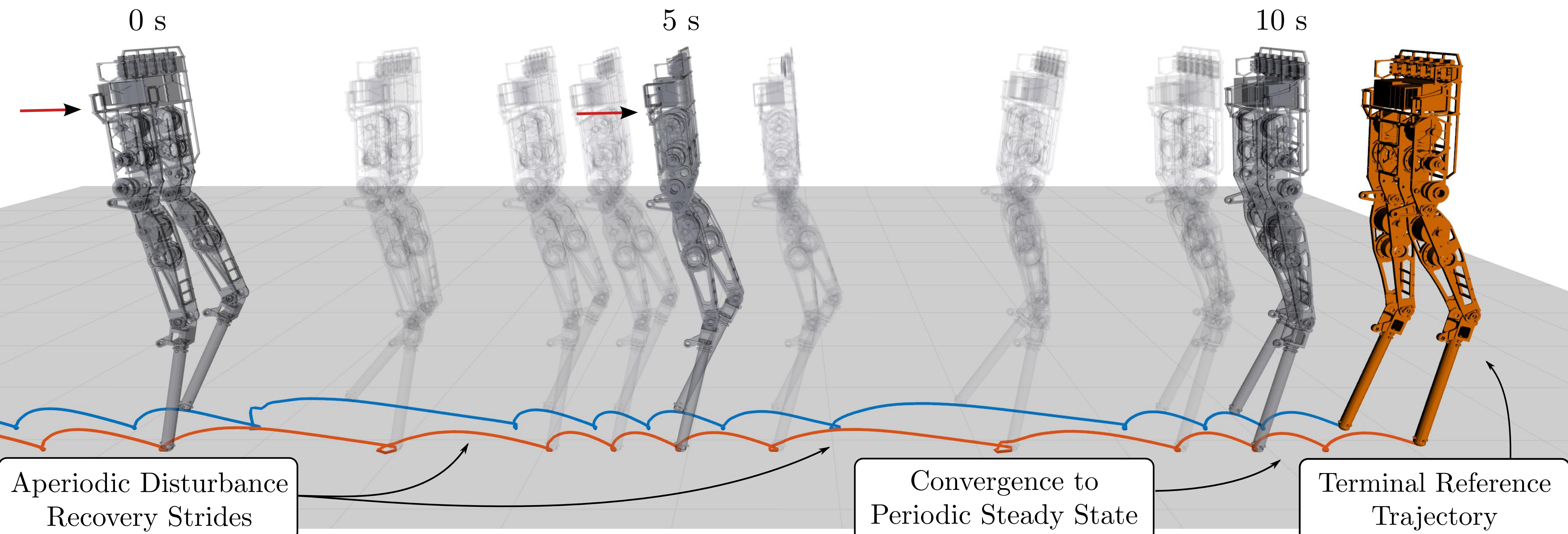# Planar Bipedal Locomotion with Nonlinear Model Predictive Control: Online Gait Generation using Whole-Body Dynamics

Manuel Y. Galliker[†], Noel Csomay-Shanklin[†],
Ruben Grandia, Andrew J. Taylor, Farbod Farshidian,
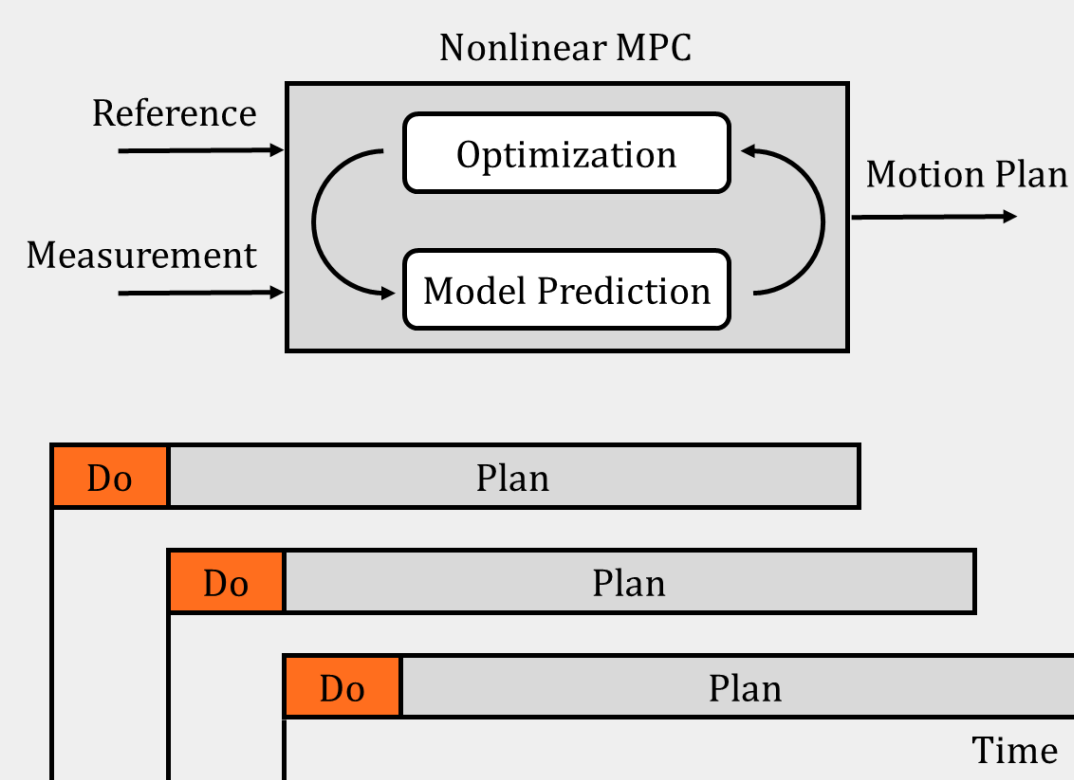Marco Hutter, Aaron D. Ames

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www.bipedalrobotics.com

RSL Robotic Systems Lab

0 s   5 s   10 s

Aperiodic Disturbance Recovery Strides

Convergence to Periodic Steady State

Terminal Reference Trajectory

Reparametrized Whole-Body MPC

Reparametriced WB Dynamics

State $\boldsymbol{x} = [\boldsymbol{q}^\top \quad \dot{\boldsymbol{q}}^\top]^\top$

Input $\boldsymbol{u} = [\ddot{\boldsymbol{q}}_j^\top \quad \boldsymbol{\lambda}^\top]^\top$

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{\boldsymbol{q}} \\ \boldsymbol{D}_{bb}^{-1}\left(-\boldsymbol{h}_b - \boldsymbol{D}_{bj}\ddot{\boldsymbol{q}}_j + \boldsymbol{J}_{c,b}^\top\boldsymbol{\lambda}\right) \\ \ddot{\boldsymbol{q}}_j \end{bmatrix}$$

Nonlinear MPC

Reference → Optimization → Motion Plan
Measurement → Model Prediction

Do | Plan
Do | Plan
Do | Plan
Time

Hybrid Zero Dynamics (HZD)

$\Delta(\mathcal{S})$

Pointwise Optimal

Optimal Over a Horizon

$\mathcal{O}_Z$

$\mathcal{S}$

$\mathcal{N}$

$\dot{\eta} = \omega(\eta, z)$

Control Architecture

High Level

Gait Sequence L R t

Target Base Pose

Reference Trajectory $(\mathbf{x}_{\text{ref}}, \mathbf{u}_{\text{ref}})$

Mid Level

AMBER-3M

MPC Motion Planner 100 Hz $\hat{\mathbf{x}}$

$(\mathbf{x}(t), \ddot{\mathbf{q}}_j(t), \boldsymbol{\lambda}(t))$

Reference Trajectory Sampler 1.5 kHz

Low Level

$\mathbf{x}_j$ → PD Controller

Friction Compensation → Σ → τ

$\ddot{\mathbf{q}}_j$

Inverse Dynamics

$\boldsymbol{\lambda}$
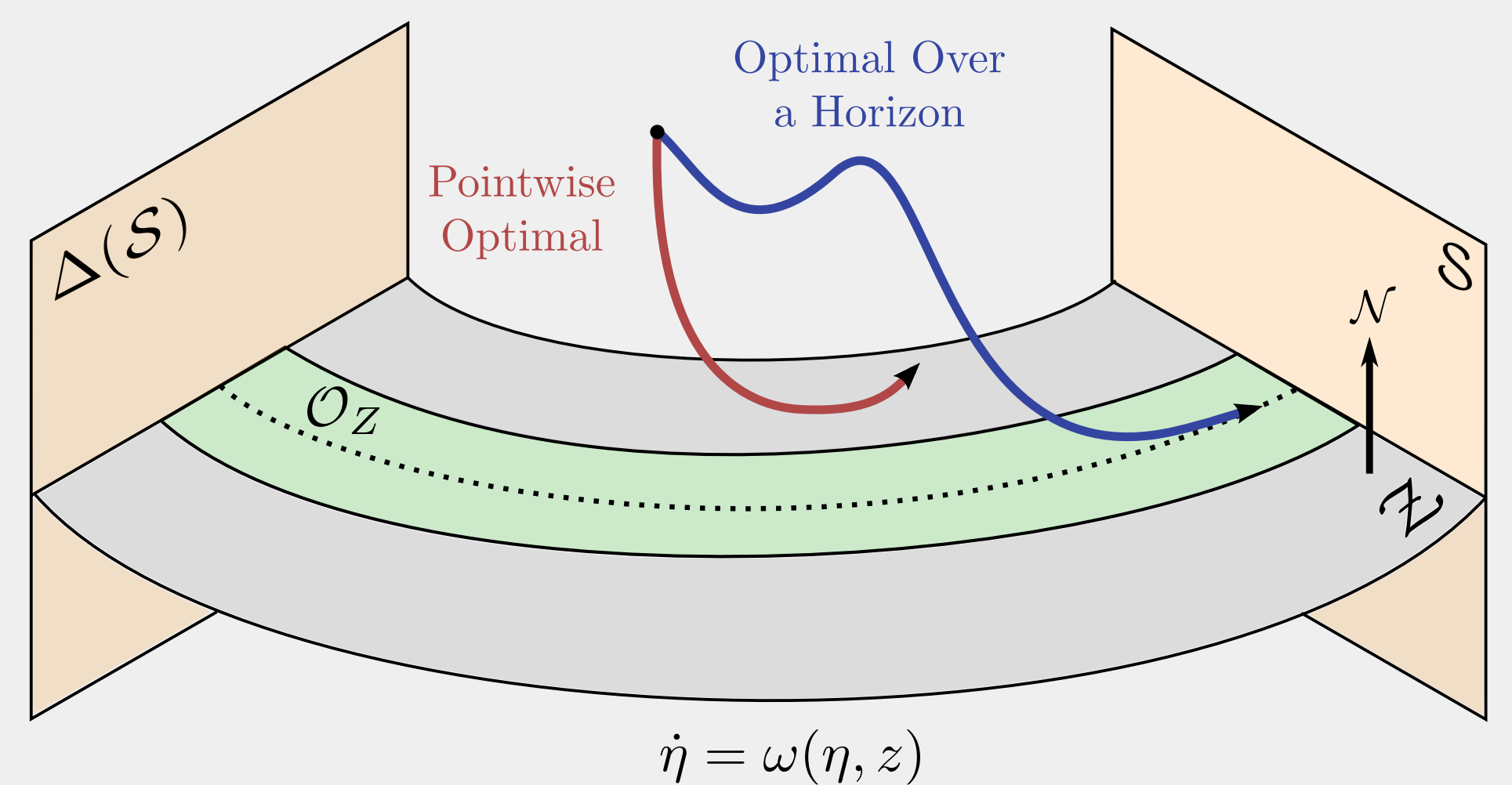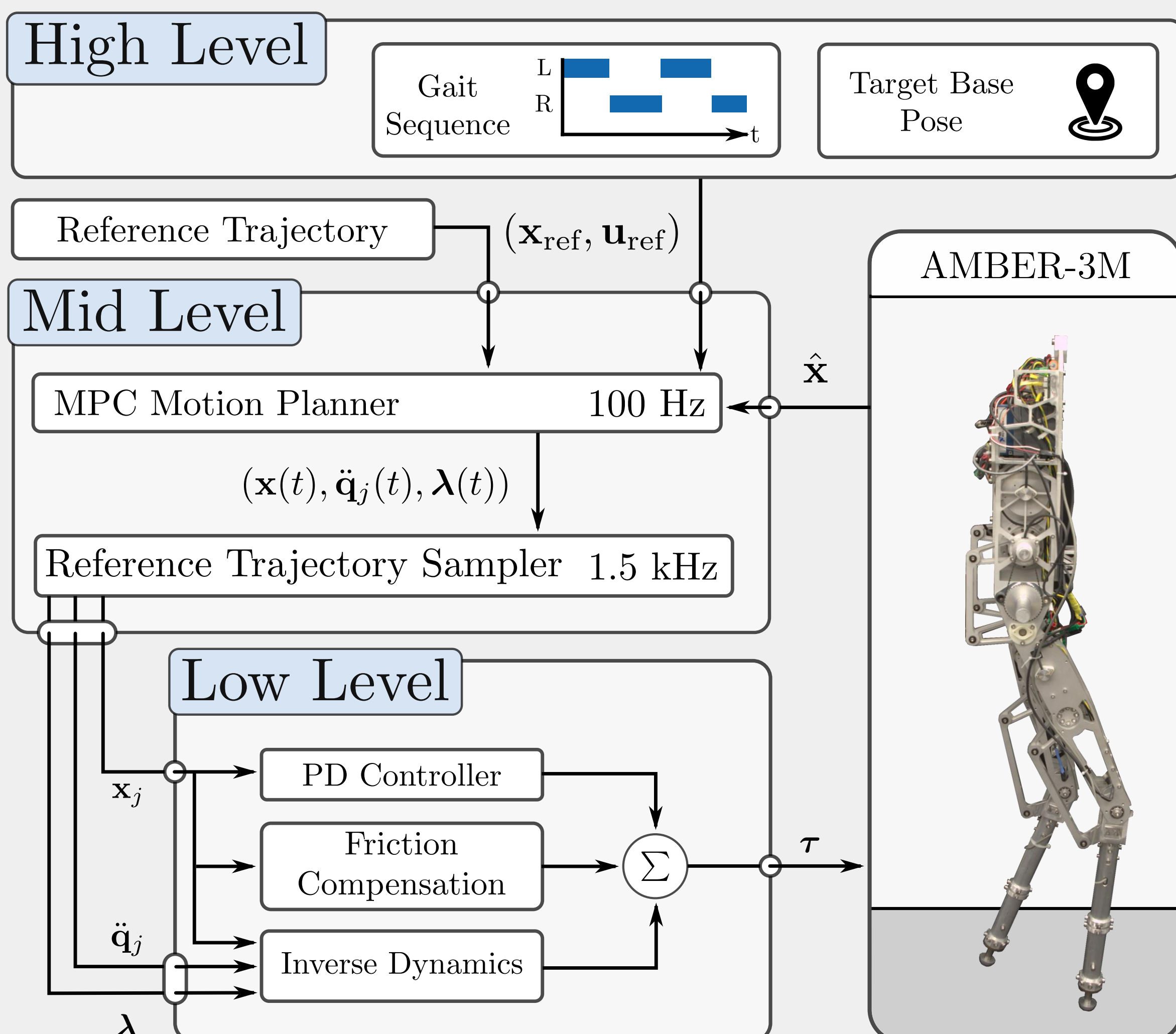
Reference & Terminal Cost

Heuristic   HZD
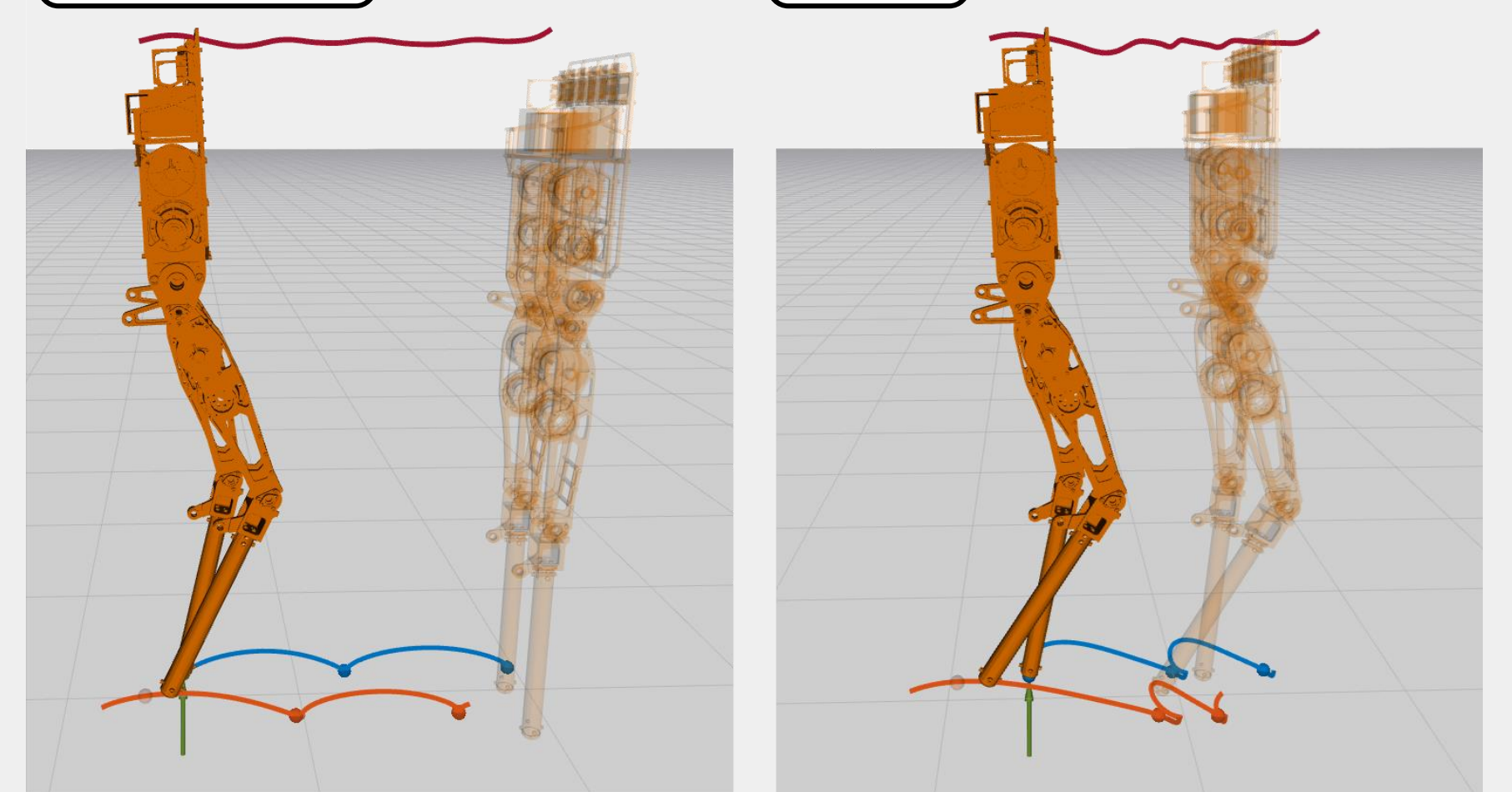
$$\underset{\mathbf{u}(\cdot)}{\text{minimize}} \quad \phi(\mathbf{x}(t_H)) + \int_0^{t_H} l(\mathbf{x}(t), \mathbf{u}(t), t)\mathrm{d}t,$$

Performance Metrics

Table 1: Maximum Disturbance Rejection for Trotting [N]

Horizon Length [s]	2	0.5	0.2
Lumped Mass MPC	2	-	-
MPC + No Terminal	22	-	-
MPC + Heuristic	22	22	-
MPC + HZD	22	22	20
HZD + PD	30		

Table II: MPC Planning Frequency (10 SQP Iterations)

Horizon Length [s]	2.0	1.0	0.5	0.2
MPC Frequency [Hz]	270	480	670	850

Paper

Video

Website