Learning and identification in DiD designs

Marx et al. (2023): "Parallel Trends and Dynamic Choices"

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Motivation (I)

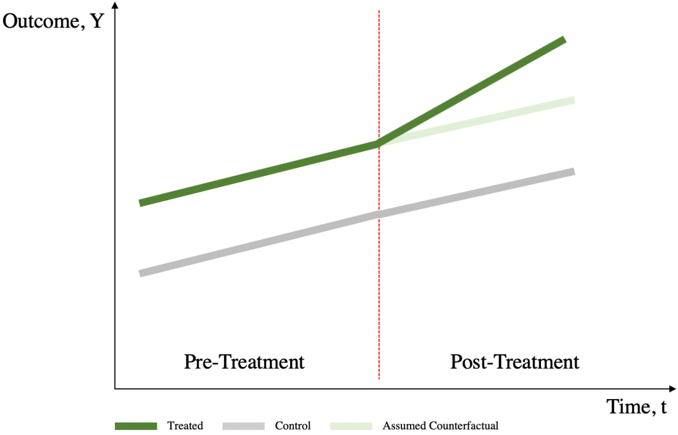

Figure: Parallel trends in sharp DiD designs.

Motivation (II)

But in many real-world settings, treatment choice is a **dynamic choice**. For instance, the present treatment choice may depend on:

- past treatments and observed levels
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Key idea: **Parallel Trends constrain agent behavior**. But how?

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Key idea: **Parallel Trends constrain agent behavior**. But how?

Past outcomes can affect future expectations in various ways, called **learning mechanisms**. We'll focus on the impact of some mechanisms on PT:

$$E[Y_1(0) - Y_0(0) | D_0 = d_0, D_1 = d_1] = \tau \in \mathbb{R} \text{ for all } d_0, d_1 \in \{0, 1\}.$$

Overview

Setup (I)

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- Random sample of $i = 1, \dots, N$ workers eligible for the program
- Treatment is participation in the program: $D_{it} = 1$ if worker i enrolls in time t, $D_{it} = 0$ if not. **Choice is flexible**: any $(d_{i0}, d_{i1}) = \mathbf{d}_i$ is possible
- Levels measured in $t = 0, 1$: wages $Y_{it} \in \mathbb{R}_+$ (or other), where $Y_{it} = D_{it} Y_{it}(1) + (1 - D_{it}) Y_{it}(0)$.

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- Levels measured in $t = 0, 1$: wages $Y_{it} \in \mathbb{R}_+$ (or other), where $Y_{it} = D_{it} Y_{it}(1) + (1 - D_{it}) Y_{it}(0)$.

As usual, $Y_{it}(\mathbf{d}_1, \ldots, \mathbf{d}_N) = Y_{it}(d_{it})$: SUTVA holds, no spillovers nor dynamic effects, etc. Population variables will be used, and subscripts i ignored.

Setup (II)

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- Utilities $V_0(d_0) = Y_0(d_0) - K(d_0)$ and $V_1(d_1) = Y_1(d_1) - K(d_1)$
- The *future* utility of the best possible choice in period 1, given d_0. Formally defined as the r.v. $W_1(d_0) \equiv \max_{d_1 \in \{0,1\}} E[V_1(d_1)|U_1(d_0)]$

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- Discount factor β reflecting the *present value* of $W_1(d_0)$. For example, $\beta = 0.5$ implies 100€ in $t = 1$ are as valuable as 50€ in $t = 0$
- Initial information U_0 with $\beta, K(d_t) \in U_0$, updated to $U_1(d_0)$ once $Y_0 = Y_0(d_0)$ is observed. Information does not decrease: $U_0 \subseteq U_1(d_0)$.

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Given initial information U_0 at the start of period 0 and accrued information $U_1(d_0)$ at the start of period 1, *treatment decisions maximize the sum of expected discounted utility*:

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where $1\{\cdot\}$ denotes the indicator function and $U_1(d_0) \equiv \{U_0, Y_0(d_0)\}$.

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- he *expects* a wage of $300 = E[Y_0(1)|U_0]$ after enrolling in $t = 0$, or a wage of $200 = E[Y_0(0)|U_0]$ if he does not enroll in $t = 0$
- he *knows* enrolling costs $200 = K(1)$, while not enrolling is free, ie. $0 = K(0)$.

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- he *knows* enrolling costs $200 = K(1)$, while not enrolling is free, ie. $0 = K(0)$.

Thus, he expects the following utilities in $t = 0$:

$$E[V_0(1)|U_0] = E[Y_0(1)|U_0] - K(1) = 300 - 200 = 100,$$

$$E[V_0(0)|U_0] = E[Y_0(0)|U_0] - K(0) = 200 - 0 = 200.$$

Moreover, he *values* utilities in $t = 1$ as half of those in $t = 0$, ie. $\beta = 0.5$.

Model illustration (II)

What's the impact of his choice on period 1? Let's **pretend** he *expects* a salary of $800 = E[Y_1(1)|U_1(d_0)]$ after enrolling in $t = 1$, or instead $300 = E[Y_1(0)|U_1(d_0)]$ if he does not enroll in $t = 1$. We then get:

$$E[W_1(d_0)|U_0] = E[Y_1(1)|U_1(d_0)] - K(1) = 800 - 200 = 600, \text{ for any } d_0.$$

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Thus, Bob is going to choose:

$$\begin{aligned} D_0 &= 1\{E[V_0(1) - V_0(0) + \beta(W_1(1) - W_1(0))|U_0] \geq 0\} \\ &= 1\{100 - 200 + 0.5 * (600 - 600) \geq 0\} \\ &= 1\{-100 \geq 0\} = 0. \end{aligned}$$

Then he observes $Y_0(0)$, updates his guesses on Y_1, and chooses D_1 *similarly*.

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(iii) the constant trends are all equal, *ie. no trend shift between time 0 and 1*.

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$$E[Y_1(0) - Y_0(0)|U_0] = \tau \in \mathbb{R}. \tag{11}$$

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In (DUM):

$$\text{assumption (11)} + \text{static selection} \implies \text{Parallel Trends},$$
$$\text{assumption (11)} + \text{dynamic selection} \not\Rightarrow \text{Parallel Trends}.$$

Sufficient initial information (I)

Example 2. Suppose (DUM) and asm (11) hold. **If the initial information is sufficient** for mean outcomes in the following sense:

$$E[Y_1(d_1)|U_0, Y_0(d_0)] = E[Y_1(d_1)|U_0] \text{ for any } d_0, d_1 \in \{0, 1\}, \tag{12}$$

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the Parallel trends condition is satisfied.

Heuristic: *PT may hold if observed outcomes do not provide additional information about the future, ie. if agents do not learn anything new about potential outcomes.*

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Proof.

In (DUM) we have the following choice rule for D_1:

$$\begin{aligned} D_1 &= 1\big\{E[V_1(d_0,d_1) - V_1(d_0,0)|U_0, U_1(d_0)] \geq 0\big\} \\ &= 1\big\{E[V_1(d_0,d_1) - V_1(d_0,0)|U_0] \geq 0\big\}, \text{per asm (12).} \end{aligned}$$

That implies D_1 is conditional on U_0 only, like D_0.

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$$E[Y_1(0) - Y_0(0)|D_0, D_1] = E\big[E[Y_1(0) - Y_0(0)|U_0]\big|D_0, D_1\big]$$

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E[Y_1(0) - Y_0(0)|D_0, D_1] &= E\big[E[Y_1(0) - Y_0(0)|U_0]\big|D_0, D_1\big] \\
&= E\big[\tau|D_0, D_1\big] = \tau, \text{ for all } (d_0, d_1).
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Since $E[Y_1(0) - Y_0(0)|D_0, D_1] = \tau \in \mathbb{R}$, PT is valid. $\square$

Sufficient initial information (III)

Interpretation.

- Asm (11) $\equiv$ const. untreated trend given $U_0 \implies$ no violation in $t = 0$
- $+$ asm (12) $\equiv$ sufficient $U_0 \implies$ no violation in $t = 1$ either.

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Is this a special case of a more general result? What if workers accurately predicted their untreated wages (*safe arm*), but could learn something about the effect of the training program (*risky arm*)?

Learning on the Treatment arm (I)

Example 3. Suppose (DUM) and asm (11) hold. If

- there is **no learning across treatment arms** ($d_0 \neq d_1$) **nor on the untreated arm** ($d_0 = d_1$):

$$E[Y_1(d_1)|U_0, Y_0(d_0)] = E[Y_1(d_1)|U_0] \text{ where } (d_0 + d_1) \in \{0, 1\}, \quad (13)$$

- and $Y_0(1)$ is **not informative** about $Y_0(0)$ given U_0:

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the Parallel trends condition is **satisfied**.

Heuristic: *PT may hold if agents can learn only from and about treated outcomes, ie. if untreated outcomes are accurately expected from the start.*

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Proof.

In (DUM) the choice rule for D_0 is conditional on U_0. By asm (13), the treatment decision $D_1(d_0 = 0)$ is also conditional on U_0 since $U_1(0) \equiv U_0$.

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&E[Y_1(0) - Y_0(0)|D_0 = 0, D_1 = d_1] \\
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Thus, the untreated trend is constant *for those initially not treated*. □

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Proof.

Choice $D_1(d_0 = 1)$ is conditional on $U_1(1) \equiv \{U_0, Y_0(1)\}$. By the LIE:

$$\begin{aligned} & E[Y_1(0) - Y_0(0) | D_0 = 1, D_1(1) = d_1] \\ = E\big[& E[Y_1(0) - Y_0(0) | U_0, Y_0(1)] \big| D_0 = 1, D_1(1) = d_1\big]. \end{aligned}$$

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Thus, the untreated trend is constant *for those who are initially treated.* □

Learning on the Treatment arm (IV)

Proof.

It is proved that (i) the conditional untreated trend is constant in d_1 for each d_0, ie. no violation in $t = 1$ given d_0.

Learning on the Treatment arm (IV)

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It is proved that (i) the conditional untreated trend is constant in d_1 for each d_0, ie. no violation in $t = 1$ given d_0.
As asm (11) implies, (ii) the conditional untreated trend is also constant in d_0 and (iii) with the same value:

$$\begin{aligned} & E[Y_1(0) - Y_0(0) | D_0 = d_0] \\ &= E\big[E[Y_1(0) - Y_0(0) | U_0] \big| D_0 = d_0\big] \\ &= \tau, \text{ for all } d_0 \in \{0, 1\}, \end{aligned}$$

ie. no violation in $t = 0$ and there is no trend shift in between.

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ie. no violation in $t = 0$ and there is no trend shift in between.
Since (i), (ii) and (iii) from (Observation 1) are equivalent to PT, the condition is valid. □

Learning on the Control arm (I)

Example 4. Suppose (DUM) and asm (11) hold. Let Ω_{vl} be the set of *valuable learners*, ie. agents whose optimal D_1 is decisively influenced by Y_0.

If $Y_0(0)$ is informative about $Y_1(0)$ but no other form of learning is possible, then PT is satisfied if and only if:

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- valuable learning is impossible (ie. $P(\Omega_{vl}) = 0$), or
- valuable learning occurs only where untreated outcomes are constant almost surely (ie. $P(Y_0(0) = Y_1(0)|\Omega_{vl}) = 1$) and $\tau = 0$.

Heuristic: *PT may not hold if valuable information can be learned from and about untreated outcomes, unless said outcomes are stable over time.*

Learning on the Control arm (II)

Interpretation. Since $D_1(0)$ is not conditional on U_0 alone in this case, PT is generally violated in $t = 1$ by the untreated in $t = 0$. Tipically:

$$E[Y_1(0) - Y_0(0)|D_0 = 0] \neq E[Y_1(0) - Y_0(0)|D_0 = 1].$$

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But PT is valid if:

- $Y_0(0)$ does not determine $D_1 = D_1(0)$, ie. U_0 is practically sufficient, or
- $Y_0(0) \equiv Y_1(0)$ in Ω_{vl}, implying that the untreated trend is zero among all valuable learners, regardless of their treatment choices.

Review of findings

Since PT *constrains agent behavior*, it's not compatible with every kind of dynamic choice mechanism. Our examples specifically deal with **learning**:

Example 2. PT may hold if observed outcomes do not provide additional information about the future, i.e., if agents do not learn anything new about potential outcomes. *No dynamic selection in this case.*

Example 3. PT may hold if agents can learn only from and about treated outcomes, i.e., if untreated outcomes are accurately expected from the start. *Learning on treated outcomes does not affect untreated trends.*

Example 4. PT may not hold if valuable information can be learned from and about untreated outcomes, unless said outcomes are stable over time. *Learning on untreated outcomes may lead to differences in conditional untreated trends.*

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Example 3. PT may hold if agents can learn only from and about treated outcomes, i.e., if untreated outcomes are accurately expected from the start. *Learning on treated outcomes does not affect untreated trends.*

Example 4. PT may not hold if valuable information can be learned from and about untreated outcomes, unless said outcomes are stable over time. *Learning on untreated outcomes may lead to differences in conditional untreated trends.*

Review of findings

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Thank you

Appendix (Alternative identification strategies)

From the last example, we can see that the causal effect of treatment is still identified for the always-treated and the switchers out-of-treatment since:

$$E[Y_1(0) - Y_0(0)|D_0 = 1, D_1(0) = d_1] = \tau \text{ for all } d_1 \in \{0, 1\}.$$

This would be an instance of **partial identification**, but is it meaningful?

The last example also suggests another approach, **mean stationarity** of untreated outcomes: $E[Y_1(0) - Y_0(0)] = 0$. It implies, for $D_0 = 0$:

$$E[Y_1(1) - Y_1(0)|D_1 = 1] = \frac{E[Y_1] - E[Y_0]}{P(D_1 = 1)},$$

ie. the ATT is identified in sharp designs if the untreated trend is zero.