1 Model Explanation

The SIRDRe model is encapsulated by the following adjustment to the Kendrick-McCormack equations:

$$\frac{dS}{dt} = -\beta SI + \gamma I$$
$$\frac{dI}{dt} = \beta SI - \gamma I - \nu I + \mu R$$
$$\frac{dR}{dt} = \gamma I - \mu dR$$
$$\frac{dD}{dt} = \nu I$$
$$\frac{dRe}{dt} = \mu R$$

The equilibrium point for this expression is clearly,

$$\begin{bmatrix} S_0 \\ I_0 \\ R_0 \\ D_0 \\ Re_0 \end{bmatrix} = span \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Therfore, the initial population can be arbitrary in the analysis of equilbirium points.
Using the equilibrium point $S = s_0$, the Jacobian is:

$$J = \begin{bmatrix} 0 & \gamma & 0 & 0 & 0 \\ 0 & \beta s_0 - \gamma - \nu & \mu & 0 & 0 \\ 0 & \gamma & -\mu & 0 & 0 \\ 0 & \nu & 0 & 0 & 0 \\ 0 & 0 & \mu & 0 & 0 \end{bmatrix}$$

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syms m n g b

% Jacobian of SIRRe model, b = beta * s_0
J(g, b, n, m) = [0  -b+g   0 0 0;
                 0 -b-g-n  m 0 0;
                 0    g   -m 0 0;
                 0    n    0 0 0;
                 0    0    m 0 0;];
[V, D] = eig(J)
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We obtain the following eigenvalues from this analysis:

$$\lambda = \left\{0, 0, 0, -\frac{1}{2}\left(\beta s_0 + \gamma + \mu + \nu \pm \sqrt{D}\right)\right\}$$

where $D = (\beta s_0 + \gamma + \nu + \mu)^2 - 4\mu(\beta s_0 + \nu)$

For most practical discussions, since s_0 describes a population,

$s_0 >> \beta, \gamma, \mu, \nu$

Therefore, we can approximate our initial estimate of non-trivial values of λ to:

$$\lambda \approx -\frac{1}{2}\beta s_0 \pm \sqrt{(\beta s_0)^2 - 4\mu\beta s_0}$$

Since the rate and population are both non-negative quantities, and non-zero for non-trivial cases, the quantity $-\frac{1}{2}\beta s_0$ is negative.

1.1 Real Eigenvalues

There are three possible eigenvalue cases, determined by values of D.

$\Rightarrow D \geq 0$

$\Rightarrow s_0 \geq 4\mu$

1.1.1 Both Negative

We add the further constraint

$\frac{1}{2}\beta s_0 > D$

$\Rightarrow \frac{1}{4}(\beta s_0)^2 > (\beta s_0)^2 - 4\mu\beta s_0$

$\Rightarrow \mu > \frac{3}{16}\beta s_0$

But, as brought up earlier, s_0, in most practical cases, is much larger than β and μ. Thus, a factor of $\frac{3}{16}$ will not be enough to make μ ever outweigh the RHS. Thus, two negative eigenvalues is impractical.

1.1.2 One Zero, One Negative

With the same thought process as the previous case, $\mu = \frac{3}{16}\beta s_0$ is also an impractical situation.

1.1.3 One Positive, One Negative

This is most likely, in the situation of real eigenvalues, that will arise.

1.2 Complex Eigenvalues

The contraint $s_0 < 4\mu$ cannot be practically achieved, hence the situation of complex eigenvalues will not arise.