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Stat 5444: Model Selection

Consider the ('semi') linear model of the form:

$$y_i = \sum_{j=1}^{p} f_j(x_{i,j})\beta_j + \epsilon_i, \qquad i = 1, \ldots, N, \tag{1}$$

where $\epsilon_i \sim Normal(0, \sigma^2)$. More specifically, we can write $Y = X\beta + \epsilon$, where $Y = [y_1, \ldots, y_n]^T$, $\beta = [\beta_1, \ldots, \beta_p]^T$, and $X_{i,j} = f_j(x_{i,j})$. Note: In general, figuring out the functions $f_j(\cdot)$ can be difficult, but as a cursory step it is common to look at polynomial evaluations of $x_{i,j}$ (which can potentially lead to the dimensionality of the problem (p) being very large).

Problem 1

Work out the Bayes factor for comparing 2 different models, where each model is of the form given in equation (1), but differ by the number of free parameters (i.e. the number of coefficients where $\beta_j \neq 0$).

$$BF = \frac{e^{-\frac{1}{2\sigma^2}\left(Y^TY\right)} \int\limits_{\beta\ \varepsilon\ M_1} e^{-\frac{1}{2}\left(-\frac{2\beta_{\setminus j}^T X_{\setminus j}^T Y}{\sigma^2} + \beta_{\setminus j}^T \left(\frac{X_{\setminus j}^T X_{\setminus j}}{\sigma^2} + \frac{I}{\psi^2}\right)\beta_{\setminus j} + \mu^T E^{-1}\mu - \mu^T E^{-1}\mu\right)} d\beta}{\left(\frac{1}{\sqrt{2\pi}\psi}\right) \int\limits_{\beta\ \varepsilon\ M_2} e^{-\frac{1}{2}\left(-\frac{2\beta^T X^T Y}{\sigma^2} + \beta^T\left(\frac{X^TX}{\sigma^2} + \frac{I}{\psi^2}\right)\beta + \mu^T E^{-1}\mu - \mu^T E^{-1}\mu\right)} d\beta}$$

$$BF = \frac{\sqrt{2\pi}\psi \ \times e^{\frac{1}{2}\left(\mu_{\setminus j}^T E_{\setminus j}^{-1}\mu_{\setminus j}\right)}(2\pi)^{k/2}|E_{\setminus j}|^{1/2}}{e^{\frac{1}{2}(\mu^T E^{-1}\mu)}(2\pi)^{\frac{k+1}{2}}|E|^{1/2}}$$

$$E_{\setminus j} = \left(\frac{X_{\setminus j}^T X_{\setminus j}}{\sigma^2} + \frac{I}{\psi^2}\right)^{-1}$$

$$E = \left(\frac{X^TX}{\sigma^2} + \frac{I}{\psi^2}\right)^{-1}$$

$$log(BF) = log(numerator) - log(denominator)$$

$$log(numerator) = log(\psi) + 0.5 * \frac{Y^T X_{\setminus j}}{\sigma^2}\left(\left(\frac{X_{\setminus j}^T X_{\setminus j}}{\sigma^2} + \frac{I}{\psi^2}\right)^{-1}\right)^T \frac{X_{\setminus j}^T Y}{\sigma^2} + 0.5 * log(|E_{\setminus j}|)$$

$$log(den) = 0.5 * \frac{Y^T X}{\sigma^2}\left(\left(\frac{X^TX}{\sigma^2} + \frac{I}{\psi^2}\right)^{-1}\right)^T \frac{X^TY}{\sigma^2} + 0.5 * log(|E|)$$

Problem 2

Using the data provided in **Model1_5444.txt**, search for the 'true' model using the following selection procedures:

- **Least Absolute Shrinkage Selection Operator** (LASSO)

$$\hat{\beta}_{LASSO} = ||Y - X\beta||_2^2 + \lambda\sum_{j=1}^{p}|\beta_j|,$$

- **Bayesian Information Criterion (BIC)**: 'Deviance' $+ \log(N)\Delta_p$, where Δ_p denotes the difference in the number of parameters used in the compared models,

- **Akaike Information Criterion (AIC)**: 'Deviance' $+ 2\Delta_p$, where Δ_p denotes the difference in the number of parameters used in the compared models,

- **Stochastic Search Variable Selection** (SSVS):

$$\pi(\beta_j) = \pi_0 \delta(\beta_j = 0) + (1 - \pi_0) N(\beta_j | 0, \psi^2).$$

In your comparison between methods, describe in detail how you 'tuned' your method (if tuning is required), and report your selected model (including coefficient estimates).

Homework 6

Problem 2:

Stochastic Search Variable Selection:

The implementation of the code is similar to what was taught in the class. The main steps of the algorithm is as explained below,

Gibbs sampling steps:

- For t=1:T

  For j=1:p

$$BF^{-1} = \frac{N(0|0,\psi^2)}{N(0|E,V)}$$

$$\hat{p} = \left(1 + \frac{1-\pi}{\pi} BF^{-1}\right)^{-1}$$

  Where $V = \left(\frac{1}{\sigma^2} X_j^T X_j + \frac{1}{\psi^2}\right)^{-1}, E = \left(\frac{1}{\sigma^2} X_j^T X_j + \frac{1}{\psi^2}\right)^{-1} X_j^T (Y - X_{-j}\beta_{-j})$

  If BERNOULI($\hat{p}$) $= 1$

  $\beta_j^t = 0$

  Else

  $\beta_j^t \sim N(\beta_j|E,V)$

  End

  Sample $\phi \sim Gamma(\frac{N}{2}, \frac{Residual}{2})$

  END

The most important factor affecting the algorithm are

1.) Tuning the σ, standard deviation of response
2.) Tuning the ψ, the standard deviation for the beta coeff model

To address the ψ part, standardization is recommended for the predictor variables and hence $\psi = 1.69 \; or \; 1.96 \; or \; 2.25$

In the present exercise the comparative study is also done.

The code is attached in appendix 1

```
clear all
clc
close all
%% Stochastic Variable selection algorithm matlab script
% This code is based on conjugate priors

%% Generate pseudo random data

init_data.order=3;

% init_data.predictor=randn([init_data.N,init_data.predictor_np]);


% init_data.noofterms=floor(size(init_data.predictor_column,2)/2);
% rng(1,'v5uniform');
%
init_data.actualmodelterms=randi([1,size(init_data.predictor_column,2)],[init
_data.noofterms,1]);
%
init_data.actualmodelterms=sort(randsample(size(init_data.predictor_column,2)
,[init_data.noofterms]));
%
init_data.actualmodelpredictors=init_data.predictor_column(:,init_data.actual
modelterms);
% beta_predictors=4*randn([length(init_data.actualmodelterms),1]);
%
init_data.response=init_data.actualmodelpredictors*beta_predictors+0.3*randn(
[init_data.N,1]);

data=xlsread('model1.xlsx');
%% SVSS algorithm initialization

data_available.response=data(:,1);
init_data.predictor=data(:,2:end);
init_data=generate_data(init_data);
% init_data.predictor_column=[ones(length(data_available.response),1)
init_data.predictor_column];
% data_available.response=zscore(init_data.response);
% data_available.predictor_library=init_data.predictor_column;
data_available.predictor_library=zscore(init_data.predictor_column);
data_available.predictor_library=[ones(length(data_available.response),1)
data_available.predictor_library];

data_available.init_pi0=0.5;
data_available.Xvar=1.96^2;
% M1=[10:-1:2]%init_data.actualmodelterms;
% M2=[10:-1:2 1];
%
%
```

```
%
% [bf,pr]=compute_bayes_fac(M1,M2,data_available);
% pr.m1_m2
%% MCMC starting

disp('Gibbs sampling running');
disp('%%%%%%%%%%%%%%%');

T=40000;flg=0;
p=size(data_available.predictor_library,2);
model.heatmap=cell(T,1);
model.global_indicator=cell(T,1);
model.global_indicator{1,1}=ones(1,p);
model.selected=cell((p),1);
model.sumterms=zeros(p,1);
N=length(data_available.response);
data_available.var_Y=0.3;
list_var=1:1:p;
for t=2:T

    model.indicator=model.global_indicator{t-1,1};
    data_available.beta=model.indicator;
    if (rem(t*100/T,20)==0)
        flg=flg+1;
        fprintf('Gibbs sampling: %d percent done\n',20*flg);
    end

    for j=1:p
        M1=j;
        M2=find(list_var~=j);
        [bf_inv,pr]=compute_bayes_fac_V2(M1,M2,data_available);

        if binornd(1,pr.H0)==1
            model.indicator(1,j)=0;
            model.betaind(1,j)=0;
        else
            model.indicator(1,j)=normrnd(pr.E,sqrt(pr.V));
            model.betaind(1,j)=1;
        end
        data_available.beta=model.indicator;
    end
    model.residuals=data_available.response-
data_available.predictor_library*model.indicator';
    model.B=(sum(model.residuals.^2))*0.5;
    model.phi(t,1)=gamrnd(N/2,1/model.B);
    data_available.var_Y=1/model.phi(t,1);
    model.global_indicator{t,1}=model.indicator;
    model.global_betaindicator{t,1}=model.betaind;
    model.heatmap{t,1}=find(model.indicator~=0);
```

```
    if flg>1
    if isempty(model.heatmap{t,1})==0
%         length(model.selected(length(model.heatmap{t,1})))=len;
        model.selected{length(model.heatmap{t,1}),1}(1,end+1)=t;

model.sumterms(length(model.heatmap{t,1}),1)=model.sumterms(length(model.heat
map{t,1}),1)+1;
    end
    end
end
%%
[~,Ind] = max(model.sumterms);newjj=1;jj=1;member=[];clstr=0;
while(jj<length(model.selected{Ind,1}))
    tterm=model.selected{Ind,1}(1,jj);cntr=0;
    totmember=0;clstr=clstr+1;newjj=[];
    for kk=jj:length(model.selected{Ind,1})
        tterm2=model.selected{Ind,1}(1,kk);
        comp_model=model.global_betaindicator{tterm2,1};
        if(ismember(comp_model,model.global_betaindicator{tterm,1},'rows'))
            totmember=totmember+1;
            member{clstr,1}(totmember)=tterm2;
            clstrlen(clstr,1)=totmember;
        else
%             clstr=clstr+1
            newjj(1,end+1)=kk;
        end
    end
    if isempty(newjj)
        break
    else
        jj=newjj(1);
    end
end
[~,indc]=max(clstrlen);
model.betarray=cell2mat(model.global_indicator(member{indc,1}));
model.meanbetarray=mean(model.betarray);
disp(model.meanbetarray);

%%

clear beta_var
beta_var{1}='Intercept';
for ii=1:50
    beta_var{1+ii}=(sprintf('beta-%d',ii));
end
flg=51;
for ii=1:50
    for jj=ii:ii
        flg=flg+1;
        beta_var{flg}=(sprintf('beta_%d_%d',ii,jj));
```

```
    end
end

flg=101;
for ii=1:50
    for jj=ii:ii
        for kk=jj:jj
        flg=flg+1;
        beta_var{flg}=(sprintf('beta_%d_%d_%d',ii,jj,kk));
        end
    end
end
beta_var=beta_var';
VarNames={'Beta','Value'};
T=table(beta_var,model.meanbetarray','VariableNames',VarNames);
%%


function [bf_inv,pr]=compute_bayes_fac_V2(M1,M2,data_available)
    Y=data_available.response;
    pi0=data_available.init_pi0;
    Y_var=data_available.var_Y;
    N=length(data_available.response);
    X_variance=data_available.Xvar;
    X_M1=data_available.predictor_library(:,M1); %X_j
    X_M2=data_available.predictor_library(:,M2); %X_(-j)
    Beta_M2=data_available.beta(:,M2)';

    Variance=1/((1/Y_var)*(X_M1'*X_M1)+(1/X_variance));
    Meanval=Variance*X_M1'*(Y-X_M2*Beta_M2)/Y_var;
    log_num=log(1/sqrt(X_variance));
    log_denom=log(1/sqrt(Variance))-0.5*(Meanval^2)/Variance;
    log_bayes_fac_inv=log_num-log_denom;
    bf_inv=exp(log_bayes_fac_inv);
    pr.H0=inv(1+((1-pi0)/pi0)*bf_inv);
    pr.E=Meanval;
    pr.V=Variance;
end
```

Results of SVSS for Model 1

```
clear beta_var
beta_var{1}='Intercept';
for ii=1:50
    beta_var{1+ii}=(sprintf('beta-%d',ii));
end
flg=51;
for ii=1:50
    for jj=ii:ii
        flg=flg+1;
        beta_var{flg}=(sprintf('beta_%d_%d',ii,jj));
    end
end

flg=101;
for ii=1:50
    for jj=ii:ii
        for kk=jj:jj
        flg=flg+1;
        beta_var{flg}=(sprintf('beta_%d_%d_%d',ii,jj,kk));
        end
    end
end
beta_var=beta_var';
VarNames={'Beta','Value'};
T=table(beta_var,model.meanbetarray','VariableNames',VarNames);
disp(T)
```

```
        Beta            Value
    _______________    _________

    'Intercept'          -2.8047
    'beta-1'                   0
    'beta-2'                   0
    'beta-3'              1.8132
    'beta-4'                   0
    'beta-5'                   0
    'beta-6'                   0
    'beta-7'                   0
    'beta-8'                   0
    'beta-9'                   0
    'beta-10'                  0
    'beta-11'                  0
    'beta-12'                  0
    'beta-13'                  0
    'beta-14'                  0
    'beta-15'                  0
    'beta-16'                  0
    'beta-17'             1.1262
    'beta-18'           -0.35505
    'beta-19'                  0
    'beta-20'                  0
    'beta-21'                  0
    'beta-22'                  0
    'beta-23'                  0
    'beta-24'                  0
```

```
    'beta-25'              0
    'beta-26'              0
    'beta-27'         2.5055
    'beta-28'              0
    'beta-29'              0
    'beta-30'              0
    'beta-31'              0
    'beta-32'              0
    'beta-33'              0
    'beta-34'              0
    'beta-35'              0
    'beta-36'              0
    'beta-37'              0
    'beta-38'        0.39425
    'beta-39'              0
    'beta-40'        -0.6552
    'beta-41'              0
    'beta-42'              0
    'beta-43'              0
    'beta-44'              0
    'beta-45'         4.3655
    'beta-46'              0
    'beta-47'              0
    'beta-48'              0
    'beta-49'              0
    'beta-50'       -0.72537
    'beta_1_1'             0
    'beta_2_2'             0
    'beta_3_3'             0
    'beta_4_4'     -0.076734
    'beta_5_5'             0
    'beta_6_6'             0
    'beta_7_7'             0
    'beta_8_8'             0
    'beta_9_9'             0
    'beta_10_10'     0.41249
    'beta_11_11'     0.29997
    'beta_12_12'           0
    'beta_13_13'           0
    'beta_14_14'           0
    'beta_15_15'    -0.13085
    'beta_16_16'           0
    'beta_17_17'           0
    'beta_18_18'           0
    'beta_19_19'    0.088538
    'beta_20_20'           0
    'beta_21_21'           0
    'beta_22_22'           0
    'beta_23_23'           0
    'beta_24_24'           0
    'beta_25_25'           0
    'beta_26_26'           0
    'beta_27_27'           0
    'beta_28_28'    -0.42006
    'beta_29_29'           0
    'beta_30_30'           0
    'beta_31_31'           0
    'beta_32_32'           0
    'beta_33_33'           0
    'beta_34_34'           0
    'beta_35_35'     -1.1168
```

'beta_36_36'	0
'beta_37_37'	-0.39743
'beta_38_38'	0
'beta_39_39'	0
'beta_40_40'	0
'beta_41_41'	0
'beta_42_42'	0
'beta_43_43'	0
'beta_44_44'	0
'beta_45_45'	0
'beta_46_46'	0
'beta_47_47'	0
'beta_48_48'	-0.10212
'beta_49_49'	0
'beta_50_50'	0
'beta_1_1_1'	0
'beta_2_2_2'	0.16643
'beta_3_3_3'	0
'beta_4_4_4'	-1.5549
'beta_5_5_5'	0
'beta_6_6_6'	0
'beta_7_7_7'	0
'beta_8_8_8'	0
'beta_9_9_9'	0
'beta_10_10_10'	0
'beta_11_11_11'	0
'beta_12_12_12'	0
'beta_13_13_13'	0.34192
'beta_14_14_14'	0
'beta_15_15_15'	0
'beta_16_16_16'	0
'beta_17_17_17'	0
'beta_18_18_18'	0
'beta_19_19_19'	0
'beta_20_20_20'	0
'beta_21_21_21'	0
'beta_22_22_22'	0
'beta_23_23_23'	0
'beta_24_24_24'	0
'beta_25_25_25'	0
'beta_26_26_26'	0
'beta_27_27_27'	0
'beta_28_28_28'	0
'beta_29_29_29'	0
'beta_30_30_30'	0
'beta_31_31_31'	0
'beta_32_32_32'	-0.16802
'beta_33_33_33'	-0.081993
'beta_34_34_34'	-0.27586
'beta_35_35_35'	0
'beta_36_36_36'	0
'beta_37_37_37'	0
'beta_38_38_38'	0
'beta_39_39_39'	-0.46109
'beta_40_40_40'	0
'beta_41_41_41'	-0.68102
'beta_42_42_42'	0
'beta_43_43_43'	0
'beta_44_44_44'	0
'beta_45_45_45'	0.14931
'beta_46_46_46'	0

```
    'beta_47_47_47'      0.49609
    'beta_48_48_48'            0
    'beta_49_49_49'            0
    'beta_50_50_50'      0.57743
```

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Derivation for BIC, AIC

(P) (P_{cr}) if $B_j = 0$?

Derivation for AIC, BIC

Denominator

$$\cancel{e^{-\frac{1}{2}(Y^T Y - 2B_{(-j)}^T X_{(-j)}}$$

$$e^{-\frac{1}{2}(Y - X_{(-j)}B_{(-j)} - X_j B_j)^T (Y - X_{(-j)}B_{(-j)} - X_j B_j)}$$

$$= e^{-\frac{1}{2}(\boxed{Y^T Y} - \boxed{2B_{(-j)}^T X_{-j}^T Y} - 2B_j^T X_j^T Y + \boxed{B_{-j}^T X_{-j}^T X_{-j} B_{-j}}}$$

$$+ B_j^T X_j^T X_{-j} B_{-j} + \cancel{X_j B_j}$$

$$B_{-j}^T X_{-j}^T X_j B_j + B_j^T X_j^T X_j B_j$$

Numerator

$$e^{-\frac{1}{2}(Y - X_{-j}B_{-j})^T (Y - X_{-j}B_{(-j)})}$$

$$= e^{\underbrace{-\frac{1}{2}(Y^T Y - 2B_{(-j)}^T X_{-j}^T Y + B_{-j}^T X_{-j}^T X_{-j} B_{-j}}}$$

$$= e^{-K}$$

Denominator

$$= e^{-K} \cdot e^{-\frac{1}{2}(-2B_j^T X_j^T Y + 2B_j^T X_j^T X_{-j} B_{-j} + \cancel{B_j^T X_{-j}^T X_j B_j} + B_j^T X_j^T X_j B_j)}$$

Likelihood ratio

$$-2\log \frac{L(\hat{B} \mid Y, M_{(i)}^{c}, X)}{L(B \mid Y, x, M_{(i)})}$$ without j

$$= -2\log \frac{e^{-k}}{e^{-k} \cdot e^{-\frac{1}{2}\left[B_j^T \frac{(x_j^T x_j)}{\sigma^2} B_j - 2B_j^T \frac{[x_j^T (Y - x_j B_j)]}{\sigma^2} \right]}}$$

$$= -2\log \left[\frac{1}{e^{-\frac{1}{2}\left[B_j^T V^{-1} B_j - 2B_j^T V^{-1} E\right] + E^T V^{-1} E}} e^{-\frac{1}{2}[E^T V^{-1} E]} \right]$$

$$= -2\log \left[\frac{\frac{1}{\sqrt{2\pi V}} e^{-\frac{1}{2}[E^T V^{-1} E]}}{\frac{1}{\sqrt{2\pi} V} e^{-\frac{1}{2}\left[B_j^T V^{-1} B_j - 2B_j^T V^{-1} E + E^T V^{-1} E\right]}} \right]$$

$$= -2\log \left[\frac{N(0 \mid E, V)}{N(B_j \mid E, V)} \right] \qquad V = \left(\frac{x_j^T x_j}{\sigma^2} \right)^{-1}$$

$$E = \left[\left(\frac{x_j^T x_j}{\sigma^2} \right)^{-1} x_j^T \frac{(Y - x_j B_j)}{\sigma^2} \right]$$

$$\Rightarrow \boxed{\begin{aligned} AIC &= -2\log \left[\frac{N(0 \mid E, V)}{N(B_j \mid E, V)} \right] + 2(\Delta \text{Parameter}) \\ BIC &= -2\log \left[\frac{N(0 \mid E, V)}{N(B_j \mid E, V)} \right] + \log N(\Delta \text{Parameter}) \end{aligned}}$$

```
clear all
clc
close all
data=xlsread('model1.xlsx');
%% SVSS algorithm initialization

data_available.response=data(:,1);
init_data.predictor=data(:,2:end);
init_data.order=3;
init_data=generate_data(init_data);
% init_data.predictor_column=[ones(length(data_available.response),1)
init_data.predictor_column];
% data_available.response=zscore(init_data.response);
% data_available.predictor_library=init_data.predictor_column;
data_available.predictor_library=zscore(init_data.predictor_column);
data_available.predictor_library=[ones(length(data_available.response),1)
data_available.predictor_library];

data_available.init_pi0=0.5;
data_available.Xvar=1.96^2;
% M1=[10:-1:2]%init_data.actualmodelterms;
% M2=[10:-1:2 1];

% M2=[10:-1:2 1];
%
%
%
% [bf,pr]=compute_bayes_fac(M1,M2,data_available);
% pr.m1_m2
%% MCMC starting

disp('Gibbs sampling running');
disp('%%%%%%%%%%%%%%%');

T=500;flg=0;
p=size(data_available.predictor_library,2);
model.heatmap=cell(T,1);
model.global_indicator=cell(T,1);
beta_init=pinv(data_available.predictor_library'*data_available.predictor_lib
rary)*data_available.predictor_library'*data_available.response;
model.global_indicator{1,1}=beta_init';
model.selected=cell((p),1);
model.sumterms=zeros(p,1);
model.betaind=ones(1,p);
N=length(data_available.response);
list_var=1:1:p;data_available.var_Y=var(data_available.response);
for t=2:T

    model.indicator=model.global_indicator{t-1,1};
```

```
    data_available.beta=model.indicator;
    if (rem(t*100/T,20)==0)
        flg=flg+1;
        fprintf('Gibbs sampling: %d percent done\n',20*flg);
    end

    for j=1:p

        data_available.beta=zeros(1,p);
        model.betaind(j)=1;
        X_new=data_available.predictor_library(:,(model.betaind==1));
        model.indicator=pinv(X_new'*X_new)*X_new'*data_available.response;
        model.indicator=model.indicator';
        data_available.beta(1,model.betaind==1)=model.indicator;


        M1=j;
        M2=find(list_var~=j);
        [del_BIC,pr]=compute_BIC(M1,M2,data_available);

        if del_BIC<0
            model.betaind(1,j)=0;
        else
            model.betaind(1,j)=1;
        end

     end
    data_available.beta=zeros(1,p);
    X_new=data_available.predictor_library(:,(model.betaind==1));
    model.indicator=pinv(X_new'*X_new)*X_new'*data_available.response;
    model.indicator=model.indicator';
    data_available.beta(1,model.betaind==1)=model.indicator;

    model.global_indicator{t,1}=data_available.beta;
    model.global_betaindicator{t,1}=model.betaind;
    model.heatmap{t,1}=find(model.indicator~=0);

    model.residuals=data_available.response-
data_available.predictor_library*data_available.beta';
    model.B=(sum(model.residuals.^2))*0.5;
    model.phi(t,1)=gamrnd(N/2,1/model.B);
    data_available.var_Y=1/model.phi(t,1);

    if flg>1
    if isempty(model.heatmap{t,1})==0
%         length(model.selected(length(model.heatmap{t,1})))=len;
        model.selected{length(model.heatmap{t,1}),1}(1,end+1)=t;
```

```
model.sumterms(length(model.heatmap{t,1}),1)=model.sumterms(length(model.heat
map{t,1}),1)+1;
    end
    end
end
%%
[~,Ind] = max(model.sumterms);newjj=1;jj=1;member=[];clstr=0;
while(jj<length(model.selected{Ind,1}))
    tterm=model.selected{Ind,1}(1,jj);cntr=0;
    totmember=0;clstr=clstr+1;newjj=[];
    for kk=jj:length(model.selected{Ind,1})
        tterm2=model.selected{Ind,1}(1,kk);
        comp_model=model.global_betaindicator{tterm2,1};
        if(ismember(comp_model,model.global_betaindicator{tterm,1},'rows'))
            totmember=totmember+1;
            member{clstr,1}(totmember)=tterm2;
            clstrlen(clstr,1)=totmember;
        else
%             clstr=clstr+1
            newjj(1,end+1)=kk;
        end
    end
    if isempty(newjj)
        break
    else
        jj=newjj(1);
    end
end
[~,indc]=max(clstrlen);
model.betarray=cell2mat(model.global_indicator(member{indc,1}));
model.meanbetarray=mean(model.betarray);
disp(model.meanbetarray);


function [delta_BIC,pr]=compute_BIC(M1,M2,data_available)
    Y=data_available.response;
    pi0=data_available.init_pi0;
    Y_var=data_available.var_Y;
    N=length(data_available.response);
    X_variance=data_available.Xvar;
    X_M1=data_available.predictor_library(:,M1); %X_j
    X_M2=data_available.predictor_library(:,M2); %X_(-j)
    Beta_M2=data_available.beta(:,M2)';
    Beta_M1=data_available.beta(:,M1)';

    Variance=1/((1/Y_var)*(X_M1'*X_M1));
    Meanval=Variance*X_M1'*(Y-X_M2*Beta_M2)/Y_var;

    log_num=log(1/sqrt(Variance))-0.5*(Meanval^2)/Variance;
```

```
    log_denom=log(normpdf(Beta_M1,Meanval,sqrt(Variance)));

    delta_BIC=-2*(log_num-log_denom)+log(N)*-1;
    pr.E=Meanval;
    pr.V=Variance;
end
```

Results of BIC for Model 1

```
clear beta_var
beta_var{1}='Intercept';
for ii=1:50
    beta_var{1+ii}=(sprintf('beta-%d',ii));
end
flg=51;
for ii=1:50
    for jj=ii:ii
        flg=flg+1;
        beta_var{flg}=(sprintf('beta_%d_%d',ii,jj));
    end
end

flg=101;
for ii=1:50
    for jj=ii:ii
        for kk=jj:jj
        flg=flg+1;
        beta_var{flg}=(sprintf('beta_%d_%d_%d',ii,jj,kk));
        end
    end
end
beta_var=beta_var';
VarNames={'Beta','Value'};
T=table(beta_var,model.meanbetarray','VariableNames',VarNames);
disp(T)
```

```
         Beta           Value
    _______________    ________

    'Intercept'         -2.8062
    'beta-1'                  0
    'beta-2'                  0
    'beta-3'                  0
    'beta-4'                  0
    'beta-5'                  0
    'beta-6'                  0
    'beta-7'                  0
    'beta-8'                  0
    'beta-9'                  0
    'beta-10'                 0
    'beta-11'                 0
    'beta-12'                 0
    'beta-13'                 0
    'beta-14'                 0
    'beta-15'                 0
    'beta-16'                 0
    'beta-17'                 0
    'beta-18'                 0
    'beta-19'                 0
    'beta-20'                 0
    'beta-21'                 0
    'beta-22'                 0
    'beta-23'                 0
    'beta-24'                 0
```

```
'beta-25'            0
'beta-26'            0
'beta-27'            0
'beta-28'            0
'beta-29'            0
'beta-30'            0
'beta-31'            0
'beta-32'            0
'beta-33'            0
'beta-34'            0
'beta-35'            0
'beta-36'            0
'beta-37'            0
'beta-38'            0
'beta-39'            0
'beta-40'            0
'beta-41'            0
'beta-42'            0
'beta-43'            0
'beta-44'            0
'beta-45'            0
'beta-46'            0
'beta-47'            0
'beta-48'            0
'beta-49'            0
'beta-50'            0
'beta_1_1'           0
'beta_2_2'           0
'beta_3_3'           0
'beta_4_4'           0
'beta_5_5'           0
'beta_6_6'           0
'beta_7_7'           0
'beta_8_8'           0
'beta_9_9'           0
'beta_10_10'         0
'beta_11_11'         0
'beta_12_12'         0
'beta_13_13'         0
'beta_14_14'         0
'beta_15_15'         0
'beta_16_16'         0
'beta_17_17'         0
'beta_18_18'         0
'beta_19_19'         0
'beta_20_20'         0
'beta_21_21'         0
'beta_22_22'         0
'beta_23_23'         0
'beta_24_24'         0
'beta_25_25'         0
'beta_26_26'         0
'beta_27_27'         0
'beta_28_28'         0
'beta_29_29'         0
'beta_30_30'         0
'beta_31_31'         0
'beta_32_32'         0
'beta_33_33'         0
'beta_34_34'         0
'beta_35_35'         0
```

```
    'beta_36_36'            0
    'beta_37_37'            0
    'beta_38_38'            0
    'beta_39_39'            0
    'beta_40_40'       1.0361
    'beta_41_41'            0
    'beta_42_42'            0
    'beta_43_43'            0
    'beta_44_44'            0
    'beta_45_45'            0
    'beta_46_46'      -2.5469
    'beta_47_47'            0
    'beta_48_48'            0
    'beta_49_49'            0
    'beta_50_50'            0
    'beta_1_1_1'            0
    'beta_2_2_2'            0
    'beta_3_3_3'            0
    'beta_4_4_4'            0
    'beta_5_5_5'            0
    'beta_6_6_6'            0
    'beta_7_7_7'            0
    'beta_8_8_8'            0
    'beta_9_9_9'            0
    'beta_10_10_10'         0
    'beta_11_11_11'         0
    'beta_12_12_12'         0
    'beta_13_13_13'    1.0432
    'beta_14_14_14'         0
    'beta_15_15_15'         0
    'beta_16_16_16'    1.2129
    'beta_17_17_17'         0
    'beta_18_18_18'         0
    'beta_19_19_19'         0
    'beta_20_20_20'         0
    'beta_21_21_21'         0
    'beta_22_22_22'   0.46931
    'beta_23_23_23'   0.85011
    'beta_24_24_24'         0
    'beta_25_25_25'    1.3003
    'beta_26_26_26'  -0.48432
    'beta_27_27_27'         0
    'beta_28_28_28'  -0.77138
    'beta_29_29_29'         0
    'beta_30_30_30'         0
    'beta_31_31_31'         0
    'beta_32_32_32'         0
    'beta_33_33_33'         0
    'beta_34_34_34'         0
    'beta_35_35_35'         0
    'beta_36_36_36'         0
    'beta_37_37_37'         0
    'beta_38_38_38'         0
    'beta_39_39_39'         0
    'beta_40_40_40'    2.8751
    'beta_41_41_41'         0
    'beta_42_42_42'         0
    'beta_43_43_43'         0
    'beta_44_44_44'         0
    'beta_45_45_45'         0
    'beta_46_46_46'         0
```

```
    'beta_47_47_47'           0
    'beta_48_48_48'           0
    'beta_49_49_49'           0
    'beta_50_50_50'      2.2418
```

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Results of AIC for Model 1

```
clear beta_var
beta_var{1}='Intercept';
for ii=1:50
    beta_var{1+ii}=(sprintf('beta-%d',ii));
end
flg=51;
for ii=1:50
    for jj=ii:ii
        flg=flg+1;
        beta_var{flg}=(sprintf('beta_%d_%d',ii,jj));
    end
end

flg=101;
for ii=1:50
    for jj=ii:ii
        for kk=jj:jj
        flg=flg+1;
        beta_var{flg}=(sprintf('beta_%d_%d_%d',ii,jj,kk));
        end
    end
end
beta_var=beta_var';
VarNames={'Beta','Value'};
T=table(beta_var,model.meanbetarray','VariableNames',VarNames);
disp(T)
```

```
        Beta             Value
    ______________     _________

    'Intercept'          -2.8062
    'beta-1'                   0
    'beta-2'                   0
    'beta-3'                   0
    'beta-4'                   0
    'beta-5'                   0
    'beta-6'                   0
    'beta-7'                   0
    'beta-8'                   0
    'beta-9'                   0
    'beta-10'                  0
    'beta-11'                  0
    'beta-12'                  0
    'beta-13'                  0
    'beta-14'                  0
    'beta-15'                  0
    'beta-16'                  0
    'beta-17'                  0
    'beta-18'                  0
    'beta-19'                  0
    'beta-20'                  0
    'beta-21'                  0
    'beta-22'                  0
    'beta-23'                  0
    'beta-24'                  0
```

```
    'beta-25'            0
    'beta-26'            0
    'beta-27'            0
    'beta-28'            0
    'beta-29'            0
    'beta-30'            0
    'beta-31'            0
    'beta-32'            0
    'beta-33'            0
    'beta-34'            0
    'beta-35'            0
    'beta-36'            0
    'beta-37'            0
    'beta-38'            0
    'beta-39'            0
    'beta-40'            0
    'beta-41'            0
    'beta-42'            0
    'beta-43'            0
    'beta-44'            0
    'beta-45'       4.2605
    'beta-46'            0
    'beta-47'            0
    'beta-48'            0
    'beta-49'            0
    'beta-50'            0
    'beta_1_1'           0
    'beta_2_2'           0
    'beta_3_3'           0
    'beta_4_4'           0
    'beta_5_5'           0
    'beta_6_6'           0
    'beta_7_7'           0
    'beta_8_8'           0
    'beta_9_9'           0
    'beta_10_10'         0
    'beta_11_11'         0
    'beta_12_12'         0
    'beta_13_13'         0
    'beta_14_14'         0
    'beta_15_15'         0
    'beta_16_16'         0
    'beta_17_17'         0
    'beta_18_18'         0
    'beta_19_19'         0
    'beta_20_20'         0
    'beta_21_21'         0
    'beta_22_22'         0
    'beta_23_23'         0
    'beta_24_24'         0
    'beta_25_25'         0
    'beta_26_26'         0
    'beta_27_27'         0
    'beta_28_28'         0
    'beta_29_29'         0
    'beta_30_30'         0
    'beta_31_31'         0
    'beta_32_32'         0
    'beta_33_33'         0
    'beta_34_34'         0
    'beta_35_35'         0
```

```
    'beta_36_36'              0
    'beta_37_37'              0
    'beta_38_38'              0
    'beta_39_39'              0
    'beta_40_40'              0
    'beta_41_41'              0
    'beta_42_42'              0
    'beta_43_43'              0
    'beta_44_44'              0
    'beta_45_45'              0
    'beta_46_46'        -1.0295
    'beta_47_47'       0.087889
    'beta_48_48'              0
    'beta_49_49'              0
    'beta_50_50'              0
    'beta_1_1_1'              0
    'beta_2_2_2'              0
    'beta_3_3_3'         0.8069
    'beta_4_4_4'         2.0415
    'beta_5_5_5'        0.22913
    'beta_6_6_6'              0
    'beta_7_7_7'              0
    'beta_8_8_8'       -0.70745
    'beta_9_9_9'        0.24963
    'beta_10_10_10'     0.25289
    'beta_11_11_11'     -1.1552
    'beta_12_12_12'      1.4891
    'beta_13_13_13'     0.64603
    'beta_14_14_14'           0
    'beta_15_15_15'      1.2087
    'beta_16_16_16'     0.61023
    'beta_17_17_17'     0.23245
    'beta_18_18_18'     -0.3791
    'beta_19_19_19'           0
    'beta_20_20_20'     -1.0719
    'beta_21_21_21'           0
    'beta_22_22_22'           0
    'beta_23_23_23'    -0.52537
    'beta_24_24_24'    -0.19916
    'beta_25_25_25'     0.11549
    'beta_26_26_26'     0.73257
    'beta_27_27_27'           0
    'beta_28_28_28'           0
    'beta_29_29_29'           0
    'beta_30_30_30'           0
    'beta_31_31_31'   -0.050845
    'beta_32_32_32'      0.3242
    'beta_33_33_33'           0
    'beta_34_34_34'           0
    'beta_35_35_35'           0
    'beta_36_36_36'           0
    'beta_37_37_37'           0
    'beta_38_38_38'           0
    'beta_39_39_39'           0
    'beta_40_40_40'           0
    'beta_41_41_41'           0
    'beta_42_42_42'           0
    'beta_43_43_43'           0
    'beta_44_44_44'           0
    'beta_45_45_45'           0
    'beta_46_46_46'           0
```

```
    'beta_47_47_47'           0
    'beta_48_48_48'           0
    'beta_49_49_49'           0
    'beta_50_50_50'           0
```

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PROBLEM 3

Results of SVSS for MODEL 2

```
clear beta_var
beta_var{1}='Intercept';
for ii=1:50
    beta_var{1+ii}=(sprintf('beta-%d',ii));
end
flg=51;
for ii=1:50
    for jj=ii:ii
        flg=flg+1;
        beta_var{flg}=(sprintf('beta_%d_%d',ii,jj));
    end
end

flg=101;
for ii=1:50
    for jj=ii:ii
        for kk=jj:jj
        flg=flg+1;
        beta_var{flg}=(sprintf('beta_%d_%d_%d',ii,jj,kk));
        end
    end
end
beta_var=beta_var';
VarNames={'Beta','Value'};
T=table(beta_var,model.meanbetarray','VariableNames',VarNames);
disp(T)
```

```
        Beta            Value  
    _______________    ________

    'Intercept'         -3.5022
    'beta-1'                  0
    'beta-2'                  0
    'beta-3'                  0
    'beta-4'                  0
    'beta-5'             1.7549
    'beta-6'                  0
    'beta-7'                  0
    'beta-8'                  0
    'beta-9'                  0
    'beta-10'                 0
    'beta-11'                 0
    'beta-12'                 0
    'beta-13'                 0
    'beta-14'                 0
    'beta-15'          -0.18829
    'beta-16'                 0
    'beta-17'                 0
    'beta-18'                 0
    'beta-19'                 0
    'beta-20'           0.78991
    'beta-21'                 0
    'beta-22'           0.40876
    'beta-23'                 0
    'beta-24'                 0
```

```
    'beta-25'            0
    'beta-26'            0
    'beta-27'       1.8254
    'beta-28'      -0.6037
    'beta-29'            0
    'beta-30'            0
    'beta-31'            0
    'beta-32'            0
    'beta-33'    -0.045552
    'beta-34'            0
    'beta-35'            0
    'beta-36'      0.64016
    'beta-37'            0
    'beta-38'            0
    'beta-39'    -0.082206
    'beta-40'            0
    'beta-41'    -0.064825
    'beta-42'            0
    'beta-43'            0
    'beta-44'            0
    'beta-45'       4.7117
    'beta-46'            0
    'beta-47'            0
    'beta-48'            0
    'beta-49'    -0.081427
    'beta-50'     0.031346
    'beta_1_1'     0.49304
    'beta_2_2'           0
    'beta_3_3'           0
    'beta_4_4'           0
    'beta_5_5'           0
    'beta_6_6'           0
    'beta_7_7'           0
    'beta_8_8'           0
    'beta_9_9'     -0.2001
    'beta_10_10'         0
    'beta_11_11'         0
    'beta_12_12'         0
    'beta_13_13'         0
    'beta_14_14'         0
    'beta_15_15'         0
    'beta_16_16'         0
    'beta_17_17'         0
    'beta_18_18'         0
    'beta_19_19'         0
    'beta_20_20'         0
    'beta_21_21'  -0.25226
    'beta_22_22'         0
    'beta_23_23'         0
    'beta_24_24'         0
    'beta_25_25'         0
    'beta_26_26'         0
    'beta_27_27'         0
    'beta_28_28'   0.10562
    'beta_29_29'         0
    'beta_30_30'         0
    'beta_31_31'         0
    'beta_32_32'   0.45424
    'beta_33_33'         0
    'beta_34_34'         0
    'beta_35_35'         0
```

```
    'beta_36_36'              0
    'beta_37_37'              0
    'beta_38_38'              0
    'beta_39_39'              0
    'beta_40_40'       -0.87016
    'beta_41_41'              0
    'beta_42_42'              0
    'beta_43_43'              0
    'beta_44_44'       -0.28638
    'beta_45_45'              0
    'beta_46_46'              0
    'beta_47_47'              0
    'beta_48_48'              0
    'beta_49_49'              0
    'beta_50_50'       -0.14452
    'beta_1_1_1'              0
    'beta_2_2_2'      -0.054628
    'beta_3_3_3'              0
    'beta_4_4_4'              0
    'beta_5_5_5'              0
    'beta_6_6_6'        0.47466
    'beta_7_7_7'              0
    'beta_8_8_8'              0
    'beta_9_9_9'              0
    'beta_10_10_10'           0
    'beta_11_11_11'           0
    'beta_12_12_12'           0
    'beta_13_13_13'           0
    'beta_14_14_14'           0
    'beta_15_15_15'           0
    'beta_16_16_16'           0
    'beta_17_17_17'     -0.4108
    'beta_18_18_18'           0
    'beta_19_19_19'     0.77229
    'beta_20_20_20'           0
    'beta_21_21_21'           0
    'beta_22_22_22'    -0.56375
    'beta_23_23_23'    -0.25808
    'beta_24_24_24'    -0.17031
    'beta_25_25_25'           0
    'beta_26_26_26'           0
    'beta_27_27_27'           0
    'beta_28_28_28'           0
    'beta_29_29_29'           0
    'beta_30_30_30'           0
    'beta_31_31_31'           0
    'beta_32_32_32'           0
    'beta_33_33_33'           0
    'beta_34_34_34'           0
    'beta_35_35_35'           0
    'beta_36_36_36'           0
    'beta_37_37_37'           0
    'beta_38_38_38'           0
    'beta_39_39_39'           0
    'beta_40_40_40'           0
    'beta_41_41_41'           0
    'beta_42_42_42'           0
    'beta_43_43_43'           0
    'beta_44_44_44'           0
    'beta_45_45_45'     0.12778
    'beta_46_46_46'           0
```

```
    'beta_47_47_47'        0.282
    'beta_48_48_48'            0
    'beta_49_49_49'            0
    'beta_50_50_50'            0
```

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Results of AIC for Model 2

```
clear beta_var
beta_var{1}='Intercept';
for ii=1:50
    beta_var{1+ii}=(sprintf('beta-%d',ii));
end
flg=51;
for ii=1:50
    for jj=ii:ii
        flg=flg+1;
        beta_var{flg}=(sprintf('beta_%d_%d',ii,jj));
    end
end

flg=101;
for ii=1:50
    for jj=ii:ii
        for kk=jj:jj
        flg=flg+1;
        beta_var{flg}=(sprintf('beta_%d_%d_%d',ii,jj,kk));
        end
    end
end
beta_var=beta_var';
VarNames={'Beta','Value'};
T=table(beta_var,model.meanbetarray','VariableNames',VarNames);
disp(T)
```

```
        Beta            Value
    _____________    __________

    'Intercept'         -3.5017
    'beta-1'                  0
    'beta-2'                  0
    'beta-3'                  0
    'beta-4'                  0
    'beta-5'                  0
    'beta-6'                  0
    'beta-7'                  0
    'beta-8'                  0
    'beta-9'                  0
    'beta-10'                 0
    'beta-11'                 0
    'beta-12'                 0
    'beta-13'                 0
    'beta-14'                 0
    'beta-15'                 0
    'beta-16'                 0
    'beta-17'                 0
    'beta-18'                 0
    'beta-19'                 0
    'beta-20'                 0
    'beta-21'                 0
    'beta-22'                 0
    'beta-23'                 0
    'beta-24'                 0
```

```
'beta-25'              0
'beta-26'              0
'beta-27'              0
'beta-28'              0
'beta-29'              0
'beta-30'              0
'beta-31'              0
'beta-32'              0
'beta-33'              0
'beta-34'              0
'beta-35'              0
'beta-36'              0
'beta-37'              0
'beta-38'              0
'beta-39'              0
'beta-40'              0
'beta-41'              0
'beta-42'              0
'beta-43'              0
'beta-44'              0
'beta-45'         2.7288
'beta-46'              0
'beta-47'              0
'beta-48'              0
'beta-49'              0
'beta-50'              0
'beta_1_1'             0
'beta_2_2'             0
'beta_3_3'             0
'beta_4_4'             0
'beta_5_5'             0
'beta_6_6'             0
'beta_7_7'             0
'beta_8_8'             0
'beta_9_9'             0
'beta_10_10'           0
'beta_11_11'           0
'beta_12_12'           0
'beta_13_13'           0
'beta_14_14'           0
'beta_15_15'           0
'beta_16_16'           0
'beta_17_17'           0
'beta_18_18'           0
'beta_19_19'           0
'beta_20_20'           0
'beta_21_21'           0
'beta_22_22'           0
'beta_23_23'           0
'beta_24_24'           0
'beta_25_25'           0
'beta_26_26'     -1.2892
'beta_27_27'           0
'beta_28_28'           0
'beta_29_29'           0
'beta_30_30'           0
'beta_31_31'           0
'beta_32_32'           0
'beta_33_33'           0
'beta_34_34'           0
'beta_35_35'           0
```

```
    'beta_36_36'               0
    'beta_37_37'               0
    'beta_38_38'               0
    'beta_39_39'               0
    'beta_40_40'               0
    'beta_41_41'               0
    'beta_42_42'               0
    'beta_43_43'               0
    'beta_44_44'               0
    'beta_45_45'               0
    'beta_46_46'               0
    'beta_47_47'         0.83638
    'beta_48_48'               0
    'beta_49_49'               0
    'beta_50_50'               0
    'beta_1_1_1'               0
    'beta_2_2_2'      -0.0091925
    'beta_3_3_3'         -1.4423
    'beta_4_4_4'               0
    'beta_5_5_5'               0
    'beta_6_6_6'         0.86654
    'beta_7_7_7'               0
    'beta_8_8_8'        -0.96641
    'beta_9_9_9'       -0.058233
    'beta_10_10_10'     -0.87767
    'beta_11_11_11'      0.60232
    'beta_12_12_12'        2.093
    'beta_13_13_13'            0
    'beta_14_14_14'   -0.0030683
    'beta_15_15_15'      0.86737
    'beta_16_16_16'     -0.80581
    'beta_17_17_17'     -0.68928
    'beta_18_18_18'      0.44008
    'beta_19_19_19'            0
    'beta_20_20_20'       1.6519
    'beta_21_21_21'    -0.089795
    'beta_22_22_22'     -0.74024
    'beta_23_23_23'       1.1953
    'beta_24_24_24'      -1.3925
    'beta_25_25_25'     -0.53152
    'beta_26_26_26'     -0.68827
    'beta_27_27_27'     0.088807
    'beta_28_28_28'            0
    'beta_29_29_29'      -1.6681
    'beta_30_30_30'            0
    'beta_31_31_31'       1.0072
    'beta_32_32_32'            0
    'beta_33_33_33'            0
    'beta_34_34_34'            0
    'beta_35_35_35'            0
    'beta_36_36_36'      -1.3651
    'beta_37_37_37'     -0.75567
    'beta_38_38_38'       3.0849
    'beta_39_39_39'            0
    'beta_40_40_40'        1.451
    'beta_41_41_41'            0
    'beta_42_42_42'            0
    'beta_43_43_43'     -0.47591
    'beta_44_44_44'       1.6789
    'beta_45_45_45'            0
    'beta_46_46_46'            0
```

```
    'beta_47_47_47'             0
    'beta_48_48_48'             0
    'beta_49_49_49'       -2.0835
    'beta_50_50_50'             0
```

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Results of BIC for Model 2

```
clear beta_var
beta_var{1}='Intercept';
for ii=1:50
    beta_var{1+ii}=(sprintf('beta-%d',ii));
end
flg=51;
for ii=1:50
    for jj=ii:ii
        flg=flg+1;
        beta_var{flg}=(sprintf('beta_%d_%d',ii,jj));
    end
end

flg=101;
for ii=1:50
    for jj=ii:ii
        for kk=jj:jj
        flg=flg+1;
        beta_var{flg}=(sprintf('beta_%d_%d_%d',ii,jj,kk));
        end
    end
end
beta_var=beta_var';
VarNames={'Beta','Value'};
T=table(beta_var,model.meanbetarray','VariableNames',VarNames);
disp(T)
```

```
        Beta           Value
    _______________    ________

    'Intercept'         -3.5017
    'beta-1'                  0
    'beta-2'                  0
    'beta-3'                  0
    'beta-4'                  0
    'beta-5'                  0
    'beta-6'                  0
    'beta-7'                  0
    'beta-8'                  0
    'beta-9'                  0
    'beta-10'                 0
    'beta-11'                 0
    'beta-12'                 0
    'beta-13'                 0
    'beta-14'                 0
    'beta-15'                 0
    'beta-16'                 0
    'beta-17'                 0
    'beta-18'                 0
    'beta-19'                 0
    'beta-20'                 0
    'beta-21'                 0
    'beta-22'                 0
    'beta-23'                 0
    'beta-24'                 0
```

```
'beta-25'            0
'beta-26'            0
'beta-27'            0
'beta-28'            0
'beta-29'            0
'beta-30'            0
'beta-31'            0
'beta-32'            0
'beta-33'            0
'beta-34'            0
'beta-35'            0
'beta-36'            0
'beta-37'            0
'beta-38'            0
'beta-39'            0
'beta-40'            0
'beta-41'            0
'beta-42'            0
'beta-43'            0
'beta-44'            0
'beta-45'            0
'beta-46'            0
'beta-47'            0
'beta-48'            0
'beta-49'            0
'beta-50'            0
'beta_1_1'           0
'beta_2_2'           0
'beta_3_3'           0
'beta_4_4'           0
'beta_5_5'           0
'beta_6_6'           0
'beta_7_7'           0
'beta_8_8'           0
'beta_9_9'           0
'beta_10_10'         0
'beta_11_11'         0
'beta_12_12'         0
'beta_13_13'         0
'beta_14_14'         0
'beta_15_15'         0
'beta_16_16'         0
'beta_17_17'         0
'beta_18_18'         0
'beta_19_19'         0
'beta_20_20'         0
'beta_21_21'         0
'beta_22_22'         0
'beta_23_23'         0
'beta_24_24'         0
'beta_25_25'         0
'beta_26_26'   -1.2153
'beta_27_27'         0
'beta_28_28'         0
'beta_29_29'         0
'beta_30_30'         0
'beta_31_31'         0
'beta_32_32'         0
'beta_33_33'         0
'beta_34_34'         0
'beta_35_35'         0
```

```
    'beta_36_36'             0
    'beta_37_37'             0
    'beta_38_38'             0
    'beta_39_39'       0.44337
    'beta_40_40'      -0.46521
    'beta_41_41'             0
    'beta_42_42'             0
    'beta_43_43'             0
    'beta_44_44'             0
    'beta_45_45'             0
    'beta_46_46'             0
    'beta_47_47'             0
    'beta_48_48'             0
    'beta_49_49'             0
    'beta_50_50'             0
    'beta_1_1_1'             0
    'beta_2_2_2'             0
    'beta_3_3_3'             0
    'beta_4_4_4'             0
    'beta_5_5_5'             0
    'beta_6_6_6'        0.3798
    'beta_7_7_7'       0.74895
    'beta_8_8_8'      -0.43907
    'beta_9_9_9'             0
    'beta_10_10_10'   -0.56148
    'beta_11_11_11'    0.74686
    'beta_12_12_12'          0
    'beta_13_13_13'          0
    'beta_14_14_14'          0
    'beta_15_15_15'      1.134
    'beta_16_16_16'          0
    'beta_17_17_17'          0
    'beta_18_18_18'          0
    'beta_19_19_19'          0
    'beta_20_20_20'     1.3983
    'beta_21_21_21'          0
    'beta_22_22_22'          0
    'beta_23_23_23'    0.60994
    'beta_24_24_24'          0
    'beta_25_25_25'          0
    'beta_26_26_26'          0
    'beta_27_27_27'          0
    'beta_28_28_28'          0
    'beta_29_29_29'    -0.4576
    'beta_30_30_30'          0
    'beta_31_31_31'          0
    'beta_32_32_32'          0
    'beta_33_33_33'          0
    'beta_34_34_34'          0
    'beta_35_35_35'          0
    'beta_36_36_36'          0
    'beta_37_37_37'          0
    'beta_38_38_38'          0
    'beta_39_39_39'          0
    'beta_40_40_40'     4.3362
    'beta_41_41_41'          0
    'beta_42_42_42'          0
    'beta_43_43_43'          0
    'beta_44_44_44'          0
    'beta_45_45_45'          0
    'beta_46_46_46'          0
```

```
    'beta_47_47_47'           0
    'beta_48_48_48'           0
    'beta_49_49_49'           0
    'beta_50_50_50'           0
```

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PROBLEM 4

Results of SVSS for MODEL 3

```
clear beta_var
beta_var{1}='Intercept';
for ii=1:50
    beta_var{1+ii}=(sprintf('beta-%d',ii));
end
flg=51;
for ii=1:50
    for jj=ii:ii
        flg=flg+1;
        beta_var{flg}=(sprintf('beta_%d_%d',ii,jj));
    end
end

flg=101;
for ii=1:50
    for jj=ii:ii
        for kk=jj:jj
        flg=flg+1;
        beta_var{flg}=(sprintf('beta_%d_%d_%d',ii,jj,kk));
        end
    end
end
beta_var=beta_var';
VarNames={'Beta','Value'};
T=table(beta_var,model.meanbetarray','VariableNames',VarNames);
disp(T)
```

```
        Beta           Value  
    _______________    ________

    'Intercept'         -2.4608
    'beta-1'                  0
    'beta-2'                  0
    'beta-3'              1.444
    'beta-4'                  0
    'beta-5'                  0
    'beta-6'                  0
    'beta-7'                  0
    'beta-8'                  0
    'beta-9'                  0
    'beta-10'          -0.50624
    'beta-11'                 0
    'beta-12'          -0.12246
    'beta-13'                 0
    'beta-14'          -0.21602
    'beta-15'                 0
    'beta-16'                 0
    'beta-17'            1.0265
    'beta-18'          -0.70354
    'beta-19'                 0
    'beta-20'                 0
    'beta-21'                 0
    'beta-22'                 0
    'beta-23'                 0
    'beta-24'                 0
```

```
    'beta-25'          0.65805
    'beta-26'                0
    'beta-27'           1.5223
    'beta-28'                0
    'beta-29'                0
    'beta-30'          0.10231
    'beta-31'                0
    'beta-32'                0
    'beta-33'                0
    'beta-34'                0
    'beta-35'                0
    'beta-36'         -0.22166
    'beta-37'                0
    'beta-38'                0
    'beta-39'                0
    'beta-40'                0
    'beta-41'                0
    'beta-42'                0
    'beta-43'                0
    'beta-44'                0
    'beta-45'           4.8476
    'beta-46'                0
    'beta-47'                0
    'beta-48'                0
    'beta-49'         -0.12472
    'beta-50'                0
    'beta_1_1'        -0.21892
    'beta_2_2'               0
    'beta_3_3'               0
    'beta_4_4'               0
    'beta_5_5'               0
    'beta_6_6'               0
    'beta_7_7'               0
    'beta_8_8'               0
    'beta_9_9'               0
    'beta_10_10'             0
    'beta_11_11'             0
    'beta_12_12'             0
    'beta_13_13'      -0.12009
    'beta_14_14'             0
    'beta_15_15'       0.11178
    'beta_16_16'             0
    'beta_17_17'             0
    'beta_18_18'             0
    'beta_19_19'             0
    'beta_20_20'        0.3688
    'beta_21_21'      -0.50193
    'beta_22_22'             0
    'beta_23_23'             0
    'beta_24_24'             0
    'beta_25_25'       0.72544
    'beta_26_26'             0
    'beta_27_27'             0
    'beta_28_28'             0
    'beta_29_29'             0
    'beta_30_30'             0
    'beta_31_31'             0
    'beta_32_32'             0
    'beta_33_33'             0
    'beta_34_34'             0
    'beta_35_35'             0
```

```
    'beta_36_36'        0.32062
    'beta_37_37'              0
    'beta_38_38'              0
    'beta_39_39'              0
    'beta_40_40'              0
    'beta_41_41'              0
    'beta_42_42'              0
    'beta_43_43'       0.056383
    'beta_44_44'              0
    'beta_45_45'              0
    'beta_46_46'              0
    'beta_47_47'              0
    'beta_48_48'              0
    'beta_49_49'              0
    'beta_50_50'       -0.63091
    'beta_1_1_1'              0
    'beta_2_2_2'              0
    'beta_3_3_3'              0
    'beta_4_4_4'              0
    'beta_5_5_5'              0
    'beta_6_6_6'              0
    'beta_7_7_7'              0
    'beta_8_8_8'       -0.10892
    'beta_9_9_9'              0
    'beta_10_10_10'           0
    'beta_11_11_11'           0
    'beta_12_12_12'           0
    'beta_13_13_13'           0
    'beta_14_14_14'           0
    'beta_15_15_15'    -0.42356
    'beta_16_16_16'           0
    'beta_17_17_17'           0
    'beta_18_18_18'           0
    'beta_19_19_19'           0
    'beta_20_20_20'           0
    'beta_21_21_21'           0
    'beta_22_22_22'           0
    'beta_23_23_23'           0
    'beta_24_24_24'           0
    'beta_25_25_25'           0
    'beta_26_26_26'           0
    'beta_27_27_27'           0
    'beta_28_28_28'           0
    'beta_29_29_29'           0
    'beta_30_30_30'           0
    'beta_31_31_31'           0
    'beta_32_32_32'           0
    'beta_33_33_33'           0
    'beta_34_34_34'           0
    'beta_35_35_35'           0
    'beta_36_36_36'           0
    'beta_37_37_37'    -0.11843
    'beta_38_38_38'           0
    'beta_39_39_39'           0
    'beta_40_40_40'           0
    'beta_41_41_41'           0
    'beta_42_42_42'           0
    'beta_43_43_43'           0
    'beta_44_44_44'      0.3025
    'beta_45_45_45'    0.052739
    'beta_46_46_46'           0
```

```
    'beta_47_47_47'           0
    'beta_48_48_48'           0
    'beta_49_49_49'           0
    'beta_50_50_50'     0.32223
```

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Results of BIC for Model 3

```
clear beta_var
beta_var{1}='Intercept';
for ii=1:50
    beta_var{1+ii}=(sprintf('beta-%d',ii));
end
flg=51;
for ii=1:50
    for jj=ii:ii
        flg=flg+1;
        beta_var{flg}=(sprintf('beta_%d_%d',ii,jj));
    end
end

flg=101;
for ii=1:50
    for jj=ii:ii
        for kk=jj:jj
        flg=flg+1;
        beta_var{flg}=(sprintf('beta_%d_%d_%d',ii,jj,kk));
        end
    end
end
beta_var=beta_var';
VarNames={'Beta','Value'};
T=table(beta_var,model.meanbetarray','VariableNames',VarNames);
disp(T)
```

```
       Beta           Value
    ______________    _______

    'Intercept'       -2.4541
    'beta-1'                0
    'beta-2'                0
    'beta-3'                0
    'beta-4'                0
    'beta-5'                0
    'beta-6'                0
    'beta-7'                0
    'beta-8'                0
    'beta-9'                0
    'beta-10'               0
    'beta-11'               0
    'beta-12'               0
    'beta-13'               0
    'beta-14'               0
    'beta-15'               0
    'beta-16'               0
    'beta-17'               0
    'beta-18'               0
    'beta-19'               0
    'beta-20'               0
    'beta-21'               0
    'beta-22'               0
    'beta-23'               0
    'beta-24'               0
```

```
    'beta-25'              0
    'beta-26'              0
    'beta-27'              0
    'beta-28'              0
    'beta-29'              0
    'beta-30'              0
    'beta-31'              0
    'beta-32'              0
    'beta-33'              0
    'beta-34'              0
    'beta-35'              0
    'beta-36'              0
    'beta-37'              0
    'beta-38'              0
    'beta-39'              0
    'beta-40'              0
    'beta-41'              0
    'beta-42'              0
    'beta-43'              0
    'beta-44'              0
    'beta-45'         4.9432
    'beta-46'              0
    'beta-47'              0
    'beta-48'              0
    'beta-49'              0
    'beta-50'              0
    'beta_1_1'             0
    'beta_2_2'             0
    'beta_3_3'             0
    'beta_4_4'             0
    'beta_5_5'             0
    'beta_6_6'             0
    'beta_7_7'             0
    'beta_8_8'             0
    'beta_9_9'             0
    'beta_10_10'           0
    'beta_11_11'           0
    'beta_12_12'           0
    'beta_13_13'           0
    'beta_14_14'           0
    'beta_15_15'           0
    'beta_16_16'           0
    'beta_17_17'           0
    'beta_18_18'           0
    'beta_19_19'           0
    'beta_20_20'           0
    'beta_21_21'           0
    'beta_22_22'           0
    'beta_23_23'           0
    'beta_24_24'           0
    'beta_25_25'           0
    'beta_26_26'           0
    'beta_27_27'           0
    'beta_28_28'           0
    'beta_29_29'           0
    'beta_30_30'           0
    'beta_31_31'           0
    'beta_32_32'           0
    'beta_33_33'           0
    'beta_34_34'           0
    'beta_35_35'           0
```

```
'beta_36_36'          0
'beta_37_37'          0
'beta_38_38'          0
'beta_39_39'          0
'beta_40_40'          0
'beta_41_41'          0
'beta_42_42'          0
'beta_43_43'          0
'beta_44_44'          0
'beta_45_45'   -0.70933
'beta_46_46'          0
'beta_47_47'          0
'beta_48_48'          0
'beta_49_49'          0
'beta_50_50'          0
'beta_1_1_1'   0.012363
'beta_2_2_2'          0
'beta_3_3_3'          0
'beta_4_4_4'          0
'beta_5_5_5'          0
'beta_6_6_6'          0
'beta_7_7_7'    0.39368
'beta_8_8_8'   -0.66544
'beta_9_9_9'          0
'beta_10_10_10'       0
'beta_11_11_11'       0
'beta_12_12_12'       0
'beta_13_13_13'       0
'beta_14_14_14'       0
'beta_15_15_15'       0
'beta_16_16_16'       0
'beta_17_17_17'       0
'beta_18_18_18'       0
'beta_19_19_19' -0.10556
'beta_20_20_20'       0
'beta_21_21_21'       0
'beta_22_22_22'       0
'beta_23_23_23'       0
'beta_24_24_24'       0
'beta_25_25_25'       0
'beta_26_26_26'       0
'beta_27_27_27'       0
'beta_28_28_28'       0
'beta_29_29_29'       0
'beta_30_30_30'       0
'beta_31_31_31'       0
'beta_32_32_32'       0
'beta_33_33_33'       0
'beta_34_34_34'       0
'beta_35_35_35'       0
'beta_36_36_36'       0
'beta_37_37_37'       0
'beta_38_38_38'       0
'beta_39_39_39'       0
'beta_40_40_40'       0
'beta_41_41_41'       0
'beta_42_42_42'       0
'beta_43_43_43'       0
'beta_44_44_44'       0
'beta_45_45_45'       0
'beta_46_46_46'       0
```

```
    'beta_47_47_47'           0
    'beta_48_48_48'           0
    'beta_49_49_49'           0
    'beta_50_50_50'           0
```

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Results of AIC for Model 3

```
clear beta_var
beta_var{1}='Intercept';
for ii=1:50
    beta_var{1+ii}=(sprintf('beta-%d',ii));
end
flg=51;
for ii=1:50
    for jj=ii:ii
        flg=flg+1;
        beta_var{flg}=(sprintf('beta_%d_%d',ii,jj));
    end
end

flg=101;
for ii=1:50
    for jj=ii:ii
        for kk=jj:jj
        flg=flg+1;
        beta_var{flg}=(sprintf('beta_%d_%d_%d',ii,jj,kk));
        end
    end
end
beta_var=beta_var';
VarNames={'Beta','Value'};
T=table(beta_var,model.meanbetarray','VariableNames',VarNames);
disp(T)
```

```
        Beta            Value
    _____________    ________

    'Intercept'       -2.4541
    'beta-1'                0
    'beta-2'                0
    'beta-3'                0
    'beta-4'                0
    'beta-5'                0
    'beta-6'                0
    'beta-7'                0
    'beta-8'                0
    'beta-9'                0
    'beta-10'               0
    'beta-11'               0
    'beta-12'               0
    'beta-13'               0
    'beta-14'               0
    'beta-15'               0
    'beta-16'               0
    'beta-17'               0
    'beta-18'               0
    'beta-19'               0
    'beta-20'               0
    'beta-21'               0
    'beta-22'               0
    'beta-23'               0
    'beta-24'               0
```

```
    'beta-25'               0
    'beta-26'               0
    'beta-27'               0
    'beta-28'               0
    'beta-29'               0
    'beta-30'               0
    'beta-31'               0
    'beta-32'               0
    'beta-33'               0
    'beta-34'               0
    'beta-35'               0
    'beta-36'               0
    'beta-37'               0
    'beta-38'               0
    'beta-39'               0
    'beta-40'               0
    'beta-41'               0
    'beta-42'               0
    'beta-43'               0
    'beta-44'               0
    'beta-45'          4.9261
    'beta-46'               0
    'beta-47'               0
    'beta-48'               0
    'beta-49'               0
    'beta-50'               0
    'beta_1_1'              0
    'beta_2_2'              0
    'beta_3_3'              0
    'beta_4_4'              0
    'beta_5_5'              0
    'beta_6_6'              0
    'beta_7_7'              0
    'beta_8_8'              0
    'beta_9_9'              0
    'beta_10_10'            0
    'beta_11_11'            0
    'beta_12_12'            0
    'beta_13_13'            0
    'beta_14_14'            0
    'beta_15_15'            0
    'beta_16_16'            0
    'beta_17_17'            0
    'beta_18_18'            0
    'beta_19_19'            0
    'beta_20_20'            0
    'beta_21_21'            0
    'beta_22_22'            0
    'beta_23_23'            0
    'beta_24_24'            0
    'beta_25_25'            0
    'beta_26_26'            0
    'beta_27_27'            0
    'beta_28_28'            0
    'beta_29_29'            0
    'beta_30_30'            0
    'beta_31_31'            0
    'beta_32_32'            0
    'beta_33_33'            0
    'beta_34_34'            0
    'beta_35_35'            0
```

```
    'beta_36_36'            0
    'beta_37_37'            0
    'beta_38_38'            0
    'beta_39_39'            0
    'beta_40_40'            0
    'beta_41_41'            0
    'beta_42_42'            0
    'beta_43_43'            0
    'beta_44_44'            0
    'beta_45_45'     -0.28521
    'beta_46_46'            0
    'beta_47_47'            0
    'beta_48_48'            0
    'beta_49_49'            0
    'beta_50_50'            0
    'beta_1_1_1'       1.2556
    'beta_2_2_2'       -1.579
    'beta_3_3_3'            0
    'beta_4_4_4'            0
    'beta_5_5_5'            0
    'beta_6_6_6'      0.36939
    'beta_7_7_7'      0.69033
    'beta_8_8_8'     -0.82567
    'beta_9_9_9'            0
    'beta_10_10_10'         0
    'beta_11_11_11'  -0.13872
    'beta_12_12_12'         0
    'beta_13_13_13'         0
    'beta_14_14_14'   0.90032
    'beta_15_15_15'  -0.71175
    'beta_16_16_16'         0
    'beta_17_17_17'         0
    'beta_18_18_18'         0
    'beta_19_19_19'   0.42133
    'beta_20_20_20'         0
    'beta_21_21_21'  -0.41859
    'beta_22_22_22'         0
    'beta_23_23_23'         0
    'beta_24_24_24'         0
    'beta_25_25_25'         0
    'beta_26_26_26'         0
    'beta_27_27_27'         0
    'beta_28_28_28'    -0.298
    'beta_29_29_29'         0
    'beta_30_30_30'  -0.80997
    'beta_31_31_31'         0
    'beta_32_32_32'         0
    'beta_33_33_33'         0
    'beta_34_34_34'  0.085582
    'beta_35_35_35'         0
    'beta_36_36_36'         0
    'beta_37_37_37'         0
    'beta_38_38_38'         0
    'beta_39_39_39'         0
    'beta_40_40_40'         0
    'beta_41_41_41'         0
    'beta_42_42_42'         0
    'beta_43_43_43'         0
    'beta_44_44_44'         0
    'beta_45_45_45'         0
    'beta_46_46_46'         0
```

```
    'beta_47_47_47'          0
    'beta_48_48_48'          0
    'beta_49_49_49'          0
    'beta_50_50_50'          0
```

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