STUDY OF PREDATOR PREY RELATIONSHIPS USING COMPUTATIONAL MODELING

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ABSTRACT

In this project, we explored how the population of predator and prey behave in a biological system by solving a pair of ordinary first order nonlinear differential equations describing the same and is popularly referred to as the Lotka Volterra equations. We observed that change in the prey population with respect to time is affected by how many of these organisms are available to reproduce (represented as ax) and the negative impact of the interaction between predators and prey (represented as −bx). The mathematical equations we used for the modeling are:

1. $dx/dt = ax - bxy$
2. $dy/dt = -cy + dxy$

where:

- x = prey population(for example, number of rabbits)
- dx/dt = change in prey population with respect to time
- y = predator population(for example, number of foxes)
- xy = interaction between predator and prey
- a, b, c, d = constant parameters defining the behavior of the population

Keywords: differential equations, biological systems, predator-prey modeling

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AIM

To study how the population of predator and prey behave in a biological system and to prove that change in the prey-population with respect to time is affected by how many of these organisms are available to reproduce and the negative impact of the interaction between predators and prey.

BACKGROUND

The Lotka–Volterra predator–prey model was initially proposed by Alfred J. Lotka in the theory of autocatalytic chemical reactions in 1910.This was effectively the logistic equation, originally derived by Pierre François Verhulst. In 1920 Lotka extended the model, via Andrey Kolmogorov, to "organic systems" using a plant species and a herbivorous animal species as an example and in 1925 he used the equations to analyze predator–prey interactions in his book on biomathematics.The same set of equations was published in 1926 by Vito Volterra, a mathematician and physicist, who had become interested in mathematical biology. Volterra's enquiry was inspired through his interactions with the marine biologist Umberto D'Ancona, who was courting his daughter at the time and later was to become his son-in-law. D'Ancona studied the fish catches in the Adriatic Sea and had noticed that the percentage of predatory fish caught had increased during the years of World War I (1914–18). This puzzled him, as the fishing effort had been very much reduced during the war years. Volterra developed his model independently from Lotka and used it to explain d'Ancona observation. The model was later extended to include density-dependent prey growth and a functional response of the form developed by C. S. Holling; a model that has become known as the Rosenzweig–MacArthur model. Both the Lotka–Volterra and Rosenzweig–MacArthur models have been used to explain the dynamics of natural populations of predators and prey, such as the lynx and snowshoe hare data of the Hudson's Bay Company and the moose and wolf populations in Isle Royale National Park.

In the late 1980s, an alternative to the Lotka–Volterra predator–prey model (and its common-prey-dependent generalizations) emerged, the ratio dependent or Arditi–Ginzburg model. The validity of prey- or ratio-dependent models has been much debated

THEORY

The Lotka–Volterra model is frequently used to describe the dynamics of ecological systems in which two species interact, one a predator and one its prey. The model is simplified with the following assumptions:
(1) only two species exist: a predator(say, fox) and prey(say, rabbit)
(2) prey is born naturally and then die through predation or inherent death
(3) predators are born and their birth rate is positively affected by the rate of predation, and they die naturally.
The characteristic of this model is that the population change of the predator and the prey are

explained in terms of each other. The size of the fox population has a negative effect on the rabbit population, and the size of the rabbit population has a positive effect on the fox population.
The logistic equation is a simple model of a single population within a habitat. We can expand this model to include populations which are dependent on one another, such as a predator-prey relationship. The Lotka-Volterra predator-prey model was presented as a system of two differential equations as follows:

$dx/dt=ax-bxy$
$dy/dt=-cy+exy$

Prey:

$dx/dt=ax-bxy$

The prey are assumed to have an unlimited food supply and to reproduce exponentially, unless subject to predation; this exponential growth is represented in the equation above by the term αx. The rate of predation upon the prey is assumed to be proportional to the rate at which the predators and the prey meet, this is represented above by βxy. If either x or y is zero, then there can be no predation.
With these two terms the equation above can be interpreted as follows: the rate of change of the prey's population is given by its own growth rate minus the rate at which it is preyed upon.

Predator:

$dy/dt=-cy+exy$

In this equation, exy represents the growth of the predator population. (Note the similarity to the predation rate; however, a different constant is used, as the rate at which the predator population grows is not necessarily equal to the rate at which it consumes the prey). The term $-cy$ represents the loss rate of the predators due to either natural death or emigration, it leads to an exponential decay in the absence of prey.

Hence the equation expresses that the rate of change of the predator's population depends upon the rate at which it consumes prey, minus its intrinsic death rate.

The Lotka–Volterra model makes a number of assumptions, about the environment and evolution of the predator and prey populations:

1. The prey population finds ample food at all times and reproduce exponentially
2. The food supply of the predator population depends entirely on the size of the prey population.
3. The rate of change of population is proportional to its size.

4. During the process, the environment does not change in favour of one species, and genetic adaptation is inconsequential.
5. Predators have limitless appetite.

In this case the solution of the differential equations is deterministic and continuous. This, in turn, implies that the generations of both the predator and prey are continually overlapping.

To write these equations in python we must convert them into a form attenuable by python for which we shall write it as :

$x = x + (ax - bxy)dt$
$y = y + (-cy + exy)dt$

where ;

- dt represents a very tiny time increment and both
- dx and dy are represented by x and y respectively.

In the equations, the new population values are calculated by adding the old values of x and y to the change in populations represented by the equations in parentheses multiplied by the time increment.

COMPUTATIONAL WORK

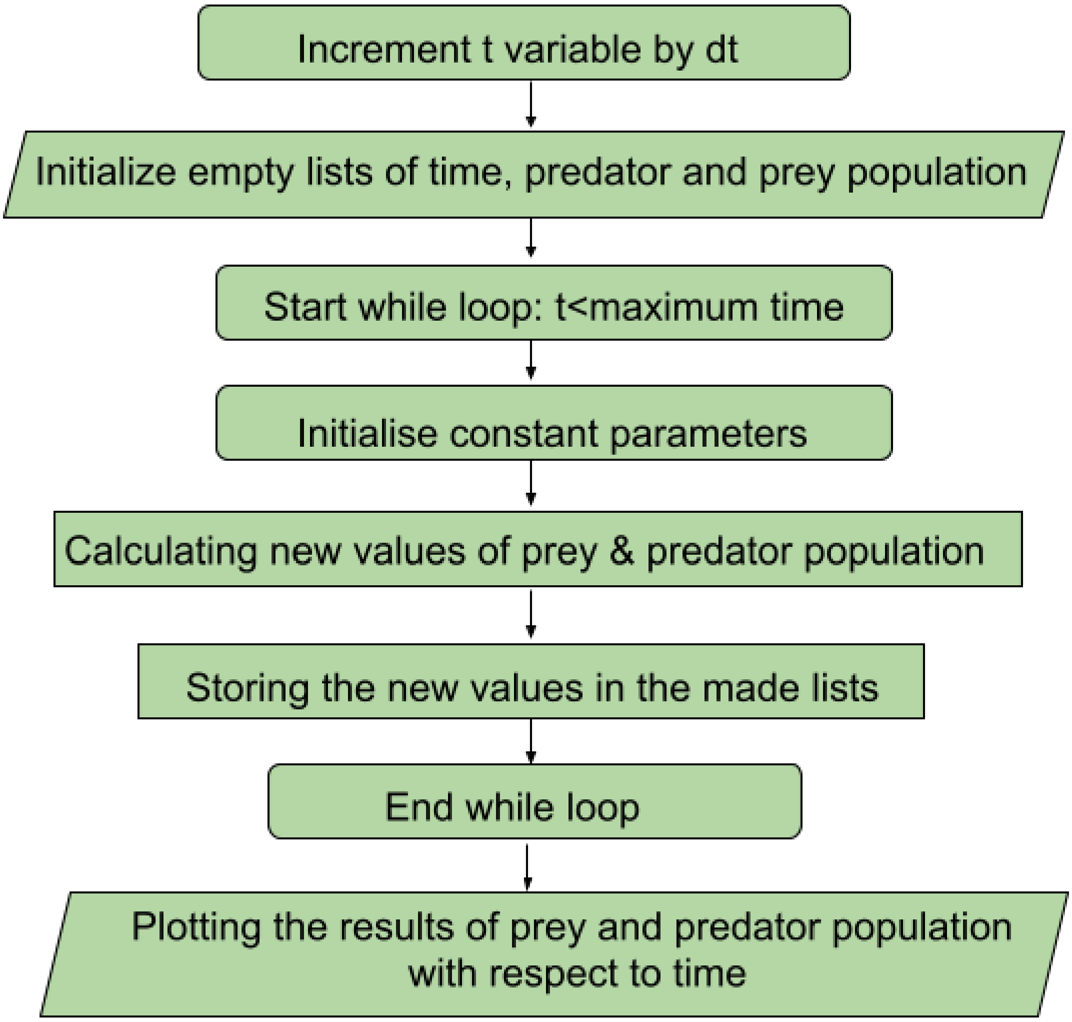
CODE USED:

```
#matplotlib inline
import matplotlib.pyplot as plt
from random import *
from numpy import *
import sys
a = 0.7; b = 0.5; c = 0.3;  e = 0.2                    # model parameters
dt = 0.001; max_time = 100
t = 0; x =1; y = 1                                     # initial time  #x-prey p. & y-predator p.
t_list = [ ]; x_list = [ ]; y_list = [ ]               # empty lists to store time and populations
t_list.append(t); x_list.append(x); y_list.append(y)  # initialize lists
while t < max_time:
    t = t + dt                                         # calc new values for t, x, y
    x = x + (a*x - b*x*y)*dt
    y = y + (-c*y + e*x*y)*dt
    t_list.append(t)                                   # store new values in lists
    x_list.append(x)
    y_list.append(y)
```

```
plt.xlabel('Prey population')                    #title
plt.ylabel('Predator Population')
plt.title('Prey-Predator Relationship')
p = plt.plot(x_list, y_list)                      # Plot the results
```

FLOW CHART:

A simple flow chart to help understand the basic structure of the code.

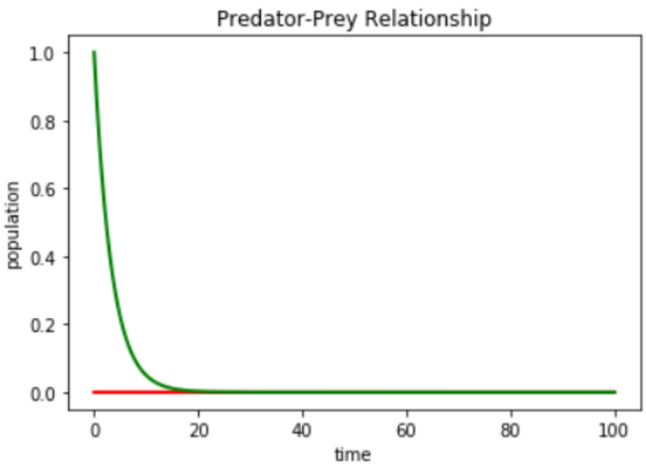


RESULTS

Population(x, y) vs time(t) graphs

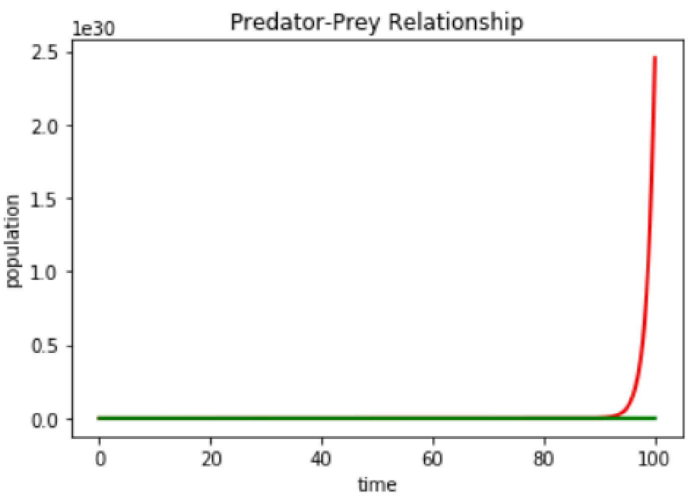
NOTE:
Population of predator is represented by the green curve and and that of prey is represented by the red curve

CASE:1 - No prey in the system(x=0,y=1)



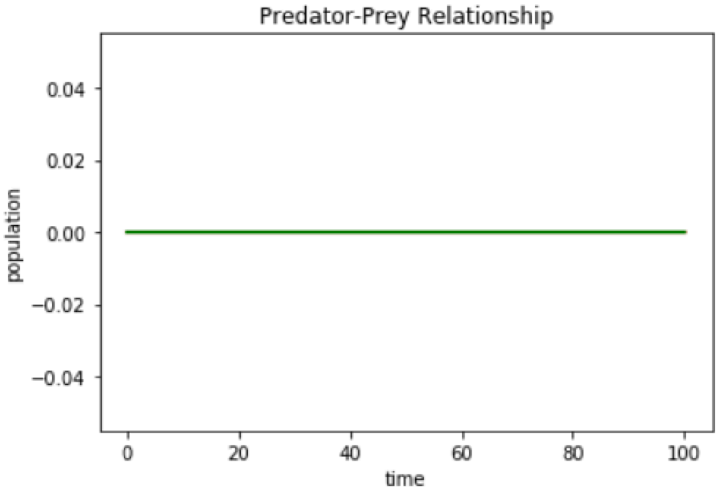
- Here, we observe that due to the lack of food resources for the predator, its population exponentially decreases to 0

CASE:2 - No predator in the system(x=1,y=0)



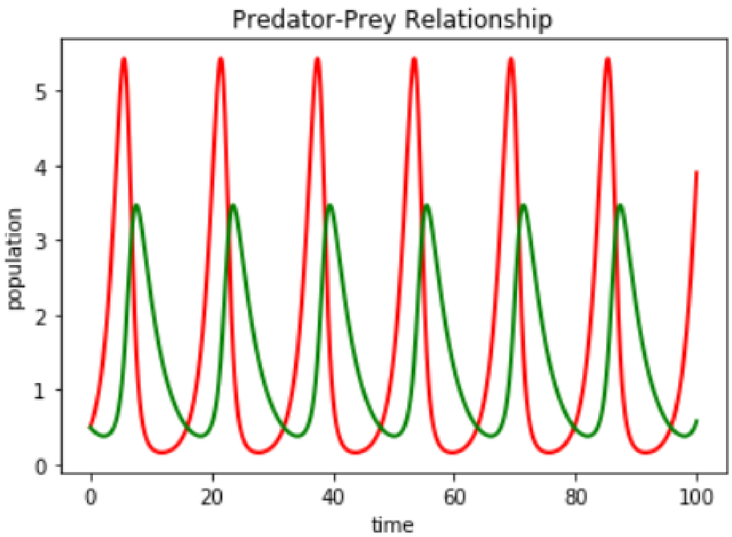
- Here, we observe that due to the lack of natural predator in the habitat, the prey population exponentially increases

CASE:3 - Neither predator nor prey are present in the system(x=0,y=0)



- This is in accordance with our initial assumptions about the system that there is no introduction of predator or prey externally into the system. If we start with 0 population of both predator and prey, it will remain zero at all times

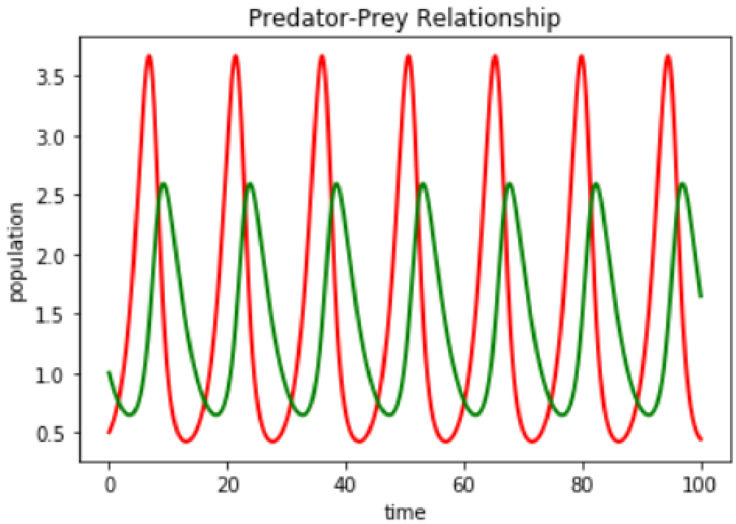
CASE:4 -Initial predator and prey populations are equal(x=0.5, y=0.5)



- Here we can clearly observe the periodic and oscillating nature of the solutions to our equations and thus the nature of the interaction between predator and prey populations
- Initially, the prey population exponentially increases and while the predator population shows a slight dip and the exponentially increases as well
- When the predator population peaks, we observe the exponential decrease of prey

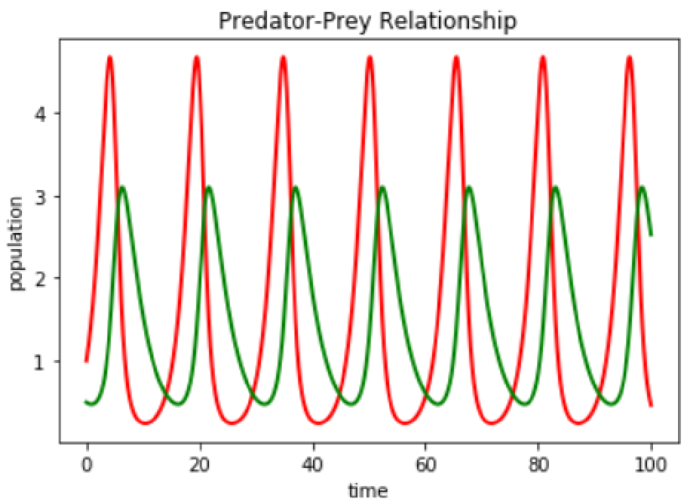
population due to over predation from that point onward thus also decreasing the food resources for the predators
- This causes the fall in predator population until we reach back at the same population state as t=0 and this cycle periodically continues
- Thus we can see how this beautifully works as a system of checks and balances to ensure the continuity of both predator and prey populations.

CASE:5 -Initial predator population is double that of prey population(x=0.5, y=1)



- We see that the trend remains the same as CASE:4 except for the fact that the peak populations(both predator and prey) are lower due to double the number of predators

CASE:6 -Initial prey population is double that of initial predator population(x=1,y=0.5)



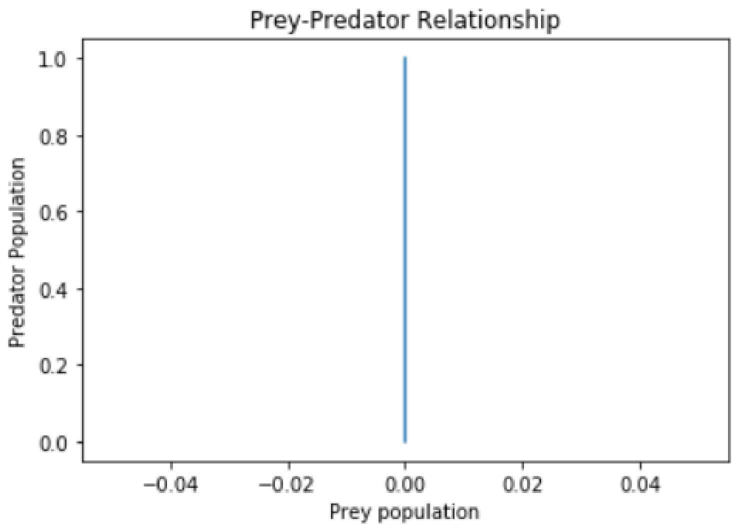
- Again, we see the same trend except for the fact that the peak populations(both predator and prey) are higher than CASE-5 but still lower than CASE-4 where there was an equal population of predators and prey.
- Thus we can conclude that the optimal condition for peak population and survival of both predator and prey harmoniously in a system is for the initial population of both of them to be equal

- Another observation from these curves is that the prey population will always be significantly higher than the predator population

Predator population vs prey population graphs

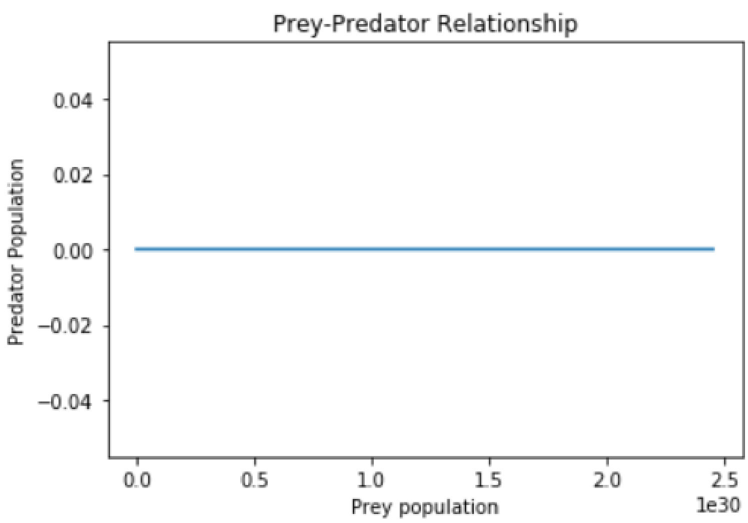
PART-1: No-predator and no prey cases

No prey(x=0,y=1)



- Here we can observe how in the absence of prey, the predator population almost instantaneously drops to zero
- This also further signifies our assumption that there is no external introduction of prey into the system

No predator(x=1,y=0)

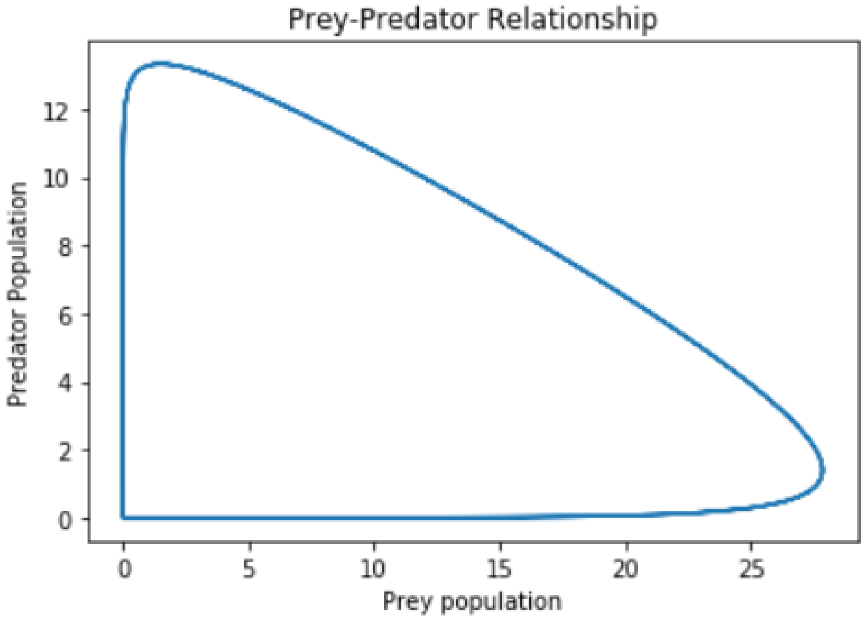


- Here we can observe how in the absence of predator, the prey population keeps

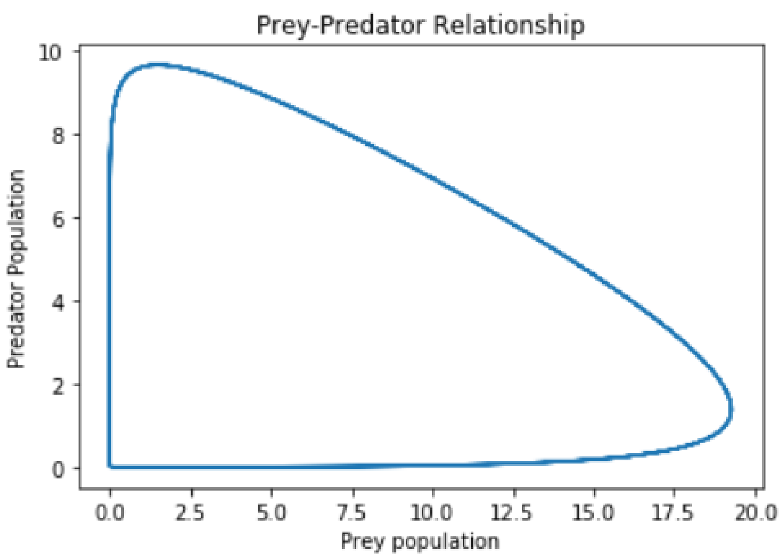
exponentially increasing without any disturbance
- This also further signifies our assumption that there is no external introduction of predators into the system once we set it in motion.

PART-2: Observing the trend as we slowly introduce predators into the system(keeping prey population constant

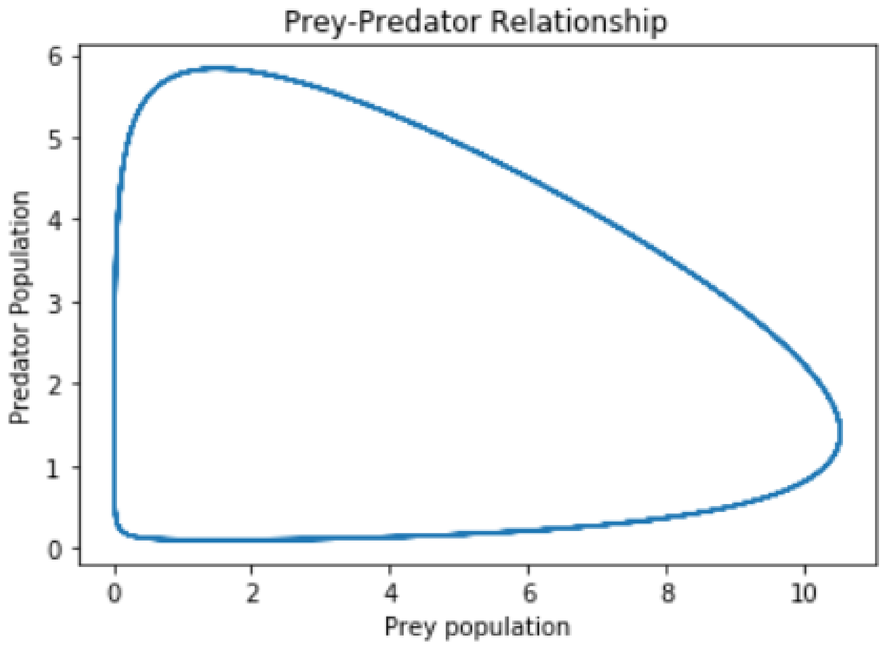
x=1,y=0.001 :



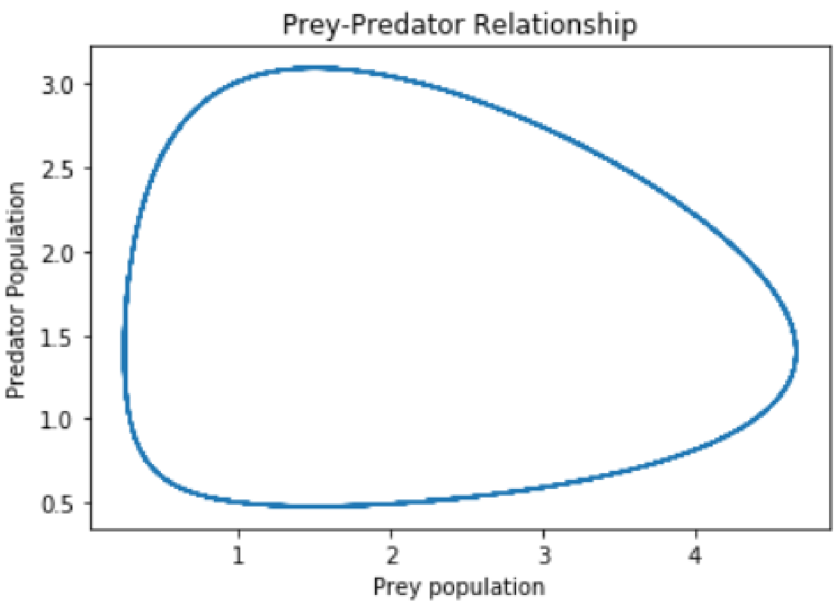
x=1,y=0.01:



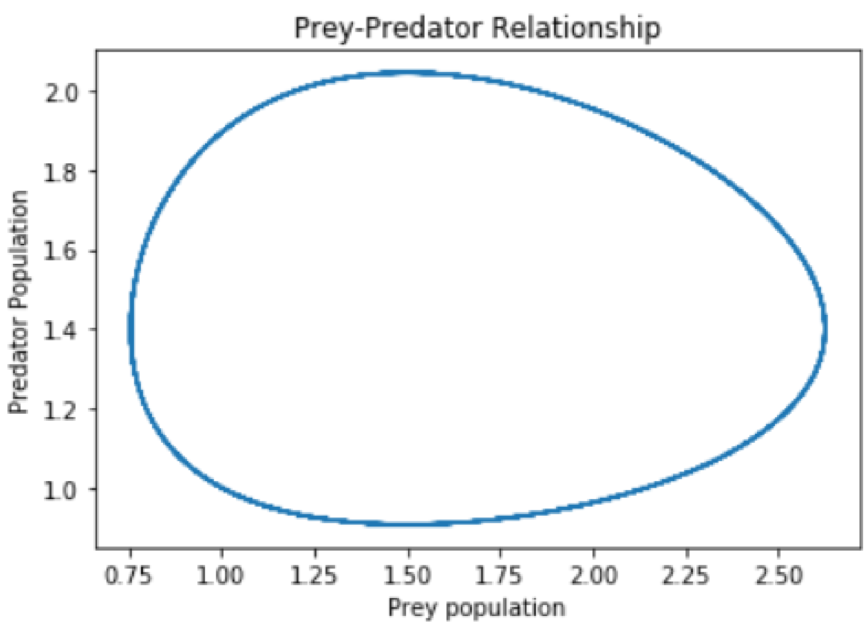
x=1,y=0.1 :


Prey-Predator Relationship
Predator Population
Prey population
0
1
2
3
4
5
6
0
2
4
6
8
10

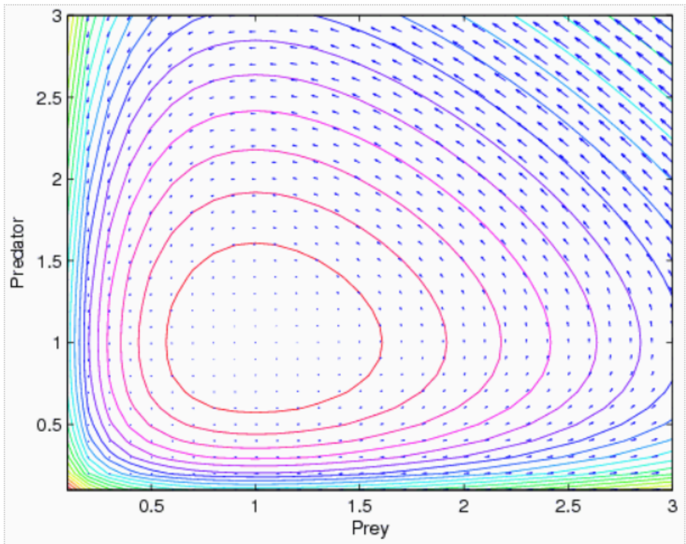
x=1,y=0.5


Prey-Predator Relationship
Predator Population
Prey population
0.5
1.0
1.5
2.0
2.5
3.0
1
2
3
4

x=1,y=1



These can also be depicted as:



- The closed loops again further depicts the cyclic nature of the predator prey interaction we observed in the population-time graphs
- Again we can observe that the most balanced output(considering both predator and prey population optimization) is when both have the same initial population from the nature of the output graph(not shifted towards either prey or predator side). Thus this is our **equilibrium condition.**

APPLICATIONS

1. This research develops a sales forecasting model that can analyze the interaction effects of two retail competing formats (convenience-oriented vs. budget-oriented formats). A traditional approach to making such a forecast is based on the Lotka–Volterra equations (also called the LV-model). The LV-model assumes that the population of each species is affected by itself-growth, internal interaction within the species, and external interaction with other species. Most prior studies in business applications directly use sales data as input to the LV-model.
2. An application of the Lotka–Volterra model to Taiwan's transition from 200 mm to 300 mm silicon wafers.the growth of the area of 200 mm and 300 mm silicon wafers manufactured in Taiwan by the competitive Lotka–Volterra model. The parameters in the Lotka–Volterra model estimated with the realistic data are obtained numerically. The dynamic growth of the competitive relationship between 200 mm silicon wafers and 300 mm silicon wafers is then analyzed.
3. Cloud is a complex distributed environment which has occupied the center stage in modern-day service computing; allowing permissive resource provisioning with minimalistic conflict and enabling on-demand, pay-per-use benefits. Provisioning of resources in a dynamic environment, such that none are under-provisioned or over-provisioned is a primal challenge. The problem is analyzed, and an optimal resource allocation strategy is formulated by the quantitative analysis of a biologically inspired model called Lotka-Volterra.

CONCLUSION

From the analysis of the observations from this experiment, we can conclude that in a biological system (where our assumption of model constraints hold true), the predator and prey follow a periodic and oscillating relationship where the equilibrium is attained when the initial prey and predator populations are the same. We also observe that in the absence of predators, the prey grows exponentially while in the absence of preys, predator populations decrease exponentially to zero thus signifying the importance of the existence of both in a balanced biological system. This modeling of predator prey competition system enables us to explore many such real life instances like stock markets and computing where the same behavior plays out thus helping us to predict the future outputs in advance given the initial parameters.

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