	3	6	7	10	11	13	30
Frequency, f	2	1	2	3	1	1	1

3, 3, 6, 7, 7, 10, 10, 10, 11, 13, 30

Median = 10

First Quartile : 3, 3, 6, 7, 7, 10

Q1 = 6.5

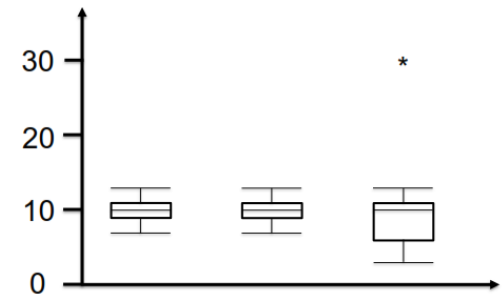
Third Quartile: 10, 10, 10, 11, 13, 30

Q3 = 10.5



Measuring Variability and Spread

- Exclude outliers scientifically – Quartiles
- Box and whisker diagram or Box plot

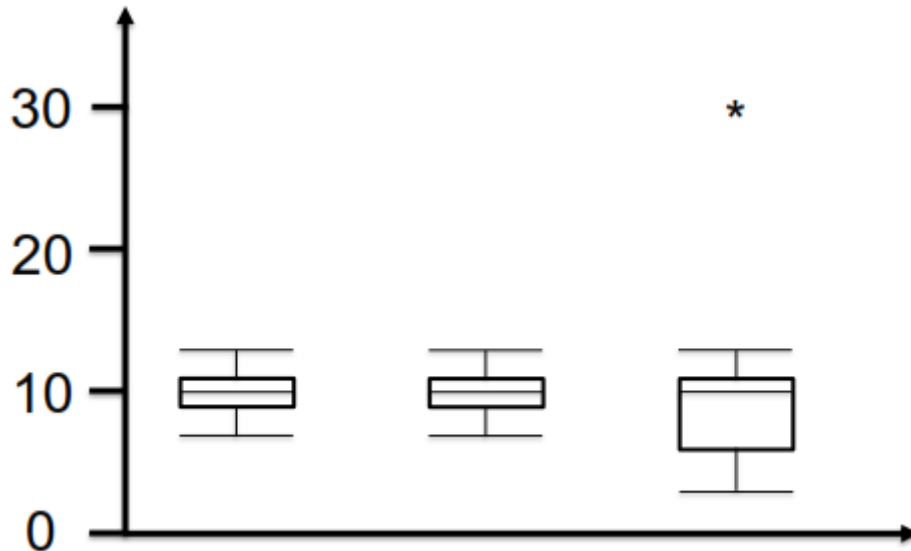


Name	Formula	Player 1	Player 2	Player 3
Lower Hinge	Q1 = 1st Quartile	9	9	6.5
Mid Line	Q2 = 2nd Quartile = Median	10	10	10
Upper Hinge	Q3 = 3rd Quartile	11	11	10.5
Body of the box	IQR = Q3 - Q1	2	2	4
Step	1.5 * IQR	3	3	6
	Lower Hinge - 1 Step	6	6	0.5
	Upper Hinge + 1 Step	14	14	16.5
Lower Fence	Smallest Actual Data Inside Fence	7	7	3
Upper Fence	Largest Actual Data Inside Fence	13	13	13
Outliers	Value beyond the Fence			30

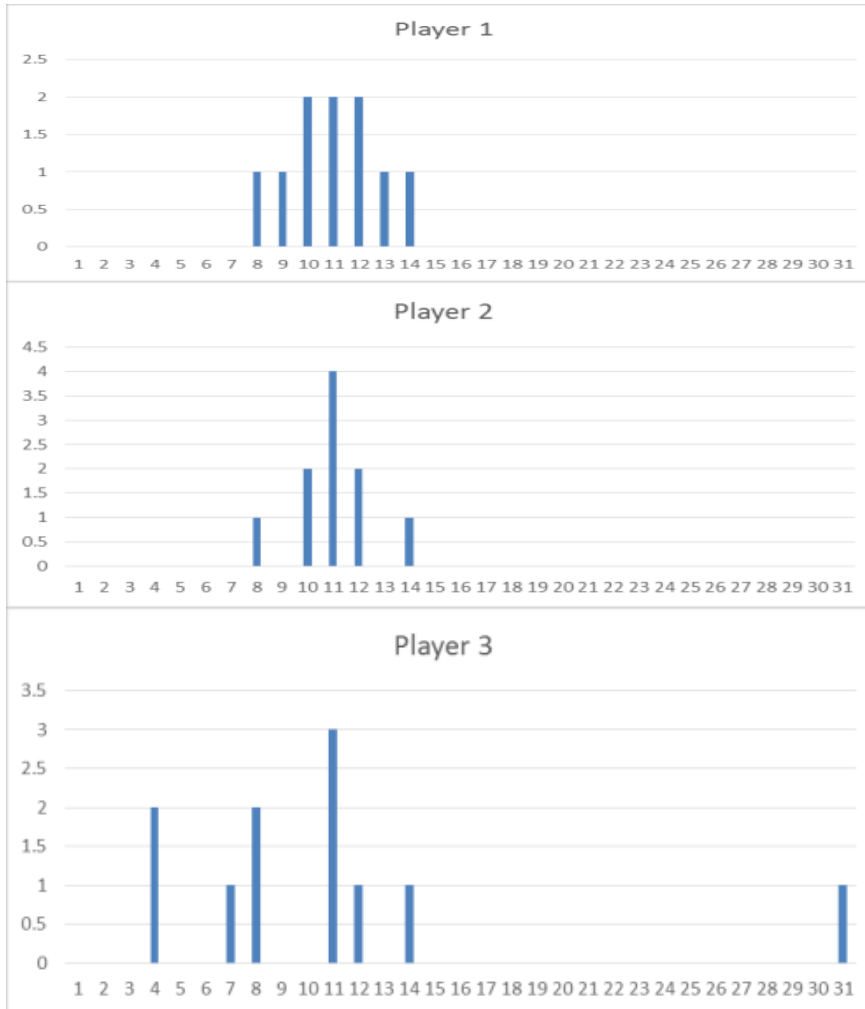


Measuring Variability and Spread

- Exclude outliers scientifically – Quartiles
- Box and whisker diagram or Box plot



Attention Check



1.73, 1.48, 7.02

Player 3 is the least reliable.



Measuring Variability and Spread

What happens to Standard Deviation if Good Heart Inc. gave all employees a Rs 2000 raise ?

No Change

What happens to Standard Deviation if Good Heart Inc. gave all employees a 10% raise ?

Increases by 1.1 times



Measuring Variability and Spread

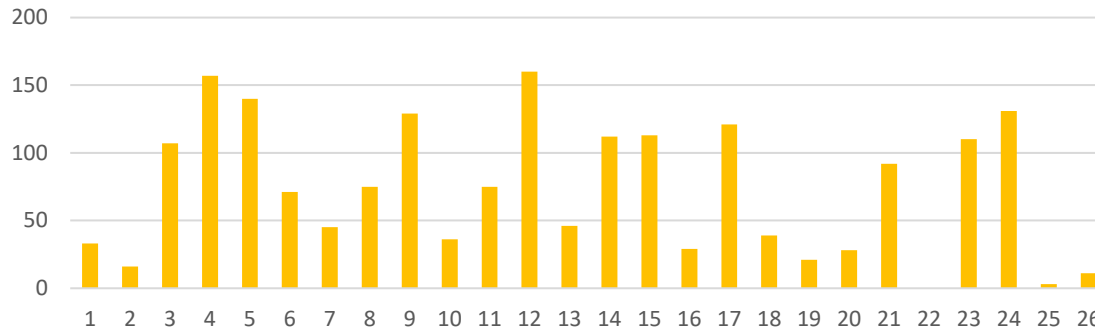
Imagine Virat Kohli and Rohit Sharma with different abilities: Virat has an average of 73 with 50 stdev and the Rohit has average of 59 with 63 stdev in past 27 matches.

In a particular match session, the Virat scores 85 runs of the time and the Rohit scores 75 Runs. Who did best against their PERSONAL track record?



Measuring Variability and Spread

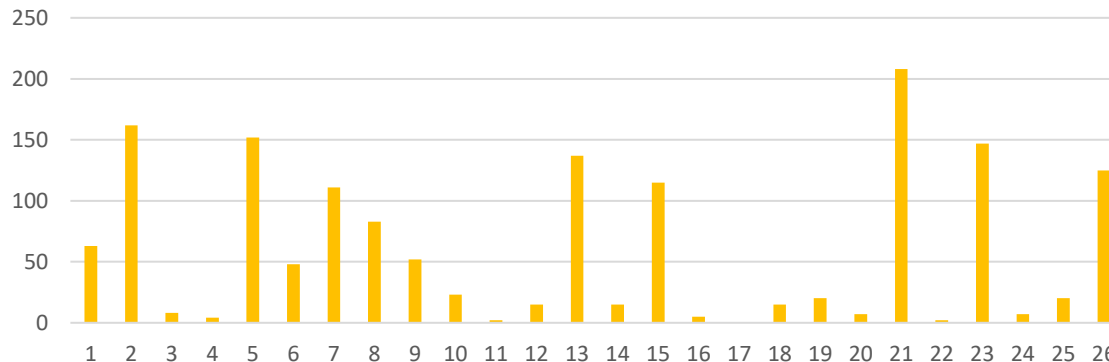
Virat Kohli



Mean = 73

Std = 50

Rohit Sharma



Mean = 59

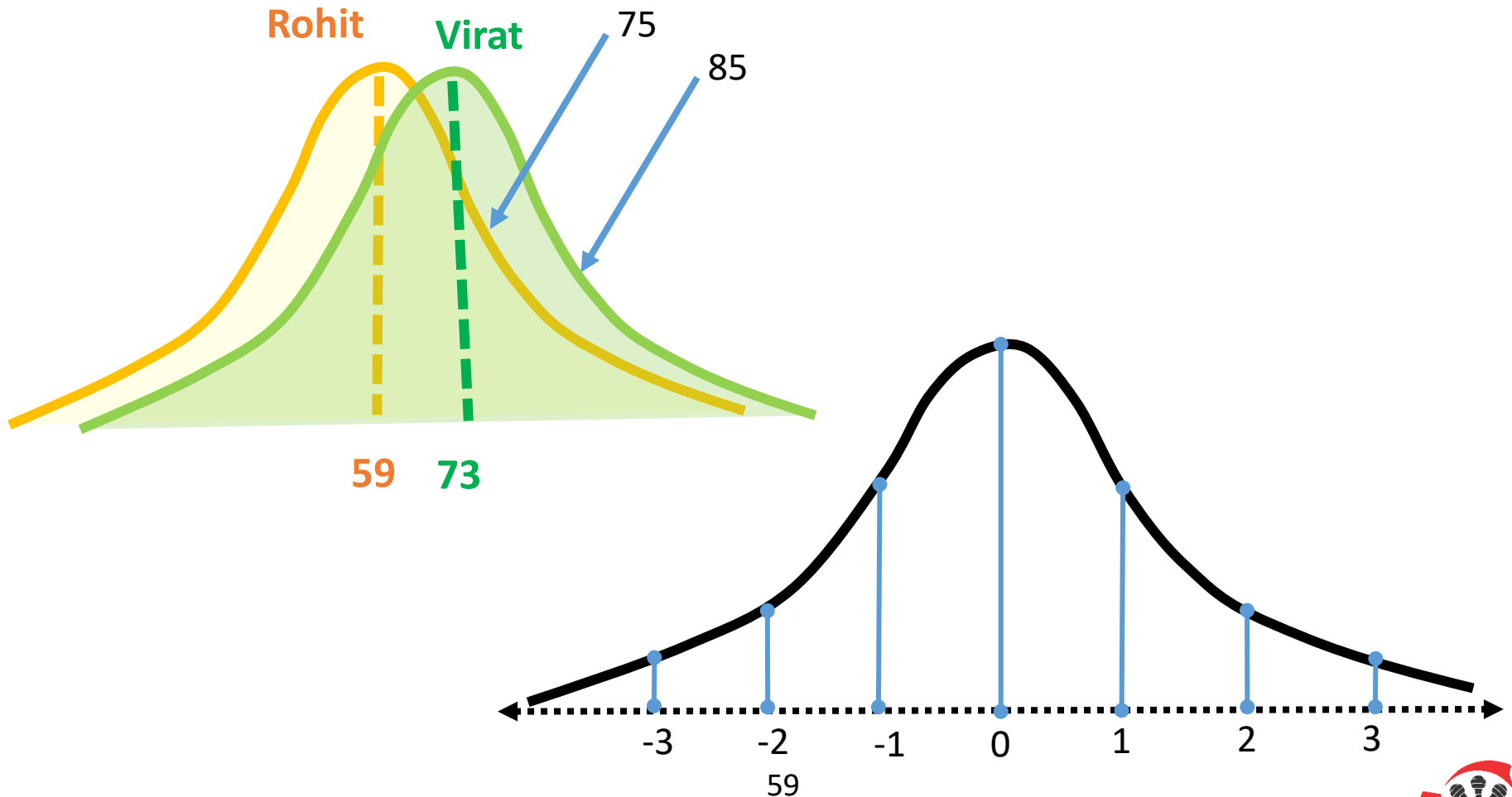
Std = 63





Measuring Variability and Spread

- Standard score, $z = \frac{x - \mu}{\sigma}$, # of stdevs from the mean



Scatter Plot and Correlation.

- Displaying Relationships: Scatterplots
- Interpreting Scatterplots.
- Measuring Linear Association: Correlation.
- Facts About Correlation



Scatter Plots - Bivariate data

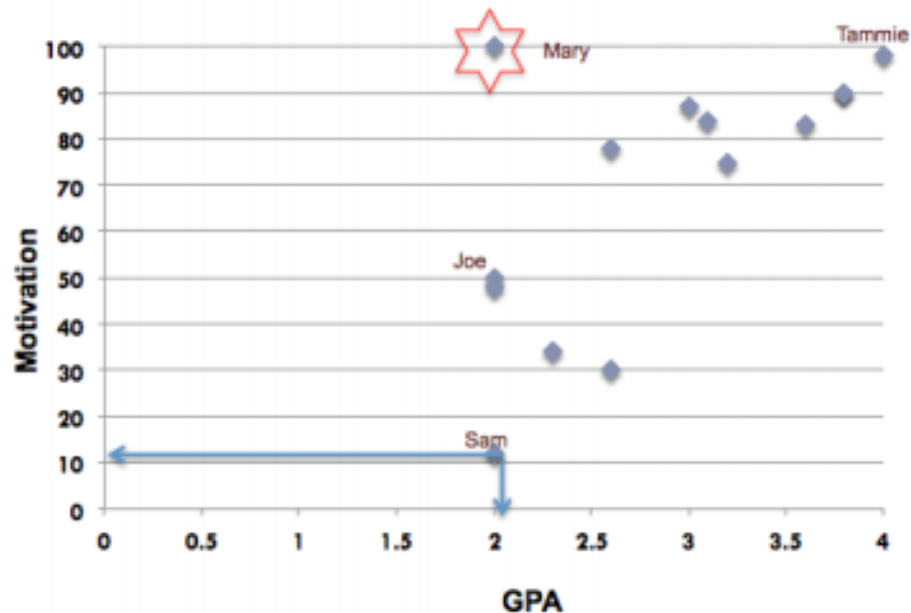
- A scatterplot shows the relationship between two quantitative variables measured for the same individuals.
- The values of one variable appear on the horizontal axis, and the values of the other variable appear on the vertical axis.
- Each individual in the data appears as a point on the graph.



Scatterplot Example

- What is the relationship between students' achievement motivation and GPA?
- the relationship between students' achievement motivation and their GPA is being investigated.

Student	Student GPA	Motivation
Joe	2.0	50
Lisa	2.0	48
Mary	2.0	100
Sam	2.0	12
Deana	2.3	34
Sarah	2.6	30
Jennifer	2.6	78
Gregory	3.0	87
Thomas	3.1	84
Cindy	3.2	75
Martha	3.6	83
Steve	3.8	90
Jamell	3.8	90
Tammie	4.0	98



Interpreting Scatterplots

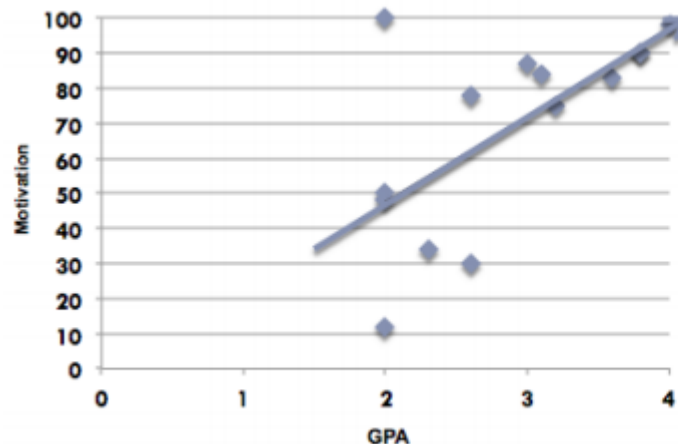
As in any graph of data, look for the overall pattern and for striking departures from that pattern.

- The overall pattern of a scatterplot can be described by the **direction, form, and strength of the relationship**.
- An important kind of departure is an outlier, an individual value that falls outside the overall pattern of the relationship



Interpreting Scatterplots: Direction

- One important component to a scatterplot is the direction of the relationship between the two variables.
- Two variables have a **positive association** when above-average values of one tend to accompany above-average values of the other, and when below-average values also tend to occur together.

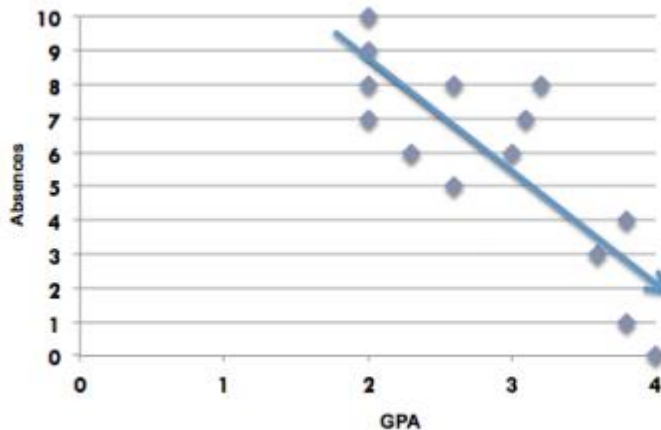


This example compares students' achievement motivation and their GPA. These two variables have a **positive association** because as GPA increases, so does motivation.



Negative Association

- Two variables have a **negative association** when above-average values of one tend to accompany below-average values of the other.

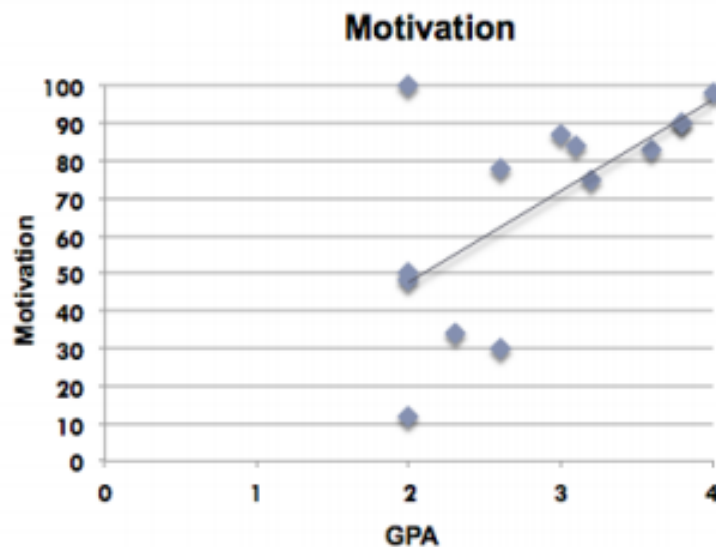


This example compares students' GPA and their number of absences. These two variables have a **negative association** because, in general, as a student's number of absences decreases, their GPA increases.



Interpreting Scatterplots: Form

- Another important component to a scatterplot is the form of the relationship between the two variables.
- Linear Relationship

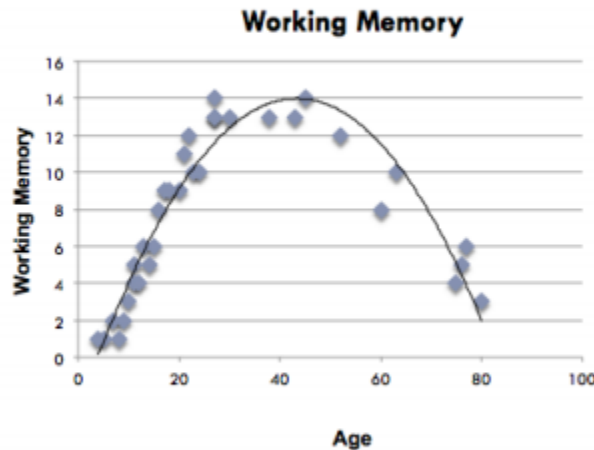


This example illustrates a linear relationship. This means that the points on the scatterplot closely resemble a straight line. A relationship is linear if one variable increases by approximately the same rate as the other variables changes by one unit.



- **Curvilinear Relationship**

Curvilinear relationship:



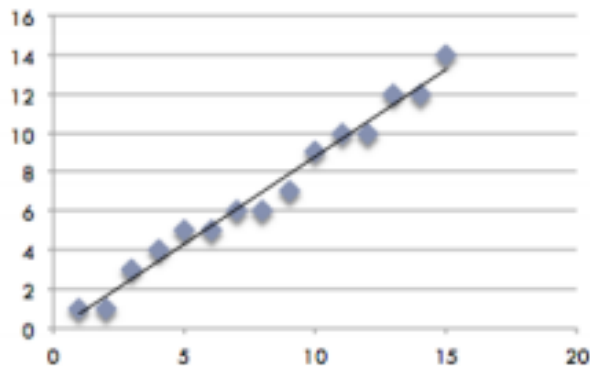
This example illustrates a relationship that has the form of a curve, rather than a straight line. This is due to the fact that one variable does not increase at a constant rate and may even start decreasing after a certain point. This example describes a curvilinear relationship between the variable “age” and the variable “working memory.” In this example, working memory increases throughout childhood, remains steady in adulthood, and begins decreasing around age 50.



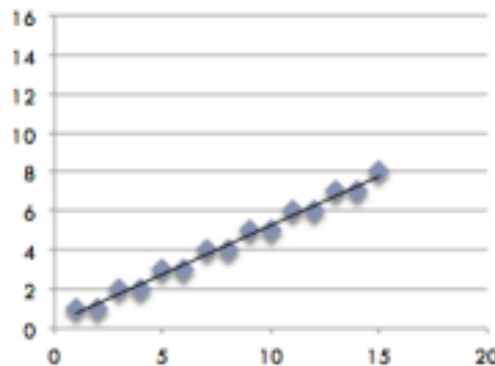
Interpreting Scatterplots: Strength

- Another important component to a scatterplot is the strength of the relationship between the two variables.
- The slope provides information on the strength of the relationship.

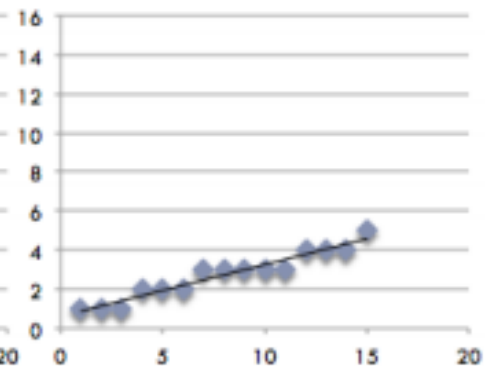
Strong relationship:



Moderate relationship:



Weak relationship:



- The strongest linear relationship occurs when the slope is 1. This means that when one variable increases by one, the other variable also increases by the same amount. This line is at **a 45 degree angle**.
- The strength of the relationship between two variables is a crucial piece of information. Relying on the interpretation of a scatterplot is too subjective. More precise evidence is needed, and this evidence is obtained by computing **a coefficient that measures the strength of the relationship under investigation**.



Measuring Linear Association

- A scatterplot displays the strength, direction, and form of the relationship between two quantitative variables.
- A correlation coefficient measures the strength of that relationship.



The **correlation r** measures the strength of the linear relationship between two quantitative variables.

Pearson r :

$$r = \frac{1}{n-1} \sum \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$

- r is always a number between -1 and 1.
- $r > 0$ indicates a positive association.
- $r < 0$ indicates a negative association.
- Values of r near 0 indicate a very weak linear relationship.
- The strength of the linear relationship increases as r moves away from 0 toward -1 or 1.
- The extreme values $r = -1$ and $r = 1$ occur only in the case of a perfect linear relationship.



- Calculating a Pearson correlation coefficient requires the assumption that the relationship between the two variables is linear.
- There is a rule of thumb for interpreting the strength of a relationship based on its r value (use the absolute value of the r value to make all values

Absolute Value of r

$r < 0.3$

$0.3 < r < 0.5$

$0.5 < r < 0.7$

$r > 0.7$

Strength of Relationship

None or very weak

Weak

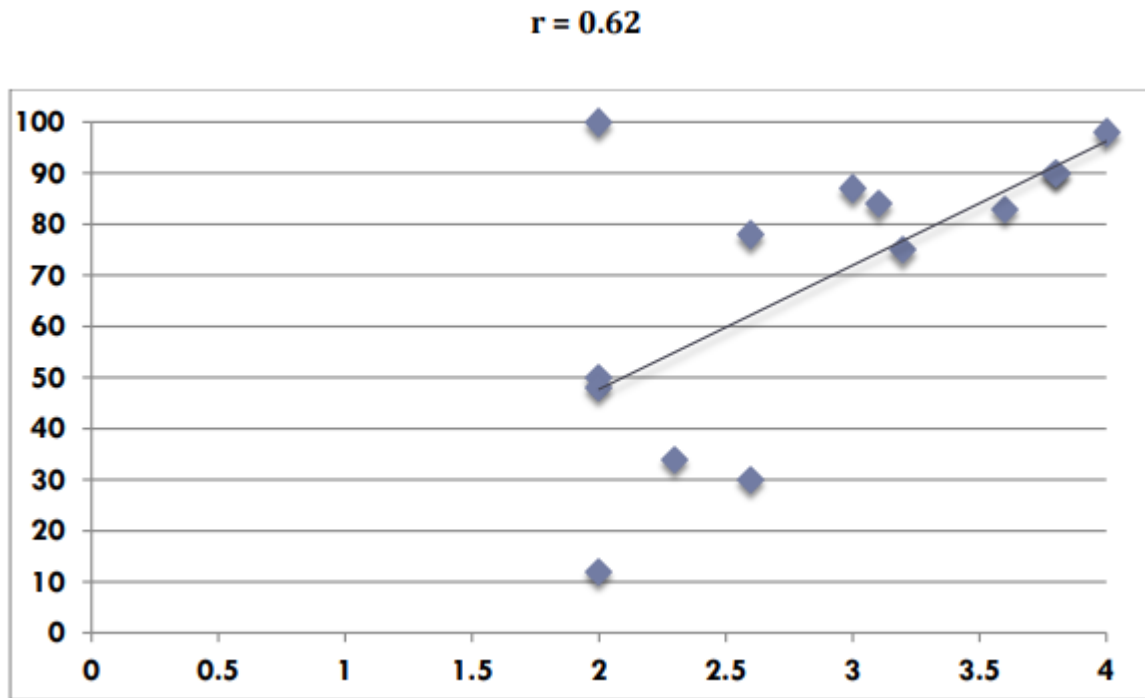
Moderate

Strong



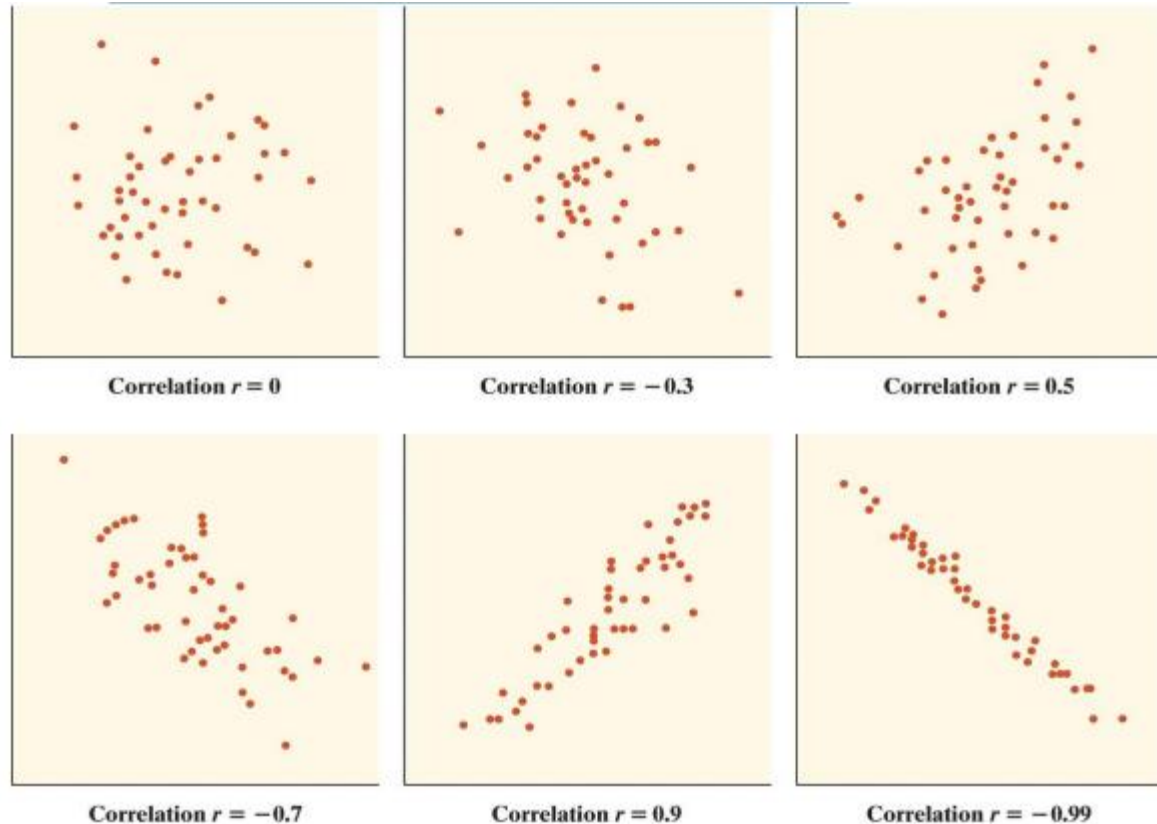
Example – GDP VS Achievement Motivation

- There is a Moderate, positive, linear Relationship between GPA and achievement Motivation.



Correlation:

- The images below illustrate what the relationships might look like at different degrees of strength (for different values of r).



- For a correlation coefficient of zero, the points have no direction, the shape is almost round, and a line does not fit to the points on the graph.
- As the correlation coefficient increases, the observations group closer together in a linear shape.
- The line is difficult to detect when the relationship is weak (e.g., $r = -0.3$), but becomes more clear as relationships become stronger (e.g., $r = -0.99$).

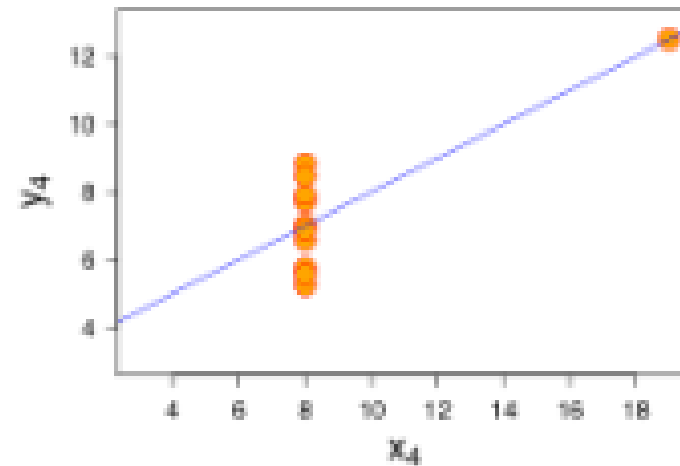
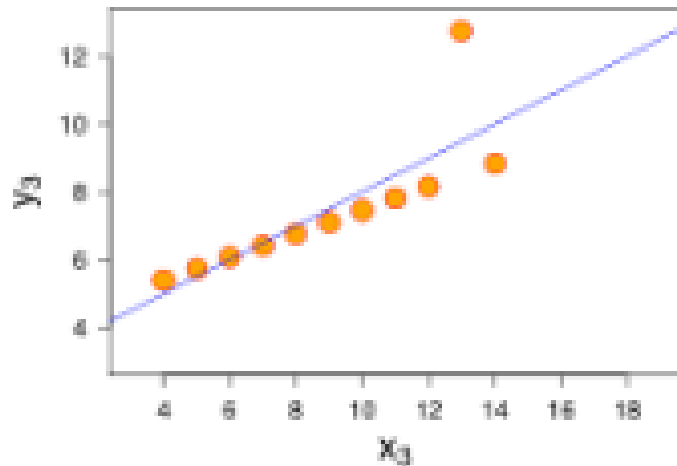
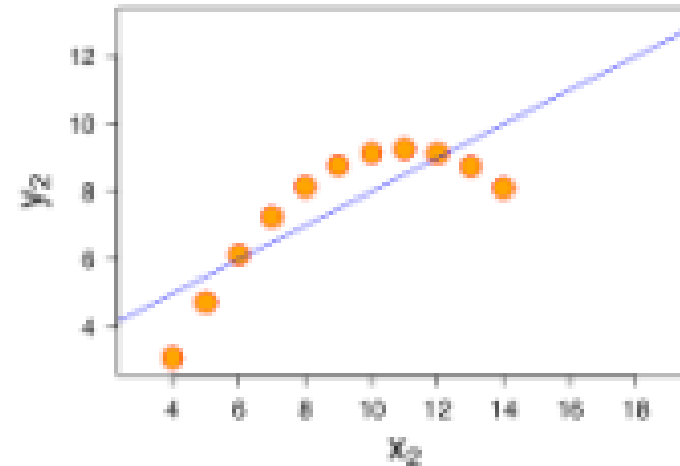
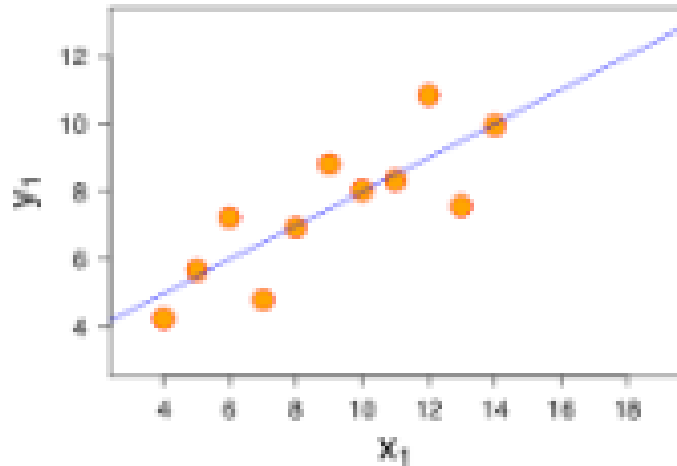


Facts About Correlation

- 1) The **order of variables in a correlation** is not important.
- 2) Correlations provide **evidence of association**, not causation.
- 3) **r has no units** and does **not change** when the units of measure of x , y , or both are changed.
- 4) Positive r values indicate positive association between the variables, and negative r values indicate negative associations.
- 5) The correlation **r is always a number between -1 and 1.**



Pattern in the data



Reference

- <https://www.mathsisfun.com/data/index.html>
- **Head First: Statistics**
- **KhanAcademy**
- **Scatterplots and Correlation by Dian mindrila Ph.D and Phoebe balentyne, M.Ed based on chapeter 4 of The basic practice of Statistics. (6th ed).**



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