

Assignment 3 : Manual Calculations

- Stochastic Gradient Descent

Data:

x	y
0.2	3.4
0.4	3.8

Step 1: $[x, y]$, epochs = 2, $\eta = 0.1$, $m = 1$, $c = -1$,

Step 2: iter = 1

Step 3: sample = 1

$$\begin{aligned}\text{Step 4: error } E &= \frac{1}{2} \times (1 - \cancel{(0.2)}) \times 3.4 - (1 \times 0.2 - 1)^2 \\ &= 0.5 \times (3.4 + 0.8)^2 \\ &= 8.82\end{aligned}$$

$$\begin{aligned}\frac{\partial E}{\partial m} &= -(y_i - mx_i - c)x_i = -(3.4 - (1)(0.2) - (-1))(0.2) \\ &= -(3.4 - 0.2 + 1)(0.2) = (4.2)(0.2) = 0.84\end{aligned}$$

$$\frac{\partial E}{\partial c} = -(y_i - mx_i - c) = -4.2$$

$$\text{Step 5: } \Delta m = -\eta \frac{\partial E}{\partial m} = -(0.1)(0.84) = 0.084$$

$$\Delta c = -\eta \frac{\partial E}{\partial c} = -(0.1)(-4.2) = 0.42$$

$$\text{Step 6: } m = m + \Delta m = 1 + 0.084 = 1.084$$

$$c = c + \Delta c = -1 + 0.42 = -0.58$$

$$\text{Step 7: } \text{sample} = \text{sample} + 1 = 1 + 1 = 2$$

Step 8: $\text{sample} < \text{total no of sample} \Rightarrow \text{True}$
go to next step 4

$$\text{eq 9: } y = (1.084)(0.4) - 0.58$$

$$y = -0.1464$$

$$E = (0.5) * (3.8 + 0.1464)^2 = 7.79$$

$$\begin{aligned} \frac{\partial E}{\partial m} &= -(y_i - mx_i - c) x_i \\ &= -(3.8 - (1.084)(0.4) + 0.58) 0.4 \\ &= -(3.8 + 0.1464) * 0.4 = -1.58 \end{aligned}$$

$$\frac{\partial E}{\partial c} = -(y_i - mx_i - c) = -3.94$$

$$\text{step 10: } \Delta m = -\eta \frac{\partial E}{\partial m} = -(0.1)(-1.58) = 0.158$$

$$\Delta c = -\eta \frac{\partial E}{\partial c} = -(0.1)(-3.94) = 0.394$$

$$\text{step 11: } m = m + \Delta m = 1.084 + 0.158 = 1.242$$

$$c = c + \Delta c = -0.58 + 0.394 = -0.186$$

$$\text{step 12: } \text{sample} = 2 + 1 = 3$$

$$\text{step 13: } \text{sample} = 3 > \text{no of samples}$$

go to next step

$$\text{step 14: } \text{iter} = \text{iter} + 1 = 1 + 1 = 2$$

$$\text{step 15: } \text{iter} < \text{epochs}$$

goto step 3

Step 16: sample = 1

Step 17: $y = (1.242)(0.2) + (-0.186) = 0.0624$

$E = \frac{1}{2} (3.4 - 0.0624) = 1.6688$

$$\frac{\partial E}{\partial m} = -(3.4 - 0.0624)0.2 = -0.66752$$

$$\frac{\partial E}{\partial c} = -3.3376$$

Step 18: $\Delta m = -\eta \left(\frac{\partial E}{\partial m} \right) = -(0.1)(-0.66752) = 0.066752$

$$\Delta c = -(0.1)(-3.3376) = 0.33376$$

Step 19: $m = m + \Delta m = 1.242 + 0.066752 = 1.90952$

$$c = c + \Delta c = -0.186 + 0.33376 = 0.14776$$

Step 20: sample = 1 + 1 = 2

Step 21: sample < no of samples
goto step 4

Step 22: $\frac{\partial E}{\partial m} = -(3.8 - ((1.90952)(0.4) - (0.14776)))(0.4)$
 $= -(2.888432)(0.4) = -1.155372$

$$\frac{\partial E}{\partial c} = -2.888432$$

Step 23: $\Delta m = -\eta \left(\frac{\partial E}{\partial m} \right) = +0.1155372$

$$\Delta c = +0.2888432$$

Step 24: $m = m + \Delta m$
 $= 2.025057$

$C = C + \Delta C = 0.4366032$

Step 25: ~~iter~~ sample = $2 + 1 = 3$

Step 26: sample > no of sample
 goto next step

Step 27: iter = iter + 1 = $2 + 1 = 3$

Step 28: iter > epoch
 goto step 29

Step 29: Print m, C
 $\Rightarrow m = 2.025057$
 $C = 0.4366032$

Step 30: Compute mse.

$$= \frac{(3.4 - 0.841614) + (3.8 - 1.246626)}{2}$$

$$= \frac{(2.558386) + (2.553374)}{2}$$

mse = 2.556063