Maximum Likelihood Estimation

Logistic Regression

Maximum Likelihood Estimation

- In statistics, maximum likelihood estimation is a method of estimating the parameters of an assumed probability distribution, given some observed data. This is achieved by maximizing a likelihood function so that, under the assumed statistical model, the observed data is most probable

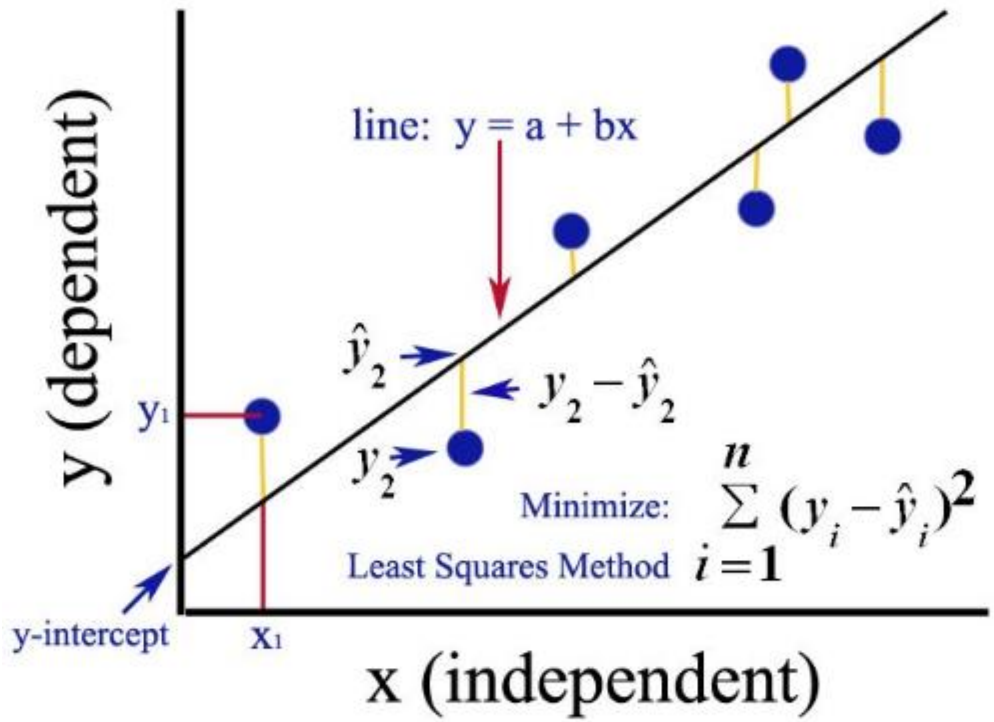
- P(X ; theta)

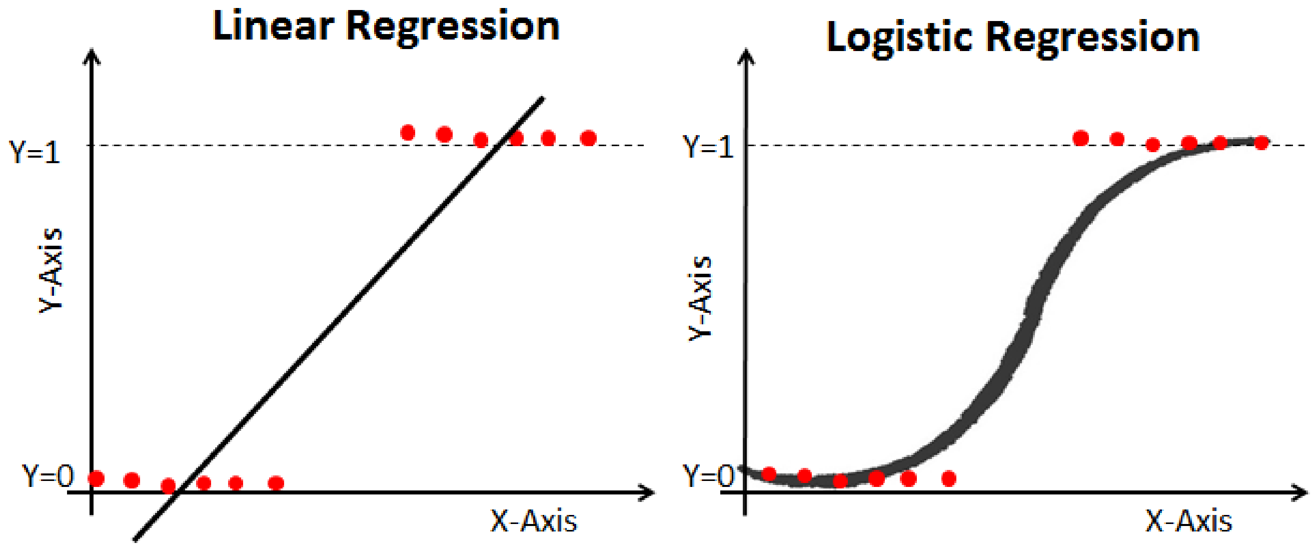
- Where *X* is, in fact, the joint probability distribution of all observations from the problem domain from 1 to *n*.

  P(x1, x2, x3, …, xn ; theta)

- This resulting conditional probability is referred to as the likelihood of observing the data given the model parameters and written using the notation *L()* to denote the likelihood function. For example:
  - L(X ; theta)

Ordinary Least Square Method



- 

$$\text{odds Ratio} = \frac{P}{1-P}$$

where

P = Probability of success

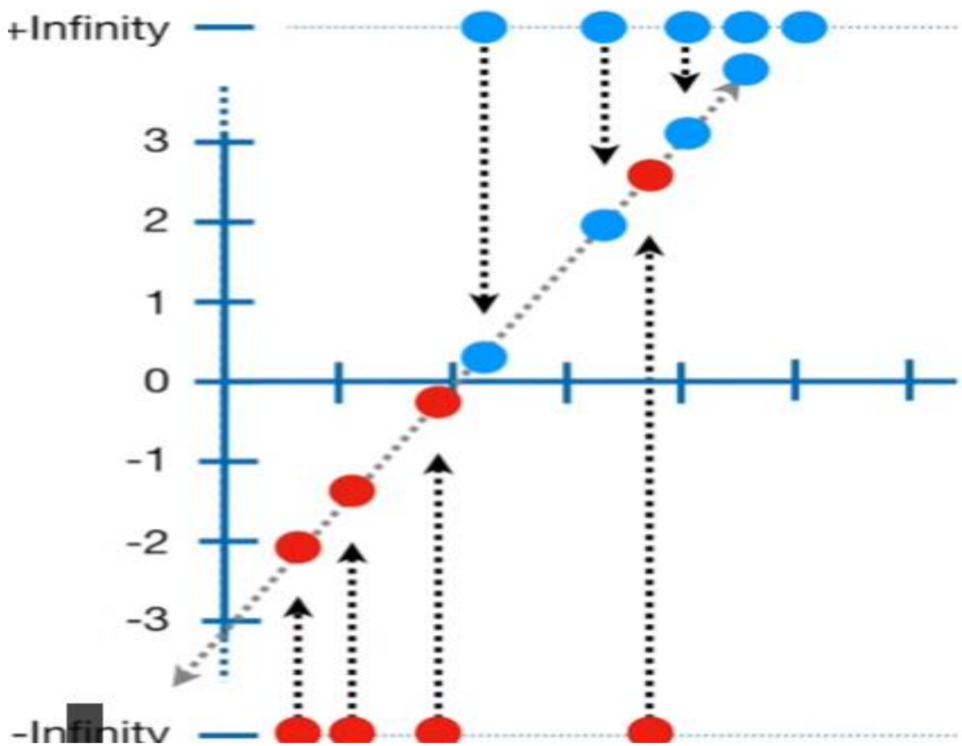
$$\log(\text{odds}) = \log\left(\frac{P}{1-P}\right) = \hat{y}$$

$\hat{y} = \beta_0 + \beta_1 x$.

$$e^{\log(\text{odds})} = \frac{P}{1-P}.$$

$$P = \frac{e^{\log(\text{odds})}}{1 + e^{\log(\text{odds})}}$$

$$P = \frac{e^{(\beta_0 + \beta_1 x)}}{1 + e^{(\beta_0 + \beta_1 x)}}$$



$$\log(\text{odds}) = \log\left(\frac{P}{1-P}\right)$$

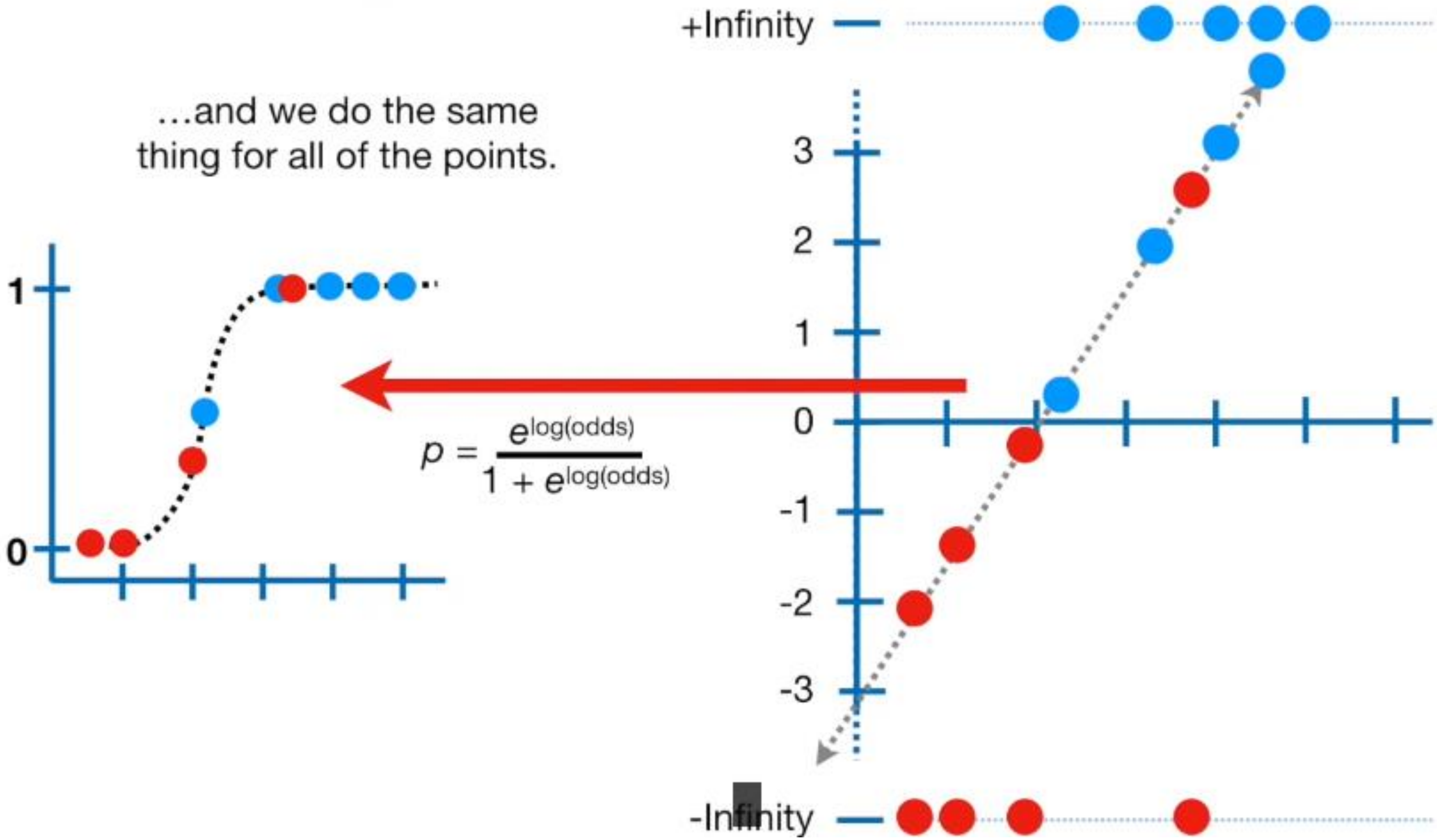
for $P = 0.5$

$$\log(\text{odds}) = \log\left(\frac{0.5}{1-0.5}\right)$$

$$= \log(1)$$

$$\log(\text{odds}) = 0$$

...and we do the same thing for all of the points.



$$p = \frac{e^{\log(\text{odds})}}{1 + e^{\log(\text{odds})}}$$

likelihood of data given the squiggle = 0.49 × 0.9 × 0.91 × 0.91 × 0.92 × (1 - 0.9) × (1 - 0.3) × (1 - 0.01) × (1 - 0.01)

$$\log(\text{likelihood of data given the squiggle}) = \log(0.49) + \log(0.9) + \log(0.91) + \log(0.91) + \log(0.92) + \log(1 - 0.9) + \log(1 - 0.3) + \log(1 - 0.01) + \log(1 - 0.01)$$

$$\boxed{\log(\text{likelihood of data given the squiggle}) = -3.77}$$

Thank you