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2ND EDITION

HOW TO PREPARE FOR

# Quantitative Aptitude for the CAT

COMMON ADMISSION TEST

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Tata McGraw-Hill

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# CONTENTS

# BLOCK 1

## CHAPTERS

- **Number Systems**
- **Progressions**

### INTRODUCTION

*Block One constitutes the most important part of the entire portion for the preparation of QA for all competitive examinations. In the context of the CAT, Block One has always had a weightage of between 12–15 marks. Seen from the perspective of the fact that the cut offs required to clear the QA section of the CAT paper have always been in the range of 12–15 marks out of 50 marks, the importance of this section can be clearly gauged.*

*It is due to this reason that this block of chapters assumes a high level of importance in your preparation for the CAT.*

*Hence, you are advised to go through this block in great depth and try to create complete concept clarity as well as question exposure for developing your ability to do well at this area.*

*We would first want you to go through the sample pre-assessment test based exclusively on questions taken from this block of chapters in order to gauge your starting level.*

***Note:*** *There is no time limit that you need to follow while you are solving this test. The objective of this test is to gauge your starting knowledge level. Hence, try to think through each question for as long as you want/need to. Try to solve the questions in this test to the best of your ability.*

*At the end of the test, we have provided a Score interpretation tool. Use the score interpretation tool in order to define your objectives while you study and go through the individual chapters of this block.*

# ...BACK TO SCHOOL

- **Chapters in this block: Number Systems and Progressions**
- **Block Importance—12–15 marks out of 50.**

As already emphasised in the preface, this block is one of the most crucial areas, when you come to think of CAT preparations. In fact, seen in the context of the fact that the sectional qualifying score in QA is just around 12–15 marks out of 50, the real importance comes out.

In fact looking at the CAT papers over the last few years alone will give you a fair idea of the importance of this block in the context of the CAT.

The following table will make things clearer for you:

| *Year* | *Number of marks from Block I* | *Estimated qualifying score* |
|---|---|---|
| CAT 2005 | 18 marks | 13 – 14 |
| CAT 2004 | 14 marks | 12 – 14 |

As you can easily see from the table above, this block alone can get you to the level of qualifying score required to clear the all important QA section for the CAT.

Hence, understanding the concepts involved in these chapters properly and creating depth in your problem solving experience might go a long way towards helping you clear the CAT.

Before we move into the individual chapters of this block, let us first organise our thinking by looking at the core concepts that we had learnt about during our school.

**PREASSESSMENT TEST**

This test consists of 25 questions based on the chapters of BLOCK ONE (Number Systems and Progressions). Do your best in trying to solve each question.

The time limit to be followed for this test is 30 minutes. However, after the 30 minutes is over continue solving till you have spent enough time and paid sufficient attention to each question. After you finish thinking about each and every question of the test, check your scores. Then go through the SCORE INTERPRETATION ALGORITHM given at the end of the test to understand the way in which you need to approach the chapters inside this block.

1. The number of integers $n$ satisfying $-n+2 \geq 0$ and $2n \geq 4$ is
   (a) 0 (b) 1
   (c) 2 (d) 3
2. The sum of two integers is 10 and the sum of their reciprocals is 5/12. Then the larger of these integers is
   (a) 2 (b) 4
   (c) 6 (d) 8
3. If $x$ is a positive integer such that $2x + 12$ is perfectly divisible by $x$, then the number of possible values of $x$ is
   (a) 2 (b) 5
   (c) 6 (d) 12
4. Let K be a positive integer such that $k+4$ is divisible by 7. Then the smallest positive integer $n$, greater than 2, such that $k+2n$ is divisible by 7 equals.
   (a) 9 (b) 7
   (c) 5 (d) 3
5. $2^{73} - 2^{72} - 2^{71}$ is the same as
   (a) $2^{69}$ (b) $2^{70}$
   (c) $2^{71}$ (d) $2^{72}$
6. Three times the first of three consecutive odd integers is 3 more than twice the third. What is the third integer?
   (a) 15 (b) 9
   (c) 11 (d) 5
7. $x$, $y$ and $z$ are three positive integers such that $x > y > z$. Which of the following is closest to the product $xyz$?
   (a) $(x-y)xy$ (b) $x(y-1)z$
   (c) $xy(z-1)$ (d) $x(y+1)z$
8. A positive integer is said to be a prime number if it is not divisible by any positive integer other than itself and 1. Let $p$ be a prime number greater than 5, then $(p^2-1)$ is
   (a) never divisible by 6.
   (b) always divisible by 6, and may or may not be divisible by 12.
   (c) always divisible by 12, and may or may not be divisible by 24.
   (d) always divisible by 24.
9. Iqbal dealt some cards to Mushtaq and himself from a full pack of playing cards and laid the rest aside. Iqbal then said to Mushtaq "If you give me a certain number of your cards, I will have four times as many cards as you will have. If I give you the same number of cards, I will have thrice as many cards as you will have". Of the given choices, which could represent the number of cards with Iqbal?
   (a) 9 (b) 31
   (c) 12 (d) 35
10. In Sivkasi, each boy's quota of match sticks to fill into boxes is not more than 200 per session. If he reduces the number of sticks per box by 25, he can fill 3 more boxes with the total number of sticks assigned to him. Which of the following is the possible number of sticks assigned to each boy?
    (a) 200 (b) 150
    (c) 125 (d) 175
11. Alord got an order from a garment manufacturer for 480 Denim Shirts. He bought 12 sewing machines and appointed some expert tailors to do the job. However, many didn't report for duty. As a result, each of those who did, had to stitch 32 more shirts than originally planned by Alord, with equal distribution of work. How many tailors had been appointed earlier and how many had not reported for work?
    (a) 12, 4 (b) 10, 3
    (c) 10, 4 (d) None of these
12. How many 3 - digit even numbers can you form such that if one of the digits is 5, the following digit must be 7?
    (a) 5 (b) 405
    (c) 365 (d) 495
13. To decide whether a number of $n$ digits is divisible by 7, we can define a process by which its magnitude is reduced as follows: ($i_1, i_2, i_3, \ldots i_n$ are the digits of the number, starting from the most significant digit).

$$i_1 i_2 \ldots i_n \Rightarrow i_1 . 3^{n-1} + i_2\, 3^{n-2} + \ldots + I_n 3^0.$$

e.g. $259 \Rightarrow 2.3^2 + 5.3^1 + 9.3^0 = 18 + 15 + 9 = 42$

Ultimately the resulting number will be seven after repeating the above process a certain number of times.

After how many such stages, does the number 203 reduce to 7?

(a) 2 (b) 3
(c) 4 (d) 1

14. A third standard teacher gave a simple multiplication exercise to the kids. But one kid reversed the digits of both the numbers and carried out the multiplication and found that the product was exactly the same as the one expected by the teacher. Only one of the following pairs of numbers will fit in the description of the exercise. Which one is that?

(a) 14, 22 (b) 13, 62
(c) 19, 33 (d) 42, 28

15. If 8 + 12 = 2, 7 + 14 = 3 then 10 + 18 =?

(a) 10 (b) 4
(c) 6 (d) 18

16. Find the minimum integral value of $n$ such that the division $55n/124$ leaves no remainder.

(a) 124 (b) 123
(c) 31 (d) 62

17. What is the value of $k$ for which the following system of equations has no solution:

$2x - 8y = 3$; and $kx + 4y = 10$.

(a) $-2$ (b) 1
(c) $-1$ (d) 2

18. A positive integer is said to be a prime if it is not divisible by any positive integer other than itself and one. Let $p$ be a prime number strictly greater than 3. Then, when $p^2 + 17$ is divided by 12, the remainder is

(a) 6 (b) 1
(c) 0 (d) 8

19. A man sells chocolates that come in boxes. Either full boxes or half a box of chocolates can be bought from him. A customer comes and buys half the number of boxes the seller has plus half a box. A second customer comes and buys half the remaining number of boxes plus half a box. After this, the seller is left with no chocolates box. How many chocolates boxes did the seller have before the first customer came?

(a) 2 (b) 3
(c) 4 (d) 3.5

20. X and Y are playing a game. There are eleven 50 paise coins on the table and each player must pick up at least one coin but not more than five. The person picking up the last coin loses. X starts. How many should he pick up at the start to ensure a win no matter what strategy Y employs?

(a) 4 (b) 3
(c) 2 (d) 5

21. If $a < b$, which of the following is always true?

(a) $a < (a + b) / 2 < b$
(b) $a < ab/2 < b$
(c) $a < b^2 - a^2 < b$
(d) $a < ab < b$

22. The money order commission is calculated as follows. From Rs. X to be sent by money order, subtract 0.01 and divide by 10. Get the quotient and add 1 to it, if the result is Y, the money order commission is Rs. 0.5Y. If a person sends two money orders to Aurangabad and Bhatinda for Rs. 71 and Rs. 48 respectively, the total commission will be

(a) Rs. 7.00 (b) Rs. 6.50
(c) Rs. 6.00 (d) Rs. 7.50

23. The auto fare in Ahmedabad has the following formula based upon the meter reading. The meter reading is rounded up to the next higher multiple of 4. For instance, if the meter reading is 37 paise, it is rounded up to 40 paise. The resultant is multiplied by 12. The final result is rounded off to nearest multiple of 25 paise. If 53 paise is the meter reading what will be the actual fare?

(a) Rs. 6.75 (b) Rs. 6.50
(c) Rs. 6.25 (d) Rs. 7.50

24. Juhi and Bhagyashree were playing simple mathematical puzzles. Juhi wrote a two digit number and asked Bhayashree to guess it. Juhi also indicated that the number is exactly thrice the product of its digits. What was the number that Juhi wrote?

(a) 36 (b) 24
(c) 12 (d) 48

25. It is desired to extract the maximum power of 3 from 24!, where $n! = n.(n - 1) . (n - 2) \ldots 3.2.1$. What will be the exponent of 3?

(a) 8 (b) 9
(c) 11 (d) 10

**Answers (Block 1 Preassessment Test)**

| | | | | |
|---|---|---|---|---|
| 1. (b) | 2. (c) | 3. (c) | 4. (a) | 5. (c) |
| 6. (a) | 7. (c) | 8. (d) | 9. (b) | 10. (b) |
| 11. (c) | 12. (a) | 13. (a) | 14. (b) | 15. (a) |
| 16. (a) | 17. (c) | 18. (a) | 19. (b) | 20. (b) |
| 21. (a) | 22. (b) | 23. (a) | 24. (b) | 25. (d) |

**Step Five:** Move to LOD 2 and repeat the process that you followed in LOD 1—first in the chapter of Number Systems, then with the Chapter on Progressions. Concentrate on understanding each and every question and its underlying concept.

**Step Six:** Go to the second review test given at the end of the block and solve it. Again, while doing so measure your score within the provided time limit first and then continue to solve the test further without a time limit and try to evaluate the improvement that you have had in your score.

**Step Seven:** Move to LOD 3 only after you have solved and understood each of the questions in LOD 1 and LOD 2. Repeat the process that you followed in LOD 1 – first in the chapter of Number Systems, then with the Chapter on Progressions.

### If You Scored: 7–15 (In Unlimited Time)

Although you are better than the person following the instructions above, obviously there is a lot of scope for the development of your score. You will need to work both on your concepts as well as speed. Initially emphasize more on the concept development aspect of your preparations, then move your emphasis onto speed development. The following process is recommended for you:

**Step One:** Go through the **block one Back to School Section** carefully. Revise each of the concepts explained in that part. Going through your 8th, 9th and 10th standard books will be an optional exercise for you. It will be recommended in case you scored in single digits, while if your score is in two digits, I leave the choice to you.

**Step Two:** Move into the first chapter of the block. Viz Number Systems. When you do so, concentrate on clearly understanding each of the concepts explained in the chapter theory.

Then move onto the LOD 1 exercises. These exercises will provide you with the first level of challenge. Try to solve each and every question provided under LOD 1 of Number Systems. Once you finish solving LOD 1, revise the questions and their solution processes.

**Step Three:** After finishing LOD 1 of number systems, move into Chapter 2 of this block, ( Progressions) and repeat the process, viz: Chapter theory comprehensively followed by solving LOD 1 questions.

**Step Four:** Go to the first review test given at the end of the block and solve it. While doing so, first look at the score you get within the time limit mentioned. Then continue to solve the test further without a time limit and try to evaluate the improvement in your score.

**Step Five:** Move to LOD 2 and repeat the process that you followed in LOD 1—first in the chapter on Number Systems, then with the Chapter on Progressions.

**Step Six:** Go to the second review test given at the end of the block and solve it. Again while doing so measure your score within the provided time limit first and then continue to solve the test further without a time limit and try to evaluate the improvement that you have had in your score.

*In case the growth in your score is not significant, go back to the theory of both the chapters and re-solve LOD 1 and LOD 2 of both the chapters. While doing so concentrate more on the LOD 2 questions.*

**Step Seven:** Move to LOD 3 and repeat the process that you followed in LOD 1—first in the chapter on Number Systems, then with the Chapter on Progressions.

### If You Scored 15+ (In Unlimited Time)

Obviously you are much better than the first two categories of students. Hence unlike them, your focus should be on developing your speed by picking up the shorter processes explained in this book. Besides, you might also need to pick up concepts that might be hazy in your mind. The following process of development is recommended for you:

**Step One:** Quickly review the concepts given in the block one Back to School Section. Only go deeper into a concept in case you find it new. Going back to school level books is not required for you.

**Step Two:** Move into the first chapter of the block: Number Systems. Go through the theory explained there carefully. Concentrate specifically on clearly understanding the concepts which are new to you. Work out the short cuts and in fact try to expand your thinking by trying to think of alternative (and expanded) lines of questioning with respect to the concept you are studying

## CORE CONCEPTS

I. The concept of the number line is one of the most crucial concepts in Quantitative Aptitude.

The number line is a line that starts from zero and goes towards positive infinity when it moves to the right and towards negative infinity when it moves to the left.


The difference between the values of any two points on the number line also gives the distance between the points.

Thus, for example if we look at the distance between the points + 3 and – 2 it will be given by their difference. $3-(-2)=3+2=5$.

II. Types of numbers –

We will be looking at the types of numbers in details, again when we go into the chapter of number systems. Let us first work out in our minds the various types of numbers. While doing so do not fail to notice that most of these number types occur in pairs (i.e. the definition of one of them, defines the other automatically).

**Natural Numbers and Whole Numbers:** Natural numbers, also called counting numbers, are the first numbers we learnt. They are the number set 1, 2, 3, 4 ……. ∞.

On the other hand, whole numbers is the set of natural numbers plus, the number zero.

Thus, 0, 1, 2, 3 ……..∞ is the set of whole number.

**Integers and Decimals:** All numbers that do not have a decimal in them are called integers. Thus, –3, –17, + 4, + 13, + 1473, 0 etc are all integers.

Obviously, decimal numbers are number which have a decimal value attached to them. Thus, 1.3, 14.76, – 12.24 etc are all decimal numbers, since they have certain values after the decimal point.

Before we move ahead, let us pause a brief while, to further understand decimals. As you shall see, the concept of decimals is closely related to the concept of division and divisibility. Suppose, I have 4 pieces of bread which I want to divide equally between two people. It is easy for me to do this, since I can give two whole pieces to each of them.

However, if we alter the situation in such a way, that I now have 5 pieces of bread to distribute equally amongst 2 people. What do I do?

I give two whole pieces each, to each of them. The 5th piece has to be divided equally between the two. I can no longer do this, without in some way breaking the 5th piece into 2 parts. This is the elementary situation that gives rise to the need for decimals in mathematics.

Going back to the situation above, my only option is to divide the 5th piece into two equal parts (which in quants are called as halves).

This concept has huge implications for problem solving especially once you recognise that a half (i.e. a .5 in the decimal) only comes when you divide a whole into two parts.

Thus, in fact, all standards decimals emerge out of certain fixed divisors.

Hence, for example, the divisor 2, gives rise to the decimal ·5.

Similarly the divisor 3 gives rise to the decimals ·33333 and ·66666 etc.

**Prime numbers and Composite numbers** Amongst natural numbers, there are three broad divisions–

***Unity*** It is representative of the number 1.

***Prime numbers*** These are numbers which have no divisors/ factors apart from 1 and itself.

***Composite numbers*** On the other hand, are numbers, which have at least one more divisor apart from 1 and itself.

**Note:** A brief word about factors/ division – A number X is said to divide Y (or is said to be a divisor or factor of Y). When the division of Y/X leaves no remainder.

All composite numbers have the property that they can be written as a product of their prime factors.

Thus, for instance, the number 40 can be represented as: $40 = 2 \times 2 \times 2 \times 5$ or $40 = 2^3 \times 5^1$

This form of writing is called as the **standard form** of the composite number.

**The difference between Rational and Irrational numbers:** This difference is one of the critical but unfortunately one of the less well understood differences in elementary Mathematics.

**The definition of rational numbers:** Numbers which can be expressed in the form $p/q$ where $q \neq 0$ are called rational numbers.

Obviously, numbers which cannot be represented in the form $p/q$ are called as irrational numbers.

However, one of the less well understood issues in this regard is what does this mean?

The difference becomes clear when the values of decimals are examined in details:

Consider the following numbers

(1) 4.2,

(2) 4.333….,

(3) 4.1472576345…….

What is the difference between the decimal values of the three numbers above?

To put it simply, the first number has what can be described as a finite decimal value. Such numbers can be expressed in the form *p/q* easily. Since 4.2 can be first written as 42/10 and then converted to 21/5.

Similarly, number like 4.5732 can be represented as 45732/10000. Thus, numbers having a finite terminating decimal value are rational.

Now, let us consider the decimal value: 4.3333……..

Such decimal values will continue endlessly i.e. they have no end. Hence, they are called **infinite decimals** (or non-terminating decimals).

But, we can easily see that the number 4.333.... can be represented as 13/3. Hence, this number is also rational. In fact, all numbers which have infinite decimal values, but have any recurring form within them can be represented in the *p/q* form.

For example the value of the number: 1.14814814814.... is 93/81.

(What I mean to say is that whenever you have any recurring decimal number, even if the value of '*q*' might not be obvious, but it will always exist.)

Thus, we can conclude that all numbers whose decimal values are infinite (non-terminating) but which have a recurring pattern within them are rational numbers.

This leaves us with the third kind of decimal values, viz. **Infinite non-recurring decimal values.** These decimals neither have a recurring pattern, nor do they have an end—they go on endlessly. For such numbers it is not possible to find the value of a denominator '*q*' which can be used in order to represent them as *p/q*. Hence, such numbers are called as irrational numbers.

In day to day mathematics, we come across numbers like $\sqrt{3}$, $\sqrt{5}$, $3\sqrt{7}$, $\pi$, $e$, etc. which are irrational numbers since they do not have a *p/q* representation.

[**Note:** $\sqrt{3}$ can also be represented as $3^{1/2}$, just as $3\sqrt{7}$ can be represented as $7^{1/3}$.]

## An important Tip:

**Rational and Irrational numbers do not mix.:** This means that in case you get a situation where an irrational number has appeared while solving a question, it will remain till the end of the solution. It can only be removed from the solution if it is multiplied or divided by the same irrational number.

**Consider an example:** The area of an equilateral triangle is given by the formula $(\sqrt{3}/4) \times a^2$ (where $a$ is the side of the equilateral triangle). Since, $\sqrt{3}$ is an irrational number, it remains in the answer till the end. Hence, the area of an equilateral triangle will always have a $\sqrt{3}$ as part of the answer.

Before we move ahead we need to understand one final thing about recurring decimals.

As I have already mentioned, recurring decimals have the property of being able to be represented in the *p/q* form. The question that arises is—Is there any process to convert a recurring decimal into a proper fraction?

Yes, there is. In fact, in order to understand how this operates, you first need to understand that there are two kinds of recurring decimals. The process for converting an infinite recurring decimal into a fraction basically varies for both of these types. Let's look at these one by one.

**Type 1–Pure recurring decimals:** These are recurring decimals where the recurrence starts immediately after the decimal point.

For example : $0.5555\ldots = 0.\overline{5}$

$3.242424\ldots = 3.\overline{24}$

$5.362362\ldots = 5.\overline{362}$

The process for converting these decimals to fractions can be illustrated as:

$0.5555 = 5/9$

$3.242424 = 3 + (24/99)$

$5.362362 = 5 + (362/999)$

A little bit of introspection will tell you that what we have done is nothing but to put down the recurring part of the decimal as it is and dividing it by a group of 9's. Also the number of 9's in this group equals the number of digits in the recurring part of the decimal.

Thus, in the second case, the fraction is derived by dividing 24 by 99. (24 being the recurring part of the decimal and 99 having 2 nines because the number of digits in 24 is 2.)

$$\text{Similarly, } 0.43576254357625\ldots = \frac{4357625}{9999999}$$

**Type 2–Impure recurring decimals:** Unlike pure recurring decimals, in these decimals, the recurrence occurs after a certain number of digits in the decimal. The process to convert these into a fraction is also best illustrated by an example:

Consider the decimal 0.435424242

$= 0.435\overline{42}$

The fractional value of the same will be given by: (43542 – 435)/99000. This can be understood in two steps.

*Step 1*: Subtract the non-recurring initial part of the decimal (in this case, it is 435) from the number formed by writing down the starting digits of the decimal value upto the digit where the recurring decimals are written for the first time;

Expanding the meaning—

**Note**: For 0.435424242, subtract 435 from 43542

*Step 2*: The number thus obtained, has to be divided by a number formed as follows; Write down as many 9's as the number of digits in the recurring part of the decimal. (in this case, since the recurring part '42' has 2 digits, we write down 2 9's.) These nines have to be followed by as many zeroes as the number of digits in the non recurring part of the decimal value. (In this case, the non recurring part of the decimal value is '435'. Since, 435 has 3 digits, attach three zeroes to the two nines to get the number to divide the result of the first step.)

Hence divide 43542 – 435 by 99000 to get the fraction.

Similarly, for 3.436213213 we get $\frac{436213-436}{999000}$.

Let us now move onto our next topic –

**Tables and their visualization:** Imagine the number line and a frog sitting at point zero of the number line. Let us say that the frog always jumps an equal distance (say 2 units.)

Imagine that the frog sitting at the origin (point 0) of the number line starts jumping to its right through equidistant jumps of exactly 2 units. It will first land on the point represented by the number 2 on the number line. It's next jump will make it land on the number 4, then 6, then 8 and so on.

This is how you should visualise the table of any number.

Thus, a frog starting from 0 and jumping 7 units to the right will land on 7, then 14, 21, 28 and so forth. This frog's jumps represent the table of the number 7.

*The meaning of 2n and 2n + 1*: $2n$ means a number which is a multiple of the number 2. Since, this can be visualised as a frog starting from the origin and jumping 2 units to the right in every jump, you can also say that this frog represents $2n$.

(**Note**: Multiples of 2, are even numbers. Hence, $2n$ is also used to denote even numbers.)

So, what does $2n + 1$ mean?

Well, simply put, if you place the above frog on the point represented by the number 1 on the number line then the frog will reach points such as 3, 5, 7, 9, 11 .....and so on. This essentially means that the points the frog now reaches are displaced by 1 unit to the right of the $2n$ frog. In mathematical terms, this is represented as $2n + 1$.

In other words, $2n + 1$ also represents numbers which leave a remainder of 1, when divided by 2. (**Note**: This is also the definition of an odd number. Hence, in Mathematics $(2n + 1)$ is used to denote an odd number. Also note that taken together $2n$ and $2n + 1$ denote the entire set of integers. i.e. all integers from $-\infty$ to $+\infty$ on the number line can be denoted by either $2n$ or $2n + 1$. This happens because when we divide any integer by 2, there are only two results possible with respect to the remainder obtained, viz: A remainder of zero ($2n$) or a remainder of one ($2n + 1$).

This concept can be expanded to represent integers with respect to any number. Thus, in terms of 3, we can only have three types of integers $3n$, $3n + 1$ or $3n + 2$ (depending on whether the integer leaves a remainder 0, 1 or 2 respectively when divided by 3.) Similarly, with respect to 4, we have 4 possibilities – $4n$, $4n + 1$, $4n + 2$ or $4n + 3$.

This form of representation of integers is extremely crucial in logically solving QA.

*Rules of Indices*: Indices means the power on a number. Many mathematical situations require us to be able to use the rules of indices. Hence, understanding these rules and their appropriate use might go a long way towards helping you in developing your skills in Quants.

The following rules apply for indices.

(1) $a^m \times a^n = a^{(m+n)}$. Thus, $2^3 \times 2^5 = 2^8$

(2) $a^m/a^n = a^{m-n}$. Thus $2^5/2^2 = 2^3$

(3) $a^m = 1/a^{-m}$ or $a^{-m} = 1/a^m$. Thus, $3^{-4} = 1/3^4$

(4) $(a^b)^c = a^{bc}$. Thus, $(5^2)^4 = 5^8$

(5) $a^0 = 1$ for all values of a. Thus, $7^0 = 1$

Besides, the following principles apply for indices:

(1) If $a^m = n$, then $a = n^{1/m}$

(2) $a^n/b^n = (a/b)^n$ or vice versa

(3) $a^{b^c} \neq a^{bc}$. Thus $2^{3^4} = 2^{81}$ and not $2^{12}$.

***Squares and square roots*** When any number is multiplied by itself, it is called as the square of the number.

Thus, $3 \times 3 = 3^2 = 9$

Squares have a very important role to play in mathematics. In the context of preparing for CAT and other Management exams, it might be a good idea to be able to recollect the squares of 2 digit numbers.

Let us now go through the following Table 1.1 carefully:

**Table 1.1**

| *Number* | *Square* | *Number* | *Square* | *Number* | *Square* |
|---|---|---|---|---|---|
| 1 | 1 | 35 | 1225 | 69 | 4761 |
| 2 | 4 | 36 | 1296 | 70 | 4900 |
| 3 | 9 | 37 | 1369 | 71 | 5041 |
| 4 | 16 | 38 | 1444 | 72 | 5184 |
| 5 | 25 | 39 | 1521 | 73 | 5329 |
| 6 | 36 | 40 | 1600 | 74 | 5476 |
| 7 | 49 | 41 | 1681 | 75 | 5625 |
| 8 | 64 | 42 | 1764 | 76 | 5776 |
| 9 | 81 | 43 | 1849 | 77 | 5929 |
| 10 | 100 | 44 | 1936 | 78 | 6084 |
| 11 | 121 | 45 | 2025 | 79 | 6241 |
| 12 | 144 | 46 | 2116 | 80 | 6400 |
| 13 | 169 | 47 | 2209 | 81 | 6561 |
| 14 | 196 | 48 | 2304 | 82 | 6724 |
| 15 | 225 | 49 | 2401 | 83 | 6889 |
| 16 | 256 | 50 | 2500 | 84 | 7056 |
| 17 | 289 | 51 | 2601 | 85 | 7225 |
| 18 | 324 | 52 | 2704 | 86 | 7396 |
| 19 | 361 | 53 | 2809 | 87 | 7561 |
| 20 | 400 | 54 | 2916 | 88 | 7744 |
| 21 | 441 | 55 | 3025 | 89 | 7921 |
| 22 | 484 | 56 | 3136 | 90 | 8100 |
| 23 | 529 | 57 | 3249 | 91 | 8281 |
| 24 | 576 | 58 | 3364 | 92 | 8464 |
| 25 | 625 | 59 | 3481 | 93 | 8649 |
| 26 | 676 | 60 | 3600 | 94 | 8836 |
| 27 | 729 | 61 | 3721 | 95 | 9025 |
| 28 | 784 | 62 | 3844 | 96 | 9216 |
| 29 | 841 | 63 | 3969 | 97 | 9409 |
| 30 | 900 | 64 | 4096 | 98 | 9604 |
| 31 | 961 | 65 | 4225 | 99 | 9801 |
| 32 | 1024 | 66 | 4356 | 100 | 10000 |
| 33 | 1089 | 67 | 4489 | | |
| 34 | 1156 | 68 | 4624 | | |

So, how does one get these numbers onto one's finger tips? Does one memorize these values or is there a simpler way?

Yes indeed! There is a very convenient process when it comes to memorising the squares of the first 100 numbers.

First of all, you are expected to memorise the squares of the first 30 numbers. In my experience, I have normally seen that most students already know this. The problem arises with numbers after 30. You do not need to worry about that. Just follow the following processes and you'll know all squares upto 100.

*Trick 1*: For squares from 51 to 80 – (**Note**: This method depends on your memory of the first thirty squares.)

The process is best explained through an example.

Suppose, you have to get an answer for the value of $67^2$. Look at 67 as $(50 + 17)$. The 4 digit answer will have two parts as follows:

$\underbrace{\_\_\ \_\_}_{\text{First 2 digits}} \qquad \underbrace{\_\_\ \_\_}_{\text{Last 2 digits}}$

The last two digits will be the same as the last two digits of the square of the number 17. (The value 17 is derived by looking at the difference of 67 with respect to 50.)

Since, $17^2 = 289$, you can say that the last two digits of $67^2$ will be 89. (i.e. the last 2 digits of 289.) Also, you will need to carry over the '2' in the hundreds place of 289 to the first part of the number.

The first two digits of the answer will be got by adding 17 (which is got from $67 - 50$) and adding the carry over (2 in this case) to the number 25. (Standard number to be used in all cases.) Hence, the first two digits of the answer will be given by $25 + 17 + 2 = 44$.

Hence, the answer is $67^2 \quad = 4489$.

Similarly, suppose you have to find $76^2$.

*Step 1*: $76 = 50 + 26$.

*Step 2*: $26^2$ is 676. Hence, the last 2 digits of the answer will be 76 and we will carry over 6.

*Step 3*: The first two digits of the answer will be $25 + 26 + 6 = 57$.

Hence, the answer is 5776.

This technique will take care of squares from 51 to 80 (if you remember the squares from 1 to 30). You are advised to use this process and see the answers for yourself.

**Squares for Numbers from 31 to 50** Such numbers can be treated in the form $(50 - x)$ and the above process modified to get the values of squares from 31 to 50. Again, to explain we will use an example. Suppose you have to find the square of 41.

*Step 1*: Look at 41 as $(50 - 9)$.

Again, similar to what we did above, realise that the answer has two parts—the first two and the last two digits.

*Step 2*: The last two digits are got by the last two digits in the value of $(-9)^2 = 81$. Hence, 81 will represent the last two digits of $41^2$.

*Step 1*: Write down the number 7016 as a product of its Prime factors. $7016 = 2 \times 2 \times 2 \times 2 \times 21 \times 21$

$= 2^4 \times 21^2$

*Step 2*: The required square root is obtained by halving the values of the powers.

Hence, $\sqrt{7056} = 2^2 \times 21^1$

## CUBES AND CUBE ROOTS

When a number is multiplied with itself two times, we get the cube of the number.

Thus, $x \times x \times x = x^3$

**Method to find out the cubes of 2 digit numbers:** The answer has to consist of 4 parts, each of which has to be calculated separately.

The first part of the answer will be given by the cube of the ten's digit.

Suppose you have to find the cube of 28.

The first step is to find the cube of 2 and write it down.

$2^3 = 8.$

The next three parts of the number will be derived as follows.

Derive the values 32, 128 and 512.

(by creating a G. P. of 4 terms with the first term in this case as 8, and a common ratio got by calculating the ratio of the unit's digit of the number with its tens digit. In this case the ratio is 8/2 = 4.)

Now, write the 4 terms in a straight line as below. Also, to the middle two terms add double the value.

|   |   |   |   |   |
|---|---|---|---|---|
|   | 8 | 32 | 128 | 512 |
| + |   | 64 | 256 |   |
|   | 21 | 9 | 5 | 2 |

(carry over 51)

(8 + 13) (32 + 64 + 43 = 139) (128 + 256 + 51 = 435)

Carry over 13 (Carry over 43)

Hence, $28^3 = 21952$

### Properties of Cubes

1. When a perfect cube is written in its standard form the values of the powers on each prime factor will be a multiple of 3.
2. In order to find the cube root of a number, first write it in its standard form and then divide all powers by 3.

   Thus, the cube root of $3^6 \times 5^9 \times 17^3 \times 2^6$ is given by $3^2 \times 5^3 \times 17 \times 2^2$
3. The cubes of all numbers (integers and decimals) greater than 1 are greater than the number itself.
4. $0^3 = 0$, $1^3 = 1$ and $-1^3 = -1$. These are the only three instances where the cube of the number is equal to the number itself.
5. The value of the cubes of a number between 0 and 1 is lower than the number itself. Thus, $0.5^3 < 0.5^2 < 0.5$.
6. The cube of a number between 0 and – 1 is greater than the number itself. $(-0.2)^3 > -0.2$.
7. The cube of any number less than –1, is always lower than the number. Thus, $(-1.5^3 < (-1.5)$.

**The BODMAS Rule:** It is used for the ordering of mathematical operations in a mathematical situation:

In any mathematical situation, the first thing to be considered is Brackets followed by Division, Multiplication, Addition and Subtraction in that order.

Thus $3 \times 5 - 2 = 15 - 2 = 13$

Also, $3 \times 5 - 6 \div 3 = 15 - 2 = 13$

Also, $3 \times (5 - 6) + 3 = 3 \times (-1) + 3 = -1.$

*Operations on Odd and Even numbers*

*ODDS*

| | |
|---|---|
| Odd × odd | = Odd |
| Odd + odd | = Even |
| Odd – odd | = Even |
| Odd ÷ odd | = odd |

*EVENS*

| |
|---|
| Even × Even = Even |
| Even + Even = Even |
| Even – even = Even |
| Even ÷ even = Even or odd |

*ODDS & EVENS*

| | |
|---|---|
| Odd × Even | = Even |
| Odd + Even | = Odd |
| Odd – Even | = Odd |
| Even ÷ odd | = Even |

Odd ÷ Even → Not divisible

## SERIES OF NUMBERS

In many instances in Mathematics we are presented with a series of numbers formed simply when a group of numbers is written together. The following are examples of series:

(1) 3, 5, 8, 12, 17…

(2) 3, 7, 11, 15, 19…(Such series where the next term is derived by adding a certain fixed value to the previous number are called as Arithmetic Progressions).

# 1

# NUMBER SYSTEMS

## INTRODUCTION

The chapter of number systems is amongst the most important chapters in the whole of mathematics syllabus for the CAT examination. Students are advised to go through this chapter with utmost care understanding each and every question type on this topic. The CAT has consistently set between 10–15 marks based on the concepts of this chapter. Hence, going through this chapter and its concepts properly is very imperative for you. Seen in the context of the qualifying score in the CAT being in the range of 12–14 marks (for Q A), this leaves us with a fair idea of the critical importance of this chapter. It would be a good idea to first go through the basic definitions of all types of numbers (something I have found to be surprisingly very less known about). The student is also advised to go through the solutions of the various questions illustrated in this chapter. Besides, while solving this chapter, try to maximise your learning experience with every problem that you solve. To start off, the following pictorial representation of the types of numbers will help you improve year quality of comprehension of different types of numbers.

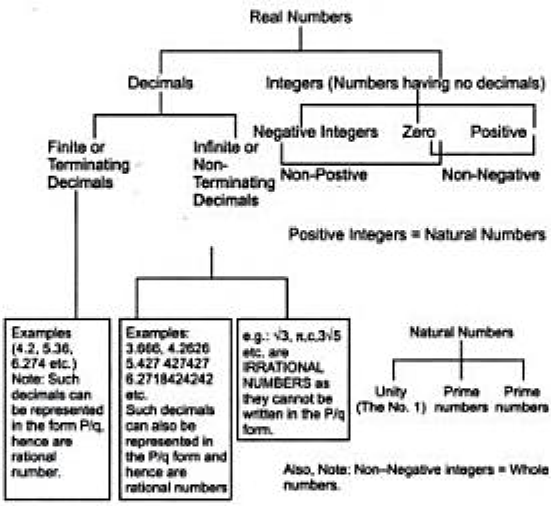
## DEFINITIONS

**Natural Numbers** These are the numbers (1, 2, 3 etc.) that are used for counting. In other words, all positive integers are natural numbers.

There are infinite natural numbers and the number 1 is the least natural number.

Examples of natural numbers: 1, 2, 4, 8, 32, 23, 4321 and so on.

The following numbers are examples of numbers that are not natural: –2, –31, 2.38, 0 and so on.




Based on divisibility, there could be two types of natural numbers: *Prime and Composite*.

**Prime Numbers** A natural number larger than unity is a prime number if it does not have other divisors except for itself and unity.

*Note:* Unity (i.e. 1) is not a prime number.

### Some Properties of Prime Numbers

- The lowest prime number is 2.
- 2 is also the only even prime number.
- The lowest odd prime number is 3.

- The remainder when a prime number $p \geq 5$ is divided by 6 is 1 or 5. However, if a number on being divided by 6 gives a remainder of 1 or 5 the number need not be prime.
- The remainder of the division of the square of a prime number $p \geq 5$ divided by 24 is 1.
- For prime numbers $p > 3$, $p^2 - 1$ is divisible by 24.
- Prime Numbers between 1 to 100 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.
- Prime Numbers between 100 to 200 are: 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199.
- If $a$ and $b$ are any two odd primes then $a^2 - b^2$ is composite. Also, $a^2 + b^2$ is composite.
- The remainder of the division of the square of a prime number $p \geq 5$ divided by 12 is 1.

## SHORT CUT PROCESS

### To Check Whether a Number is Prime or Not

To check whether a number $N$ is prime, adopt the following process.

(a) Take the square root of the number.

(b) Round of the square root to the immediately lower integer. Call this number $z$. For example if you have to check for 181, its square root will be 13.... Hence, the value of $z$, is this case will be 13.

(c) Check for divisibility of the number $N$ by all prime numbers below $z$. If there is no prime number below the value of $z$ which divides $N$ then the number $N$ will be prime.

To illustrate :-

The value of $\sqrt{239}$ lies between 15 to 16. Hence, take the value of $z$ as 16.

Prime numbers less than 16 are 2, 3, 5, 7, 11 and 13, 239 is not divisible by any of these. Hence you can conclude that 239 is a prime number.

### A Brief look into why this works?

Suppose you are asked to find the factors of the number 40.

An untrained mind will find the factors as : 1, 2, 4, 5, 8, 10, 20 and 40.

The same task will be performed by a trained mind as follows:

| | | | |
|---|---|---|---|
| | 1 | × | 40 |
| | 2 | × | 20 |
| | 4 | × | 10 |
| and | 5 | × | 8 |

i.e., The discovery of one factor will automatically yield the other factor. In other words, factors will appear in terms of what can be called as factor pairs. The locating of one factor, will automatically pinpoint the other one for you. Thus, in the example above, when you find 5 as a factor of 40, you will automatically get 8 too as a factor.

Now take a look again at the pairs in the example above. If you compare the values in each pair with the square root of 40 (i.e. 6.___) you will find that for each pair the number in the left column is lower than the square root of 40, while the number in the right column is higher than the square root of 40.

This is a property for all numbers and is always true.

Hence, we can now phrase this as: Whenever, you have to find the factors of any number $N$, you will get the factors in pairs (i.e. factor pairs). Further, the factor pairs will be such that in each pair of factors, one of the factors will be lower than the square root of $N$ while the other will be higher than the square root of $N$.

As a result of this fact one need not make any effort to find the factors of a number above the square root of the number. These come automatically. All you need to do is to find the factors below the square root of the number.

***Extending this logic, we can say that if we are not able to find a factor of a number upto the value of its square root, we will not be able to find any factor above the square root and the number under consideration will be a prime number.*** This is the reason why when we need to check whether a number is prime, we have to check for factors only below the square root.

But, we have said that you need to check for divisibility only with the prime numbers below (and including) the square root of the number. What logic will explain this:

Let us look at an example to understand why you need to look only at prime numbers below the square root.

Uptil now, we have deduced that in order to check whether a number is prime, we just need to do a factor search below (and including) the square root.

Thus, for example, in order to find whether 181 is a prime number, we need to check with the numbers = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and 13.

The first thing you will realise, when you first look at the list above is that all even numbers will get eliminated automatically (since no even number can divide an odd number and of course you will check a number for being prime only if it is odd!)

This will leave you with the numbers 3, 5, 7, 9, 11 and 13 to check 181.

**Why do we not need to check with composite numbers below the square root?** This will again be understood best if explained in the context of the example above. The only composite number in the list above is 9. You do not need to check with 9, because when you checked $N$ for divisibility with 3 you would get either of two cases:

**Case I: If N is divisible by 3:** In such a case, $N$ will automatically become non-prime and we can stop our checking. Hence, you will not need to check for the divisibility of the number by 9.

**Case II: N is not divisible by 3:** If $N$ is not divisible by 3, it is obvious that it will not be divisible by 9. Hence, you will not need to check for the divisibility of the number by 9.

Thus, in either case, checking for divisibility by a composite number (9 in this case) will become useless. This will be true for all composite numbers.

Hence, when we have to check whether a number $N$ is prime or not, we need to only check for its divisibility by prime factors below the square root of $N$.

**Integers** A set which consists of natural numbers, negative integers $(-1, -2, -3 \ldots -n \ldots)$ and zero is known as the set of integers. The numbers belonging to this set are known as integers.

**Composite Numbers** It is a natural number that has at least one divisor different from unity and itself.

Every composite number $n$ can be factored into its prime factors. (This is sometimes called the canonical form of a number.)

In mathematical terms: $n = p_1^m \cdot p_2^n \ldots p_k^t$, where $p_1, p_2 \ldots p_k$ are prime numbers called factors and $m, n \ldots k$ are natural numbers.

Thus, $24 = 2^3 \cdot 3$, $84 = 7 \cdot 3 \cdot 2^2$ etc.

This representation of a composite number is known as the standard form of a composite number. It is an extermely useful form of seeing a composite number as we shall see.

**Whole Numbers** The set of numbers that includes all natural numbers and the number zero are called whole numbers. Whole number are also called as Non-negative integers.

**The Concept of the Number Line** The number line is a straight line between negative infinity on the left to infinity to the right.


The distance between any two points on the number line is got by subtracting the lower value from the higher value. Alternately, we can also start with the lower number and find the required addition to reach the higher number.

For example: The distance between the points 7 and $-4$ will be $7 - (-4) = 11$.

**Real Numbers** All numbers that can be represented on the number line are called real numbers. Every real number can be approximately replaced with a terminating decimal.

The followning operations of addition, subtraction, multiplication and division are valid for both whole numbers and real numbers: [For any real or whole numbers $a$, $b$ and $c$].

(a) Commutative property of addition: $a + b = b + a$.
(b) Associative property of addition: $(a + b) + c = a + (b + c)$.
(c) Commutative property of multiplication: $a \cdot b = b \cdot a$.
(d) Associative property of multiplication: $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.
(e) Distributive property of multiplication with respect to addition: $(a + b)\, c = ac + bc$.
(f) Subtraction and division are defined as the inverse operations to addition and multiplication respectively.

Thus if $a + b = c$, then $c - b = a$ and if $q = a/b$ then $b \cdot q = a$ (where $b \neq 0$).

Division by zero is not possible since there is no number $q$ for which $b \cdot q$ equals $a$ non zero number $a$.

**Rational Numbers** A rational number is defined as number of the form $a/b$ where $a$ and $b$ are integers and $b \neq 0$.

The set of rational numbers encloses the set of integers and fractions. The rules given above for addition, subtraction, multiplication and division also apply on rational numbers.

Rational numbers that are not integral will have decimal values. These values can be of two types:

(a) **Terminating (or finite) decimal fractions:** For example, $17/4 = 4.25$, $21/5 = 4.2$ and so forth.

(b) **Non-terminating decimal fractions:** Amongst non-terminating decimal fractions there are two types of decimal values:

(i) *Non-terminating periodic fractions*: These are non-terminating decimal fractions of the type $x \cdot a_1a_2a_3a_4 \ldots a_na_1a_2a_3a_4 \ldots a_na_1a_2a_3a_4 \ldots a_n$. For example $\frac{16}{3}$ = 5.3333, 15.23232323, 14.287628762 876 … and so on.

(ii) *Non-terminating non-periodic fractions*: These are of the form $x \cdot b_1b_2b_3b_4 \ldots b_nc_1c_2c_3 \ldots c_n$. For example: 5.27316871437251 86....

Of the above categories, terminating decimal and non-terminating periodic decimal fractions belong to the set of rational numbers.

**Irrational Numbers** Fractions, that are non-terminating, non-periodic fractions, are irrational numbers.

Some examples of irrational numbers are $\sqrt{2}$, $\sqrt{3}$ etc. In other words, all square and cube roots of natural numbers that are not squares and cubes of natural numbers are irrational. Other irrational numbers include $\pi$, $e$ and so on.

Every positive irrational number has a negative irrational number corresponding to it.

All operations of addition, subtraction, multiplication and division applicable to rational numbers are also applicable to irrational numbers.

As briefly stated in the Back to school section, whenever an expression contains a rational and an irrational number together, the two have to be carried together till the end. In other words, an irrational number once it appears in the solution of a question will continue to appear till the end of the question. This concept is particularly useful in Geometry For example: If you are asked to find the ratio of the area of a circle to that of an equilateral triangle, you can expect to see a $\alpha/\sqrt{3}$ in the answer. This is because the area of a circle will always have a p component in it, while that of an equilateral triangle will always have $\sqrt{3}$.

You should realise that once an irrational number appears in the solution of a question, it can only disappear if it is multiplied or divided by the same irrational number.

## THE CONCEPT OF GCD (GREATEST COMMON DIVISOR OR HIGHEST COMMON FACTOR)

Consider two natural numbers $n_1$ and $n_2$.

If the numbers $n_1$ and $n_2$ are exactly divisible by the same number $x$, then $x$ is a common divisor of $n_1$ and $n_2$.

The highest of all the common divisors of $n_1$ and $n_2$ is called as the GCD or the HCF. This is denoted as GCD ($n_1$, $n_2$).

### Rules for Finding the GCD of Two Numbers $n_1$ and $n_2$

(a) Find the standard form of the numbers $n_1$ and $n_2$.

(b) Write out all prime factors that are common to the standard forms of the numbers $n_1$ and $n_2$.

(c) Raise each of the common prime factors listed above to the lesser of the powers in which it appears in the standard forms of the numbers $n_1$ and $n_2$.

(d) The product of the results of the previous step will be the GCD of $n_1$ and $n_2$.

**Illustration:** Find the GCD of 150, 210, 375.

***Step 1:*** Writting down the standard form of numbers

$$150 = 5 \times 5 \times 3 \times 2$$
$$210 = 5 \times 2 \times 7 \times 3$$
$$375 = 5 \times 5 \times 5 \times 3$$

***Step 2:*** Writing Prime factors common to all the three numbers is $5^1 \times 3^1$

***Step 3:*** This will give the same result, i.e. $5^1 \times 3^1$

***Step 4:*** Hence, the HCF will be $5 \times 3 = 15$

For practice, find the HCF of the following:

(a) 78, 39, 195
(b) 440, 140, 390
(c) 198, 121, 1331

## THE CONCEPT OF LCM (LEAST COMMON MULTIPLE)

Let $n_1$, and $n_2$ be two natural numbers distinct from each other. The smallest natural number $n$ that is exactly divisible by $n_1$ and $n_2$ is called the Least Common Multiple (LCM) of $n_1$ and $n_2$ and is designated as LCM ($n_1$, $n_2$).

### Rule for Finding the LCM of 2 numbers $n_1$ and $n_2$

(a) Find the standard form of the numbers $n_1$ and $n_2$.

(c) $2^2 \times 3^2 \times 5^1$ (d) None of these

Answer: (c) Use HCF × LCM = product of the numbers. Hence $(2^3 \times 3^2 \times 5^1 \times 103 \times 107) \times (2^2 \times 3^1) = 2^3 \times 3^1 \times 103 \times 2^2 \times 107 \times 3^1 \times N$.

10. Two equilateral triangles have the sides of lengths 34 and 85 respectively.
    (a) The greatest length of tape that can measure both of them exactly is:

    Answer: HCF of 34 and 85 is 17.

    (b) How many such equal parts can be measured?

    Answer: $34/17 + 85/17 = 2 + 5 = 7$

11. Two numbers are in the ratio 17:13. If their HCF is 15, what are the numbers?

    Answer: $17 \times 15$ and $13 \times 15$ i.e. 255 and 195 respectively.

12. A forester wants to plant 44 apple trees, 66 banana trees and 110 mango trees in equal rows (in terms of number of trees). Also, he wants to make distinct rows of trees (i.e. only one type of tree in one row). The number of rows (minimum) that are required are:

    (a) 2 (b) 3
    (c) 10 (d) 11

    Answer: (c) 44/22 + 66/22 + 110/22 (Since 22 is the HCF)

13. Three runners running around a circular track, can complete one revolution in 2, 4 and 5.5 hours respectively. When will they meet at the starting point?

    (a) 22 (b) 33
    (c) 11 (d) 44

    (The answer will be the LCM of 2, 4 and 11/2. This will give you 44 as the answer).

14. The HCF and LCM of two numbers are 33 and 264 respectively. When the first number is divided by 2, the quotient is 33. The other number is?

    (a) 66 (b) 132
    (c) 198 (d) 99

    (Answer: $33 \times 264 = 66 \times n$. Hence, $n = 132$)

15. The greatest number which will divide: 4003, 4126 and 4249, leaving the same remainder in each case:

    (a) 43 (b) 41
    (c) 45 (d) None of these

    The answer will be the HCF of the three numbers. (41 in this case)

16. Which of the following represents the largest 4 digit number which can be added to 7249 in order to make the derived number divisible by each of 12, 14, 21, 33, and 54.

    (a) 9123 (b) 9383
    (c) 8727 (d) None of these

    Answer: The LCM of the numbers 12, 14, 21, 33 and 54 is 8316. Hence, in order for the condition to be satisfied we need to get the number as:

    $$7249 + n = 8316 \times 2$$

    Hence, $n = 9383$.

17. Find the greatest number of 5 digits, that will give us a remainder of 5, when divided by 8 and 9 respectively.

    (a) 99931 (b) 99941
    (c) 99725 (d) None of these

    Answer: The LCM of 8 and 9 is 72. The longest 5 digit multiple of 72 is 99936. Hence, the required answer is 99941.

18. The least perfect square number which is divisible by 3, 4, 6, 8, 10 and 11 is:

    Solution: The number should have at least one 3, three 2's, one 5 and one 11 for it to be divisible by 3, 4, 6, 8, 10 and 11.

    Further, each of the prime factors should be having an even power. Thus, the correct answer will be: $3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 11 \times 11$

19. Find the greatest number of four digits which when divided by 10, 11, 15 and 22 leaves 3, 4, 8 and 15 as remainders respectively.

    (a) 9907 (b) 9903
    (c) 9893 (d) None of these

    Answer: First find the greatest 4 digit multiple of the LCM of 10, 11, 15 and 22. (In this case it is 9900). Then, subtract 7 from it to give the answer.

20. Find the HCF of $(3^{125} - 1)$ and $(3^{35} - 1)$.

    Answer: The solution of this question is based on the rule that:

    The HCF of $(a^m - 1)$ and $(a^n - 1)$ is given by $(a^{\text{HCF of } m,\, n} - 1)$

    Thus, in this question the answer is: $(3^5 - 1)$. Since 5 is the HCF of 35 and 125.

21. What will be the least possible number of the planks, if three pieces of timber 42 m, 49 m and 63 m long have to be divided into planks of the same length?

    (a) 7 (b) 8
    (c) 22 (d) None of these

22. Find the greatest number, which will divide 215, 167 and 135 so as to leave the same remainder in each case.

    (a) 64 (b) 32
    (c) 24 (d) 16

## Theorems of Divisibility

(a) If $a$ is divisible by $b$ then $ac$ is also divisible by $b$.

(b) If $a$ is divisible by $b$ and $b$ is divisible by $c$ then $a$ is divisible by $c$.

(c) If $a$ and $b$ are natural numbers such that $a$ is divisible by $b$ and $b$ is divisible by $a$ then $a = b$.

(d) If $n$ is divisible by $d$ and $m$ is divisible by $d$ then $(m + n)$ and $(m - n)$ are both divisible by $d$. This has an important implication. Suppose 28 and 742 are both divisible by 7. Then (742 + 28) as well as (742 – 28) are divisible by 7. (and in fact so is + 28 – 742).

(e) If $a$ is divisible by $b$ and $c$ is divisible by $d$ then $ac$ is divisible by $bd$.

(f) The highest power of a prime number $p$, which divides $n!$ exactly is given by

$$[n/p] + [n/p^2] + [n/p^3] + \ldots$$

where $[x]$ denotes the greatest integer less than or equal to $x$.

As we have already seen earlier –

Any composite number can be written down as a product of its prime factors. (Also called standard form)

Thus, for example the number 1240 can be written as $2^3 \times 31^1 \times 5^1$.

The standard form of any number has a huge amount of information stored in it. The best way to understand the information stored in the standard form of a number is to look at concrete examples. As a reader I want you to understand each of the processes defined below and use them to solve similar questions given in the exercise that follows and beyond:

1. Using the standard form of a number to find the sum and the number of factors of the number:

**(a) Sum of factors of a number:**

Suppose, we have to find the sum of factors and the number of factors of 240.

$$240 = 2^4 \times 3^1 \times 5^1$$

The sum of factors will be given by:

$$(2^0 + 2^1 + 2^2 + 2^3 + 2^4)(3^0 + 3^1)(5^0 + 5^1)$$
$$= 31 \times 4 \times 6 = 744$$

[**Note:** This is a standard process, wherein you create the same number of brackets as the number of distinct prime factors the number contains and then each bracket is filled with the sum of all the powers of the respective prime number starting from 0 to the highest power of that prime number contained in the standard form.

Thus, for 1240, we create 3 brackets—one each for 2, 3 and 5. Further in the bracket corresponding to 2 we write $(2^0 + 2^1 + 2^2 + 2^3 + 2^4)$.

Hence, for example for the number $40 = 2^3 \times 5^1$, the sum of factors will be given by: $(2^0 + 2^1 + 2^2 + 2^3)(5^0 + 5^1)$ {2 brackets since 40 has 2 distinct prime factors 2 and 5}

**(b) Number of factors of the number:**

Let us explore the sum of factors of 40 in a different context.

$$(2^0 + 2^1 + 2^2 + 2^3)(5^0 + 5^1)$$
$$= 2^0 \times 5^0 + 2^0 \times 5^1 + 2^1 \times 5^0 + 2^1 \times 5^1 + 2^2 \times 5^0 + 2^2 \times 5^1 + 2^3 \times 5^0 + 2^3 \times 5^1$$
$$= 1 + 5 + 2 + 10 + 4 + 20 + 8 + 40 = 90$$

A clear look at the numbers above will make you realize that it is nothing but the addition of the factors of 40

Hence, we realise that the number of terms in the expansion of $(2^0 + 2^1 + 2^2 + 2^3)(5^0 + 5^1)$ will give us the number of factors of 40. Hence, 40 has $4 \times 2 = 8$ factors.

**Note:** The moment you realise that $40 = 2^3 \times 5^1$ the answer for the number of factors can be got by $(3 + 1)(1 + 1) = 8$

2. **Sum and Number of even and odd factors of a number.**

Suppose, you are trying to find out the number of factors of a number represented in the standard form by: $2^3 \times 3^4 \times 5^2 \times 7^3$

As you are already aware the answer to the question is $(3 + 1)(4 + 1)(2 + 1) \times (3 + 1)$ and is based on the logic that the number of terms will be the same as the number of terms in the expansion: $(2^0 + 2^1 + 2^2 + 2^3)(3^0 + 3^1 + 3^2 + 3^3 + 3^4)(5^0 + 5^1 + 5^2)(7^0 + 7^1 + 7^2 + 7^3)$.

Now, suppose you have to find out the sum of the even factors of this number. The only change you need to do in this respect will be evident below. The answer will be given by:

$(2^1 + 2^2 + 2^3)(3^0 + 3^1 + 3^2 + 3^3 + 3^4)(5^0 + 5^1 + 5^2)(7^0 + 7^1 + 7^2 + 7^3)$

**Note:** That we have eliminated $2^0$ from the original answer. By eliminating $2^0$ from the expression for the sum of all factors you are ensuring that you have only even numbers in the expansion of the expression.

Consequently, the number of even factors will be given by: $(3)(4 + 1)(2 + 1)(3 + 1)$

i.e. Since $2^0$ is eliminated, we do not add 1 in the bracket corresponding to 2.

Let us now try to expand our thinking to try to think about the number of odd factors for a number.

In this case, we just have to do the opposite of what we did for even numbers. The following step will make it clear:

Odd factors of the number whose standard form is : $2^3 \times 3^4 \times 5^2 \times 7^3$

Sum of odd factors = $(2^0)(3^0 + 3^1 + 3^2 + 3^3 + 3^4)(5^0 + 5^1 + 5^2)(7^0 + 7^1 + 7^2 + 7^3)$

i.e.: Ignore all powers of 2. The result of the expansion of the above expression will be the complete set of odd factors of the number. Consequently, the number of odd factors for the number will be given by the number of terms in the expansion of the above expression.

Thus, the number of odd factors for the number $2^3 \times 3^4 \times 5^2 \times 7^3 = 1 \times (4 + 1)(2 + 1)(3 + 1)$.

3. **Sum and number of factors satisfying other conditions for any composite number:**

These are best explained through examples:

(i) Find the sum and the number of factors of 1200 such that the factors are divisible by 15.

Solution : $1200 = 2^4 \times 5^2 \times 3^1$.

For a factor to be divisible by 15 it should compulsorily have $3^1$ and $5^1$ in it. Thus, sum of factors divisible by 15 = $(2^0 + 2^1 + 2^2 + 2^3 + 2^4) \times (5^1 + 5^2)(3^1)$ and consequently the number of factors will be given by $5 \times 2 \times 1 = 10$.

(What we have done is ensure that in every individual term of the expansion, there is a minimum of $3^1 \times 5^1$. This is done by removing powers of 3 and 5 which are below 1.)

***Task for the student: Physically verify the answers to the question above and try to convert the logic into a mental algorithm.***

**<u>NOTE FROM THE AUTHOR</u>—The need for thought algorithms:**

I have often observed that the key difference between understanding a concept and actually applying it under examination pressure, is the presence or absence of a mental thought algorithm which clarifies the concept to you in your mind. The thought algorithm is a personal representation of a concept—and any concept that you read/understand in this book (or elsewhere) will remain an external concept till it remains in someone else's words. The moment the thought becomes internalised the concept becomes yours to apply and use.

**Practice exercise on factors:**

I. For the number 2450 find.

(1) The sum and number of all factors.

(2) The sum and number of even factors.

(3) The sum and number of odd factors.

(4) The sum and number of factors divisible by 5

(5) The sum and number of factors divisible by 35.

(6) The sum and number of factors divisible by 245.

II. For the number 7200 find.

(7) The sum and number of all factors.

(8) The sum and number of even factors.

(9) The sum and number of odd factors.

(10) The sum and number of factors divisible by 25.

(11) The sum and number of factors divisible by 40.

(12) The sum and number of factors divisible by 150.

(13) The sum and number of factors not divisible by 75.

(14) The sum and number of factors not divisible by 24.

15. Find the number of divisors of 1728.

(a) 18 (b) 30
(c) 28 (d) 20

16. Find the number of divisors of 1080 excluding the throughout divisors, which are perfect squares.

(a) 28 (b) 29
(c) 30 (d) 31

17. Find the number of divisors of 544 excluding 1 and 544.

(a) 12 (b) 18
(c) 11 (d) 10

18. Find the number of divisors 544 which are greater than 3.

(a) 15 (b) 10
(c) 12 (d) None of these.

19. Find the sum of divisors of 544 excluding 1 and 544.

(a) 1089 (b) 545
(c) 589 (d) 1134

20. Find the sum of divisors of 544 which are perfect squares.

(a) 32 (b) 64
(c) 42 (d) 21

21. Find the sum of odd divisors of 544.

(a) 18 (b) 34
(c) 68 (d) 36

22. Find the sum of even divisors of 4096.

(a) 8192 (b) 6144
(c) 8190 (d) 6142

23. Find the sum the sums of divisors of 144 and 160.
(a) 589 (b) 781
(c) 735 (d) None of these

24. Find the sum of the sum of even divisors of 96 and the sum of odd divisors of 3600.
(a) 639 (b) 735
(c) 651 (d) 589

**Answers**

| | | | | |
|---|---|---|---|---|
| 15. (c) | 16. (a) | 17. (d) | 18. (b) | 19. (c) |
| 20. (d) | 21. (a) | 22. (c) | 23. (b) | 24. (c) |

## NUMBER OF ZEROES IN AN EXPRESSION

Suppose you have to find the number of zeroes in a product: $24 \times 32 \times 17 \times 23 \times 19 = (2^3 \times 3^1) \times (2^5) \times 17^1 \times 23 \times 19$.

As you can notice, this product will have no zeroes because it has no 5 in it.

However, if you have an expression like: $8 \times 15 \times 23 \times 17 \times 25 \times 22$

The above expression can be rewritten in the standard form as:

$$2^3 \times 3^1 \times 5^1 \times 23 \times 17 \times 5^2 \times 2^1 \times 11^1$$

Zeroes are formed by a combination of $2 \times 5$. Hence, the number of zeroes will depend on the number of pairs of 2's and 5's that can be formed.

In the above product, there are four twos and two fives. Hence, we shall be able to form only two pairs of $(2 \times 5)$. Hence, there will be 2 zeroes in the product.

(Refer to Solved Example No. 1.11 for another example of this)

### Finding the Number of Zeroes in a Factorial Value

Suppose you had to find the number of zeroes in 6!.

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = (3 \times 2) \times (5) \times (2 \times 2) \times (3) \times (2) \times (1)$.

The above expression will have only one pair of $5 \times 2$, since there is only one 5 and an abundance of 2's.

It is clear that in any factorial value, the number of 5's will always be lesser than the number of 2's. Hence, all we need to do is to count the number of 5's. The process for this is explained in Solved Examples 1.1 to 1.3.

## EXERCISES FOR SELF-PRACTICE

Find the number of zeroes in the following cases:

1. 47!
2. 58!
3. $13 \times 15 \times 22 \times 125 \times 44 \times 35 \times 11$
4. $12 \times 15 \times 5 \times 24 \times 13 \times 17$
5. 173!
6. $144! \times 5 \times 15 \times 22 \times 11 \times 44 \times 135$
7. 148!
8. 1093!
9. 1132!
10. $1142! \times 348! \times 17!$

**A special implication:** Suppose you were to find the number of zeroes in the value of the following factorial values:

45!, 46!, 47!, 48!, 49!

What do you notice? The number of zeroes in each of the cases will be equal to 10. Why does this happen? It is not difficult to understand that the number of fives in any of these factorials is equal to 10. The number of zeroes will only change at 50! (It will become 12).

In fact, this will be true for all factorial values between two consecutive products of 5.

Thus, 50!, 51!, 52!, 53! And 54! will have 12 zeroes (since they all have 12 fives).

Similarly, 55!, 56!, 57!, 58! And 59! will each have 13 zeroes.

Apart from this fact, did you notice another thing? That while there are 10 zeroes in 49! there are directly 12 zeroes in 50!. This means that there is no value of a factorial which will give 11 zeroes. This occurs because to get 50! we multiply the value of 49! by 50. When you do so, the result is that we introduce two 5's in the product. Hence, the number of zeroes jumps by two (since we never had any paucity of twos.)

**Note:** at 124! you will get 24 + 4 => 28 zeroes.

At 125! you will get 25 + 5 + 1 = 31 zeroes. (A jump of 3 zeroes.)

## EXERCISES FOR SELF-PRACTICE

1. $n!$ has 23 zeroes. What is the maximum possible value of $n$?
2. $n!$ has 13 zeroes. The highest and least values of $n$ are?
3. Find the number of zeroes in the product $1^1 \times 2^2 \times 3^3 \times 4^4 \times 5^5 \times 6^6 \times \ldots \times 49^{49}$

4. Find the number of zeroes in:
$100^1 \times 99^2 \times 98^3 \times 97^4 \times \ldots\ldots\ldots \times 1^{100}$
5. Find the number of zeroes in:
$1^{1!} \times 2^{2!} \times 3^{3!} \times 4^{4!} \times 5^{5!} \times \ldots\ldots 10^{10!}$
6. Find the number of zeroes in the value of:
$2^2 \times 5^4 \times 4^6 \times 10^8 \times 6^{10} \times 15^{12} \times 8^{14} \times 20^{16} \times 10^{18} \times 25^{20}$
7. What is the number of zeroes in the following:
   (a) $3200 + 1000 + 40000 + 32000 + 15000000$
   (b) $3200 \times 1000 \times 40000 \times 32000 \times 15000000$

**Solution:**

1. This can never happen.
2. 59 and 55 respectively.
3. The fives will be less than the twos. Hence, we need to count only the fives.
Thus : $5^5 \times 10^{10} \times 15^{15} \times 20^{20} \times 25^{25} \times 30^{30} \times 35^{35} \times 40^{40} \times 45^{45}$
gives us: $5 + 10 + 15 + 20 + 25 + 25 + 30 + 35 + 40 + 45$ fives. Thus, the product has 250 zeroes.
4. Again the key here is to count the number of fives. This can get done by:
$100^1 \times 95^6 \times 90^{11} \times 85^{16} \times 80^{21} \times 75^{26} \times \ldots\ldots 5^{96}$
$(1 + 6 + 11 + 16 + 21 + 26 + 31 + 36 + 41 + 46 + \ldots\ldots + 96) + (1 + 26 + 51 + 76)$
$= 20 \times 48.5 + 4 \times 38.5$ (Using sum of A.P. explained in the next chapter.)
$= 970 + 154 = 1124.$
5. The answer will be the number of 5's. Hence, it will be $5! + 10!$
6. The number of fives is again lesser than the number of twos.
The number of 5's will be given by the power of 5 in the product:
$$5^4 \times 10^8 \times 15^{12} \times 20^{16} \times 10^{18} \times 25^{20}$$
$$= 4 + 8 + 12 + 16 + 18 + 40 = 98.$$
7. A. The number of zeroes in the sum will be two, since:

$$\begin{array}{r} 3200 \\ 1000 \\ 40000 \\ 32000 \\ \underline{15076200} \\ 15152400 \end{array}$$

Thus, in such cases the number of zeroes will be the least number of zeroes amongst the numbers.
Exception: $3200 + 1800 = 5000$ (three zeroes, not two).

B. The number of zeroes will be:
$$2 + 3 + 4 + 3 + 6 = 18.$$

**An extension of the process for finding the number of zeroes.**
Consider the following questions:

1. Find the highest power of 5 which is contained in the value of 127!
2. When 127! is divided by $5^n$ the result is an integer. Find the highest possible value for $n$.
3. Find the number of zeroes in 127!

In each of the above cases, the value of the answer will be given by:
$$[127/5] + [127/25] + [127/125]$$
$$= 25 + 5 + 1 = 31$$

This process can be extended to questions related to other prime numbers. For example:
Find the highest power of:

1. 3 which completely divides 38!
Solution: $[38/3] + [38/3^2] + 38/3^3] = 12 + 4 + 1 = 17$
2. 7 which is contained in 57!
$[57/7] + [57/7^2] = 8 + 1 = 9.$

This process changes when the divisor is not a prime number. You are first advised to go through worked out problems 1.4, 1.5, 1.6 and 1.19 .

Now try to solve the following exercise:

1. Find the highest power of 7 which divides 81!
2. Find the highest power of 42 which divides 122!
3. Find the highest power of 84 which divides 342!
4. Find the highest power of 175 which divides 344!
5. Find the highest power of 360 which divides 520!

**Solution Hints:**

1. You will check only with 7.
2. You will need to check with 7 only. (Since $42 = 7 \times 3 \times 2$)
3. $84 = 7 \times 3 \times 2 \times 2$. You will need to check for the number of 7's only.
4. $175 = 5^2 \times 7^1$. In this case you need to be careful. You will need to check the number of $5^2$'s and the number of 7's.
5. $36 = 2^3 \times 3^2 \times 5^1$. In this case you will need to check for the number of $2^3$'s, the number of $3^2$'s and the number of $5^1$'s.

## ■ EXERCISES FOR SELF-PRACTICE ■

1. Find the maximum value of $n$ such that 157! is perfectly divisible by $10^n$.

The result will never be integral if the two denominators are co-prime.

**Note:** This holds true even for expressions of the nature $A/7 - B/6$ etc.

This has huge implications for problem solving especially in the case of solving linear equations related to word based problems. Students are advised to try to use these throughout Block 1, 2 and 3 of this book.

**Key Concept 2:** Two consecutive integers are always co-prime.

***Example:*** Find all five-digit numbers of the form 34 $X$ 5$Y$ that are divisible by 36.

*Solution*: 36 is a product of two co-primes 4 and 9. Hence, if 34 $X$ 5$Y$ is divisible by 4 and 9, it will also be divisible by 36. Hence, for divisibility by 4, we have that the value of $Y$ can be 2 or 6. Also, if $Y$ is 2 the number becomes 34 $X$ 52. For this to be divisible by 9, the addition of $3 + 4 + X + 5 + 2$ should be divisible by 9. For this $X$ can be 4.

Hence the number 34452 is divisible by 36.

Also for $Y = 6$, the number $34 \times 56$ will be divisible by 36 when the addition of the digits is divisible by 9. This will happen when $X$ is 0 or 9. Hence, the numbers 34056 and 34956 will be divisible by 36.

## EXERCISES FOR SELF-PRACTICE

Find all numbers of the form 56 $x$ 3y that are divisible by 36.
Find all numbers of the form 72 $xy$ that are divisible by 45.
Find all numbers of the form 135 $xy$ that are divisible by 45.
Find all numbers of the form 517 $xy$ that are divisible by 89.

## Divisibility Rules

**Divisibility by 2 or 5:** A number is divisible by 2 or 5 if the last digit is divisible by 2 or 5.

**Divisibility by 3 (or 9):** All such numbers the sum of whose digits are divisible by 3 (or 9) are divisible by 3 (or 9).

**Divisibility by 4:** A number is divisible by 4 if the last 2 digits are divisible by 4.

**Divisibility by 6:** A number is divisible by 6 if it is simultaneously divisible by 2 and 3.

**Divisibility by 8:** A number is divisible by 8 if the last 3 digits of the number are divisible by 8.

**Divisibility by 11:** A number is divisible by 11 if the difference of the sum of the digits in the odd places and the sum of the digits in the even place is zero or is divisible by 11.

**Divisibility by 12:** All numbers divisible by 3 and 4 are divisible by 12.

**Divisibility by 7, 11 or 13:** The integer $n$ is divisible by 7, 11 or 13 if and only if the difference of the number of its thousands and the remainder of its division by 1000 is divisible by 7, 11 or 13.

***Example:*** 473312 is divisible by 7 since the difference between $473 - 312 = 161$ is divisible by 7.

**Even Numbers:** All integers that are divisible by 2 are even numbers. They are also denoted by $2n$.

***Example:*** 2, 4, 6, 12, 122, –2, –4, –12. Also note that zero is an even number.
2 is the lowest positive even number.

**Odd Numbers:** All integers that are not divisible by 2 are odd numbers. Odd number leave a remainder of 1 on being divided by 2. They are denoted by $2n + 1$ or $2n - 1$.

Lowest positive odd number is 1.

***Example:*** –1, –3, –7, –35, 3, 11 etc.

*Complex Numbers*: The arithmetic combination of real numbers and imaginary numbers are called complex numbers.

*Alternately*: All numbers of the form $a + ib$, where $i = \sqrt{-1}$ are called complex number.

*Twin Primes*: A pair of prime numbers are said to be twin prime when they differ by 2.

***Example:*** 3 and 5 are Twin Primes, so also are 11 and 13.

*Perfect Numbers*: A number $n$ is said to be a perfect number if the sum of all the divisors of $n$ (including $n$) is equal to $2n$.

***Example:*** $6 = 1 \times 2 \times 3$ sum of the divisors $= 1 + 2 + 3 + 6 = 12 = 2 \times 6$

$$28 = 1, 2, 4, 7, 14, 28, = 56 = 2 \times 28$$

*Task for student:* Find all perfect numbers below 1000.
*Mixed Numbers*: A number that has both an integral and a fractional part is known as a mixed number.
*Triangular Numbers:* A number which can be represented as the sum of consecutive natural numbers starting with 1 are called as triangular numbers.
e.g.: $1 + 2 + 3 + 4 = 10$

**Learning Point:** In order to find the remainder of $17 \times 23$ when divided by 12, you need to look at the individual remainders of 17 and 23 when divided by 12. The respective remainders (5 and 11) will give you the remainder of the original expression when divided by 12.

Mathematically, this can be written as:

The remainder of the expression $[A \times B \times C + D \times E]/M$, will be the same as the remainder of the expression $[A_R \times B_R \times C_R + D_R \times E_R]/M$.

Where $A_R$ is the remainder when $A$ is divided by $M$,

$B_R$ is the remainder when $B$ is divided by $M$,

$C_R$ is the remainder when $C$ is divided by $M$

$D_R$ is the remainder when $D$ is divided by $M$ and

$E_R$ is the remainder when $E$ is divided by $M$,

We call this transformation as the remainder theorem transformation and denote it by the sign $\xrightarrow{R}$

Thus, the remainder of

$1421 \times 1423 \times 1425$ when divided by 12 can be given as:

$$\frac{1421 \times 1423 \times 1425}{12} \xrightarrow{R} \frac{5 \times 7 \times 9}{12} = \frac{35 \times 9}{12}$$

$$\xrightarrow{R} \frac{11 \times 9}{12}.$$

$\xrightarrow{R}$ gives us a remainder of 3.

In the above question, we have used a series of remainder theorem transformations (denoted by $\xrightarrow{R}$) and equality transformations to transform a difficult looking expression into a simple expression.

Try to solve the following questions on Remainder theorem:

Find the remainder in each of the following cases:

1. $17 \times 23 \times 126 \times 38$ divided by 8.
2. $243 \times 245 \times 247 \times 249 \times 251$ divided by 12.
3. $\frac{173 \times 261}{13} + \frac{248 \times 249 \times 250}{15}$.
4. $\frac{1021 \times 2021 \times 3021}{14}$.
5. $\frac{37 \times 43 \times 51}{7} + \frac{137 \times 143 \times 151}{9}$.

## USING NEGATIVE REMAINDERS

Consider the following question:

Find the remainder when: $14 \times 15$ is divided by 8.

The obvious approach in this case would be

$$\frac{14 \times 15}{8} \xrightarrow{R} \frac{6 \times 7}{8} = \frac{42}{8} \xrightarrow{R} 2 \text{ (Answer).}$$

However there is another option by which you can solve the same question:

When 14 is divided by 8, the remainder is normally seen as + 6. However, there might be times when using the negative value of the remainder might give us more convenience. Which is why you should know the following process:

**Concept Note:** Remainders by definition are always non-negative. Hence, even when we divide a number like – 27 by 5 we say that the remainder is 3 (and not – 2). However, looking at the negative value of the remainder has its own advantages in Mathematics as it results in reducing calculations.

Thus, when a number like 13 is divided by 8, the remainder being 5, the negative remainder is – 3.

(**Note:** It is in this context that we mention numbers like 13, 21, 29 etc as $8n + 5$ or $8n - 3$ numbers.)

Thus $\frac{14 \times 15}{8}$ will give us $\frac{-2 \times -1}{8}$ $R \to 2$.

Consider the advantage this process will give you in the following question:

$$\frac{51 \times 52}{53} \xrightarrow{R} \frac{-2 \times -1}{53} \xrightarrow{R} 2.$$

(The alternative will involve long calculations. Hence, the principle is that you should use negative remainders wherever you can. They can make life much simpler!!!)

### What if the answer comes out negative?

For instance, $\frac{62 \times 63 \times 64}{66} R \to \frac{-4 \times -3 \times -2}{66} R \to \frac{-24}{66}$.

But, we know that a remainder of –24, equals a remainder of 42 when divided by 66. Hence, the answer is 42.

Of course nothing stops you from using positive and negative remainders at the same time in order to solve the same question –

Thus $\frac{17 \times 19}{9} R \to \frac{(-1) \times (1)}{9}$ $R \to -1$ $R \to 8$.

**Dealing with large powers** There are two tools which are effective in order to deal with large powers –

(A) If you can express the expression in the form $\frac{(ax+1)^n}{a}$, the remainder will become 1 directly. In such a case, no matter how large the value of the power $n$ is, the remainder is 1.

For instance, $\frac{(37^{12635})}{9} \xrightarrow{R} \frac{(1^{12635})}{9} \xrightarrow{R} 1.$

In such a case the value of the power does not matter.

(B) $\frac{(mx-1)^n}{a}$. In such a case using $-1$ as the remainder it will be evident that the remainder will be $+1$ if $n$ is even and it will be $-1$ (Hence $a-1$) when $n$ is odd.

e.g.: $\frac{31^{127}}{8} \xrightarrow{R} \frac{(-1)^{127}}{8} \xrightarrow{R} \frac{(-1)}{8} \xrightarrow{R} 7$

## ANOTHER IMPORTANT POINT

Suppose you were asked to find the remainder of 14 divided by 4. It is clearly visible that the answer should be 2.

But consider the following process:

$14/4 = 7/2 \xrightarrow{R} 1$ (The answer has changed!!)

What has happened?

We have transformed 14/4 into 7/2 by dividing the numerator and the denominator by 2. The result is that the original remainder 2 is also divided by 2 giving us 1 as the remainder. In order to take care of this problem, we need to reverse the effect of the division of the remainder by 2. This is done by multiplying the final remainder by 2 to get the correct answer.

**Note:** *In any question on remainder theorem, you should try to cancel out parts of the numerator and denominator as much as you can, since it directly reduces the calculations required.*

## AN APPLICATION OF REMAINDER THEOREM

Finding the last two digits of an expression:

Suppose you had to find the last 2 digits of the expression:

$$22 \times 31 \times 44 \times 27 \times 37 \times 43$$

The remainder the above expression will give when it is divided by 100 is the answer to the above question.

Hence, to answer the question above find the remainder of the expression when it is divided by 100.

*Solution:* $\frac{22 \times 31 \times 44 \times 27 \times 37 \times 43}{100}$

$= \frac{22 \times 31 \times 11 \times 27 \times 37 \times 43}{25}$ (on dividing by 4)

$\xrightarrow{R} \frac{22 \times 6 \times 11 \times 2 \times 12 \times 18}{25} = \frac{132 \times 22 \times 216}{25}$

$\xrightarrow{R} \frac{7 \times 22 \times 16}{25}$

$= \frac{154 \times 16}{25} \xrightarrow{R} \frac{4 \times 16}{25} \xrightarrow{R} 14$

Thus the remainder being 14, (after division by 4). The actual remainder should be 56.

[Don't forget to multiply by 4 !!]

Hence, the last 2 digits of the answer will be 56.

Using negative remainders here would have helped further.

**Note:** Similarly finding the last three digits of an expression means finding the remainder when the expression is divided by 1000.

## EXERCISES

1. Find the remainder when 73 + 75 + 78 + 57 + 197 is divided by 34.
   (a) 32 (b) 30
   (c) 15 (d) 28
2. Find the remainder when $73 \times 75 \times 78 \times 57 \times 197$ is divided by 34.
   (a) 32 (b) 30
   (c) 15 (d) 28
3. Find the remainder when $73 \times 75 \times 78 \times 57 \times 197 \times 37$ is divided by 34.
   (a) 32 (b) 30
   (c) 15 (d) 28
4. Find the remainder when $43^{197}$ is divided by 7.
   (a) 2 (b) 4
   (c) 6 (d) 1
5. Find the remainder when $51^{203}$ is divided by 7.
   (a) 4 (b) 2
   (c) 1 (d) 6
6. Find the remainder when $59^{28}$ is divided by 7.
   (a) 2 (b) 4
   (c) 6 (d) 1
7. Find the remainder when $67^{99}$ is divided by 7.
   (a) 2 (b) 4
   (c) 6 (d) 1

8. Find the remainder when $75^{80}$ is divided by 7.
   (a) 4 (b) 3
   (c) 2 (d) 6
9. Find the remainder when $41^{77}$ is divided by 7.
   (a) 2 (b) 1
   (c) 6 (d) 4
10. Find the remainder when $21^{875}$ is divided by 17.
   (a) 8 (b) 13
   (c) 16 (d) 9
11. Find the remainder when $54^{124}$ is divided by 17.
   (a) 4 (b) 5
   (c) 13 (d) 15
12. Find the remainder when $83^{261}$ is divided by 17.
   (a) 13 (b) 9
   (c) 8 (d) 2
13. Find the remainder when $25^{102}$ is divided by 17.
   (a) 13 (b) 15
   (c) 4 (d) 2

**Answers**

| | | | | |
|---|---|---|---|---|
| 1. (b) | 2. (a) | 3. (d) | 4. (a) | 5. (a) |
| 6. (b) | 7. (d) | 8. (a) | 9. (c) | 10. (b) |
| 11. (a) | 12. (d) | 13. (c) | | |

## Units Digit

(A) By the logic of what we have just seen above, the unit's digit of an expression will be got by getting the remainder when the expression is divided by 10.

Thus for example if we have to find the units digit of the expression:

$$17 \times 22 \times 36 \times 54 \times 27 \times 31 \times 63$$

We try to find the remainder –

$$\frac{17 \times 22 \times 36 \times 54 \times 27 \times 31 \times 63}{10}$$

$$\xrightarrow{R} \frac{7 \times 2 \times 6 \times 4 \times 7 \times 3}{10}$$

$$= \frac{14 \times 24 \times 21}{10} \xrightarrow{R} \frac{4 \times 4 \times 1}{10} = \frac{16}{10} \xrightarrow{R} 6.$$

Hence, the required answer is 6.

This could have been directly got by multiplying: $7 \times 2 \times 6 \times 4 \times 7 \times 1 \times 3$ and only accounting for the units' digit.

(B) Unit's digits in the contexts of powers –
Study the following table carefully.

**Unit's digit when 'N' is raised to a power**

| *Number Ending With* | *Value of power* 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 4 | 8 | 6 | 2 | 4 | 8 | 6 | 2 |
| 3 | 3 | 9 | 7 | 1 | 3 | 9 | 7 | 1 | 3 |
| 4 | 4 | 6 | 4 | 6 | 4 | 6 | 4 | 6 | 4 |
| 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 |
| 7 | 7 | 9 | 3 | 1 | 7 | 9 | 3 | 1 | 7 |
| 8 | 8 | 4 | 2 | 6 | 8 | 4 | 2 | 6 | 8 |
| 9 | 9 | 1 | 9 | 1 | 9 | 1 | 9 | 1 | 9 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

In the table above, if you look at the columns corresponding to the power 5 or 9 you will realize that the unit's digit for all numbers is repeated (i.e. it is 1 for 1, 2 for, 3 for 3….9 for 9.)

This means that whenever we have any number whose unit's digit is '$x$' and it is raised to a power of the form $4n + 1$, the value of the unit's digit of the answer will be the same as the original units digit.

***Illustrations:***

$(1273)^{101}$ will give a unit's digit of 3. $(1547)^{25}$ will give a units digit of 7 and so forth.

Thus, the above table can be modified into the form –

**Value of power**

| *Number 'N' ending in* | *4n + 1* | *4n + 2* | *4n + 3* | $U_{4n}$ |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 2 | 4 | 8 | 6 |
| 3 | 3 | 9 | 7 | 1 |
| 4 | 4 | 6 | 4 | 6 |
| 5 | 5 | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 | 6 |
| 7 | 7 | 9 | 3 | 1 |
| 8 | 8 | 4 | 2 | 6 |
| 9 | 9 | 1 | 9 | 1 |

[Remember, at this point that we had said (in the back to school section of Block 1) that all natural numbers can be expressed in the form $4n + x$. Hence, with the help of the logic that helps us build this table, we can easily derive the units digit of any number when it is raised to a power.)

**A special Case**

***Question:***

What will be the Unit's digit of $(1273)^{122!}$?

*Solution:*

122! is a number of the form $4n$. Hence, the answer should be 1. [**Note:** 1 here is derived by thinking of it as 3 (for $4n + 1$), 9 (for $4n + 2$), 7 (for $4n + 3$), 1(for $4n$)]

## ■ EXERCISES FOR ALL-PRACTICE:SS ■

Find the Units digit in each of the following cases:

1. $2^2 \times 4^4 \times 6^6 \times 8^8$
2. $1^1 \times 2^2 \times 3^3 \times 4^4 \times 5^5 \times 6^6 \ldots \times 100^{100}$
3. $17 \times 23 \times 51 \times 32 + 15 \times 17 \times 16 \times 22$
4. $13 \times 17 \times 22 \times 34 + 12 \times 6 \times 4 \times 3 - 13 \times 33$
5. $37^{123} \times 43^{144} \times 57^{226} \times 32^{127} \times 52^{51}$
6. $67 \times 37 \times 43 \times 91 \times 42 \times 33 \times 42$
   (a) 2 (b) 6
   (c) 8 (d) 4
7. $67 \times 35 \times 43 \times 91 \times 47 \times 33 \times 49$
   (a) 1 (b) 9
   (c) 5 (d) 6
8. $67 \times 35 \times 45 \times 91 \times 42 \times 33 \times 81$
   (a) 2 (b) 4
   (c) 0 (d) 8
9. $67 \times 35 \times 45 + 91 \times 42 \times 33 \times 82$
   (a) 8 (b) 7
   (c) 0 (d) 5
10. $(52)^{97} \times (43)^{72}$
    (a) 2 (b) 6
    (c) 8 (d) 4
11. $(55)^{75} \times (93)^{175} \times (107)^{275}$
    (a) 7 (b) 3
    (c) 5 (d) 0
12. $(173)^{45} \times (152)^{77} \times (777)^{999}$
    (a) 2 (b) 4
    (c) 8 (d) 6
13. $81 \times 82 \times 83 \times 84 \times 86 \times 87 \times 88 \times 89$
    (a) 0 (b) 6
    (c) 2 (d) 4
14. $82^{43} \times 83^{44} \times 84^{97} \times 86^{98} \times 87^{105} \times 88^{94}$
    (a) 2 (b) 6
    (c) 4 (d) 8
15. $432 \times 532 + 532 \times 974 + 537 \times 531 + 947 \times 997$
    (a) 5 (b) 6
    (c) 9 (d) 8

**Answers**

| | | | | |
|---|---|---|---|---|
| 6. (d) | 7. (c) | 8. (c) | 9. (b) | 10. (a) |
| 11. (a) | 12. (c) | 13. (b) | 14. (c) | 15. (d) |

## WORKED-OUT PROBLEMS

**Problem 1.1** Find the number of zeroes in the factorial of the number 18.

**Solution** 18! Contains 15 and 5, which combined with one even number give zeroes. Also, 10 is also contained in 18!, which will give an additional zero. Hence, 18! Contains 3 zeroes and the last digit will always be zero.

**Problem 1.2** Find the numbers of zeroes in 27!

**Solution** $27! = 27 \times 26 \times 25 \times \ldots \times 20 \times \ldots \times 15 \times \ldots \times 10 \times \ldots \times 5 \times \ldots \times 1$.

A zero can be formed by combining any number containing 5 multiplied by any even number. Similarly, everytime a number ending in zero is found in the product, it will add an additional zero. For this problem, note that $25 = 5 \times 5$ will give 2 zeroes and zeroes will also be got by 20, 15, 10 and 5. Hence 27! Will have 6 zeroes.

*Short-cut method:* Number of zeroes is $27! \rightarrow [27/5] + [27/25]$ where $[x]$ indicates the integer just lower than the fraction Hence, $[27/5] = 5$ and $[27/5^2] = 1$, 6 zeroes

**Problem 1.3** Find the number of zeroes in 137!

**Solution** $[137/5] + [137/5^2] + [137/5^3]$
$= 27 + 5 + 1 = 33$ zeroes

(since the restriction on the number of zeroes is due to the number of fives.)

## ■ EXERCISE FOR SELF-PRACTICE ■

Find the number of zeroes in

(a) 81! (b) 100! (c) 51!

## ■ EXERCISES FOR SELF-PRACTICE ■

(a) Find the number of numbers between 140 to 259, both included, which are divisible by 7.
(b) Find the number of numbers between 100 to 200, that are divisible by 3.

**Problem 1.10** Find the number of numbers between 300 to 400 (both included), that are not divisible by 2, 3, 4, and 5.

**Solution** Total numbers: 101
*Step 1:* Not divisible by 2 = All even numbers rejected: 51
Numbers left 50.

*Step 2:* Of which: divisible by 3 = first number 300, last number 399. But even numbers have already been removed, hence count out only odd numbers between 300 and 400 divisible by 3. This gives us that:
First number 303, last number 399, common difference 6

*So, remove:* $[(399 - 303)/6] + 1 = 17$.
$\therefore \quad 50 - 17 = 33$ numbers left.

We do not need to remove additional terms for divisibility by 4 since this would eliminate only even numbers (which have already been eliminated)

*Step 3:* Remove from 33 numbers left all odd numbers that are divisible by 5 and not divisible by 3.

Between 300 to 400, the first odd number divisible by 5 is 305 and the last is 395 (since both ends are counted, we have 10 such numbers as: $[(395 - 305)/10 + 1 = 10]$.

However, some of these 10 numbers have already been removed to get to 33 numbers.

*Operation left:* Of these 10 numbers, 305, 315…395, reduce all numbers that are also divisible by 3. Quick perusal shows that the numbers start with 315 and have common difference 30.

Hence [(Last number – First number)/Difference + 1] $= [(375 - 315)/30 + 1] = 3$

These 3 numbers were already removed from the original 100. Hence, for numbers divisible by 5, we need to remove only those numbers that are odd, divisible by 5 but not by 3. There are 7 such numbers between 300 and 400.

So numbers left are: $33 - 7 = 26$.

## ■ EXERCISES FOR SELF-PRACTICE ■

Find the number of numbers between 100 to 400 which are divisible by either 2, 3, 5 and 7.

**Problem 1.11** Find the number of zeroes in the following multiplication: $5 \times 10 \times 15 \times 20 \times 25 \times 30 \times 35 \times 40 \times 45 \times 50$.

**Solution** The number of zeroes depends on the number of fives and the number of twos. Here, close scrutiny shows that the number of twos is the constraint. The expression can be written as

$5 \times (5 \times 2) \times (5 \times 3) \times (5 \times 2 \times 2) \times (5 \times 5) \times (5 \times 2 \times 3) \times (5 \times 7) \times (5 \times 2 \times 2 \times 2) \times (5 \times 3 \times 3) \times (5 \times 5 \times 2)$

Number of 5s – 12, Number of 2s – 8.
Hence: 8 zeroes.

**Problem 1.12** Find the remainder for $[(73 \times 79 \times 81)/11]$.

**Solution** The remainder for the expression: $[(73 \times 79 \times 81)/11]$ will be the same as the remainder for $[(7 \times 2 \times 4)/11]$

That is, $56/11 \Rightarrow$ remainder = 1

**Problem 1.13** Find the remainder for $(3^{560}/8)$.

**Solution** $(3^{560}/8) = [(3^2)^{280}/8] = (9^{280}/8)$
$= [9.9.9\ldots(280 \text{ times})]/8$

remainder for above expression = remainder for $[1.1.1\ldots(280 \text{ times})]/8 \Rightarrow$ remainder = 1.

**Problem 1.14** Find the remainder when $(2222^{5555} + 5555^{2222})/7$.

**Solution** This is of the form: $[(2222^{5555})/7 + (5555^{2222})/7]$

We now proceed to find the individual remainder of : $(2222^{5555})/7$. Let the remainder be $R_1$.

When 2222 is divided by 7, it leaves a remainder of 3.

Hence, for remainder purpose $(2222^{5555})/7 \approx (3^{5555}/7)$ $= (3.3^{5554})/7 = [3(3^2)^{2777}]/7 = [3.(7+2)^{2777}]/7 \approx (3.2^{2777})/7$ $= (3.2^2 \cdot 2^{2775})/7 = [3.2^2 \cdot (2^3)^{925}]/7$ $= [3.2^2 \cdot (8)^{925}]/7 \approx (12/7)$ Remainder = 5.

Similarly, $(5555^{2222})/7 \approx (4^{2222})/7 = [(2^2)^{2222}]/7 = (2)^{4444}/7 = (2.2^{4443})/7 = [2.(2^3)^{1481}]/7 = [2.(8)^{1481}]/7 \approx [2.(1)^{1481}]/7$ 2 (remainder).

Hence, $(2222^{5555})/7 + (5555^{2222})/7 \approx (5+2)/7 \Rightarrow$ Remainder $= 0$

**Problem 1.15** Find the GCD and the LCM of the numbers 126, 540 and 630.

**Solution** The standard forms of the numbers are:

$$\text{Number of threes} = \left[\frac{10200}{3}\right] + \left[\frac{10200}{9}\right] + \left[\frac{10200}{27}\right] + \left[\frac{10200}{81}\right] + \left[\frac{10200}{243}\right] + \left[\frac{10200}{729}\right] + \left[\frac{10200}{2187}\right] + \left[\frac{10200}{6561}\right]$$

$$= 3400 + 1133 + 377 + 125 + 41 + 13 + 4 + 1$$

Number of threes = 5094

$\therefore$ Number of $3^2 = 2547$

Similarly we find the number of 7s as

$$\left[\frac{10200}{7}\right] + \left[\frac{10200}{49}\right] + \left[\frac{10200}{343}\right] + \left[\frac{10200}{2401}\right]$$

$$= 1457 + 208 + 29 + 4 = 1698.$$

Thus, we have, 1698 sevens, 2547 nines and 3397 eights contained in 10200!.

The required value of $n$ will be given by the lowest of these three [The student is expected to explore why this happens]

Hence, answer = **1698**.

***Short Cut*** We will look only for the number of 7s in this case. *Reason*: $7 > 3 \times 2$. So, the number of 7s must always be less than the number of $2^3$.

And $7 > 2 \times 3$, so the number of 7s must be less than the number of $3^2$.

Recollect that earlier we had talked about the finding of powers when the divisor only had prime factors. There we had seen that we needed to check only for the highest power as the restriction had to lie there.

In cases of the divisors having composite factors, we have to be slightly careful in estimating the factor that will reflect the restriction. In the above example, we saw a case where even though 7 was the lowest factor (in relation to 8 and 9), the restriction was still placed by 7 rather than by 9 (as would be expected based on the previous process of taking the highest number).

**Problem 1.20** Find the units digit of the expression: $78^{5562} \times 56^{256} \times 97^{1250}$.

**Solution** We can get the units digits in the expression by looking at the patterns followed by 78, 56 and 97 when they are raised to high powers.

In fact, for the last digit we just need to consider the units digit of each part of the product.

A number (like 78) having 8 as the units digit will yield units digit as

$78^1 \to 8$ $\quad 78^5 \to 8$

$78^2 \to 4$ $\quad 78^6 \to 4$

$78^3 \to 2$ $\quad 78^7 \to 2$

$78^4 \to 6$ $\quad 78^8 \to 6$

> $8^{4n+1} \to 8$
> $8^{4n+2} \to 4$
> Hence $78^{5562}$ will yield four as the units digit

Similarly, $56^1 \to 6$, $56^2 \to 6$, $56^3 \to 6$ $\quad \to 56^{256}$ will yield 6 as the units digit.

Similarly,

$97^1 \to 7$

$97^2 \to 9$

$97^3 \to 3$

$97^4 \to 1$

> $7^{4n+1} \to 7$
> $7^{4n+2} \to 9$
> Hence, $97^{1250}$ will yield a units digit of 9.

Hence, the required units digit is given by $4 \times 6 \times 9 \to$ **6** (answer).

**Problem 1.21** Find the GCD and the LCM of the numbers $P$ and $Q$ where $P = 2^3 \times 5^3 \times 7^2$ and $Q = 3^3 \times 5^4$.

**Solution** GCD or HCF is given by the lowest powers of the common factors.

Thus, $\quad$ GCD $= 5^3$.

LCM is given by the highest powers of all factors available.

Thus, $\quad$ LCM $= 2^3 \times 3^3 \times 5^4 \times 7^2$

**Problem 1.22** A school has 378 girl students and 675 boy students. The school is divided into strictly boys or strictly girls sections. All sections in the school have the same number of students. Given this information, what are the number of sections in the school.

**Solution** The answer will be given by the HCF of 378 and 675.

$$378 = 2 \times 3^3 \times 7$$
$$675 = 3^3 \times 5^2$$

Hence, HCF of the two is $3^3 = 27$.

Hence, the number of sections is given by: $\frac{378}{27} + \frac{675}{27} = 14 + 25 =$ **39 sections.**

## Level of Difficulty (LOD) I

1. The last digit of the number obtained by multiplying the numbers $81 \times 82 \times 83 \times 84 \times 85 \times 86 \times 87 \times 88 \times 89$ will be
   (a) 0 (b) 9 (c) 7 (d) 2
   (e) 8
2. The sum of the digits of a two-digit number is 10, while when the digits are reversed, the number decreases by 54. Find the changed number.
   (a) 28 (b) 19 (c) 37 (d) 46
   (e) 82
3. When we multiply a certain two-digit number by the sum of its digits, 405 is achieved. If you multiply the number written in reverse order of the same digits by the sum of the digits, we get 486. Find the number.
   (a) 81 (b) 45
   (c) 36 (d) 54
   (e) None of these
4. The sum of two numbers is 15 and their geometric mean is 20% lower than their arithmetic mean. Find the numbers.
   (a) 11, 4 (b) 12, 3
   (c) 13, 2 (d) 10, 5
   (e) 9, 6
5. The difference between two numbers is 48 and the difference between the arithmetic mean and the geometric mean is two more than half of 1/3 of 96. Find the numbers.
   (a) 49, 1 (b) 12, 60
   (c) 50, 2 (d) 36, 84
   (e) None of these
6. If $A381$ is divisible by 11, find the value of the smallest natural number $A$?
   (a) 5 (b) 6
   (c) 7 (d) 9
   (e) None of these
7. If $381A$ is divisible by 9, find the value of smallest natural number $A$?
   (a) 5 (b) 5
   (c) 7 (d) 9
   (e) 6
8. What will be the remainder obtained when $(9^6 + 1)$ will be divided by 8?
   (a) 0 (b) 3 (c) 7 (d) 2
   (e) 1
9. Find the ratio between the LCM and HCF of 5, 15 and 20?
   (a) 8 : 1 (b) 14 : 3 (c) 12 : 2 (d) 12 : 1
   (e) 1 : 12
10. Find the LCM of 5/2, 8/9, 11/14.
    (a) 280 (b) 360
    (c) 420 (d) 220
    (e) None of these
11. If the number A is even, which of the following will be true?
    (a) 3A will always be divisible by 6
    (b) 3A + 5 will always be divisible by 11
    (c) $(A^2 + 3)/4$ will be divisible by 7
    (d) All of these
    (e) None of these
12. A five-digit number is taken. Sum of the first four digits (excluding the number at the units digit) equals sum of all the five digits. Which of the following will not divide this number necessarily?
    (a) 10 (b) 2 (c) 4 (d) 5
    (e) None of these
13. A number $15B$ is divisible by 6. Which of these will be true about the positive integer $B$?
    (a) $B$ will be even
    (b) $B$ will be odd
    (c) $B$ will be divisble by 6
    (d) Both (a) and (c)
    (e) None of these
14. Two numbers $P = 2^3.3^{10}.5$ and $Q = 2^5.3^1.7^1$ are given. Find the GCD of $P$ and $Q$.
    (a) 2.3.5.7 (b) $3.2^2$
    (c) $2^2.3^2$ (d) $2^3.3$
    (e) $2^3 \times 3^{10} \times 5^1 \times 7^1$
15. Find the units digit of the expression $25^{6251} + 36^{528} + 73^{54}$.
    (a) 4 (b) 0 (c) 6 (d) 5
    (e) 1
16. Find the units digit of the expression $55^{725} + 73^{5810} + 22^{853}$.
    (a) 4 (b) 0 (c) 6 (d) 5
    (e) 8

17. Find the units digit of the expression $11^1 + 12^2 + 13^3 + 14^4 + 15^5 + 16^6$.
(a) 1 (b) 9 (c) 7 (d) 0
(e) 8

18. Find the units digit of the expression $11^1.12^2.13^3.14^4.15^5.16^6$.
(a) 4 (b) 3
(c) 7 (d) 0
(e) None of these

19. Find the number of zeroes at the end of 1090!
(a) 270 (b) 268
(c) 269 (d) 271
(e) None of these

20. If 146! is divisible by $5^n$, then find the maximum value of $n$.
(a) 34 (b) 35
(c) 36 (d) 37
(e) None of these

21. Find the number of divisors of 1420.
(a) 14 (b) 15
(c) 13 (d) 12
(e) None of these

22. Find the HCF and LCM of the polynomials $(x^2 - 5x + 6)$ and $(x^2 - 7x + 10)$.
(a) $(x - 2), (x - 2)(x - 3)(x - 5)$
(b) $(x - 2), (x - 2)(x - 3)$
(c) $(x - 3), (x - 2)(x - 3)(x - 5)$
(d) $(x - 2), (x - 2)(x - 3)(x - 5)^2$
(e) None of these

***Directions for Questions 23–25:*** Given two different prime numbers $P$ and $Q$, find the number of divisors of the following:

23. $P.Q$
(a) 2 (b) 4 (c) 6 (d) 8
(e) 5

24. $P^2Q$
(a) 2 (b) 4 (c) 6 (d) 8
(e) 9

25. $P^3Q^2$
(a) 2 (b) 4 (c) 6 (d) 12
(e) 8

26. The sides of a pentagonal field (not regular) are 1737 metres, 2160 metres, 2358 metres, 1422 metres and 2214 metres respectively. Find the greatest length of the tape by which the five sides may be measured completely?
(a) 7 (b) 13 (c) 11 (d) 9
(e) 10

27. There are 576 boys and 448 girls in a school that are to be divided into equal sections of either boys or girls alone. Find the total number of sections thus formed?
(a) 24 (b) 32
(c) 16 (d) 20
(e) None of these

28. A milkman has three different qualities of milk. 403 gallons of 1st quality, 465 gallons of 2nd quality and 496 gallons of 3rd quality. Find the least possible number of bottles of equal size in which different milk of different qualities can be filled without mixing?
(a) 34 (b) 46 (c) 26 (d) 44
(e) 45

29. What is the greatest number of 4 digits that when divided by any of the numbers 6, 9, 12, 17 leaves a remainder of 1?
(a) 9997 (b) 9793
(c) 9895 (d) 9487
(e) 9897

30. Find the least number that when divided by 16, 18 and 20 leaves a remainder 4 in each case, but is completely divisible by 7.
(a) 364 (b) 2254
(c) 2964 (d) 3234
(e) 2884

31. Four bells ring at the intervals of 6, 8, 12 and 18 seconds. They start ringing together at 12'O' clock. After how many seconds will they ring together again?
(a) 72 (b) 84 (c) 60 (d) 48
(e) 144

32. For question 31 find how many times will they ring together during the next 12 minutes? (including the 12 minute mark)
(a) 9 (b) 10
(c) 11 (d) 12
(e) None of these

33. The units digit of the expression $125^{813} \times 553^{3703} \times 4532^{828}$ is
(a) 4 (b) 2
(c) 0 (d) 5
(e) None of these

34. Which of the following is not a perfect square?
(a) 1,00,856 (b) 3,25,137
(c) 9,45,729 (d) All of these
(e) None of these

35. Which of the following can never be in the ending of a perfect square?
(a) 6 (b) 00 (c) 000 (d) 1
(e) 9

36. The LCM of 5, 8,12, 20 will not be a multiple of
(a) 3 (b) 9
(c) 8 (d) 5
(e) None of these

37. Find the number of divisors of 720 (including 1 and 720).
(a) 25 (b) 28 (c) 29 (d) 30
(e) 32

38. The LCM of $(16 - x^2)$ and $(x^2 + x - 6)$ is
(a) $(x - 3)(x + 3)(4 - x^2)$
(b) $4(4 - x^2)(x + 3)$
(c) $(4 - x^2)(x - 3)$
(d) $(4 - x)(x - 3)$
(e) None of these

39. GCD of $x^2 - 4$ and $x^2 + x - 6$ is
(a) $x + 2$ (b) $x - 2$
(c) $x^2 - 2$ (d) $x^2 + 2$
(e) None of these

40. The number $A$ is not divisible by 3. Which of the following will not be divisible by 3?
(a) $9 \times A$ (b) $2 \times A$
(c) $18 \times A$ (d) $24 \times A$
(e) None of these

41. Find the remainder when the number $9^{100}$ is divided by 8?
(a) 1 (b) 2 (c) 0 (d) 4
(e) 2

42. Find the remainder of $2^{1000}$ when divided by 3?
(a) 1 (b) 2 (c) 4 (d) 6
(e) 0

43. Decompose the number 20 into two terms such that their product is the greatest.
(a) $x_1 = x_2 = 10$ (b) $x_1 = 5, x_2 = 15$
(c) $x_1 = 16, x_2 = 4$ (d) $x_1 = 8, x_2 = 12$
(e) None of these

44. Find the number of zeroes at the end of 50!
(a) 13 (b) 11 (c) 5 (d) 12
(e) 10

45. Which of the following can be a number divisible by 24?
(a) 4,32,15,604 (b) 25,61,284
(c) 13,62,480 (d) All of these
(e) None of these

46. For a number to be divisible by 88, it should be
(a) Divisible by 22 and 8
(b) Divisible by 11 and 8
(c) Divisible by 11 and thrice by 2
(d) Both (b) and (c)
(e) All of these

47. Find the number of divisors of 10800.
(a) 57 (b) 60
(c) 72 (d) 64
(e) None of these

48. Find the GCD of the polynomials $(x + 3)^2(x - 2)(x + 1)^2$ and $(x + 1)^3(x + 3)(x + 4)$.
(a) $(x + 3)^3(x + 1)^2(x - 2)(x + 4)$
(b) $(x + 3)(x - 2)(x + 1)(x + 4)$
(c) $(x + 3)(x + 1)^2$
(d) $(x + 1)(x + 3)^2$
(e) None of these

49. Find the LCM of $(x + 3)(6x^2 + 5x + 4)$ and $(2x^2 + 7x + 3)(x + 3)$
(a) $(2x + 1)(x + 3)(3x + 4)$
(b) $(4x^2 - 1)(x + 3)^2(3x + 4)$
(c) $(4x^2 - 1)(x + 3)(3x + 4)$
(d) $(2x - 1)(x + 3)(3x + 4)$
(e) None of these

50. The product of three consecutive natural numbers, the first of which is an even number, is always divisible by
(a) 12 (b) 24
(c) 6 (d) All of these
(e) None of these

51. Some birds settled on the branches of a tree. First, they sat one to a branch and there was one bird too many. Next they sat two to a branch and there was one branch too many. How many branches were there?
(a) 3 (b) 4 (c) 5 (d) 6
(e) 2

52. The square of a number greater than 1000 that is not divisible by three, when divided by three, leaves a remainder of
(a) 1 always (b) 2 always
(c) 0 (d) either 1 or 2
(e) Cannot be said

(a) 4 (b) 5
(c) 6 (d) 7

70. In the famous Bel Air Apartments in Ranchi, there are three watchmen meant to protect the precious fruits in the campus. However, one day a thief got in without being noticed and stole some precious mangoes. On the way out however, he was confronted by the three watchmen, the first two of whom asked him to part with $1/3^{rd}$ of the fruits and one more. The last asked him to part with $1/5^{th}$ of the mangoes and 4 more. As a result he had no mangoes left. What was the number of mangoes he had stolen?
(a) 12 (b) 13
(c) 15 (d) None of these

71. A hundred and twenty digit number is formed by writing the first x natural numbers in front of each other as 12345678910111213… Find the remainder when this number is divided by 8.
(a) 6 (b) 7
(c) 2 (d) 0

72. A test has 80 questions. There is one mark for a correct answer, while there is a negative penalty of $-1/2$ for a wrong answer and $-1/4$ for an unattempted question. What is the number of questions answered correctly, if the student has scored a net total of 34.5 marks.
(a) 45 (b) 48
(c) 54 (d) Cannot be determined

73. For the question 72, if it is known that he has left 10 questions unanswered, the number of correct answers are:
(a) 45 (b) 48
(c) 54 (d) Cannot be determined

74. Three mangoes, four guavas and five watermelons cost Rs. 750. Ten watermelons, six mangoes and 9 guavas cost Rs.1580. What is the cost of six mangoes, ten watermelons and 4 guavas?
(a) 1280 (b) 1180
(c) 1080 (d) Cannot be determined

75. From a number $M$ subtract 1. Take the reciprocal of the result to get the value of '$N$'. Then which of the following is necessarily true?
(a) $M^N \leq 2$ (b) $M^N > 3$
(c) $1 < M^N < 3$ (d) $1 < M^N < 5$

76. The cost of four mangoes, six guavas and sixteen watermelons is Rs. 500, while the cost of seven magoes, nine guavas and nineteen watermelons is Rs. 620. What is the cost of one mango, one guava and one watermelon?
(a) 120 (b) 40
(c) 150 (d) Cannot be determined

77. For the question above, what is the cost of a mango?
(a) 20 (b) 14
(c) 15 (d) Cannot be determined

78. The following is known about three real numbers, $x$, $y$ and $z$.
$-4 \leq x \leq 4$, $-8 \leq y \leq 2$ and $-8 \leq z \leq 2$. Then the range of values that $M = xz/y$ can take is best represented by:
(a) $-16 \leq x \leq 16$ (b) $-16 \leq x \leq 8$
(c) $-8 \leq x \leq 8$ (d) $-4 \leq x \leq 4$

79. A man sold 38 pieces of clothing (combined in the form of shirts, trousers and ties). If he sold at least 11 pieces of each item and he sold more shirts than trousers and more trousers than ties, then the number of ties that he must have sold is:
(a) Exactly 11 (b) At least 11
(c) At least 12 (d) Cannot be determined

80. For the question 79, find the number of shirts he must have sold?
(a) At least 13 (b) At least 14
(c) At least 15 (d) At most 16.

81. Find the least number which when divided by 12, 15, 18 or 20 leaves in each case a remainder 4.
(a) 124 (b) 364
(c) 184 (d) None of these

82. What is the least number by which 2800 should be multiplied so that the product may be a perfect square?
(a) 2 (b) 7
(c) 14 (d) None of these

83. The least number of 4 digits which is a perfect square is:
(a) 1064 (b) 1040
(c) 1024 (d) 1012

84. The least multiple of 7 which leaves a remainder of 4 when divided by 6, 9, 15 and 18 is
(a) 94 (b) 184
(c) 364 (d) 74

85. What is the least 3 digit number that when divided by 2, 3, 4, 5 or 6 leaves a remainder of 1?
(a) 131 (b) 161
(c) 121 (d) None of these

86. The highest common factor of 70 and 245 is equal to
(a) 35 (b) 45
(c) 55 (d) 65

(a) $2^{24}$ to $2^{25}$ (b) $2^{25}$ to $2^{26}$
(c) $2^{26}$ to $2^{27}$ (d) $2^{29}$ to $2^{30}$
(e) $2^{23}$ to $2^{24}$

10. *xy* is a number that is divided by *ab* where $xy < ab$ and gives a result 0.*xyxyxy*… then *ab* equals
(a) 11 (b) 33 (c) 99 (d) 66
(e) 88

11. A number *xy* is multiplied by another number *ab* and the result comes as *pqr*, where $r = 2y$, $q = 2(x + y)$ and $p = 2x$ where $x, y < 5$, $q \neq 0$. The value of *ab* may be:
(a) 11 (b) 13
(c) 31 (d) 22
(e) None of these

12. $[x]$ denotes the greatest integer value just below $x$ and $\{x\}$ its fractional value. The sum of $[x]^3$ and $\{x\}^2$ is $-7.91$. Find $x$.
(a) $-2.03$ (b) $-1.97$
(c) $-2.97$ (d) $-1.7$
(e) None of these

13. $16^5 + 2^{15}$ is divisible by
(a) 31 (b) 13 (c) 27 (d) 33
(e) 11

14. If $AB + XY = 1XP$, where $A \neq 0$ and all the letters signify different digits from 0 to 9, then the value of $A$ is:
(a) 6 (b) 7
(c) 9 (d) 8
(e) Any value above 6

***Directions for Questions number 15–16:*** Find the possible integral values of $x$.

15. $|x - 3| + 2|x + 1| = 4$
(a) 1 (b) $-1$
(c) 3 (d) 2
(e) There are many solutions

16. $x^2 + |x - 1| = 1$
(a) 1 (b) $-1$
(c) 0 (d) 1 or 0
(e) $-2$

17. If $4^{n+1} + x$ and $4^{2n} - x$ are divisible by 5, $n$ being an even integer, find the least value of $x$.
(a) 1 (b) 2
(c) 3 (d) 0
(e) None of these

18. If the sum of the numbers $(a25)^2$ and $a^3$ is divisible by 9, then which of the following may be a value for $a$ ?
(a) 1 (b) 7
(c) 9 (d) 8
(e) There is no value

19. If $|x - 4| + |y - 4| = 4$, then how many integer values can the set $(x, y)$ have?
(a) Infinite (b) 5
(c) 16 (d) 9
(e) 25

20. $[3^{32}/50]$ gives remainder and $\{.\}$ denotes the fractional part of that. The fractional part is of the form $(0 \cdot bx)$. The value of $x$ could be
(a) 2 (b) 4
(c) 6 (d) 8
(e) None of these

21. The sum of two numbers is 20 and their geometric mean is 20% lower than their arithmetic mean. Find the ratio of the numbers.
(a) 4 : 1 (b) 9 : 1
(c) 1 : 1 (d) 17 : 3
(e) 5 : 1

22. The highest power on 990 that will exactly divide 1090! is
(a) 101 (b) 100 (c) 108 (d) 109
(e) 110

23. If 146! is divisible by $6^n$, then find the maximum value of $n$.
(a) 74 (b) 70 (c) 76 (d) 75
(e) 73

24. The last two digits in the multiplication of $35 \cdot 34 \cdot 33 \cdot 32 \cdot 31 \cdot 30 \cdot 29 \cdot 28 \cdot 27 \cdot 26$ is
(a) 00 (b) 40
(c) 30 (d) 10
(e) None of these

25. The expression $333^{555} + 555^{333}$ is divisible by
(a) 2 (b) 3
(c) 37 (d) 11
(e) All of these

26. $[x]$ denotes the greatest integer value just below $x$ and $\{x\}$ its fractional value. The sum of $[x]^2$ and $\{x\}^1$ is 25.16. Find $x$.
(a) 5.16 (b) $-4.84$
(c) Both (a) and (b) (d) 4.84
(e) Cannot be determined

27. If we add the square of the digit in the tens place of a positive two-digit number to the product of the digits of that number, we shall get 52, and if we add the square of the digit in the units place to the same product of the digits, we shall get 117. Find the two-digit number.
(a) 18 (b) 39 (c) 49 (d) 28
(e) 30

28. Find two numbers such that their sum, their product and the differences of their squares are equal.
(a) $\left(\frac{3+\sqrt{3}}{2}\right)$ and $\left(\frac{1+\sqrt{2}}{2}\right)$ or $\left(\frac{3+\sqrt{2}}{2}\right)$ and $\left(\frac{1+\sqrt{2}}{2}\right)$
(b) $\left(\frac{3+\sqrt{7}}{2}\right)$ and $\left(\frac{1+\sqrt{7}}{2}\right)$ or $\left(\frac{3+\sqrt{6}}{2}\right)$ and $\left(\frac{1-\sqrt{6}}{2}\right)$
(c) $\left(\frac{3-\sqrt{5}}{2}\right)$ and $\left(\frac{1-\sqrt{5}}{2}\right)$ or $\left(\frac{3+\sqrt{5}}{2}\right)$ and $\left(\frac{1+\sqrt{5}}{2}\right)$
(d) All of these
(e) None of these

29. The sum of the digits of a three-digit number is 17, and the sum of the squares of its digits is 109. If we subtract 495 from that number, we shall get a number consisting of the same digits written in the reverse order. Find the number.
(a) 773 (b) 863
(c) 683 (d) 944
(e) 684

30. Find the number of zeros in the product: $1^1 \times 2^2 \times 3^3 \times 4^4 \times \ldots\ldots 98^{98} \times 99^{99} \times 100^{100}$
(a) 1200 (b) 1300
(c) 1050 (d) 1225
(e) None of these

31. Find the pairs of a natural number whose greatest common divisor is 5 and the least common multiple is 105.
(a) 5 and 105 or 15 and 35
(b) 6 and 105 or 16 and 35
(c) 5 and 15 or 15 and 135
(d) 5 and 20 or 15 and 35
(e) None of these

32. The denominator of an irreducible fraction is greater than the numerator by 2. If we reduce the numerator of the reciprocal fraction by 3 and subtract the given fraction from the resulting one, we get 1/15. Find the given fraction.
(a) $\frac{2}{4}$ (b) $\frac{3}{5}$ (c) $\frac{5}{7}$ (d) $\frac{7}{9}$
(e) $\frac{8}{9}$

33. A two-digit number exceeds by 19 the sum of the squares of its digits and by 44 the double product of its digits. Find the number.
(a) 72 (b) 62 (c) 22 (d) 12
(e) 15

34. The sum of the squares of the digits constituting a two-digit positive number is 2.5 times as large as the sum of its digits and is larger by unity than the trebled product of its digits. Find the number.
(a) 13 and 31 (b) 12 and 21
(c) 22 and 33 (d) 14 and 41
(e) None of these

35. The units digit of a two-digit number is greater than its tens digit by 2, and the product of that number by the sum of its digits is 144. Find the number.
(a) 14 (b) 24 (c) 46 (d) 35
(e) 20

36. Find the number of zeroes in the product: $5 \times 10 \times 25 \times 40 \times 50 \times 55 \times 65 \times 125 \times 80$
(a) 8 (b) 9 (c) 12 (d) 13
(e) 10

37. The power of 45 that will exactly divide 123! is
(a) 28 (b) 30 (c) 31 (d) 59
(e) 29

38. Three numbers are such that the second is as much lesser than the third as the first is lesser than the second. If the product of the two smaller numbers is 85 and the product of two larger numbers is 115 find the middle number.
(a) 9 (b) 8 (c) 12 (d) 30
(e) 10

39. Find the smallest natural number $n$ such that $n!$ is divisible by 990.
(a) 3 (b) 5

(c) 11 (d) 12

(e) None of these

40. $\sqrt{x}\sqrt{y}=\sqrt{xy}$ is true only when

(a) $x > 0, y > 0$ (b) $x > 0$ and $y < 0$

(c) $x < 0$ and $y > 0$ (d) All of these

(e) None of these

***Directions for Questions 41–60:*** Read the instructions below and solve the questions based on this.

In an examination situation, always solve the following type of questions by substituting the given options, to arrive at the solution.

However, as you can see, there are no options given in the questions here since these are meant to be an exercise in equation writing (which I believe is a primary skill required to do well in aptitude exams testing mathematical aptitude). Indeed, if these questions had options for them, they would be rated as LOD 1 questions. But since the option-based solution technique is removed here, I have placed these in the LOD 2 category.

41. Find the two-digit number that meets the following criteria. If the number in the units place exceeds, the number in its tens by 2 and the product of the required number with the sum of its digits is equal to 144.
42. The product of the digits of a two-digit number is twice as large as the sum of its digits. If we subtract 27 from the required number, we get a number consisting of the same digits written in the reverse order. Find the number?
43. The product of the digits of a two-digit number is one-third that number. If we add 18 to the required number, we get a number consisting of the same digits written in the reverse order. Find the number?
44. The sum of the squares of the digits of a two-digit number is 13. If we subtract 9 from that number, we get a number consisting of the same digits written in the reverse order. Find the number?
45. A two-digit number is thrice as large as the sum of its digits, and the square of that sum is equal to the trebled required number. Find the number?
46. Find a two-digit number that exceeds by 12 the sum of the squares of its digits and by 16 the doubled product of its digits.
47. The sum of the squares of the digits constituting a two-digit number is 10, and the product of the required number by the number consisting of the same digits written in the reverse order is 403. Find the 2 numbers that satisfy these conditions?
48. If we divide a two-digit number by the sum of its digits, we get 4 as a quotient and 3 as a remainder. Now, if we divide that two-digit number by the product of its digits, we get 3 as a quotient and 5 as a remainder. Find the two-digit number.
49. There is a natural number that becomes equal to the square of a natural number when 100 is added to it, and to the square of another natural number when 169 is added to it. Find the number?
50. Find two natural numbers whose sum is 85 and whose least common multiple is 102.
51. Find two-three digit numbers whose sum is a multiple of 504 and the quotient is a multiple of 6.
52. The difference between the digits in a two-digit number is equal to 2, and the sum of the squares of the same digits is 52. Find all the possible numbers?
53. If we divide a given two-digit number by the product of its digits, we obtain 3 as a quotient and 9 as a remainder. If we subtract the product of the digits constituting the number, from the square of the sum of its digits, we obtain the given number. Find the number.
54. Find the three-digit number if it is known that the sum of its digits is 17 and the sum of the squares of its digits is 109. If we subtract 495 from this number, we obtain a number consisting of the same digits written in reverse order.
55. The sum of the cubes of the digits constituting a two-digit number is 243 and the product of the sum of its digits by the product of its digits is 162. Find the two two-digit number?
56. The difference between two numbers is 16. What can be said about the total numbers divisible by 7 that can lie in between these two numbers.
57. Arrange the following in descending order:

$$111^4, 110.109.108.107, 109.110.112.113$$

58. If $3 \le x \le 5$ and $4 \le y \le 7$. Find the greatest value of $xy$ and the least value of $x/y$.
59. Which of these is greater:

(a) $200^{300}$ or $300^{200}$ or $400^{150}$

(b) $5^{100}$ and $2^{200}$

(c) $10^{20}$ and $40^{10}$

60. The sum of the two numbers is equal to 15 and their arithmetic mean is 25 per cent greater than its geometric mean. Find the numbers.
61. Define a number $K$ such that it is the sum of the squares of the first $M$ natural numbers.(i.e. $K = 1^2 + 2^2 + \ldots + M^2$)

where $M < 55$. How many values of $M$ exist such that $K$ is divisible by 4?

(a) 10 (b) 11
(c) 12 (d) None of these

62. $M$ is a two digit number which has the property that: The product of factorials of it's digits > sum of factorials of its digits
How many values of $M$ exist?
(a) 56 (b) 64
(c) 63 (d) None of these

63. A natural number when increased by 50% has its number of factors unchanged. However, when the value of the number is reduced by 75%, the number of factors is reduced by 66.66%. One such number could be:
(a) 32 (b) 84
(c) 126 (d) None of these

64. Find the 28383[rd] term of the series: 123456789101112....
(a) 3 (b) 4
(c) 9 (d) 7

65. If you form a subset of integers chosen from between 1 to 3000, such that no two integers add up to a multiple of nine, what can be the maximum number of elements in the subset.
(a) 1668 (b) 1332
(c) 1333 (d) 1334

66. The series of numbers (1,1/2,1/3,1/4 ................1/1972) is taken. Now two numbers are taken from this series (the first two) say $x$, $y$. Then the operation $x + y + x.y$ is performed to get a consolidated number. The process is repeated. What will be the value of the set after all the numbers are consolidated into one number.
(a) 1970 (b) 1971
(c) 1972 (d) None of these

67. $K$ is a three digit number such that the ratio of the number to the sum of its digits is least? What is the difference between the hundreds and the tens digits of $K$?
(a) 9 (b) 8
(c) 7 (d) None of these

68. In the question 67, what can be said about the difference between the tens and the units digit?
(a) 0 (b) 1
(c) 2 (d) None of these

69. For the above question, for how many values of $K$ will the ratio be the highest?
(a) 9 (b) 8
(c) 7 (d) None of these

70. A triangular number is defined as a number which has the property of being expressed as a sum of consecutive natural numbers starting with 1. How many triangular numbers less than 1000, have the property that they are the difference of squares of two consecutive natural numbers?
(a) 20 (b) 21
(c) 22 (d) 23

71. $x$ and $y$ are two positive integers. Then what will be the sum of the coefficients of the expansion of the expression $(x + y)^{44}$? Answer: $2^{44}$
(a) $2^{43}$ (b) $2^{43} + 1$
(c) $2^{44}$ (d) $2^{44} - 1$

72. What is the remainder when $9 + 9^2 + 9^3 + \ldots 9^{2n+1}$ is divided by 6?
(a) 1 (b) 2
(c) 3 (d) 4

73. The remainder when the number 123456789101112 ......484950 is divided by 16 is:
(a) 3 (b) 4
(c) 5 (d) 6

74. What is the highest power of 3 available in the expression $58! - 38!$
(a) 17 (b) 18
(c) 19 (d) None of these

75. Find the remainder when the number represented by 22334 raised to the power $(1^2 + 2^2 + .. + 66^2)$ is divided by 5?
(a) 2 (b) 4
(c) 0 (d) None of these

76. What is the total number of divisors of the number $12^{33} \times 34^{23} \times 2^{47}$?
(a) 4658. (b) 9316
(c) 2744 (d) None of these

77. For the question 76, which of the following will represent the sum of factors of the number (such that only odd factors are counted)?
(a) $\frac{(3^{34} - 1)}{2} \times \frac{(17^{24} - 1)}{16}$ (b) $(3^{34} - 1) \times (17^{24} - 1)$
(c) $\frac{(3^{34} - 1)}{33}$ (d) None of these

78. What is the remainder when $(1!)^3 + (2!)^3 + (3!)^3 + (4!)^3 + \ldots (1152!)^3$ is divided by 1152?
(a) 125 (b) 225
(c) 325 (d) 205

79. A set $S$ is formed by including some of the first One thousand natural numbers. $S$ contains the maximum

questions in three levels of difficulty—LOD1, LOD2 and LOD3.

The following table gives the details of the positive and negative marks attached to each question type:

| *Difficulty level* | *Positive marks for answering the question correctly* | *Negative marks for answering the question wrongly* |
|---|---|---|
| LOD 1 | 4 | 2 |
| LOD 2 | 3 | 1.5 |
| LOD 3 | 2 | 1 |

The test had 200 questions with 80 on LOD 1 and 60 each on LOD 2 and LOD 3.

97. If a student has solved 100 questions exactly and scored 120 marks, the maximum number of incorrect questions that he/she might have marked is:
(a) 44 (b) 56
(c) 60 (d) None of these

98. If Amit attempted the least number of questions and got a total of 130 marks, and if it is known that he attempted at least one of every type, then the number of questions he must have attempted is:
(a) 34 (b) 35
(c) 36 (d) None of these

99. In the above question, what is the least number of questions he might have got incorrect?
(a) 0 (b) 1
(c) 2 (d) None of these

100. Amitabh has a certain number of toffees, such that if he distributes them amongst ten children he has nine left, if he distributes amongst 9 children he would have 8 left, if he distributes amongst 8 children he would have 7 left ... and so on until if he distributes amongst 5 children he should have 4 left. What is the second highest number of toffees he could have with him?
(a) 2519 (b) 7559
(c) 8249 (d) None of these

## Level of Difficulty (LOD) III

1. What two-digit number is less than the sum of the square of its digits by 11 and exceeds their doubled product by 5?
(a) 15, 95 (b) 95
(c) Both (a) and (b) (d) 15, 95 and 12345
(e) None of these

2. Find the lower of the two successive natural numbers if the square of the sum of those numbers exceeds the sum of their squares by 112.
(a) 6 (b) 7 (c) 8 (d) 9
(e) 10

3. First we increased the denominator of a positive fraction by 3 and then we decreased it by 5. The sum of the resulting fractions proves to be equal to 2/3. Find the denominator of the fraction if its numerator is 2.
(a) 7 (b) 8 (c) 12 (d) 9
(e) 13

4. Find the last two digits of: $15 \times 37 \times 63 \times 51 \times 97 \times 17$.
(a) 35 (b) 45 (c) 55 (d) 85
(e) 75

5. Let us consider a fraction whose denominator is smaller than the square of the numerator by unity. If we add 2 to the numerator and the denominator, the fraction will exceed 1/3. If we subtract 3 from the numerator and the denominator, the fraction will be positive but smaller than 1/10. Find the value?
(a) $\frac{3}{8}$ (b) $\frac{4}{15}$
(c) $\frac{5}{24}$ (d) $\frac{6}{35}$
(e) None of these

6. Find the sum of all three-digit numbers that give a remainder of 4 when they are divided by 5.
(a) 98,270 (b) 99,270
(c) 1,02,090 (d) 90,270
(e) None of these

7. Find the sum of all two-digit numbers that give a remainder of 3 when they are divided by 7.
(a) 686 (b) 676
(c) 666 (d) 656
(e) None of these

8. Find the sum of all odd three-digit numbers that are divisible by 5.
(a) 50,500 (b) 50,250
(c) 50,000 (d) 49,500
(e) 51,250

9. The product of a two-digit number by a number consisting of the same digits written in the reverse order is equal to 2430. Find the lower number.
(a) 54 (b) 54 (c) 63 (d) 65
(e) 45

(c) 197 (d) 159
(e) Both (a) and (b)

23. A three-digit positive integer $abc$ is such that $a^2 + b^2 + c^2 = 74$. $a$ is equal to the doubled sum of the digits in the tens and units places. Find the number if it is known that the difference between that number and the number written by the same digits in the reverse order is 495.
(a) 813 (b) 349
(c) 613 (d) 713
(e) None of these

24. Represent the number 1.25 as a product of three positive factors so that the product of the first factor by the square of the second is equal to 5 if we have to get the lowest possible sum of the three factors.
(a) $x_1 = 2.25, x_2 = 5, x_3 = 0.2$
(b) $x_1 = 1.25, x_2 = 4, x_3 = 4.5$
(c) $x_1 = 1.25, x_2 = 2, x_3 = 0.5$
(d) $x_1 = 1.25, x_2 = 4, x_3 = 2$
(e) None of these

25. Find a number $x$ such that the sum of that number and its square is the least.
(a) –0.5 (b) 0.5 (c) –1.5 (d) 1.5
(e) 1

26. When $2222^{5555} + 5555^{2222}$ is divided by 7, the remainder is
(a) 0 (b) 2 (c) 4 (d) 5

27. If $x$ is a number of five-digits which when divided by 8, 12, 15 and 20 leaves respectively 5, 9, 12 and 17 as remainders, then find $x$ such that it is the lowest such number?
(a) 10017 (b) 10057 (c) 10097 (d) 10137
(e) None of these

28. $3^{2n} - 1$ is divisible by $2^{n+3}$ for $n =$
(a) 1 (b) 2
(c) 3 (d) 4
(e) None of these

29. $10^n - \left(5+\sqrt{17}\right)^n$ is divisible by $2^{n+2}$ for what whole number value of $n$?
(a) 2 (b) 3 (c) 7 (d) 8
(e) None of these

30. $\dfrac{{}^{32}32^{32}}{9}$ will leave a remainder:
(a) 4 (b) 7 (c) 1 (d) 2

31. Find the remainder that the number $1989 \cdot 1990 \cdot 1992^3$ gives when divided by 7;
(a) 0 (b) 1 (c) 5 (d) 2
(e) 2

32. Find the remainder of $2^{100}$ when divided by 3.
(a) 3 (b) 0 (c) 1 (d) 2
(e) None of these

33. Find the remainder when the number $3^{1989}$ is divided by 7.
(a) 1 (b) 5 (c) 6 (d) 4
(e) 3

34. Find the last digit of the number $1^2 + 2^2 + \ldots + 99^2$.
(a) 0 (b) 1 (c) 2 (d) 3
(e) 5

35. Find gcd $(2^{100} - 1, 2^{120} - 1)$.
(a) $2^{20} - 1$ (b) $2^{40} - 1$
(c) $2^{60} - 1$ (d) $2^{10} - 1$
(e) $2^{40} - 1$

36. Find the gcd (111…11 hundred ones ; 11…11 sixty ones).
(a) 111…forty ones (b) 111…twenty five ones
(c) 111…twenty ones (d) 111…sixty ones
(e) None of these

37. Find the last digit of the number $1^3 + 2^3 + 3^3 + 4^3 \ldots + 99^3$.
(a) 0 (b) 1 (c) 2 (d) 5
(e) 8

38. Find the GCD of the numbers $2n + 13$ and $n + 7$.
(a) 1 (b) 2 (c) 3 (d) 4
(e) 5

39. $\dfrac{{}^{32}32^{32}}{7}$
(a) 4 (b) 2 (c) 1 (d) 3
(e) 5

40. The remainder when $10^{10} + 10^{100} + 10^{1000} + \ldots + 10^{10000000000}$ is divided by 7 is
(a) 0 (b) 1 (c) 2 (d) 5
(e) 4

41. $n$ is a number, such that $2n$ has 28 factors and $3n$ has 30 factors. $6n$ has?
(a) 35 (b) 32
(c) 28 (d) None of these

42. Suppose the product of $n$ consecutive integers is $x$. $(x + 1)(x + 2)(x + 3)\ldots(x + (n - 1)) = 1000$, then which

of the following cannot be true about the number of terms $n$

1. The number of terms can be 16
2. The number of terms can be 5
3. The number of terms can be 25
4. The number of terms can be 20

43. The remainder when $2^2 + 22^2 + 222^2 + 2222^2 + \ldots\ldots.(222\ldots..49 \text{ twos})^2$ is divided by 9 is:
(a) 2 (b) 5
(c) 6 (d) 7
44. $N = 202 \times 20002 \times 200000002 \times 20000000000000002 \times 200000000\ldots.2$ (31 zeroes) The sum of digits in this multiplication will be:
(a) 112 (b) 160
(c) 144 (d) Cannot be determined
45. Twenty five sets of problems on Data Interpretation–one each for the DI sections of 25 CATALYST tests were prepared by the AMS research team. The DI section of each CATALYST contained 50 questions of which exactly 35 questions were unique, i.e. they had not been used in the DI section of any of the other 24 CATALYSTs. What could be the maximum possible number of questions prepared for the DI sections of all the 25 CATALYSTs put together?
(a) 1100 (b) 975
(c) 1070 (d) 1055
46. In the above question, what could be the minimum possible number of questions prepared?
(a) 890 (b) 875
(c) 975 (d) None of these

***Directions for Questions 11–13:*** At a particular time in the twenty first century there were seven bowlers in the Indian cricket team's list of 16 players short listed to play the next world cup. Statisticians discovered that that if you looked at the number of wickets taken by any of the 7 bowlers of the current Indian cricket team, the number of wickets taken by them had a strange property. The numbers were such that for any team selection of 11 players (having 1 to 7 bowlers) by using the number of wickets taken by each bowler and attaching coefficients of +1, 0, or –1 to each value available and adding the resultant values, any number from 1 to 1093, both included could be formed. If we denote $W_1$, $W_2$, $W_3$, $W_4$, $W_5$, $W_6$ and $W_7$ as the 7 values in the ascending order what could be the answer to the following questions:

47. Find the value of $W_1 + 2W_2 + 3W_3 + 4W_4 + 5W_5 + 6W_6$.
(a) 2005 (b) 1995
(c) 1985 (d) None of these
48. Find the index of the largest power of 3 contained in the product $W_1\, W_2\, W_3\, W_4\, W_5\, W_6\, W_7$.
(a) 15 (b) 10
(c) 21 (d) 6
49. If the sum of the seven coefficients is 0, find the smallest number that can be obtained.
(a) – 1067 (b) – 729
(c) – 1041 (d) – 1054

***Directions for Questions 50 and 51:*** Answer these questions on the basis of the information given below.

In the ancient game of Honololo the task involves solving a puzzle asked by the chief of the tribe. Anybody answering the puzzle correctly is given the hand of the most beautiful maiden of the tribe. Unfortunately, for the youth of the tribe, solving the puzzle is not a cakewalk since the chief is the greatest mathematician of the tribe.

In one such competition the chief called everyone to attention and announced openly:

"A three-digit number '*mnp*' is a perfect square and the number of factors it has is also a perfect square. It is also known that the digits $m$, $n$ and $p$ are all distinct. Now answer my questions and win the maiden's hand."

50. If $(m + n + p)$ is also a perfect square, what is the number of factors of the six-digit number *mnpmnp*?
(a) 32 (b) 72
(c) 48 (d) Cannot be determined
51. If the fourth power of the product of the digits of the number *mnp* is not divisible by 5, what is the number of factors of the nine-digit number, *mnpmnpmnp*?
(a) 32 (b) 72
(c) 48 (d) Cannot be determined
52. In a cricket tournament organised by the ICC, a total of 15 teams participated. Australia, as usual won the tournament by scoring the maximum number of points. The tournament is organised as a single round robin tournament—where each team plays with every other team exactly once. 3 points are awarded for a win, 2 points are awarded for a tie/washed out match and 1 point is awarded for a loss. Zimbabwe had the lowest score (in terms of points) at the end of the tournament. Zimbabwe scored a total of 21 points. All the 15 national teams got a distinct score (in terms of points scored). It is also known that at least one match played by the Australian team was tied/washed out. Which of the following is always true for the Australian team?
(a) It had at least two ties/washouts.
(b) It had a maximum of 3 losses.

(c) It had a maximum of 9 wins.
(d) All of the above.

53. What is the remainder when $128^{1000}$ is divided by 153
(a) 103 (b) 145 (c) 118 (d) 52

54. Find the remainder when $50^{51^{52}}$ is divided by 11.
(a) 6 (b) 4 (c) 7 (d) 3

55. Find the remainder when $32^{33^{34}}$ is divided by 11.
(a) 5 (b) 4 (c) 10 (d) 1

56. Find the remainder when $30^{72^{87}}$ is divided by 11.
(a) 5 (b) 9 (c) 6 (d) 3

57. Find the remainder when $50^{56^{52}}$ is divided by 11.
(a) 7 (b) 5 (c) 9 (d) 10

58. Find the remainder when $33^{34^{35}}$ is divided by 7.
(a) 5 (b) 4 (c) 6 (d) 2

59. Let $S_m$ denote the sum of the squares of the first $m$ natural numbers. For how many values of $m < 100$, is $S_m$ a multiple of 4?
(a) 50 (b) 25 (c) 36 (d) 24

60. For the above question, for how many values will the sum of cubes of the first $m$ natural numbers be a multiple of 5 (if $m < 50$)?
(a) 20 (b) 21
(c) 22 (d) None of these

61. How many integer values of $x$ and $y$ satisfy the expression $4x + 7y = 3$ where $|x| < 1000$ and $|y| < 1000$.
(a) 284 (b) 285
(c) 286 (d) None of these

## Hints and Solutions II

1. If $a$ and $b$ are two numbers, then their $AM = (a + b)/2$ and $GM = (ab)^{0.5}$. Use the options to answer the question.
2. Use the principal of counting given in the theory of the chapter. Start with 101 numbers and reduce all the numbers which are divisible with 2, 3 and 5. Ensure that there is no double counting in this process.
4. Find the units digits individually and subtract.
5. $(10^{25} - 7) - (10^{24} + x) = 10^{24}(10 - 1) - (7 + x) \rightarrow$ for this expression to be divisible by 3, the value of $x$ has to be 2.
6. Solve using options.
7. Solve using options
8. $12^{55}/3^{11} = 3^{44}.4^{55} \rightarrow$ 4 as units place.
Similarly, $8^{48}/16^{18} = 2^{72} \rightarrow$ 6 as the units place.
Hence, 0 is the answer.
9. Use the formula for the sum of a GP, with $a = 1$ and $r = 2$.
10. This is a property of the number 99.
11. The value of $b$ has to be 2 since, $r = 2y$. Hence, option $d$ is the only choice.
12. Solve through options to get integral value and fractional value. Remember that –7.91 has an integral value of –8.
13. $16^5 + 2^{15} = 2^{20} + 2^{15} = 2^{15}(2^5 + 1) \rightarrow$ Hence, is divisible by 33.
14. For $A + X$ to be $X$, the only possible situation is that the value of a should be either 0 or 9.

15&16. Use the method of physical counting of all possible values of $x$.

17. Use options and try to fit in the values one by one.
18. Check through options.
19. The two modulus values have to add up to 4 together. This can happen by (0 + 4), (1 + 3), (2 + 2), (3 + 1) or (4 + 0). Find the individual values of $x$ and $y$ for which these set of values are satisfied.
20. The last digit of $3^{32}$ is 1. Also, any number having the unit's digit as 1, will give 2 at the last place in the quotient.
21. Use standard formulae for Am and GM.
25. This is a property of the number 37. Besides, the addition is odd + odd = even. Hence, it will be divisible by 2. You don't need to check for 3 to mark the answer as (d).

26–35. Solve through options.

36. The number of zeroes depends on the number of 5's and the number of 2's, whichever is less. Here, the constraint is the number of 2's and not 5's (the usual case).
37. Check the powers of 5 and $3^2$ contained in 123! The lower value amongst these will be the answer.
38. Solve through options.
39. Find the largest prime factor contained in 990 and check its factorial value for divisibility by 990.
40. Check for different positive and negative values of $x$ and $y$ according to the options.

41–56. Write simple equations for each of the questions and solve.

57. 110.109.108.107 is obviously smaller than 109.110.112.113 and also $111^4$.
Hence, it is the lowest. Also, $111^4 >$ 109.110.112.113 (property of squares)

58. Both $x$ and $y$ should be highest for $xy$ to be maximum. Similarly $x$ should be minimum and $y$ should be maximum for $x/y$ to be minimum.
59. Resolve to a common base for comparing.

## Hints and Solutions III

1–5. Solve through options.
6. Use AP with first term 104 and last term 999 and common difference 5.
7. Find the first 2 digit number which gives a remainder of 3 when divided by 7 and then find the largest such number (10 and 94 respectively). Use Arithmetic Progression formulae to add the numbers.
8. Use AP with first term 105 and last term 995 and common difference is 10.
10. The cubes of the numbers are $x-3$, $x+2$ and $x+3$. Use options and you will see that (a) is the answer.
11. $(x^2-y^2)=45$. i.e. $(x-y)(x+y)=45$. The factors of 45 possible are, 15, 3; 9, 5 and 45, 1.
Hence, the numbers are 9 and 6, or 7 and 2 or 23 and 22.
12–18. Use options to check the given conditions.
19. The answer will be 50 since, 125*122 will give 50 as the last two digits.
20. The remainder theorem is to be used.
21. The unit's digit will be $1 \times 5 = 5$ (no carry over.) The tens digit will be ( 4*1 + 5*2) = 4 (carry over 1). The hundreds digit will be (3*1 + 4*2 + 5*1) = 6 + 1 (carried over) = 7. Hence, answer is 745.
22–25. Use options to solve.
26. Use the rule of indices and remainder theorem.
27. Options are not provided as it is an LOD 3 question. If they were there you should have used options.
28. Use trial and error
29. Use options.
30. Use remainder theorem and look at patterns by applying the rules of indices.
We get the value as:

$$5 \xrightarrow{\frac{32.32.32..........32 \text{ times}}{9}}$$

$$7 \xrightarrow{\frac{32.32.32..........31 \text{ times}}{9}}$$

$$4 \xrightarrow{\frac{32.32.32..........30 \text{ times}}{9}}$$

$$7 \xrightarrow{\frac{32.32....29 \text{ times}}{9}}$$

$\rightarrow$ Looking at the pattern we will get 4 as the final remainder.
31. Use the remainder theorem and get the remainder as: $1 \times 2 \times 4 \times 4 \times 4/7 = 128/7 \rightarrow 2$ is the remainder. 32.
32. $2^{100}/3 = (2^4)^{25}/3 \rightarrow 1$.
33. Use the remainder theorem and try finding the patterns.
34. Find the last digit of the number got by adding $1^2 + 2^2 + \ldots 9^2$ (you will get 5 here). Then multiply by 10 to get zero as the answer.
39. Use remainder theorem and look at patterns by applying the rules of indices.

## Thought Processes, Solutions and Short Cuts to Select Questions

### LOD 1

3. The two numbers should be factors of 405. A factor search will yield the factors. (look only for 2 digit factors of 405 with sum of digits between 1 to 19).
Also $405 = 5 \times 3^4$. Hence: $15 \times 27$
$45 \times 9$ are the only two options.
From these factors pairs only the second pair gives us the desired result.
i.e. Number × sum of digits = 405.
Hence, the answer is 45.
26. The sides of the pentagon being 1422, 1737, 2160, 2214 and 2358, the least difference between any two numbers is 54. Hence, the correct answer will be a factor of 54.
Further, since there are some odd numbers in the list, the answer should be an odd factor of 54.
Hence, check with 27, 9 and 3 in that order. You will get 9 as the HCF.
51. When the birds sat one on a branch, there was one extra bird. When they sat 2 to a branch one branch was extra.
To find the number of branches, go through options.
Checking option (a)
If three were 3 branches, there would be 4 birds. (this would leave one bird without branch as per the question.)

# ANSWER KEY

**Number System LOD I**

| | |
|---|---|
| 1. | a |
| 2. | a |
| 3. | b |
| 4. | b |
| 5. | a |
| 6. | c |
| 7. | e |
| 8. | d |
| 9. | d |
| 10. | e |
| 11. | a |
| 12. | c |
| 13. | d |
| 14. | d |
| 15. | b |
| 16. | c |
| 17. | b |
| 18. | d |
| 19. | a |
| 20. | b |
| 21. | d |
| 22. | a |
| 23. | b |
| 24. | c |
| 25. | d |
| 26. | d |
| 27. | c |
| 28. | d |
| 29. | b |
| 30. | e |
| 31. | a |
| 32. | b |
| 33. | c |
| 34. | d |
| 35. | c |
| 36. | b |
| 37. | d |
| 38. | e |
| 39. | b |
| 40. | b |
| 41. | a |
| 42. | a |
| 43. | a |
| 44. | d |
| 45. | c |
| 46. | e |
| 47. | b |
| 48. | c |
| 49. | c |
| 50. | d |
| 51. | a |
| 52. | a |
| 53. | b |
| 54. | c |
| 55. | d |
| 56. | b |
| 57. | a |
| 58. | b |
| 59. | d |
| 60. | d |
| 61. | a |
| 62. | a $\rightarrow (21)^{12}$<br>b $\rightarrow (0.8)^3$ |
| 63. | b |
| 64. | 1. $\rightarrow$ 0, 2, 4, 6, 8<br>2. $\rightarrow$ 1, 4, 7<br>3. $\rightarrow$ 0, 4, 8<br>4. $\rightarrow$ 0, 5<br>5. $\rightarrow$ 4<br>6. $\rightarrow$ 7<br>7. $\rightarrow$ 0 |
| 65. | LCM $\rightarrow$ 17010<br>HCF $\rightarrow$ 27<br>LCM $\rightarrow$ 2340<br>HCF $\rightarrow$ 13<br>LCM $\rightarrow$ 245700<br>HCF $\rightarrow$ 30 |
| 66. | b |
| 67. | a |
| 68. | b |
| 69. | c |
| 70. | c |
| 71. | a |
| 72. | d |
| 73. | b |
| 74. | b |
| 75. | a |
| 76. | b |
| 77. | d |
| 78. | a |
| 79. | a |
| 80. | a |
| 81. | c |
| 82. | d |
| 83. | c |
| 84. | c |
| 85. | c |
| 86. | a |
| 87. | c |
| 88. | b |
| 89. | b |
| 90. | b |
| 91. | d |
| 92. | d |
| 93. | a |
| 94. | c |
| 95. | 2 |

**Number System LOD III**

| | | |
|---|---|---|
| 1. a | 33. c | |
| 2. b | 34. a | |
| 3. d | 35. a | |
| 4. a | 36. c | |
| 5. b | 37. a | |
| 6. b | 38. a | |
| 7. b | 39. a | |
| 8. d | 40. d | |
| 9. e | 41. a | |
| 10. a | 42. d | |
| 11. c | 43. c | |
| 12. a | 44. b | |
| 13. e | 45. c | |
| 14. b | 46. a | |
| 15. e | 47. a | |
| 16. e | 48. c | |
| 17. b | 49. c | |
| 18. c | 50. d | |
| 19. b | 51. c | |
| 20. d | 52. b | |
| 21. b | 53. d | |
| 22. b | 54. a | |
| 23. a | 55. c | |
| 24. c | 56. a | |
| 25. a | 57. b | |
| 26. a | 58. d | |
| 27. d | 59. d | |
| 28. e | 60. a | |
| 29. e | 61. a | |
| 30. a | | |
| 31. d | | |
| 32. c | | |

# 2

# PROGRESSIONS

The chapter on progressions essentially yields common-sense based questions in examinations.

Questions in the CAT and other aptitude exams mostly appear from either Arithmetic Progressions (more common) or from Geometric Progressions.

The chapter of progressions is a logical and natural extension of the chapter on Number Systems, since there is such a lot of commonality of logic between the problems associated with these two chapters. As already stated Block 1 of the six blocks of Chapters in QA accounts for anything between 12–18 marks in the CAT. This pattern has been consistently observed over the past decade.

## ARITHMETIC PROGRESSIONS

Quantities are said to be in arithmetic progression when they increase or decrease by a common difference.

Thus each of the following series forms an arithmetic progression:

$$3, 7, 11, 15, \ldots$$
$$8, 2, -4, -10, \ldots$$
$$a, a + d, a + 2d, a + 3d, \ldots$$

*The common difference is found by subtracting any term of the series from the next term.*

That is, common difference of an $AP = (t_N - t_{N-1})$.

In the first of the above examples the common difference is 4; in the second it is −6; in the third it is $d$.

If we examine the series $a, a + d, a + 2d, a + 3d, \ldots$ we notice that *in any term the coefficient of d is always less by one than the position of that term in the series.*

Thus the $r$th term of an arithmetic progression is given by $T_r = a + (r - 1)d$.

If $n$ be the number of terms, and if $L$ denotes the last term or the $n$th term, we have

$$\boxed{L = a + (n - 1)d}$$

### To Find the Sum of the given Number of Terms in an Arithmetic Progression

Let $a$ denote the first term $d$, the common difference, and $n$ the total number of terms. Also, let $L$ denote the last term, and $S$ the required sum; then

$$S = \frac{n(a + L)}{2} \tag{1}$$

$$L = a + (n - 1)d \tag{2}$$

$$S = \frac{n}{2} \times [2a + (n - 1)d] \tag{3}$$

If any two terms of an arithmetical progression be given, the series can be completely determined; for this data results in two simultaneous equations, the solution of which will give the first term and the common difference.

When three quantities are in arithmetic progression, the middle one is said to be the **arithmetic mean** of the other two.

Thus $a$ is the arithmetic mean between $a - d$ and $a + d$. So, when it is required to arbitrarily consider three numbers in $AP$ take $a - d$, $a$ and $a + d$ as the three numbers as this reduces one unknown thereby making the solution easier.

### To Find the Arithmetic Mean between any Two Given Quantities

Let $a$ and $b$ be two quantities and $A$ be their arithmetic mean. Then since $a, A, b$, are in $AP$. We must have

$$b - A = A - a$$

Each being equal to the common difference;

This gives us $A = \dfrac{(a+b)}{2}$

Between two given quantities it is always possible to insert any number of terms such that the whole series thus formed shall be in *AP*. The terms thus inserted are called the **arithmetic means**.

## To Insert a given Number of Arithmetic Means between Two Given Quantities

Let $a$ and $b$ be the given quantities and $n$ be the number of means.

Including the extremes, the number of terms will then be $n + 2$ so that we have to find a series of $n + 2$ terms in *AP*, of which $a$ is the first, and $b$ is the last term.

Let $d$ be the common difference;

then $b$ = the $(n + 2)$th term $= a + (n + 1)d$

Hence, $d = \dfrac{(b-a)}{(n+1)}$

and the required means are

$$a + \frac{(b-a)}{n+1},\ a + \frac{2(b-a)}{n+1},\ \ldots\ a + \frac{n(b-a)}{n+1}$$

Till now we have studied APs in their mathematical context. This was important for you to understand the basic mathematical construct of A.Ps. However, you need to understand that questions on A.P. are seldom solved on a mathematical basis, (Especially under the time pressure that you are likely to face under the CAT and other aptitude exams). In such situations the mathematical processes for solving progressions based questions are likely to fail. Hence, understanding the following logical aspects about Arithmetic Progressions is likely to help you solve questions based on APs in the context of an aptitude exam.

Let us look at these issues one by one:

**1. Process for finding the nth term of an A.P:**
Suppose you have to find the 17th term of the
A.P. 3, 7, 11.......
The conventional mathematical process for this question would involve using the formula.

$$T_n = a + (n - 1)\ d$$

Thus, for the 17th term we would do

$$T_{17} = 3 + (17 - 1) \times 4 = 3 + 16 \times 4 = 67$$

Most students would mechanically insert the values for $a$, $n$ and $d$ and get this answer.

However, if you replace the above process with a thought algorithm, you will get the answer much faster. The algorithm goes like this:

In order to find the 17th term of the above sequence add the common difference to the first term, sixteen times. (**Note:** Sixteen, since it is one less than 17).
Similarly, in order to find the 37th term of the A.P. 3, 11 ...., All you need to do is add the common difference (8 in this case), 36 times.

Thus, the answer is 288 + 3 = 291.
(**Note:** You ultimately end up doing the same thing, but you are at an advantage since the entire solution process is reactionary.)

**2. Average of an A.P. and Corresponding terms of the A.P:**
Consider the A.P., 2, 6, 10, 14, 18, 22. If you try to find the average of these six numbers you will get: Average = 2 + 6 + 10 + 14 + 18 + 22/6 = 12
Notice that 12 is also the average of the first and the last terms of the A.P. In fact, it is also the average of 6 and 18 (which correspond to the second and 5th terms of the A.P.). Further, 12 is also the average of the 3rd and 4th terms of the A.P.
(**Note:** In this A.P. of six terms, the average was the same as the average of the 1st and 6th terms. It was also given by the average of the 2nd and the 5th terms, as well as that of the 3rd and 4th terms. )
We can call each of these pairs as "CORRESPONDING TERMS" in an A.P.

**What you need to understand is that every AP has an average.**
**And for any A.P., the average of any pair of corresponding terms will also be the average of the A.P.**
If you try to notice the sum of the term numbers of the pair of corresponding terms given above:

1st and 6th (so that 1 + 6 = 7)
2nd and 5th(hence, 2 + 5 =7)
3rd and 4th (hence, 3 + 4 = 7)

**Note that:** In each of these cases, the sum of the term numbers for the terms in a corresponding pair is one greater than the number of terms of the A.P.

This rule will hold true for all A.P.s.
For example, if an A.P. has 23 terms then for instance, you can predict that the 7th term will have the 17th term as its corresponding term, or for that matter the 9th term will

have the 15th term as its corresponding term. (Since 24 is one more than 23 and 7 + 17 = 9 + 15 = 24.)

**3. Process for Finding the Sum of an A.P.**

Once you can find a pair of corresponding terms for any A.P., you can easily find the sum of the A.P. by using the property of averages:

i.e. Sum = Number of terms × Average.

In fact, this is the best process for finding the sum of an A.P. It is much more superior than the process of finding the sum of an A.P. using the expression $\frac{n}{2}(2a+(n-1)d)$.

**4. Finding the common difference of an A.P., given 2 terms of an A.P.**

Suppose you were given that an A.P. had its 3rd term as 8 and its 8th term as 28. You should visualize this A.P. as

–, –, 8, –, –, –, –, 28.

From the above figure, you can easily visualize that to move from the third term to the eighth term, (8 to 28) you need to add the common difference five times. The net addition being 20, the common difference should be 4.

*Illustration*: Find the sum of an A.P. of 17 terms, whose 3rd term is 8 and 8th term is 28.

*Solution*: Since we know the third term and the eighth term, we can find the common difference as 4 by the process illustrated above.

The total = 17 × Average of the A.P.

Our objective now shifts into the finding of the average of the A.P. In order to do so, we need to identify either the 10th term (which will be the corresponding term for the 8th term) or the 15th term (which will be the corresponding term for the 3rd term.)

Again: Since the 8th term is 28 and $d = 4$, the 10th term becomes 28 + 4 + 4 = 36.

Thus, the average of the A.P. = Average of 8th and 10th terms

= (28 + 36)/2 = 32.

Hence, the required answer is sum of the A.P. = 17 × 32 = 544.

The logic that has applied here is that the difference in the term numbers will give you the number of times the common difference is used to get from one to the other term.

For instance, if you know that the difference between the 7th term and 12th term of an AP is –30, you should realize that 5 times the common difference will be equal to –30. (Since 12 – 7 = 5).

Hence, $d = -6$.

**Note:** Replace this algorithmic thinking in lieu of the mathematical thinking of:

$$12^{th} \text{ term} = a + 11d$$
$$7^{th} \text{ term} = a + 6d$$
$$\text{Hence, difference} = -30 = (a + 11d) - (a + 6d)$$
$$-30 = 5d$$
$$\therefore \quad d = -6.$$

**5. Types of APs:Increasing and Decreasing A.P.s.**

Depending on whether '$d$' is positive or negative, an A.P. can be increasing or decreasing.

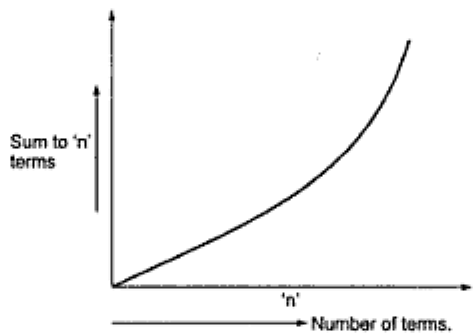
Let us explore these two types of A.P.s further:

**(A) Increasing A.P.s:**

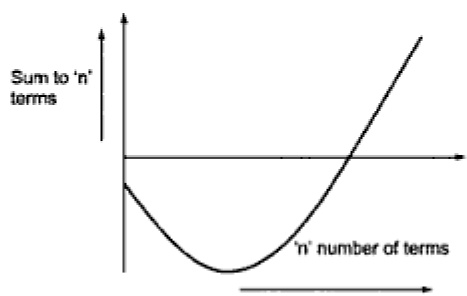
Every term of an increasing AP is greater than the previous term.

Depending on the value of the first term, we can construct two graphs for sum of an increasing A.P.

**Case 1:** When the first term of the increasing A.P. is positive. In such a case the sum of the A.P. will show a continuously increasing graph which will look like the one shown in the figure below:




**Case 2:** When the first term of the increasing A.P. is negative. In such a case, the Sum of the A.P. plotted against the number of terms will give the following figure:



### To Insert a given Number of Geometric Means between Two Given Quantities

Let $a$ and $b$ be the given quantities and $n$ the required number of means to be inserted. In all there will be $n + 2$ terms so that we have to find a series of $n + 2$ terms in GP of which $a$ is the first and $b$ the last.

Let $r$ be the common ratio;

Then $\quad b = \text{the } (n + 2)\text{th term} = ar^{n+1};$

$$\therefore \quad r^{(n+1)} = \frac{b}{a}$$

$$\therefore \quad r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}} \qquad (1)$$

Hence the required number of means are $ar, ar^2, \ldots ar^n$, where $r$ has the value found in (1).

### To Find the Sum of a Number of Terms in a Geometric Progression

Let $a$ be the first term, $r$ the common ratio, $n$ the number of terms, and $S_n$ be the sum to $n$ terms.

If $r > 1$, then

$$S_n = \frac{a(r^n - 1)}{(r - 1)} \qquad (1)$$

If $r < 1$, then

$$S_n = \frac{a(1 - r^n)}{(1 - r)} \qquad (2)$$

*Note:* It will be convenient to remember both forms given above for $S$. Number (2) will be used in all cases except when $r$ is positive and greater than **one**.

Sum of an infinite geometric progression when $r < 1$

$$S_\infty = \frac{a}{(1 - r)}$$

Obviously, this formula is used only when the common ratio of the GP is less than one.

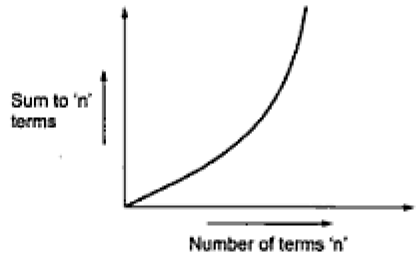
Similar to APs, GPs can also be logically viewed. Based on the value of the common ratio and its first term a G.P. might have one of the following structures:

**(1) Increasing GPs type 1:**

A G.P. with first term positive and common ratio greater than 1. This is the most common type of G.P.

e.g: 3, 6, 12, 24…(A G.P. with first term 3 and common ratio 2)

The plot of the sum of the series with respect to the number of terms in such a case will appear as follows:



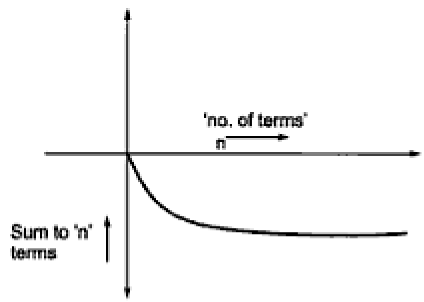

**(2) Increasing GPs type 2:**

A G.P. with first term negative and common ratio less than 1.

e.g: –8, –4,–2, –1, – ……

As you can see in this GP all terms are greater than their previous terms.

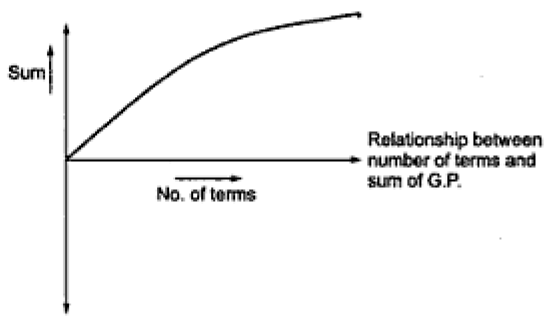
[The following figure will illustrate the relationship between the number of terms and the sum to '$n$' terms in this case]




**(3) Decreasing G.Ps type 1:**

These GPs have their first term positive and common ratio less than 1.

e.g: 12, 6, 3, 1.5, 0.75 …..




**(4) Decreasing GPs type 2:**

First term negative and common ratio greater than 1.

e.g: –2, –6, –18 ….

In this case the relationship looks like.

Consider the following question which appeared in CAT 2003.

Find the infinite sum of the series:

$$1 + \frac{4}{7} + \frac{9}{7^2} + \frac{16}{7^3} + \frac{25}{7^4} + \ldots\ldots$$

(a) 27/14 (b) 21/13 (c) 49/27 (d) 256/147

*Solution:* Such questions have two alternative widely divergent processes to solve them.

The first relies on mathematics using algebraic solving. Unfortunately this process being overly mathematical requires a lot of writing and hence is not advisable to be used in an aptitude exam.

The other process is one where we try to predict the approximate value of the sum by taking into account the first few significant terms. (This approach is possible to use because of the fact that in such series we invariably reach the point where the value of the next term becomes insignificant and does not add substantially to the sum). After adding the significant terms we are in a position to guess the approximate value of the sum of the series.

Let us look at the above question in order to understand the process.

In the given series the values of the terms are:

First term = 1
Second term = 4/7 = 0.57
Third term = 9/63 = 0.14
Fourth term = 16/343 = 0.04
Fifth term = 25/2401 = 0.01

Addition upto the fifth term is approximately 1.76

Options 2 and 4 are smaller than 1.76 in value and hence cannot be correct.

That leaves us with options 1 and 3

Option 1 has a value of 1.92 approximately while option 3 has a value of 1.81 approximately.

At this point you need to make a decision about how much value the remaining terms of the series would add to 1.76 (sum of the first 5 terms)

Looking at the pattern we can predict that the sixth term will be

$$36/7^5 = 36/16807 = 0.002 \text{ (approx.)}$$

And the seventh term would be $49/7^6 = 49/117649 = 0.0004$ (approx.).

The eighth term will obviously become much smaller.

It can be clearly visualized that the residual terms in the series are highly insignificant. Based on this judgement you realize that the answer will not reach 1.92 and will be restricted to 1.81. Hence the answer will be option 3.

Try using this process to solve other questions of this nature whenever you come across them. (There are a few such questions inserted in the LOD exercises of this chapter)

## USEFUL RESULTS

1. If the same quantity be added to, or subtracted from, all the terms of an AP, the resulting terms will form an *AP*, but with the same common difference as before.
2. If all the terms of an AP be multiplied or divided by the same quantity, the resulting terms will form an AP, but with a new common difference, which will be the multiplication/division of the old common difference. (as the case may be)
3. If all the terms of a GP be multiplied or divided by the same quantity, the resulting terms will form a GP with the same common ratio as before.
4. If $a, b, c, d, \ldots$ are in GP, they are also in continued proportion, since, by definition,

   $$a/b = b/c = c/d = \ldots = 1/r$$

   Conversely, a series of quantities in continued proportion may be represented by $x, xr, xr^2, \ldots$
5. If you have to assume 3 terms in *AP*, assume them as

   $a - d, a, a + d$ or as $a, a + d$ and $a + 2d$

   For assuming 4 terms of an AP we use: $a - 3d$, $a - d$, $a + d$ and $a + 3d$

   For assuming 5 terms of an *AP*, take them as:

   $$a - 2d, a - d, a, a + d, a + 2d.$$

   These are the most convenient in terms of problems solving.
6. For assuming three terms of a GP assume them as

   $a, ar$ and $ar^2$ or as $a/r, a$ and $ar$
7. To find the sum of the first $n$ natural numbers

   Let the sum be denoted by $S$; then

   $S = 1 + 2 + 3 + \ldots + n$, is given by

   $$S = \frac{n(n+1)}{2}$$
8. To find the sum of the squares of the first $n$ natural numbers

Let the sum be denoted by $S$; then

$$S = 1^2 + 2^2 + 3^2 + \ldots + n^2$$

This is given by : $S = \left\{\frac{n(n+1)(2n+1)}{6}\right\}$

9. To find the sum of the cubes of the first $n$ natural numbers.

   Let the sum be denoted by $S$; then

$$S = 1^3 + 2^3 + 3^3 + \ldots + n^3$$

$$S = \left[\frac{n(n+1)}{2}\right]^2$$

   Thus, the sum of the cubes of the first $n$ natural numbers is equal to the square of the sum of these numbers.

10. To find the sum of the first $n$ odd natural numbers.

$$S = 1 + 3 + 5 + \ldots + (2n - 1) \rightarrow n^2$$

11. To find the sum of the first $n$ even natural numbers.

$$S = 2 + 4 + 6 + \ldots + 2n \rightarrow n(n + 1)$$
$$= n^2 + n$$

12. To find the sum of odd numbers $\leq n$ where $n$ is a natural number:

   *Case A:* If $n$ is odd $\rightarrow [(n + 1)/2]^2$
   *Case B:* If $n$ is even $\rightarrow [n/2]^2$

13. To find the sum of even numbers $\leq n$ where $n$ is a natural number:

   *Case A:* If $n$ is odd $\rightarrow \{(n/2)[(n/2) + 1]\}$
   *Case B:* If $n$ is even $\rightarrow [(n - 1)/2][(n + 1)/2]$

14. Number of terms in a count:

- If we are counting in steps of 1 from $n_1$ to $n_2$ including both the end points, we get $(n_2 - n_1) + 1$ numbers.
- If we are counting in steps of 1 from $n_1$ to $n_2$ including only one end, we get $(n_2 - n_1)$ numbers.
- If we are counting in steps of 1 from $n_1$ to $n_2$ excluding both ends, we get $(n_2 - n_1) - 1$ numbers.

*Example:* Between 16 and 25 both included there are $9 + 1 = 10$ numbers.

Between 100 and 200 both excluded there are $100 - 1 = 99$ numbers.

- If we are counting in steps of 2 from $n_1$ to $n_2$ including both the end points, we get $[(n_2 - n_1)/2] + 1$ numbers.
- If we are counting in steps of 2 from $n_1$ to $n_2$ including only one end, we get $[(n_2 - n_1)/2]$ numbers.
- If we are counting in steps of 2 from $n_1$ to $n_2$ excluding both ends, we get $[(n_2 - n_1)/2] - 1$ numbers.

*Example:* Number of even numbers between 20 and 100 (both included) is $100 - 20 \rightarrow (80/2) + 1 = 41$

- If we are counting in steps of 3 from $n_1$ to $n_2$ including both the end points, we get $[(n_2 - n_1)/3] + 1$ numbers.
- If we are counting in steps of 3 from $n_1$ to $n_2$ including only one end, we get $[(n_2 - n_1)/3]$ numbers.
- If we are counting in steps of 3 from $n_1$ to $n_2$, excluding both ends, we get $[(n_2 - n_1)/3] - 1$ numbers.

*Example:* Number of numbers between 100 and 200 divisible by three.

*Solution:* The first number is 102 and the last number is 198. Hence, answer = $(96/3) + 1 = 33$ (since both 102 and 198 are included).

Alternately, highest number below 100 that is divisible by 3 is 99, and the lowest number above 200 which is divisible by 3 is 201.

Hence, $201 - 99 = 102 \rightarrow 102/3 = 34 \rightarrow$ Answer = $34 - 1 = 33$ (Since both ends are not included.)

**In general**

- If we are counting in steps of $x$ from $n_1$ to $n_2$ including both the end points, we get $[(n_2 - n_1)/x] + 1$ numbers.
- If we are counting in steps of "$x$" from $n_1$ to $n_2$ including only one end, we get $(n_2 - n_1)/x$ numbers.
- If we are counting in steps of "$x$" from $n_1$ to $n_2$ excluding both ends, we get $[(n_2 - n_1)/x] - 1$ numbers.

For instance, if we have to find how many terms are there in the series 107, 114, 121, 128 ... 254, then we have

$(254 - 107)/7 + 1 = 147/7 + 1 = 21 + 1 = 22$ terms in the series

*Of course, an appropriate adjustment will have to be made when $n_2$ does not fall into the series. This will be done as follows:*

For instance, if we have to find how many terms of the series 107, 114, 121, 128 ... are below 258, then we have by the formula:

$(258 - 107)/7 + 1 = 151/7 + 1 = 21.57 + 1 = 22.57$. This will be adjusted by taking the lower integral value = 22. → The number of terms in the series below 258.

The student is advised to try and experiment on these principles to get a clear picture.

## WORKED-OUT PROBLEMS

**Problem 2.1** Two persons Ramu Dhobi and Kalu Mochi have joined Donkey-work Associates. Ramu Dhobi and Kalu Mochi started with an initial salary of Rs. 500 and Rs. 640 respectively with annual increments of Rs. 25 and Rs. 20 each respectively. In which year will Ramu Dhobi start earning more salary than Kalu Mochi?

**Solution** The current difference between the salaries of the two is Rs. 140. The annual rate of reduction of this dfference is Rs. 5 per year. At this rate, it will take Ramu Dhobi 28 years to equalise his salary with Kalu Dhobi's salary.

Thus, in the 29th year he will earn more.

This problem should be solved while reading and the thought process should be 140/5 = 28. Hence, answer is 29th year.

**Problem 2.2** Find the value of the expression

$1 - 6 + 2 - 7 + 3 - 8 + \ldots\ldots\ldots$ to 100 terms

(a) −250 (b) −500 (c) −450 (d) −300

**Solution** The series ($1 - 6 + 2 - 7 + 3 - 8 + \ldots\ldots\ldots$ to 100 terms) can be rewritten as:

⇒ $(1 + 2 + 3 + \ldots$ to 50 terms$) - (6 + 7 + 8 + \ldots$ to 50 terms$)$

Both these are *AP*'s with values of $a$ and $d$ as →

$a = 1$, $n = 50$ and $d = 1$ and $a = 6$, $n = 50$ and $d = 1$ respectively.

Using the formula for sum of an *AP* we get:

$$25(2 + 49) - 25(12 + 49)$$

$$\rightarrow 25(51 - 61) = -250$$

Alternatively, we can do this faster by considering $(1 - 6)$, $(2 - 7)$, and so on as one unit or one term.

$1 - 6 = 2 - 7 = \ldots = -5$. Thus the above series is equivalent to a series of fifty −5's added to each other.

So, $(1 - 6) + (2 - 7) + (3 - 8) + \ldots$ 50 terms $= -5 \times 50 = -250$

**Problem 2.3** Find the sum of all numbers divisible by 6 in between 100 to 400.

**Solution** Here 1st term = $a$ = 102 (which is the 1st term greater than 100 that is divisible by 6.

The last term less than 400, which is divisible by 6 is 396.

The number of terms in the *AP*; 102, 108, 114…396 is given by $[(396 - 102)/6] + 1 = 50$ numbers.

Common difference = $d$ = 6

So, $S = 25\,(204 + 294) = 12450$

**Problem 2.4** If $x$, $y$, $z$ are in *GP*, then $1/(1 + \log_{10}x)$, $1/(1 + \log_{10}y)$ and $1/(1 + \log_{10}z)$ will be in:

(a) AP (b) GP

(c) HP (d) Cannot be said

**Solution** Go through the options.

Checking option (a), the three will be in AP if the 2nd expression is the average of the 1st and 3rd expressions. This can be mathematically written as

$$2/(1 + \log_{10}y) = [1/(1 + \log_{10}x)] + [1/(1 + \log_{10}z)]$$

$$= \frac{[1 + (1 + \log_{10}x) + 1 + (1 + \log_{10}z)]}{[(1 + \log_{10}x)(1 + \log_{10}z)}$$

$$= \frac{[2 + \log_{10}xz]}{(1 + \log_{10}x)\,(1 + \log_{10}z)}$$

Applying our judgement, there seems to be no indication that we are going to get a solution.

Checking option (b)

$$[1/(1 + \log_{10}y)]^2 = [1/(1 + \log_{10}x)]\,[1/(1 + \log_{10}z)]$$

$$= [1/(1 + \log_{10}(x + z) + \log_{10}xz)]$$

Again we are trapped and any solution is not in sight.

Checking option (c).

$1/(1 + \log_{10}x)$, $1/(1 + \log_{10}y)$ and $1/(1 + \log_{10}z)$ are in *HP* then $1 + \log_{10}x$, $1 + \log_{10}y$ and $1 + \log_{10}z$ will be in AP.

So, $\log_{10}x$, $\log_{10}y$ and $\log_{10}z$ will also be in AP.

Hence, $2\log_{10}y = \log_{10}x + \log_{10}z$

⇒ $y^2 = xz$ which is given.

So, (c) is the correct option.

Alternatively, you could have solved through the following process.

$x$, $y$ and $z$ are given as logarithmic functions.

Assume $x = 1$, $y = 10$ and $z = 100$ as $x$, $y$, $z$ are in GP

So, $1 + \log_{10}x = 1$, $1 + \log_{10}y = 2$ and $1 + \log_{10}z = 3$

⇒ Thus we find that since 1, 2 and 3 are in AP, we can assume that

$1 + \log_{10}x$ , $1 + \log_{10}y$ and $1 + \log_{10}z$ are in AP

⇒ Hence, by definition of an HP we have that $1/(1 + \log_{10}x)$, $1/(1 + \log_{10}y)$ and $1/(1 + \log_{10}z)$ are in *HP*. Hence, option (c) is the required answer.

*Author's Note:* In my experience I have always found that the toughest equations and factorisations get solved very easily when there are options, by assuming values in place of the variables in the equation. The values of the variables should be taken in such a manner that the basic restrictions put on the variables should be respected. For example, if an expression in three variables *a*, *b* and *c* is given and it is mentioned that $a + b + c = 0$ then the values that you assume for *a*, *b* and *c* should satisfy this restriction. Hence, you should look at values like 1, 2 and –3 or 2, –1, –1 etc.

This process is especially useful in the case where the question as well as the options both contain expressions. Factorisation and advanced techniques of maths are then not required. This process will be very beneficial for students who are weak at Mathematics.]

**Problem 2.5** Find $t_{10}$ and $S_{10}$ for the following series:

1, 8, 15,...

**Solution** This is an AP with first term 1 and common difference 7.

$$t_{10} = a + (n - 1)\,d = 1 + 9 \times 7 = 64$$

$$S_{10} = \frac{n[2a + (n-1)d]}{2}$$

$$= \frac{10[21 + (10-1)7]}{2} = 325$$

Alternatively, if the number of terms is small, you can count it directly.

**Problem 2.6** Find $t_{18}$ and $S_{18}$ for the following series:

2, 8, 32, ...

**Solution** This is a GP with first term 2 and common ratio 4.

$$t_{18} = ar^{n-1} = 2.4^{17}$$

$$S_{18} = \frac{a(r^n - 1)}{r - 1} = \frac{2(4^{18} - 1)}{(4 - 1)}$$

**Problem 2.7** Is the series 1, 4,... to *n* terms an *AP*, or a GP, or an HP, or a series which cannot be determined?

**Solution** *To determine any progression, we should have at least three terms.*

If the series is an *AP* then the next term of this series will be 7

Again, if the next term is 16, then this will be a GP series (1, 4, 16 ...)

So, *we cannot determine* the nature of the progression of this series.

**Problem 2.8** Find the sum to 200 terms of the series $1 + 4 + 6 + 5 + 11 + 6 + \ldots$

(a) 30,200 (b) 29,800
(c) 30,200 (d) None of these

**Solution** Spot that the above series is a combination of two *AP*s.

The 1st *AP* is (1 + 6 + 11 +...) and the 2nd *AP* is (4 + 5 + 6 +...)

Since the terms of the two series alternate, $S = (1 + 6 + 11 + \ldots$ to 100 terms) + (4 + 5 + 6 + ... to 100 terms)

$$= \frac{100[2 \times 1 + 99 \times 5]}{2} + \frac{100[2 \times 4 + 99 \times 1]}{2} \rightarrow \text{(Using}$$

the formula for the sum of an *AP*)

$= 50[497 + 107] = 50[604] = 30200$

Alternatively, we can treat every two consecutive terms as one.

So we will have a total of 100 terms of the nature:

$$(1 + 4) + (6 + 5) + (11 + 6) \ldots \rightarrow 5, 11, 17\ldots$$

Now, $a = 5$, $d = 6$ and $n = 100$

Hence the sum of the given series is

$$S = \frac{100}{2} \times [2 \times 5 + 99 \times 6]$$
$$= 50[604] = 30200$$

**Problem 2.9** How many terms of the series –12, –9, –6,... must be taken that the sum may be 54?

**Solution** Here $S = 54$, $a = -12$, $d = 3$, $n$ is unknown and has to be calculated. To do so we use the formula for the sum of an *AP* and get.

$$54 = \frac{[2(-12) + (n-1)3]n}{2}$$

(a) Rs. 800 (b) Rs. 900
(c) Rs. 1150 (d) Rs. 1000
(e) Rs. 1200

10. A number 15 is divided into three parts which are in *AP* and the sum of their squares is 83. Find the smallest number.
(a) 5 (b) 3 (c) 6 (d) 8
(e) 7

11. The sum of the first 16 terms of an *AP* whose first term and third term are 5 and 15 respectively is
(a) 600 (b) 765 (c) 640 (d) 680
(e) 690

12. The number of terms of the series 54 + 51 + 48 +... such that the sum is 513 is
(a) 18 (b) 19
(c) Both *a* and *b* (d) 15
(e) None of these

13. The least value of *n* for which the sum of the series 5 + 8 + 11... *n* terms is not less than 670 is
(a) 20 (b) 19 (c) 22 (d) 21
(e) 18

14. A man receives Rs. 60 for the first week and Rs. 3 more each week than the preceding week. How much does he earn by the 20th week?
(a) Rs. 1770 (b) Rs. 1620
(c) Rs. 1890 (d) Rs. 1790
(e) None of these

15. How many terms are there in the GP 5, 20, 80, 320,... 20480?
(a) 6 (b) 5 (c) 7 (d) 8
(e) 9

16. A boy agrees to work at the rate of one rupee on the first day, two rupees on the second day, four rupees on the third day and so on. How much will the boy get if he starts working on the 1st of February and finishes on the 20th of February?
(a) $2^{20}$ (b) $2^{20} - 1$
(c) $2^{19} - 1$ (d) $2^{19}$
(e) None of these

17. If the fifth term of a GP is 81 and first term is 16, what will be the 4th term of the GP?
(a) 36 (b) 18 (c) 54 (d) 24
(e) 27

18. The seventh term of a GP is 8 times the fourth term. What will be the first term when its fifth term is 48?
(a) 4 (b) 3 (c) 5 (d) 2
(e) 6

19. The sum of three numbers in a GP is 14 and the sum of their squares is 84. Find the largest number.
(a) 8 (b) 6
(c) 4 (d) 12
(e) None of these

20. The first term of an arithmetic progression is unity and the common difference is 4. Which of the following will be a term of this AP?
(a) 4551 (b) 10091
(c) 7881 (d) 13531
(e) 13537

21. How many natural numbers between 300 to 500 are multiples of 7?
(a) 29 (b) 28
(c) 27 (d) 30
(e) None of these

22. The sum of the first and the third term of a geometric progression is 20 and the sum of its first three terms is 26. Find the progression.
(a) 2, 6, 18,... (b) 18, 6, 2,...
(c) Both of these (d) None of these
(e) Cannot be determined

23. If a man saves Rs. 4 more each year than he did the year before and if he saves Rs. 20 in the first year, after how many years will his savings be more than Rs. 1000 altogether?
(a) 19 years (b) 20 years
(c) 21years (d) 18 years
(e) 17 years

24. A man's salary is Rs. 800 per month in the first year. He has joined in the scale of 800–40–1600. After how many years will his savings be Rs. 64,800?
(a) 8 years (b) 7 years
(c) 6 years (d) None of these
(e) Cannot be determined

25. The 4th and 10th term of an GP are 1/3 and 243 respectively. Find the 2nd term.
(a) 3 (b) 1 (c) 1/27 (d) 1/9
(e) 9

26. The 7th and 21st terms of an AP are 6 and −22 respectively. Find the 26th term.
(a) −34 (b) −32 (c) −12 (d) −10
(e) −16

(c) 2550 (d) 2650
(e) None of these

9. Find the sum of all integers of 3 digits that are divisible by 7.
(a) 69,336 (b) 71,336
(c) 70,336 (d) 72,336
(e) None of these

10. The first and the last terms of an AP are 107 and 253. If there are five terms in this sequence, find the sum of sequence.
(a) 1080 (b) 720
(c) 900 (d) 620
(e) 1020

11. Find the value of 1 – 2 – 3 + 2 – 3 – 4 +... + upto 100 terms.
(a) –694 (b) –626
(c) –624 (d) –549
(e) –676

12. What will be the sum to $n$ terms of the series 8 + 88 + 888 +...?
(a) $\frac{8(10^n - 9n)}{81}$ (b) $\frac{8(10^{n+1} - 10 - 9n)}{81}$
(c) $8(10^{n-1} - 10)$ (d) $8(10^{n+1} - 10)$
(e) None of these

13. If $a$, $b$, $c$ are in GP, then log $a$, log $b$, log $c$ are in
(a) AP (b) GP
(c) HP (d) None of these
(e) Cannot be said

14. After striking the floor, a rubber ball rebounds to 4/5th of the height from which it has fallen. Find the total distance that it travels before coming to rest if it has been gently dropped from a height of 120 metres.
(a) 540 metres (b) 960 metres
(c) 1080 metres (d) 1020 metres
(e) 1120 metres

15. If $x$ be the first term, $y$ be the $n$th term and $p$ be the product of $n$ terms of a GP, then the value of $p^2$ will be
(a) $(xy)^{n-1}$ (b) $(xy)^n$
(c) $(xy)^{1-n}$ (d) $(xy)^{n/2}$
(e) None of these

16. The sum of an infinite GP whose common ratio is numerically less than 1 is 32 and the sum of the first two terms is 24. What will be the third term?
(a) 2 (b) 16 (c) 8 (d) 12
(e) 4

17. What will be the value of $x^{1/2}x^{1/4}x^{1/8}$... to infinity.
(a) $x^2$ (b) $x$
(c) $x^{3/2}$ (d) $x^3$
(e) None of these

18. Find the sum to $n$ terms of the series
1.2.3 + 2.3.4 + 3.4.5 +...
(a) $(n + 1)(n + 2)(n + 3)/3$
(b) $n(n + 1)(2n + 2)(n + 2)/4$
(c) $n(n + 1)(n + 2)$
(d) $n(n + 1)(n + 2)(n + 3)/4$
(e) None of these

19. Determine the first term of the geometric progression, the sum of whose first term and third term is 40 and the sum of the second term and fourth term is 80.
(a) 12 (b) 16 (c) 8 (d) 4
(e) 6

20. Find the second term of an AP if the sum of its first five even terms is equal to 15 and the sum of the first three terms is equal to –3.
(a) –3 (b) –2 (c) –1 (d) 0
(e) 1

21. The sum of the second and the fifth term of an AP is 8 and that of the third and the seventh term is 14. Find the eleventh term.
(a) 19 (b) 17 (c) 15 (d) 16
(e) 18

22. How many terms of an AP must be taken for their sum to be equal to 120 if its third term is 9 and the difference between the seventh and the second term is 20?
(a) 6 (b) 9 (c) 7 (d) 8
(e) 5

23. Four numbers are inserted between the numbers 4 and 39 such that an AP results. Find the biggest of these four numbers.
(a) 31.5 (b) 31 (c) 32 (d) 30
(e) 33

24. Find the sum of all three-digit natural numbers, which on being divided by 5, leave a remainder equal to 4.
(a) 57,270 (b) 96,780
(c) 49,680 (d) 99,270
(e) 90270

25. The sum of the first three terms of the arithmetic progression is 30 and the sum of the squares of the first term and the second term of the same progression is

116. Find the seventh term of the progression if its fifth term is known to be exactly divisible by 14.

(a) 36 (b) 40 (c) 43 (d) 22

(e) 24

26. $A$ and $B$ set out to meet each other from two places 165 km apart. $A$ travels 15 km the first day, 14 km the second day, 13 km the third day and so on. $B$ travels 10 km the first day, 12 km the second day, 14 km the third day and so on. After how many days will they meet?

(a) 8 days (b) 5 days

(c) 6 days (d) 7 days

(e) 9 days

27. If a man saves Rs. 1000 each year and invests at the end of the year at 5% compound interest, how much will the amount be at the end of 15 years?

(a) Rs. 21,478 (b) Rs. 21,578

(c) Rs. 22,578 (d) Rs. 22,478

(e) Rs. 22178

28. If sum to $n$ terms of a series is given by $(n + 8)$, then its second term will be given by

(a) 10 (b) 9

(c) 8 (d) 1

(e) None of these

29. If $A$ is the sum of the $n$ terms of the series $1 + 1/4 + 1/16 + \ldots$ and $B$ is the sum of $2n$ terms of the series $1 + 1/2 + 1/4 + \ldots$, then find the value of $A/B$.

(a) 1/3 (b) 1/2 (c) 2/3 (d) 3/4

(e) 4/5

30. A man receives a pension starting with Rs. 100 for the first year. Each year he receives 90% of what he received the previous year. Find the maximum total amount he can receive even if he lives forever.

(a) Rs. 1100 (b) Rs. 1000

(c) Rs. 1200 (d) Rs. 900

(e) Rs. 1250

31. The sum of the series represented as:

$1/1\times5 + 1/5\times9 + 1/9\times13 \ldots\ldots + 1/221\times225$

is

(a) 28/221 (b) 56/221

(c) 56/225 (d) None of these

32. The sum of the series

$1/(\sqrt{2} + \sqrt{1}) + 1/(\sqrt{2} + \sqrt{3}) + \ldots\ldots\ldots\ldots + 1/(\sqrt{120} + \sqrt{121})$ is:

(a) 10 (b) 11

(c) 12 (d) None of these

33. Find the infinite sum of the series $1/1 + 1/3 + 1/6 + 1/10 + 1/15 \ldots$

(a) 2 (b) 2.25

(c) 3 (d) 4

34. The sum of the series $5 \times 8 + 8 \times 11 + 11 \times 14$ upto $n$ terms will be:

(a) $(n + 1)[3(n + 1)^2 + 6(n + 1) + 1] - 10$

(b) $(n + 1)[3(n + 1)^2 + 6(n + 1) + 1] + 10$

(c) $(n + 1)[3(n + 1) + 6(n + 1)^2 + 1] - 10$

(d) $(n + 1)[3(n + 1) + 6(n + 1)^2 + 1] + 10$

35. The sum of the series: $½ + 1/6 + 1/12 + 1/20 + \ldots 1/156 + 1/182$ is:

(a) 12/13 (b) 13/14

(c) 14/13 (d) None of these

36. For the above question 35, what is the sum of the series if taken to infinite terms:

(a) 1.1 (b) 1

(c) 14/13 (d) None of these

***Directions for Questions 37-39:*** Answer the questions based on the following information.

There are 250 integers $a_1, a_2 \ldots\ldots a_{250}$, not all of them necessarily different. Let the greatest integer of these 250 integers be referred to as Max, and the smallest integer be referred to as Min. The integers $a_1$ through $a_{124}$ form sequence $A$, and the rest form sequence $B$. Each member of $A$ is less than or equal to each member of $B$.

37. All values in $A$ are changed in sign, while those in $B$ remain unchanged. Which of the following statements is true?

(a) Every member of $A$ is greater than or equal to every member of $B$.

(b) Max is in $A$.

(c) If all numbers originally in $A$ and $B$ had the same sign, then after the change of sign, the largest number of $A$ and $B$ is in $A$.

(d) None of these

38. Elements of $A$ are in ascending order, and those of $B$ are in descending order. $a_{124}$ and $a_{125}$ are interchanged. Then which of the following statements is true?

(a) A continues to be in ascending order.

(b) B continues to be in descending order.

(c) A continues to be in ascending order and 52 in descending order.

(d) None of the above

39. Every element of $A$ is made greater than or equal to every element of $B$ by adding to each element of $A$ an integer $x$. Then, $x$ cannot be less than:

(a) $2^{10}$
(b) the smallest value of $B$
(c) the largest value of $B$
(d) (Max-Min)

40. Rohit drew a rectangular grid of 529 cells, arranged in 23 Rows and 23 columns, and filled each cell with a number. The numbers with which he filled each cell were such that the numbers of each row taken from left to right formed an arithmetic series and the numbers of each column taken from top to bottom also formed an arithmetic series. The seventh and the seventeenth numbers of the fifth row were 47 and 63 respectively, while the seventh and the seventeenth numbers of the fifteenth row were 53 and 77 respectively. What is the sum of all the numbers in the grid?
(a) 32798 (b) 65596
(c) 52900 (d) None of these

41. How many three digit numbers have the property that their digits taken from left to right form an Arithmetic or a Geometric Progression?
(a) 15 (b) 18
(c) 20 (d) None of these

***Directions for Questions 42 and 43:*** These questions are based on the following data.

At Burger King—a famous fast food centre on Main Street in Pune, burgers are made only on an automatic burger making machine. The machine continuously makes different sorts of burgers by adding different sorts of fillings on a common bread. The machine makes the burgers at the rate of 1 burger per half a minute. The various fillings are added to the burgers in the following manner. The 1st, 5th, 9th, ...... burgers are filled with a chicken patty; the 2nd, 9th, 16th, ........ burgers with vegetable patty; the 1st, 5th, 9th, ...... burgers with mushroom patty; and the rest with plain cheese and tomato fillings.

The machine makes exactly 660 burgers per day.

42. How many burgers per day are made with cheese and tomato as fillings?
(a) 424 (b) 236
(c) 237 (d) None of these

43. How many burgers are made with all three fillings Chicken, vegetable and mushroom?
(a) 23 (b) 24
(c) 25 (d) 26

44. An arithmetic progression P consists of $n$ terms. From the progression three different progressions $P_1$ $P_2$ and $P_3$ are created such that $P_1$ is obtained by the 1st, 4th ,7th...... terms of $P$, $P_2$ has the 2nd, 5th, 8th,.......terms of $P$ and $P_3$ has the 3rd, 6th, 9th,.......terms of P. It is found that of $P_1$, $P_2$ and $P_3$ two progressions have the property that their average is itself a term of the original Progression $P$. Which of the following can be a possible value of $n$?
(a) 20 (b) 26
(c) 36 (d) Both 1 and 2

45. For the above question, if the Common Difference between the terms of $P_1$ is 6, what is the common difference of $P$?
(a) 2 (b) 3
(c) 6 (d) Cannot be determined

## Level of Difficulty (LOD) III

1. If in any decreasing arithmetic progression, sum of all its terms, except for the first term, is equal to –36, the sum of all its terms, except for the last term, is zero, and the difference of the tenth and the sixth term is equal to – 16, then what will be first term of this series?
(a) 16 (b) 20 (b) –16 (d) –20
(e)–24

2. The sum of all terms of the arithmetic progression having ten terms except for the first term, is 99, and except for the sixth term, 89. Find the third term of the progression if the sum of the first and the fifth term is equal to 10.
(a) 15 (b) 5
(c) 8 (d) 10
(e) None of these

3. Product of the fourth term and the fifth term of an arithmetic progression is 456. Division of the ninth term by the fourth term of the progression gives quotient as 11 and the remainder as 10. Find the first term of the progression.
(a) – 52 (b) – 42 (c) – 56 (d) – 66
(e) –50

4. A number of saplings are lying at a place by the side of a straight road. These are to be planted in a straight line at a distance interval of 10 meters between two consecutive saplings. Mithilesh, the country's greatest forester, can carry only one sapling at a time and has to move back to the original point to get the next sap-

ling. In this manner he covers a total distance of 1.32 kms. How many saplings does he plant in the process if he ends at the starting point?

(a) 15 (b) 14 (c) 13 (d) 12
(e) 10

5. A geometric progression consists of 500 terms. Sum of the terms occupying the odd places is $P_1$ and the sum of the terms occupying the even places is $P_2$. Find the common ratio.

(a) $P_2/P_1$ (b) $P_1/P_2$
(c) $P_2 + P_1/P_1$ (d) $P_2 + P_1/P_2$
(e) None of these

6. The sum of the first ten terms of the geometric progression is $S_1$ and the sum of the next ten terms (11th through 20th) is $S_2$. Find the common ratio.

(a) $(S_1/S_2)^{1/10}$ (b) $-(S_1/S_2)^{1/10}$
(c) $\pm \sqrt[10]{S_2/S_1}$ (d) $(S_1/S_2)^{1/5}$
(e) None of these

7. The first and the third terms of an arithmetic progression are equal, respectively, to the first and the third term of a geometric progression, and the second term of the arithmetic progression exceeds the second term of the geometric progression by 0.25. Calculate the sum of the first five terms of the arithmetic progression if its first term is equal to 2.

(a) 2.25 or 25 (b) 2.5 or 27.5
(c) 1.5 (d) 3.25
(e) None of these

8. If (2 + 4 + 6 +... 50 terms)/(1 + 3 + 5 +... $n$ terms) = 51/2, then find the value of $n$ .

(a) 12 (b) 13 (c) 9 (d) 10
(e) 11

9. (666.... $n$ digits)$^2$ + (888... $n$ digits) is equal to

(a) $(10^n - 1) \times \frac{4}{9}$ (b) $(10^{2n} - 1) \times \frac{4}{9}$
(c) $\frac{4(10^n - 10^{n-1} - 1)}{9}$ (d) $\frac{4(10^n + 1)}{9}$
(e) None of these

10. The interior angles of a polygon are in $AP$. The smallest angle is 120° and the common difference is 5°. Find the number of sides of the polygon.

(a) 7 (b) 8 (c) 9 (d) 10
(e) 11

11. Find the sum to $n$ terms of the series 11 + 103 + 1005 + ...

(a) $\frac{10(10^n - 1)}{9} + 1$ (b) $\frac{10(10^n - 1)}{9} + n$
(c) $\frac{10(10^n - 1)}{9} + n^2$ (d) $\frac{10(10^n + 1)}{11} + n^2$
(e) None of these

12. The sum of the first term and the fifth term of an $AP$ is 26 and the product of the second term by the fourth term is 160. Find the sum of the first seven terms of this $AP$.

(a) 110 (b) 114 (c) 112 (d) 116
(e) 120

13. The sum of the third and the ninth term of an $AP$ is 10. Find a possible sum of the first 11 terms of this $AP$.

(a) 55 (b) 44
(c) 66 (d) 48
(e) None of these

14. The sum of the squares of the fifth and the eleventh term of an $AP$ is 3 and the product of the second and the fourteenth term is equal it $P$. Find the product of the first and the fifteenth term of the $AP$.

(a) $(58P - 39)/45$ (b) $(98P + 39)/72$
(c) $(116P - 39)/90$ (d) $(98P + 39)/90$
(e) None of these

15. If the ratio of harmonic mean of two numbers to their geometric mean is 12 : 13, find the ratio of the numbers.

(a) 4/9 or 9/4 (b) 2/3 or 3/2
(c) 2/5 or 5/2 (d) 3/4 or 4/3
(e) 3/4 or 4/5

16. Find the sum of the series $1.2 + 2.2^2 + 3.2^3 + \ldots + 100.\ 2^{100}$.

(a) $100.2^{101} + 2$ (b) $99.2^{100} + 2$
(c) $99.2^{101} + 2$ (d) $100.2^{100} + 2$
(e) None of these

17. The sequence $[x_n]$ is a $GP$ with $x_2/x_4 = 1/4$ and $x_1 + x_4 = 108$. What will be the value of $x_3$?

(a) 42 (b) 48 (c) 44 (d) 56
(e) 52

18. If $x$, $y$, $z$ are in GP and $a^x$, $b^y$ and $c^z$ are equal, then $a$, $b$, $c$ are in

(a) $AP$ (b) $GP$
(c) $HP$ (d) None of these
(e) Cannot be determined

19. Find the sum of all possible whole number divisors of 720.

32. In a certain colony of cancerous cells, each cell reproduce by giving birth to two new cells every hour. If there is a single productive cell at the start and this process continues for 9 hours, how many cells will the colony have at the end of 9 hours? It is known that the life of an individual cell is 20 hours and only 50% of the cells are capable enough of producing next generation cells?
    (a) $2^9 - 1$ (b) $2^{10}$
    (c) $2^9$ (d) $2^{10} - 1$
    (e) $2^{11} - 1$
33. Find the sum of all three-digit whole numbers less than 500 that leave a remainder of 2 when they are divided by 3.
    (a) 49637 (b) 39767
    (c) 49634 (d) 39770
    (e) 38770
34. If $a$ be the arithmetic mean and $b$, $c$ be the two geometric means between any two positive numbers, then $(b^3 + c^3)/abc$ equals
    (a) $(ab)^{1/2}/c$ (b) 1
    (c) $a^2c/b$ (d) $2a$
    (e) None of these
35. If $p, q, r$ are three consecutive distinct natural numbers then the expression $(q + r - p)(p + r - q)(p + q - r)$ is
    (a) Positive (b) Negative
    (c) Non-positive (d) Non-negative
    (e) Either (c) or (d)

## Hints and Solutions II

2. $a + (a + d) + (a + 2d) + (a + 3d) = 20$
   and $a(a + 3d) = (a + d)(a + 2d)$
4. Calculate the sum of an AP with first term 1, common difference 1 and last term 12. Multiply this sum by 4 for 2 days.
5. The maximum sum will occur when the last term is either 2 or 0.
6. Visualise the AP as 7, 14…196.
9. The AP is 105, 112 …994.
10. The common difference is $\frac{146}{5} = 29.2$.
11. See the terms of the series in 33 blocks of 3 each. This will give the AP – 4, –5, –6…–33. Further, the hundredth term will be 34.
12. Solve through options.
14. The first drop is 120 metres. After this the ball will rise by 96 metres and fall by 96 meters. This process will continue in the form of an infinite GP with common ratio 0.8 and first term 96.
    The required answer will be got by
    $$120 + 96 * 1.25 * 2$$
15. Take any GP and solve by using values.
18. Solve by using values to check options.
22. The difference between the seventh and third term is given by
    $$(a + 6d) - (a + d)$$
23. $\frac{(39 - 4)}{5} = 7.$
27. The required answer will be by adding 20 terms of the GP starting with the first term as 1000 and the common ratio as 1.05.
30. Visualise it as an infinite GP with common ratio 0.9.

## Hints and Solutions III

1. Difference between the tenth and the sixth term = –16
   or $(a + 9d) - (a + 5d) = -16$
   $\rightarrow d = -4$
2. Sum of the first term and the fifth term = 10
   or $a + a + 4d = 10$
   or $a + 2d = 5$ (1)
   and, the sum of all terms of the AP except for the 1st term = 99
   or $9a + 45d = 99$
   $a + 5d = 11$ (2)
   Solve (1) and (2) to get the answer.
3. The second statement gives the equation as $a + 8d = 2(a + 3d) + 6$
   or $a - 2d = 6$
   Now, use the options to find the value of $d$, and put these values to check the equation obtained from the first statement.
   i.e. $(a + 2d)(a + 5d) = 406$
4. To plant the 1st sapling, Mithilesh will cover 20 m; to plant the 2nd sapling he will cover 40 m and so

While the second series is 3, 6, 9 ..... 240.
Hence, the last common term is 237.

Thus our answer becomes $\frac{237-3}{6} + 1 = 40$

7. Trying Option (a),
We get least term 5 and largest term 30 (since the largest term is 6 times the least term).
The average of the A.P becomes $(5 + 30)/2 = 17.5$
Thus, $17.5 \times n = 105$ gives us:
to get a total of 105 we need $n = 6$ i.e. 6 terms in this A.P. That means the A.P. should look like:
5, _ , _ , _, _, 30.
It can be easily seen that the common difference should be 5. The A.P. 5, 10, 15, 20, 25, 30 fits the situation.
The same process used for option (b) gives us the A.P. 10, 35, 60. (10 + 35 + 60 = 105) and in the third option 15, 90 (15 + 90 = 105).
Hence, all the three options are correct.

9. The difference between the amounts at the end of 4 years and 10 years will be the simple interest on the initial capital for 6 years.
Hence, 360/6 = 60 =(simple interest.)
Also, the Simple Interest for 4 years when added to the sum gives 1240 as the amount.
Hence, the original sum must be 1000.

16. Sum of a G.P. with first term 1 and common ratio 2 and no. of terms 20.

$$\frac{1 \times (2^{20} - 1)}{(2-1)} = 2^{20} - 1$$

18. In the case of a G.P. the 7th term is derived by multiplying the fourth term thrice by the common ratio. (**Note:** this is very similar to what we had seen in the case of an A.P.)
Since, the seventh term is derived by multiplying the fourth term by 8, the relationship.
$r^3 = 8$ must be true.
Hence, $r = 2$
If the fifth term is 48, the series in reverse from the fifth to the first term will look like:
48, 24, 12, 6, 3. Hence, option (b) is correct.

21. The series will be 301, 308, ........ 497

Hence, Answer = $\frac{196}{7} + 1 = 29$

25. Similar to what we saw in question 18,
4th term $\times r^6$ = 10th term.
The 4th term here is $3^{-1}$ and the tenth term is $3^5$.
Hence $3^{-1} \times r^6 = 3^5$
Gives us: $r = 3$.
Hence, the second term will be given by (Second term/$r^2$)

$$\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$$

[**Note:** To go forward in a G.P. you multiply by the common ratio, to go backward in a G.P. you divide by the common ratio.]

28. The answer will be given by:
[10 + 11 + 12 + ........ + 50] – [16 + 24 + ... + 48]
$= 41 \times 30 - 32 \times 5$
$= 1230 - 160 = 1070$.

29. Think like this:
The average of the first 4 terms is 7, while the average of the first 8 terms must be 11.
Now visualize this :

1st $\underbrace{\text{2nd 3rd}}_{\text{average = 7}}$ $\underbrace{\text{4th 5th}}_{\text{average = 11}}$ 6th 7th 8th

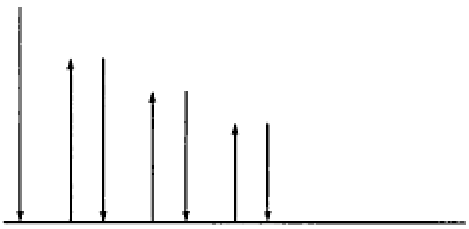
Hence, $d = 4/2 = 2$ [Note: understand this as a property of an A.P.]
Hence, the average of the 6th and 7th terms = 15 and the average of the 8th and 9th term = 19
But this (19) also represents the average of the 16 term A.P.
Hence, required answer = $16 \times 19 = 304$.

30. Go through options. The correct option should give value as 1, when $n = 3$ and as 8 when $n = 8$.
Only option (a) Satisfies both conditions.

## LOD 2

1. Identify an A.P. which satisfies the given condition. Suppose we are talking about the second and third terms of the A.P.
Then an A.P. with second term 3 and third term 2 satisfies the condition.
a times the $a$th term = $b$ times the $b$th term.
In this case the value of $a = 2$ and $b = 3$.
Hence, for the $(a + b)^{th}$ term, we have to find the fifth term.
It is clear that the fifth term of this A.P. must be zero.
Check the other three options to see whether any option gives 0 when $a = 2$ and $b = 3$.
Since none of the options $b$, $c$ or $d$ gives zero for this particular value, the option a is correct.

3. View: $1 - 4 + 5 - 8 + 9 - 12$ . . . . . 50 terms as $(1 - 4) + (5 - 8) + (9 - 12)$ …… 25 terms.
Hence, $-3 + -3 + -3$ . . . . 25 terms
$= 25 \times -3 = -75$.
5. Since this is a decreasing A.P. with first term positive, the maximum sum will occur upto the point where the progression remains non – negative.
44, 42, 40 ….. 0
Hence, 23 terms $\times$ 22 = 506.
7. A little number juggling would give you 2nd term is 1/3 and 3rd term is ½ is a possible situation that satisfies the condition.
The A.P. will become:
1/6, 1/3, 1/2, 2/3, 5/6, 1
or in decimal terms, 0.166, 0.333, 0.5, 0.666, 0.833, 1
Sum to 6 terms = 3.5
Check the option with $m = 2$ and $n = 3$. Only option (c) gives 3.5. Hence, must be the answer.
11. The first 100 terms of this series can be viewed as:
$(1 - 2 - 3) + (2 - 3 - 4) + \ldots + (33 - 34 - 35) + 34$
The first 33 terms of the above series (indicated inside the brackets) will give an A.P.: $-4, -5, -6 \ldots -36$
Sum of this A.P. $= 33 \times -20 = -660$
Answer $= 660 + 34 = -626$
14. The path of the rubber ball is:



In the figure above, every bounce is 4/5th of the previous drop.
In the above movement, there are two infinite G.Ps (The GP representing the falling distances and the GP representing the Rising distances.)
The required answer: (Using $a/(1-r)$ formula)

$$\frac{120}{1/5} + \frac{96}{1/5} = 1080$$

18. For $n = 1$, the sum should be 6. Option (b), (c) and (d) all give 6 as the answer.
For $n = 2$, the sum should be 30.
Only option $d$ gives this value. Hence must be the answer.
24. Find sum of the series:
104, 109, 114 . . . . 999
Average $\times\, n = 551.5 \times 180 = 99270$
28. Since, sum to n terms is given by (n + 8),
Sum to 1 terms = 9
Sum to 2 terms = 10
Thus, the 2nd term must be 1.

## LOD 3

9. For 1 term, the value should be:
$$6^2 + 8 = 44$$
Only option (b) gives 44 for $n = 1$
16. The solution (from the options) has got something to do with either $2^{100}$ or $2^{101}$ for 100 terms. Hence, for 3 terms recreate the options and crosscheck with the actual sum.
For 3 terms: Sum = 2 + 8 + 24 = 34.
(a) $100 \times 2^{101} + 2$ for 100 terms becomes $3 \times 2^4 + 2$ for 3 terms.
$= 48 + 2 = 50 \neq 34$. Hence is not correct.
(b) $99 \times 2^{100} + 2$ for 100 terms becomes $2 \times 2^3 + 2$ for 3 terms.
But this does not give 34. Hence is not correct.
(c) $99 \times 2^{101} + 2 \rightarrow 2 \times 2^4 + 2 = 34$
(d) $100 \times 2^{100} + 2 \rightarrow 3 \times 2^3 + 2 \neq 34$.
Hence, option (c) is correct.

# ANSWER KEY

**LOD I**

| | | |
|---|---|---|
| 1. b | 31. a | |
| 2. a | 32. d | |
| 3. c | 33. c | |
| 4. b | 34. b | |
| 5. d | 35. b | |
| 6. c | | |
| 7. e | | |
| 8. c | | |
| 9. d | | |
| 10. b | | |
| 11. d | | |
| 12. c | | |
| 13. a | | |
| 14. a | | |
| 15. c | | |
| 16. b | | |
| 17. c | | |
| 18. b | | |
| 19. a | | |
| 20. c | | |
| 21. a | | |
| 22. c | | |
| 23. a | | |
| 24. e | | |
| 25. c | | |
| 26. b | | |
| 27. b | | |
| 28. a | | |
| 29. c | | |
| 30. a | | |

**LOD II**

| | |
|---|---|
| 1. a | 33. a |
| 2. c | 34. a |
| 3. b | 35. b |
| 4. b | 36. b |
| 5. c | 37. d |
| 6. b | 38. a |
| 7. c | 39. d |
| 8. a | 40. a |
| 9. c | 41. b |
| 10. c | 42. a |
| 11. b | 43. b |
| 12. b | 44. d |
| 13. a | 45. a |
| 14. c | |
| 15. b | |
| 16. e | |
| 17. b | |
| 18. d | |
| 19. c | |
| 20. c | |
| 21. a | |
| 22. d | |
| 23. c | |
| 24. d | |
| 25. b | |
| 26. c | |
| 27. b | |
| 28. d | |
| 29. c | |
| 30. b | |
| 31. c | |
| 32. a | |

# BLOCK REVIEW TESTS

## REVIEW TEST ONE

1. Lata has the same number of sisters as she has brothers, but her brother Shyam has twice as many sisters as he has brothers. How many children are there in the family?
(a) 7 (b) 6 (c) 5 (d) 3
2. How many times does the digit 6 appear when you count from 11 to 100?
(a) 9 (b) 10 (c) 19 (d) 20
3. If m < $n$, then
(a) $m$ :$m$ < $n$ . $n$
(b) $m.m > n . n$
(c) $m . n. n < n. m . m$
(d) $m . m . m < n. n . n$
4. A square is drawn by joining the midpoints of the side of a given square. A third square is drawn in side the second square in the same way and this process is continued indefinitely. If a side of the first square is 8 cm, the sum of the areas of all the squares (in sq. cm) is
(a) 128 (b) 120
(c) 96 (d) None of these
5. Find the least number which when divided by 6, 15, 17 leaves a remainder 1, but when divided by 7 leaves no remainder.
(a) 211 (b) 511 (c) 1022 (d) 86
6. The number of positive integers not greater than 100, which are not divisible by 2, 3 or 5 is
(a) 26 (b) 18 (c) 31
(d) None of these
7. The smallest number which when divided by 4, 6 or 7 leaves a remainder of 2, is
(a) 44 (b) 62 (c) 80 (d) 86
8. An intelligence agency decides on a code of 2 digits selected from 0, 1, 2,....9. But the slip on which the code is hand-written allows confusion between top and bottom, because these are indistinguishable. Thus, for example, the code 91 could be confused with 16. How many codes are there such that there is no possibility of any confusion?
(a) 25 (b) 75
(c) 80 (d) None of these
9. Suppose one wishes to find distinct positive integers $x$, $y$ such that $(x + y)/xy$ is also a positive integer. Identify the correct alternative.
(a) This is never possible
(b) This is possible and the pair $(x, y)$ satisfying the stated condition is unique.
(c) This is possible and there exist more than one but a finite number of ways of choosing the pair$(x, y)$.
(d) This is possible and the pair $(x, y)$ can be chosen in infinite ways.
10. A young girl counted in the following way on the fingers of her left hand. She started calling the thumb 1, the index finger 2, middle finger 3, ring finger 4, little finger 5, then reversed direction, calling the ring finger 6, middle finger 7, index finger 8, thumb 9, then back to the index finger for 10, middle finger for 11, and so on. She counted up to 1994. She ended on her.
(a) thumb (b) index finger
(c) middle finger (d) ring finger
11. 139 persons have signed up for an elimination tournament. All players are to be paired up for the first round, but because 139 is an odd number one player gets a bye, which promotes him to the second round, without actually playing in the first round. The pairing continues on the next round, with a bye to any player left over. If the schedule is planned so that a minimum number of matches is required to determine the champion, the number of matches which must be played is
(a) 136 (b) 137 (c) 138 (d) 139
12. The product of all integers from 1 to 100 will have the following numbers of zeros at the end.
(a) 20 (b) 24 (c) 19 (d) 22
13. There are ten 50 paise coins placed on a table. Six of these show tails four show heads. A coin chosen at random and flipped over (not tossed). This operation is performed seven times. One of the coins is then covered. Of the remaining nine coins five show tails and four show heads. The covered coin shows
(a) a head (b) a tail
(c) more likely a head (d) more likely a tail
14. A five digit number is formed using digits 1, 3, 5, 7 and 9 without repeating any one of them. What is the sum of all such possible numbers?
(a) 6666600 (b) 6666660
(c) 6666666 (d) None

10. $P$, $Q$ and $R$ are three consecutive odd numbers in ascending order. If the value of three times $P$ is three less than two times $R$, find the value of $R$.
(a) 5 (b) 7 (c) 9 (d) 11

11. $ABC$ is a three-digit number in which $A > 0$. The value of $ABC$ is equal to the sum of the factorials of its three digits. What is the value of $B$?
(a) 9 (b) 7 (c) 4 (d) 2

12. $A$, $B$ and $C$ are defined as follows:
$$A = (2.000004) \div [(2.000004)^2 + (4.000008)$$
$$B = (3.000003) \div [(3.000003)^2 + (9.000009)$$
$$C = (4.000002) \div [(4.000002)^2 + (8.000004)$$
Which of the following is true about the value of the above three expressions?
(a) All of them lie between 0.18 and 0.20
(b) $A$ is twice $C$
(c) $C$ is the smallest
(d) $B$ is the smallest

13. Let $x < 0.50$, $0 < y < 1$, $z > 1$. Given a set of numbers, the middle number, when they arranged in ascending order is called the median. So the median of the numbers $x$, $y$ and $z$ would be
(a) less than one (b) between 0 and 1
(c) greater than one (d) cannot say

14. let $a$, $b$, $c$ $d$, and $e$ be integers such that $a = 6b = 12c$, and $2b = 9d = 12e$. Then which of the following pairs contains a number that is not an integer?.
(a) $\left(\frac{a}{27}, \frac{b}{e}\right)$ (b) $\left(\frac{a}{36}, \frac{c}{e}\right)$
(c) $\left(\frac{a}{3}, \frac{bd}{9}\right)$ (d) $\left(\frac{a}{7}, \frac{c}{d}\right)$

15. If $a$, $a + 2$, and $a + 4$ are prime numbers, then the number of possible solutions for $a$ is:
(a) One (b) Two
(c) Three (d) More than three

16. Let $x$ and $y$ be positive integers such that $x$ is prime and $y$ is composite. Then,
(a) $y - x$ cannot be an even integer
(b) $xy$ cannot be an even integer
(c) $(x + y)$ cannot be an even integer
(d) None of the above statement is true

17. Let $n$ (>1) be a composite natural number such that the square root of $n$ is not an integer. Consider the following statements:
A: $n$ has a factor which is greater than 1 and less than square root $n$
B: $n$ has a factor which is greater than square root $n$ but less than $n$
Then
(a) Both A and B are false
(b) Both A and B are true$
(c) A is false but B is true
(d) Both A and B are true

18. What is the remainder when $4^{96}$ is divided by 6?
(a) 0 (b) 2 (c) 3 (d) 4

19. What is the sum of all two-digit numbers that give a remainder of 3 when they are divided by 7?
(a) 646 (b) 676 (c) 683 (d) 797

20. The infinite sum $1+\frac{4}{7}+\frac{9}{7^2}+\frac{16}{7^3}+\frac{25}{7^4}+\text{-----}$
(a) $\frac{27}{14}$ (b) $\frac{29}{13}$
(c) $\frac{49}{27}$ (d) $\frac{256}{147}$

21. If the product of $n$ positive real numbers is unity, then their sum is necessarily:
(a) a multiple of $n$ (b) equal to $n+\frac{1}{n}$
(c) never less than $n$ (d) None of these

22. How many three digit positive integer, with digits $x$, $y$ and $z$ in the hundred's, ten's and unit's place respectively, exist such that $x < y$, $z < y$ and $x \neq 0$?
(a) 245 (b) 285 (c) 240 (d) 320

23. How many even integers $n$, where $100 \leq n \leq 200$, are divisible neither by seven nor by nine?
(a) 40 (b) 37 (c) 39 (d) 38

24. A positive whole number $M$ less than 100 is represented in base 2 notation, base 3 notation, and base 5 notation. It is found that in all three cases the last digit is 1, while in exactly two out of the three cases the leading digit in 1. Then $M$ equals:
(a) 31 (b) 63 (c) 75 (d) 91

25. In a certain examinations paper, there are $n$ questions. For $j = 1, 2 \ldots.n$. there are $2^{n-j}$ students who answered $j$ or more questions wrongly. If the total number of wrong answers is 4095, then the value of $n$ is:
(a) 12 (b) 11 (c) 10 (d) 9

# ANSWER KEY

**Review Test One**

| | | |
|---|---|---|
| 1. d | | |
| 2. c | | |
| 3. d | | |
| 4. a | | |
| 5. b | | |
| 6. a | | |
| 7. d | | |
| 8. c | | |
| 9. c | | |
| 10. c | | |
| 11. c | | |
| 12. b | | |
| 13. a | | |
| 14. a | | |
| 15. a | | |
| 16. c | | |
| 17. c | | |
| 18. d | | |
| 19. d | | |
| 20. b | | |
| 21. d | | |
| 22. a | | |
| 23. b | | |
| 24. d | | |
| 25. b | | |

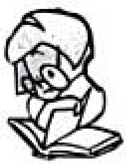
**Review Test Two**

| | |
|---|---|
| 1. | d |
| 2. | a |
| 3. | b |
| 4. | c |
| 5. | c |
| 6. | a |
| 7. | d |
| 8. | c |
| 9. | b |
| 10. | c |
| 11. | c |
| 12. | d |
| 13. | b |
| 14. | d |
| 15. | a |
| 16. | d |
| 17. | b |
| 18. | d |
| 19. | b |
| 20. | c |
| 21. | c |
| 22. | c |
| 23. | c |
| 24. | d |
| 25. | d |

# BLOCK 2

## CHAPTERS

- **Averages**
- **Alligations**



## ...BACK TO SCHOOL

- **The Relevance of Averages**

Average is one of the most important mathematical concepts that we use in our day to day life. In fact, even the most non mathematical individuals regularly utilize the concept of averages on a day to day basis.

So, we use averages in all the following and many more instances.

- How a class of students fared in an exam is assessed by looking at the average score.
- What is the average price of items purchased by an individual.
- A person might be interested in knowing his average telephone expenditure, electricity expenditure, petrol expenditure etc.
- A manager might be interested in finding out the average sales per territory or even the average growth rate month to month.
- Clearly there can be immense application of averages that you might be able to visualize on your own.

- **The Meaning of an Average**

The average is best seen as a representative value which can be used to represent the value of the general term in a group of values.

For instance, suppose that a cricket team had 10 partnerships as follows:

| | |
|---|---|
| 1$^{st}$ wicket 28 | 2$^{nd}$ wicket 42 |
| 3$^{rd}$ wicket 112 | 4$^{th}$ wicket 52 |
| 5$^{th}$ wicket 0 | 6$^{th}$ wicket 23 |
| 7$^{th}$ wicket 41 | 8$^{th}$ wicket 18 |
| 9$^{th}$ wicket 9 | 10$^{th}$ wicket 15 |

On adding the ten values above, we get a total of 340—which gives an average of 34 runs per wicket, i.e. the average partnership of the team was 34 runs.

*Contd.*

In other words, if we were to replace the value of all the ten partnerships by 34 runs, we would get the same total score. Hence, 34 represents the average partnership value for the team.

Suppose, in a cricket series of 5 matches between 2 countries, you are given that Team A had an average partnership of 58 Runs per wicket while Team B had an average partnership of 34 runs per wicket. What conclusion can you draw about the performance of the two teams, given that both the teams played 5 complete test matches?

Obviously, Team B would have performed much worse than Team A: For that matter, if I tell you that the average daytime high temperature of Lucknow was 18° C for a particular month, you can easily draw some kind of conclusion in your mind about the month we could possibly be talking about.

Thus, you should realize that the beauty of averages lies in the fact that it is one single number that tells you a lot about the group of numbers – hence, it is one number that represents an entire group of numbers.

But one of the key concepts that you need to understand before you move into the chapters of this block is the Concept of WEIGHTED AVERAGES.

As always, the concept is best explained through a concrete example.

Suppose I had to buy a shirt and a trouser and let us say that the average cost of a shirt was Rs. 1200 while that of a trouser was Rs. 900.

In such a case, the average cost of a shirt and a trouser would be given by (1200 + 900)/2 = 1050.

This can be visualized on the number line as:

(midpoint) = answer


As you can easily see in the figure, the average occurs at the midpoint of the two numbers.

Now, let us try to modify the situation:

Suppose, I were to buy 2 trousers and 1 shirt. In such a case I would endup spending (900 + 900 + 1200) = Rs. 3000 in buying a total of 3 items. What would be my average in this case?

Obviously, 3000/3 = Rs. 1000!!

Clearly, the average has shifted!!

On the number line we could visualize this as follows:

(Answer) (midpoint)


It is clearly visible that the average has shifted towards 900 (which was the cost price of the trousers—the larger purchased item.)

In a way, this shift is similar to the way a two pan weighing balance shifts on weights being put on it. The balance shifts towards the pan containing the larger weight.

Similarly, in this case, the correct average (1000) is closer to 900 than it is to 1200. This has happened because the number of elements in the group of average 900 is greater than the number of elements in the group having average 1200. Since, this is very similar to the system of weights, we call this as a weighted average situation.

At this stage, you should realize that weighted averages are not solely restricted to two groups. We can also come up with a weighted average situation for three groups (although in such a case the representation of the weighted average on the number line might not be so easily possible.) In fact, it is the number line representation of a weighted average situation that is defined as alligation (when 2 groups are involved).

The number of educated employees working in the organisation are:

(a) 15 (b) 20
(c) 10 (d) 25

16. Mr.Akhilesh Bajpai while going from Lucknow to Jamshedpur covered half the distance by train at the speed of 96 km/hr, half the rest of the distance by his scooter at the speed of 60 km/hr and the remaining distance at the speed of 40 km/hr by car. The average speed at which he completed his journey is:

(a) 64 km/hr (b) 56 km/hr
(c) 60 km/hr (d) 36 km/hr

17. There are four types of candidates in AMS Learning Systems preparing for the CAT. The number of students of Engineering, Science, Commerce and Humanities is 400, 600, 500 and 300 respectively and the respective percentage of students who qualified the CAT is 80%, 75%, 60% and 50% respectively the overall percentage of successful candidates in our institute is:

(a) 67.77% (b) 66.66%
(c) 68.5% (d) None of these

18. Mr. Jagmohan calculated the average of 10 'Three digit numbers'. But due to mistake he reversed the digits of a number and thus his average increased by 29.7. The difference between the unit digit and hundreds digit of that number is:

(a) 4 (b) 3
(c) 2 (d) can't be determined

*Directions for questions 19-20:* Answer the questions based on the following Information.

Production pattern for number of units (in cubic feet) per day.

| Days | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Numbers of units | 150 | 180 | 120 | 250 | 160 | 120 | 150 |

For a truck that can carry 2,000 cubic feet, hiring cost per day is Rs. 1,000. Storing cost per cubic feet is Rs. 5 per day. Any residual material left at the end of the seventh day has to be transferred.

19. If all the units should be sent to the market, then on which days should the trucks be hired to minimize the cost:

(a) $2^{nd}$, $4^{th}$, $6^{th}$, $7^{th}$ (b) $7^{th}$
(c) $2^{nd}$, $4^{th}$, $5^{th}$, $7^{th}$ (d) None of these

20. If the storage cost is reduced to Rs. 0.9 per cubic feet per day, then on which day/days, should the truck be hired?

(a) $4^{th}$ (b) $7^{th}$
(c) $4^{th}$ and $7^{th}$ (d) None of these

**Answers (Block 2 Preassessment Test)**

| | | | | |
|---|---|---|---|---|
| 1. (a) | 2. (a) | 3. (a) | 4. (b) | 5. (a) |
| 6. (b) | 7. (b) | 8. (c) | 9. (c) | 10. (e) |
| 11. (b) | 12. (c) | 13. (b) | 14. (c) | 15. (c) |
| 16. (a) | 17. (a) | 18. (b) | 19. (a) | 20. (2) |

## SCORE INTERPRETATION ALGORITHM

### Block 2 Preassessment Test

### If You Scored: <12: (In Unlimited Time)

**Step One** Go through the first chapter of the block. Viz Averages. When you do so, concentrate on clearly understanding each of the concepts explained in the chapter theory.

Then move onto the LOD 1 exercises. These exercises will provide you with the first level of challenge. Try to solve each and every question provided under LOD 1. While doing so do not think about the time requirement .Once you finish solving LOD 1, revise the questions and their solution processes.

Also at this stage study the concept of averages from your school text books (Class 8, 9 & 10) and solve all the questions which are available to you in those books.

**Step Two** After finishing LOD 1 of Averages, move into LOD 2 and then LOD 3 of this chapter.

**Step Three** Go to the chapter of alligations and study the shortcuts provided carefully. Understand the use of the alligation process as clearly as possible.

Then move to the LOD 1 exercise of the same. (Note: The chapter of alligations does not have an LOD2 & LOD 3 exercise)

**Step Four** Go to the practice test given at the end of the block and solve it. While doing so, first look at the score you get within the time limit mentioned. Then continue to solve the test further without a time limit and try to evaluate the improvement in your unlimited time score.

In case the growth in your score is not significant, go back to the theory of each chapter and review each of the questions you have solved for both the chapters.

### If You Scored:> 12 (In Unlimited Time)

Follow the same process as above. The only difference is that the school book work is optional – do it only if you feel you need to. However, your concentration during the solving of the two chapters has to be on developing your speed at solving questions on this chapter.

# 3

# AVERAGES

## INTRODUCTION

The concept of Averages is important since questions based on this chapter regularly appear in all aptitude exams. While most lower level MBA exams as well as Bank PO, NLS, NIFT exams test the concept of averages through direct questions, the CAT and the XLRI examinations have used the concept to set conceptual problems that will not be found in most textbooks. Such problems have been given in this chapter in LOD II and LOD III questions.

While studying this chapter, make sure that you understand the techniques of faster illustrated computation in the text. These techniques are essentially based on certain concepts and understanding them will, in turn, help you get a better grip on these concepts.

## THEORY

The average of a number is a measure of the central tendency of a set of numbers. In other words, it is an estimate of where the center point of a set of numbers lies.

The basic formula for the average of $n$ numbers $x_1, x_2, x_3, \ldots x_n$ is

$A_n = (x_1 + x_2 + x_3 + \ldots + x_n)/n$ = (Total of set of $n$ numbers)/$n$

This also means $A_n \times n$ = total of the set of numbers.

The average is always calculated for a set of numbers.

**Concept of Weighted Average:** When we have 2 or more groups whose individual averages are known, then to find the combined average of all the elements of all the groups we use weighted average. Thus, if we have $k$ groups with averages $A_1, A_2 \ldots A_k$ and having $n_1, n_2 \ldots n_k$ elements then the weighted average is given by the formula:

$$A_w = \frac{n_1 A_1 + n_2 A_2 + n_3 A_3 + \ldots + n_k A_k}{n_1 + n_2 + n_3 + \ldots + n_k}$$

**Another Meaning of Average** The average [also known as *arithmetic mean* (AM)] of a set of numbers can also be defined as the number by which we can replace each and every number of the set without changing the total of the set of numbers.

**Properties of Average (AM)** The properties of averages [arithmetic mean] can be elucidated by the following examples:

***Example 1:*** The average of 4 numbers 12, 13, 17 and 18 is:

*Solution:* Required average = (12 + 13 + 17 + 18)/4 = 60/4 = 15

This means that if each of the 4 numbers of the set were replaced by 15 each, there would be no change in the total.

This is an important way to look at averages. In fact, whenever you come across any situation where the average of a group of '$n$' numbers is given, you should visualise that there are '$n$' numbers, each of whose value is the average of the group. This view is a very important way to visualise averages.

This can be visualised as

$$\begin{array}{c} 12 \rightarrow +3 \rightarrow 15 \\ 13 \rightarrow +2 \rightarrow 15 \\ 17 \rightarrow -2 \rightarrow 15 \\ 18 \rightarrow -3 \rightarrow 15 \\ \hline 60 \rightarrow +0 \rightarrow 60 \\ \hline \end{array}$$

***Example 2:*** In Example 1, visualise addition of a fifth number, which increases the average by 1.

15 + 1 = 16
15 + 1 = 16
15 + 1 = 16
15 + 1 = 16

The +1 appearing 4 times is due to the fifth number, which is able to maintain the average of 16 first and then 'give one' to each of the first 4.

Hence, the fifth number in this case is 20

***Example 3:*** The average always lies above the lowest number of the set and below the highest number of the set.

***Example 4:*** The net deficit due to the numbers below the average always equals the net surplus due to the numbers above the average.

***Example 5:*** *Ages and averages:* If the average age of a group of persons is $x$ years today then after $n$ years their average age will be $(x + n)$.

Also, $n$ years ago their average age would have been $(x - n)$. This happens due to the fact that for a group of people, 1 year is added to each person's age every year.

***Example 6:*** A man travels at 60 kmph on the journey from $A$ to $B$ and returns at 100 kmph. Find his average speed for the journey.

*Solution:*

Average speed = (total distance)/(total time)

If we assume distance between 2 points to be $d$

Then

Average speed = $2d/[(d/60) + (d/100)] = (2 \times 60 \times 100)/(60 + 100) = (2 \times 60 \times 100)/160 = 75$

Average speed = $(2S_1 \cdot S_2)/(S_1 + S_2)$
[$S_1$ and $S_2$ are speeds]

of going and coming back respectively.

***Short Cut*** The average speed will always come out by the following process:

The ratio of speeds is 60 : 100 = 3 : 5 (say $r_1 : r_2$)

Then, divide the difference of speeds (40 in this case) by $r_1 + r_2$ (3 + 5 = 8, in this case) to get one part. (40/8 = 5, in this case)

The required answer will be three parts away (i.e. $r_1$ parts away) from the lower speed.

Check out how this works with the following speeds:

$$S_1 = 20 \quad \text{and} \quad S_2 = 40$$

***Step 1:*** Ratio of speeds = 20 : 40 = 1 : 2

***Step 2:*** Divide difference of 20 into 3 parts $(r_1 + r_2) \rightarrow = 20/3 = 6.66$

Required average speed = $20 + 1 \times 6.66$

*Note:* This process is essentially based on alligations and we shall see it again in the next chapter.

## ■ EXERCISES FOR SELF-PRACTICE ■

Find the average speed for the above problem if

(1) $S_1 = 20$ $S_2 = 200$
(2) $S_1 = 60$ $S_2 = 120$
(3) $S_1 = 100$ $S_2 = 50$
(4) $S_1 = 60$ $S_2 = 180$

# WORKED-OUT PROBLEMS

**Problem 2.1** The average of a batsman after 25 innings was 56 runs per innings. If after the 26th inning his average increased by 2 runs, then what was his score in the 26th inning?

**Solution** *Normal process:*

Runs in 26th inning = Runs total after 26 innings – Runs total after 25 innings

$$= 26 \times 58 - 25 \times 56$$

For mental calculation use:

$$(56 + 2) \times 26 - 56 \times 25$$
$$= 2 \times 26 + (56 \times 26 - 56 \times 25)$$
$$= 52 + 56 = 108$$

***Short Cut*** Since the average increases by 2 runs per innings it is equivalent to 2 runs being added to each score in the first 25 innings. Now, since these runs can only be added by the runs scored in the 26th inning, the score in the 26th inning must be 25 × 2 = 50 runs higher than the average after 26 innings (i.e. new average = 58).

Hence, runs scored in 26th inning = New Average + Old innings × Change in average

$$= 58 + 25 \times 2 = 108$$

Visualise this as

| Average in first 25 innings | Average after 26 innings |
|---|---|
| 56 | 58 |
| 56 | 58 |
| 56 | 58 |
| ... | ... |
| ... | ... |
| 25 times... | 26 times... |

Difference in total is two, 25 times and 58 once, that is, 58 + 25 × 2.

**Problem 2.2** The average age of a class of 30 students and a teacher reduces by 0.5 years if we exclude the teacher. If the initial average is 14 years, find the age of the class teacher.

**Solution** *Normal process:*

$$\begin{aligned}\text{Age of teacher} &= \text{Total age of (students + teacher)} \\ &\quad - \text{Total age of students} \\ &= 31 \times 14 - 30 \times 13.5 \\ &= 434 - 405 \\ &= 29 \text{ years}\end{aligned}$$

***Short Cut*** The teacher after fulfilling the average of 14 (for the group to which he belonged) is also able to give 0.5 years to the age of each of the 30 students. Hence, he has $30 \times 0.5 \to 15$ years to give over and above maintaining his own average age of 14 years.

$$\text{Age of teacher} = 14 + 30 \times 0.5 = 29 \text{ years}$$

(*Note:* This problem should be viewed as change of average from 13.5 to 14 when teacher is included.)

**Problem 2.3** The average marks of a group of 20 students on a test is reduced by 4 when the topper who scored 90 marks is replaced by a new student. How many marks did the new student have?

**Solution** *Normal process:*

Let initial average be $x$.

Then the initial total is $20x$

New average will be $(x - 4)$ and the new total will be $20(x - 4) = 20x - 80$.

The reduction of 80 is created by the replacement.

Hence, the new student has 80 marks less than the student he replaces. Hence, he must have scored 10 marks.

***Short Cut*** The replacement has the effect of reducing the average marks for each of the 20 students by 4. Hence, the replacement must be $20 \times 4 = 80$ marks below the original.

Hence, answer = 10 marks.

**Problem 2.4** The average age of 3 students $A$, $B$ and $C$ is 48 marks. Another student $D$ joins the group and the new average becomes 44 marks. If another student $E$, who has 3 marks more than $D$, joins the group, the average of the 4 students $B$, $C$, $D$ and $E$ becomes 43 marks. Find how many marks $A$ got in the exam.

**Solution** Solve while reading. The first sentence gives you a total of 144 for $A$, $B$ and $C$'s marks. *Second sentence*: When $D$ joins the group, the total becomes $44 \times 4 = 176$. Hence $D$ must get 32 marks.

Alternatively, you can reach this point by considering the first 2 statements together as:

$D$'s joining the group reduces the average from 48 to 44 marks (i.e. 4 marks).

This means that to maintain the average of 44 marks, $D$ has to take 4 marks from $A$, 4 from $B$ and 4 from $C \to$ A total of 12 marks. Hence, he must have got 32 marks.

From here:

The first part of the third sentence gives us information about $E$ getting 3 marks more than 32 $\to$ Hence, $E$ gets 35 marks.

Now, it is further stated that when $A$ is replaced by $E$, the average marks of the students reduces by 1 to 43.

Mathematically this can be shown as

$$A + B + C + D = 44 \times 4 = 176 \text{ while, } B + C + D + E = 43 \times 4 = 172$$

Subtracting the two equations, we get $A - E = 4$ marks.

Hence, $A$ would have got 39 marks.

Alternatively, you can think of this as:

The replacement of $A$ with $E$ results in the reduction of 1 mark from each of the 4 people who belong to the group. Hence, the difference is 4 marks. Hence, $A$ would get 4 marks more than $E$ i.e. A gets 39 marks.

**Problem 2.5** The mean temperature of Monday to Wednesday was 27 °C and of Tuesday to Thursday was 24 °C. If the temperature on Thursday was 2/3rd of the temperature on Monday, what was the temperature on Thursday?

As soon as you get into the second line of the question, get back to the first sentence and get the total number of passed students = 2 + 6 + 12 and you are through with the problem.

## Level of Difficulty (LOD) I

1. The average age of 24 students and the principal is 15 years. When the principal's age is excluded, the average age decreases by 1 year. What is the age of the principal?
   (a) 38 (b) 40
   (c) 39 (d) 37
   (e) Data inadequate
2. The average weight of 3 men *A*, *B* and *C* is 84 kg. Another man *D* joins the group and the average now becomes 80 kg. If another man *E*, whose weight is 3 kg more than that of *D*, replaces *A* then the average weight of *B*, *C*, *D* and *E* becomes 78 kg. The weight of *A* is
   (a) 70 kg (b) 72 kg
   (c) 79 kg (d) 78 kg
   (e) 80 kg
3. The mean temperature of Monday to Wednesday was 37 °C and of Tuesday to Thursday was 34 °C. If the temperature on Thursday was 4/5 that of Monday, the temperature on Thursday was
   (a) 38 °C (b) 36 °C
   (c) 40 °C (d) 39 °C
   (e) None of these
4. Three years ago, the average age of *A*, *B* and *C* was 27 years and that of *B* and *C* 5 years ago was 20 years. *A*'s present age is
   (a) 30 years (b) 35 years
   (c) 40 years (d) 48 years
   (e) None of these
5. Ajit Tendulkar has a certain average for 9 innings. In the tenth inning, he scores 100 runs thereby increasing his average by 8 runs. His new average is
   (a) 20 (b) 24 (c) 28 (d) 32
   (e) 36
6. The average of the first five multiples of 7 is
   (a) 20 (b) 21 (c) 28 (d) 30
   (e) 31
7. There are three fractions *A*, *B* and *C*. If $A = \frac{1}{4}$ and $B = 1/6$ and the average of *A*, *B* and *C* is 1/12. What is the value of *C*?
   (a) –1/2 (b) – 1/6
   (c) –1/3 (d) – 1/4
   (e) None of these
8. The marks obtained by Hare Rama in Mathematics, English and Biology are respectively 93 out of 100, 78 out of 150 and 177 out of 200. Find his average score in percent.
   (a) 87.83 (b) 86.83 (c) 76.33 (d) 75.33
   (e) 77.83
9. The average monthly expenditure of a family was Rs. 2750 for the first 3 months, Rs. 3150 for the next three months and Rs. 6750 for the next three months. Find the average income of the family for the 9 months, if they save Rs. 650 per month.
   (a) 4866.66 (b) 5123.33
   (c) 4666.66 (d) 4216.66
   (e) None of these
10. The average age of a family of 6 members is 22 years. If the age of the youngest member be 7 years, what was the average age of the family at the birth of the youngest member?
    (a) 15 (b) 18 (c) 21 (d) 12
    (e) None of these
11. The average age of 8 persons in a committee is increased by 2 years when two men aged 35 years and 45 years are substituted by two women. Find the average age of the two women.
    (a) 48 (b) 45 (c) 51 (d) 42
    (e) 46
12. The average temperature for Wednesday, Thursday and Friday was 40 °C. The average for Thursday, Friday and Saturday was 41 °C. If the temperature on Saturday was 42 °C, what was the temperature on Wednesday?
    (a) 39 °C (b) 44 °C (c) 38 °C (d) 41 °C
    (e) None of these
13. The speed of the train in going from Nagpur to Allahabad is 100 km/hr while when coming back from Allahabad to Nagpur, its speed is 150 km/hr. Find the average speed during the whole journey.
    (a) 125 (b) 75 (c) 135 (d) 140
    (e) 120

14. The average weight of a class of 29 students is 40 kg. If the weight of the teacher be included, the average rises by 500 gm. What is the weight of the teacher?
    (a) 40.5 kg (b) 50.5 kg
    (c) 45 kg (d) 55 kg
    (e) 52 kg
15. The average of 3 numbers is 17 and that of the first two is 16. Find the third number.
    (a) 15 (b) 16 (c) 17 (d) 19
    (e) 18
16. The average weight of 19 men in a ship is increased by 3.5 kg when one of the men, who weighs 79 kg, is replaced by a new man. Find the weight of the new man upto 2 decimal places
    (a) 105.75 (b) 107.55
    (c) 145.50 (d) 140.50
    (e) None of these
17. The age of Shaurya and Kauravki is in the ratio 2 : 6. After 5 years, the ratio of their ages will become 6 : 8. Find the average of their ages after 10 years.
    (a) 12 (b) 13 (c) 17 (d) 24
    (e) None of these
18. Find the average of the first 97 natural numbers.
    (a) 47 (b) 37 (c) 48 (d) 49
    (e) 49.5
19. Find the average of all prime numbers between 30 and 50.
    (a) 39.8 (b) 38.8 (c) 37.8 (d) 41.8
    (e) 40.8
20. If we take four numbers, the average of the first three is 16 and that of the last three is 15. If the last number is 18, the first number is
    (a) 20 (b) 21 (c) 23 (d) 25
    (e) 24
21. The average of 5 consecutive numbers is $n$. If the next two numbers are also included, the average will.
    (a) increase by 1 (b) remain the same
    (c) increase by 1.4 (d) increase by 2
    (e) None of these
22. The average of 50 numbers is 38. If two numbers, namely, 45 and 55 are discarded, the average of the remaining numbers is
    (a) 36.5 (b) 37 (c) 37.6 (d) 38
    (e) 37.5
23. The average of ten numbers is 7. If each number is multiplied by 12, then the average of the new set of numbers is
    (a) 7 (b) 19 (c) 82 (d) 83
    (e) 84
24. In a family of 8 males and a few ladies, the average monthly consumption of grain per head is 10.8 kg. If the average monthly consumption per head be 15 kg in the case of males and 6 kg in the case of females, find the number of females in the family.
    (a) 8 (b) 7 (c) 9 (d) 15
    (e) 16
25. Average marks obtained by a student in 3 papers is 52 and in the fourth paper he obtains 60 marks. Find his new average.
    (a) 54 (b) 52 (c) 55 (d) 53.5
    (e) 54.5
26. The average earning of Shambhu Nath Pandey for the initial three months of the calendar year 2002 is Rs. 1200. If his average earning for the second and third month is Rs. 1300 find his earning in the first month?
    (a) 900 (b) 1100 (c) 1000 (d) 1200
    (e) 1300
27. In a hotel where rooms are numbered from 101 to 130, each room gives an earning of Rs. 3000 for the first fifteen days of a month and for the latter half, Rs. 2000 per room. Find the average earning per room per day over the month. (Assume 30 day month)
    (a) 2250 (b) 2500 (c) 2750 (d) 2466.66
    (e) 2483.33
28. The average weight of 5 men is decreased by 3 kg when one of them weighing 150 kg is replaced by another person. Find the weight of the new person?
    (a) 165 kg (b) 135 kg
    (c) 138 kg (d) 162 kg
    (e) 165 kg
29. The average age of a group of men is increased by 5 years when a person aged 18 years is replaced by a new person of aged 38 years. How many men are there in the group?
    (a) 3 (b) 4 (c) 5 (d) 6
    (e) 7
30. The average score of a cricketer in three matches is 22 runs and in two other matches, it is 17 runs. Find the average in all the five matches.

(a) 64 years (b) 48 years
(c) 45 years (d) 40 years
(e) None of these

45. The average salary of 20 workers in an office is Rs.1900 per month. If the manager's salary is added, the average salary becomes Rs. 2000 per month. What is the manager's annual salary?
(a) Rs. 24,000 (b) Rs. 25,200
(c) Rs. 45,600 (d) RS. 46,000
(e) None of these

46. If $a$, $b$, $c$, $d$ and $e$ are five consecutive odd numbers, then their average is
(a) $5(a + b)$ (b) $(a\ b\ c\ d\ e)/5$
(c) $5(a + b + c + d + e)$ (d) $\frac{a+b}{5}$
(e) None of these

47. The average of first five multiples of 3 is
(a) 3 (b) 9 (c) 12 (d) 15
(e) None of these

48. The average weight of a class of 40 students is 40 kg. If the weight of the teacher be included, the average weight increases by 500 gm. The weight of the teacher is
(a) 40.5 kg (b) 60 kg
(c) 62 kg (d) 60.5 kg
(e) 64 kg

49. In a management entrance test, a student scores 2 marks for every correct answer and loses 0.5 marks for every wrong answer. A student attempts all the 100 questions and scores 120 marks. The number of questions he answered correctly was
(a) 50 (b) 45 (c) 60 (d) 68
(e) None of these

50. The average age of four children is 8 years, which is increased by 4 years when the age of the father is included. Find the age of the father.
(a) 32 (b) 28 (c) 16 (d) 24
(e) 30

51. The average of the first ten natural numbers is
(a) 5 (b) 5.5 (c) 6.5 (d) 6

52. The average of the first ten whole numbers is
(a) 4.5 (b) 5 (c) 5.5 (d) 4

53. The average of the first ten even numbers is
(a) 18 (b) 22 (c) 9 (d) 11

54. The average of the first ten odd numbers is
(a) 11 (b) 10 (c) 17 (d) 9

55. The average of the first ten prime numbers is
(a) 15.5 (b) 12.5 (c) 10 (d) 12.9

56. The average of the first ten composite numbers is
(a) 12.9 (b) 11 (c) 11.2 (d) 10

57. The average of the first ten prime numbers, which are odd, is
(a) 12.9 (b) 13.8 (c) 17 (d) 15.8

58. The average weight of a class of 30 students is 40 kg. If, however, the weight of the teacher is included, the average become 41 kg. The weight of the teacher is
(a) 31 kg (b) 62 kg (c) 71 kg (d) 70 kg

59. Ram bought 2 toys for Rs. 5.50 each, 3 toys for Rs. 3.66 each and 6 toys for Rs. 1.833 each. The average price per toy is
(a) Rs. 3 (b) Rs. 10 (c) Rs. 5 (d) Rs. 9

60. 30 oranges and 75 apples were purchased for Rs. 510. If the price per apple was Rs. 2, then the average price of oranges was
(a) Rs. 12 (b) Rs. 14 (c) Rs. 10 (d) Rs. 15

61. The average income of Sambhu and Ganesh is Rs. 3,000 and that of Arun and Vinay is Rs. 500. What is the average income of Sambhu, Ganesh, Arun and Vinay?
(a) Rs. 1750 (b) Rs. 1850
(c) Rs. 1000 (d) Rs. 2500

62. A batsman made an average of 40 runs in 4 innings, but in the fifth inning, he was out on zero. What is the average after fifth inning?
(a) 32 (b) 22 (c) 38 (d) 49

63. The average weight of a school of 40 teachers is 80 kg. If, however, the weight of the principal be included, the average decreases by 1 kg. What is the weight of the principal?
(a) 109 kg (b) 29 kg
(c) 39 kg (d) None of these

64. The average temperature of 1st, 2nd and 3rd December was 24.4 °C. The average temperature of the first two days was 24 °C. The temperature on the 3rd of December was
(a) 20 °C (b) 25 °C
(c) 25.2 °C (d) None of these

65. The average age of Ram and Shyam is 20 years. Their average age 5 years hence will be

(a) 25 years (b) 22 years
(c) 21 years (d) 20 years

66. The average of 20 results is 30 and that of 30 more results is 20. For all the results taken together, the average is
(a) 25 (b) 50 (c) 12 (d) 24

67. The average of 5 consecutive numbers is 18. The highest of these numbers will be
(a) 24 (b) 18 (c) 20 (d) 22

68. The average of 6 students is 11 years. If 2 more students of age 14 and 16 years join, their average will become
(a) 12 years (b) 13 years
(c) 21 years (d) 19 years

69. The average of 8 numbers is 12. If each number is increased by 2, the new average will be
(a) 12 (b) 14 (c) 13 (d) 15

70. Three years ago, the average age of a family of 5 members was 17 years. A baby having been born, the average of the family is the same today. What is the age of the baby?
(a) 1 year (b) 2 years
(c) 6 months (d) 9 months

71. Sambhu's average daily expenditure is Rs. 10 during May, Rs. 14 during June and Rs. 15 during July. His approximate daily expenditure for the 3 months is
(a) Rs. 13 approximately (b) Rs. 12
(c) Rs. 12 approximately (d) Rs. 10

72. A ship sails out to a mark at the rate of 15 km per hour and sails back at the rate of 20 km/h. What is its average rate of sailing?
(a) 16.85 km (b) 17.14 km
(c) 17.85 km (d) 18 km

73. The average temperature on Monday, Tuesday and Wednesday was 41 °C and on Tuesday, Wednesday and Thursday it was 40 °C. If on Thursday it was exactly 39 °C, then on Monday, the temperature was
(a) 42 °C (b) 46 °C (c) 23 °C (d) 26 °C

74. The average of 20 results is 30 out of which the first 10 results are having an average of 10. The average of the rest 10 results is
(a) 50 (b) 40 (c) 20 (d) 25

75. A man had seven children. When their average age was 12 years a child aged 6 years died. The average age of the remaining 6 children is
(a) 6 years (b) 13 years
(c) 17 years (d) 15 years

76. The average income of Ram and Shyam is Rs. 200. The average income of Rahul and Rohit is Rs. 250. The average income of Ram, Shyam, Rahul and Rohit is
(a) Rs. 275 (b) Rs. 225
(c) Rs. 450 (d) Rs. 250

77. The average weight of 35 students is 35 kg. If the teacher is also included, the average weight increases to 36 kg. The weight of the teacher is
(a) 36 kg (b) 71 kg (c) 70 kg (d) 45 kg

78. The average of $x$, $y$ and $z$ is 45. $x$ is as much more than the average as $y$ is less than the average. Find the value of $z$.
(a) 45 (b) 25 (c) 35 (d) 15

79. Find the average of four numbers $2\frac{3}{4}, 5\frac{1}{3}, 4\frac{1}{6}, 8\frac{1}{2}$.
(a) $5\frac{3}{16}$ (b) $3\frac{3}{16}$ (c) $16\frac{5}{3}$ (d) $3\frac{16}{5}$

80. The average salary per head of all the workers in a company is Rs. 95. The average salary of 15 officers is Rs. 525 and the average salary per head of the rest is Rs. 85. Find the total number of workers in the workshop.
(a) 660 (b) 580 (c) 650 (d) 460

81. The average age of 8 men is increased by 2 years when one of them whose age is 24 years is replaced by a woman. What is the age of the woman?
(a) 35 years (b) 28 years
(c) 32 years (d) 40 years

82. The average monthly expenditure of Ravi was Rs. 1100 during the first 3 months, Rs. 2200 during the next 4 months and Rs. 4620 during the subsequent five months of the year. If the total saving during the year was Rs. 2100, find Ravi's average monthly income.
(a) Rs. 1858 (b) Rs. 3108.33
(c) Rs. 3100 (d) None of these

83. Shyam bought 2 articles for Rs. 5.50 each, and 3 articles for Rs. 3.50 each, and 3 articles for Rs. 5.50 each and 5 articles for Rs. 1.50 each. The average price for one article is
(a) Rs. 3 (b) Rs. 3.10
(c) Rs. 3.50 (d) Rs. 2

84. In a bag, there are 150 coins of Re. 1, 50 p and 25 p denominations. If the total value of coins is Rs. 150, then find how many rupees can be constituted by 50 p coins.
(a) 16 (b) 20
(c) 28 (d) None of these

## Level of Difficulty (LOD) II

1. With an average speed of 40 km/h, a train reaches its destination in time. If it goes with an average speed of 35 km/h, it is late by 15 minutes. The length of the total journey is:
   (a) 40 km (b) 70 km (c) 30 km (d) 80 km
   (e) 60 km
2. In the month of July of a certain year, the average daily expenditure of an organisation was Rs. 68. For the first 15 days of the month, the average daily expenditure was Rs. 85 and for the last 17 days, Rs. 51. Find the amount spent by the organisation on the 15th of the month.
   (a) Rs. 42 (b) Rs. 36
   (c) Rs. 34 (d) Rs. 52
   (e) Rs. 40
3. In 1919, W. Rhodes, the Yorkshire cricketer, scored 891 runs for his county at an average of 34.27; in 1920, he scored 949 runs at an average of 28.75; in 1921, 1329 runs at an average of 42.87 and in 1922, 1101 runs at an average of 36.70. What was his county batting average for the four years?
   (a) 36.23 (b) 37.81 (c) 35.58 (d) 28.72
   (e) None of these
4. A train travels with a speed of 20 m/s in the first 10 minutes, goes 8.5 km in the next 10 minutes, 11 km in the next 10, 8.5 km in the next 10 and 6 km in the next 10 minutes. What is the average speed of the train in kilometer per hour for the journey described?
   (a) 42 kmph (b) 35.8 kmph
   (c) 55.2 kmph (d) 46 kmph
   (e) 52 kmph
5. One-fourth of a certain journey is covered at the rate of 25 km/h, one-third at the rate of 30 km/h and the rest at 50 km/h. Find the average speed for the whole journey.
   (a) 600/53 km/h (b) 1200/53 km/h
   (c) 1800/53 km/h (d) 1600/53
   (e) None of these
6. Typist $A$ can type a sheet in 6 minutes, typist $B$ in 7 minutes and typist $C$ in 9 minutes. The average number of sheets typed per hour per typist for all three typists is
   (a) 265/33 (b) 530/63
   (c) 655/93 (d) 530/33
   (e) None of these
7. Find the average increase rate if increase in the population in the first year is 30% and that in the second year is 40%.
   (a) 41 (b) 56 (c) 40 (d) 38
   (e) 39
8. The average income of a person for the first 6 days is Rs. 29, for the next 6 days it is Rs. 24, for the next 10 days it is Rs. 32 and for the remaining days of the month it is Rs. 30. Find the average income per day.
   (a) Rs. 31.64 (b) Rs. 30.64
   (c) Rs. 29.26 (d) Rs. 31.22
   (e) Cannot be determined
9. In hotel Jaysarmin, the rooms are numbered from 101 to 130 on the first floor, 221 to 260 on the second floor and 306 to 345 on the third floor. In the month of June 2002, the room occupancy was 60% on the first floor, 40% on the second floor and 75% on the third floor. If it is also known that the room charges are Rs. 200, Rs. 100 and Rs. 150 on each of the floors, then find the average income per room for the month of June 2002.
   (a) Rs. 151.5 (b) Rs. 88.18
   (c) Rs. 78.3 (d) Rs. 65.7
   (e) None of these
10. A salesman gets a bonus according to the following structure: If he sells articles worth Rs. $x$ then he gets a bonus of Rs. $(x/100 - 1)$. In the month of January, his sales value was Rs. 100, in February it was Rs. 200, from March to November it was Rs. 300 for every month and in December it was Rs. 1200. Apart from this, he also receives a basic salary of Rs. 30 per month from his employer. Find his average income per month during the year.
    (a) Rs. 31.25 (b) Rs. 30.34
    (c) Rs. 32.58 (d) Rs. 34.5
    (e) Rs. 33.5
11. A man covers half of his journey by train at 60 km/h, half of the remainder by bus at 30 km/h and the rest by cycle at 10 km/h. Find his average speed during the entire journey.
    (a) 36 kmph (b) 30 kmph
    (c) 24 kmph (d) 18 kmph
    (e) 42 km/h

(a) 100 (b) 90 (c) 110 (d) 105

(e) 120

25. The average salary of the entire staff in an office is Rs. 3200 per month. The average salary of officers is Rs. 6800 and that of non-officers is Rs. 2000. If the number of officers is 5, then find the number of non-officers in the office?

(a) 8 (b) 12 (c) 15 (d) 5

(e) 10

26. A person travels three equal distances at a speed of $x$ km/h, $y$ km/h and $z$ km/h respectively. What will be the average speed during the whole journey?

(a) $xyz/(xy + yz + zx)$ (b) $(xy + yz + zx)/xyz$

(c) $3xyz/(xy + yz + xz)$ (d) None of these

***Directions for Questions 27–30:*** Read the following passage and answer the questions that follow.

In a family of five persons *A*, *B*, *C*, *D* and *E*, each and everyone loves one another very much. Their birthdays are in different months and on different dates. *A* remembers that his birthday is between 25th and 30th, of *B* it is between 20th and 25th, of *C* it is between 10th and 20th, of *D* it is between 5th and 10th and of *E* it is between 1st to 5th of the month. The sum of the date of birth is defined as the addition of the date and the month, for example 12th January will be written as 12/1 and will add to a sum of the date of 13. (Between 25th and 30th includes both 25 and 30).

27. What may be the maximum average of their sum of the dates of birth?

(a) 24.6 (b) 15.2

(c) 28 (d) 32

28. What may be the minimum average of their sum of the dates of births?

(a) 24.6 (b) 15.2

(c) 28 (d) 32

29. If it is known that the dates of birth of three of them are even numbers then find maximum average of their sum of the dates of birth.

(a) 24.6 (b) 15.2

(c) 27.6 (d) 28

30. If the date of birth of four of them are prime numbers, then find the maximum average of the sum of their dates of birth.

(a) 27.2 (b) 26.4

(c) 28 (d) None of these

31. The average age of a group of persons going for a picnic is 16.75 years. 20 new persons with an average age of 13.25 years join the group on the spot due to which the average of the group becomes 15 years. Find the number of persons initially going for the picnic?

(a) 24 (b) 20 (c) 15 (d) 18

32. A school has only four classes that contain 10, 20, 30 and 40 students respectively. The pass percentage of these classes are 20%, 30%, 60% and 100% respectively. Find the pass % of the entire school.

(a) 56% (b) 76% (c) 34% (d) 66%

33. Find the average of $f(x)$, $g(x)$, $h(x)$, $d(x)$ at $x = 10$. $f(x)$ is equal to $x^2 + 2$, $g(x) = 5x^2 - 3$, $h(x) = \log x^2$ and $d(x) = (4/5)x^2$

(a) 170 (b) 170.25 (c) 70.25 (d) 70

34. Find the average of $f(x) - g(x)$, $g(x) - h(x)$, $h(x) - d(x)$, $d(x) - f(x)$

(a) 0 (b) –2.25 (c) 4.5 (d) 2.25

35. $\sum_{r=1}^{n} (n+1)r$ where $r = n$.

(a) $\frac{(n-1)(n)(n+1)}{2}$ (b) $\frac{n(n+1)^2}{2}$

(c) $\frac{n(n-1)^2}{2}$ (d) $\frac{n^2}{2}$

36. The average of '$n$' numbers is $z$. If the number $x$ is replaced by the number $x^1$, then the average becomes $z^1$. Find the relation between $n$, $z$, $z^1$, $x$ and $x^1$.

(a) $\left[\frac{z^1 - 2}{x^1 - x} = \frac{1}{n}\right]$ (b) $\left[\frac{x^1 - x}{z^1} = \frac{1}{n}\right]$

(c) $\left[\frac{z - z^1}{x - x^1} = \frac{1}{n}\right]$ (d) $\left[\frac{x - x^1}{z - z^1} = \frac{1}{n}\right]$

37. The average salary of workers in AMS careers is Rs. 2,000, the average salary of faculty being Rs. 4,000 and the management trainees being Rs. 1,250. The total number of workers could be

(a) 450 (b) 300 (c) 110 (d) 500

***Directions for Questions 38–41:*** Read the following and answer the questions that follows.

During a criket match, India playing against NZ scored in the following manner:

| *Partnership* | *Runs scored* |
|---|---|
| 1st wicket | 112 |

| | |
|---|---|
| 2nd wicket | 58 |
| 3rd wicket | 72 |
| 4th wicket | 92 |
| 5th wicket | 46 |
| 6th wicket | 23 |

38. Find the average runs scored by the first four batsmen.
    (a) 83.5 (b) 60.5
    (c) 66.8 (d) Cannot be determined
39. The maximum average runs scored by the first five batsmen could be
    (a) 80.6 (b) 66.8
    (c) 76 (d) Cannot be determined.
40. The minimum average runs scored by the last five batsmen to get out could be
    (a) 53.6 (b) 44.4 (c) 66.8 (d) 0
41. If the fifth down batsman gets out for a duck, then find the average runs scored by the first six batsmen.
    (a) 67.1 (b) 63.3
    (c) 48.5 (d) Cannot be determined
42. The weight of a body as calculated by the average of 7 different experiments is 53.735 gm. The average of the first three experiments is 54.005 gm, of the fourth is 0.004 gm greater than the fifth, while the average of the sixth and seventh experiment was 0.010 gm less than the average of the first three. Find the weight of the body obtained by the fourth experiment.
    (a) 49.353 gm (b) 51.712 gm
    (c) 53.072 gm (d) 54.512 gm
43. A man's average expenditure for the first 4 months of the year was Rs. 251.25. For the next 5 months the average monthly expenditure was Rs. 26.27 more than what it was during the first 4 months. If the person spent Rs. 760 in all during the remaining 3 months of the year, find what percentage of his annual income of Rs. 3000 he saved in the year.
    (a) 14% (b) –5.0866%
    (c) 12.5% (d) None of these
44. A certain number of trucks were required to transport 60 tons of steel wire from the TISCO factory in Jamshedpur. However, it was found that since each truck could take 0.5 tons of cargo less, another 4 trucks were needed. How many trucks were initially planned to be used?
    (a) 10 (b) 15 (c) 20 (d) 25
45. One collective farm got an average harvest of 21 tons of wheat and another collective farm that had 12 acres of land less given to wheat, got 25 tons from a hectare. As a result, the second farm harvested 300 tons of wheat more than the first. How many tons of wheat did each farm harvest?
    (a) 3150, 3450 (b) 3250, 3550
    (c) 2150, 2450 (d) None of these

## Level of Difficulty (LOD) III

***Directions for Questions 1–28:*** Read the following:

There are 3 classes having 20, 25 and 30 students respectively having average marks in an examination as 20, 25 and 30 respectively. If the three classes are represented by *A*, *B* and *C* and you have the following information about the three classes, answer the questions that follow:

$A \rightarrow$ Highest score 22, Lowest score 18
$B \rightarrow$ Highest score 31, Lowest score 23
$C \rightarrow$ Highest score 33, Lowest score 26

If five students are transferred from *A* to *B*.

1. What will happen to the average score of *B*?
   (a) Definitely increase (b) Definitely decrease
   (c) Remain constant (d) Cannot say
2. What will happen to the average score of *A*?
   (a) Definitely increase (b) Definitely decrease
   (c) Remain constant (d) Cannot say

In a transfer of 5 students from *A* to *C*

3. What will happen to the average score of *C*?
   (a) Definitely increase (b) Definitely decrease
   (c) Remain constant (d) Cannot say
4. What will happen to the average score of *A*?
   (a) Definitely increase (b) Definitely decrease
   (c) Remain constant (d) Cannot say

In a transfer of 5 students from *B* to *C* (Questions 5–6)

5. What will happen to the average score of *C*?
   (a) Definitely increase (b) Definitely decrease
   (c) Remain constant (d) Cannot say
6. Which of these can be said about the average score of *B*?
   (a) Increases if *C* decreases

(c) 29.6 (d) 28

(e) Cannot say

22. The minimum possible average of group *B* after this set of operation is

(a) 21.6 (b) 21.4

(c) 21.8 (d) 21.2

(e) Cannot say

23. The minimum possible average of group *A* after the set of 3 operation is

(a) 20 (b) 20.3

(c) 20.4 (d) 19.8

(e) Cannot say

24. Which of these will definitely not constitute an operation for getting the minimum possible average value for group *A*:

(a) Transfer of five 31s from *B* to *A*

(b) Transfer of five 26s from *C* to *B*

(c) Transfer of five 22s from *A* to *B*

(d) Transfer of five 33s from *C* to *B*

(e) None of these

25. For getting the lowest possible value of *C*'s average, the sequence of operations could be

(a) Transfer five 33s from *C* to *B*, five 23s from *B* to *A*, five 18s from *A* to *B*, five 18s from *B* to *C*

(b) Transfer five 33s from *C* to *B*, 31s from *B* to *A*.

(c) Both a and b

(d) None of the above

(e) Cannot say

26. If we set the highest possible average of class *C* as the primary objective and want to achieve the highest possible value for class *B* as the secondary objective, what is the maximum value of class *B*'s average that is attainable?

(a) 27 (b) 26 (c) 25 (d) 24

(e) 23

27. For question 26, if the secondary objective is changed to achieving the minimum possible average value of class *B*'s average, the lowest value of class *B*'s average that could be attained is

(a) 22.2 (b) 23 (c) 22.6 (d) 22

(e) 24

28. For question 27, what can be said about class *A*'s average?

(a) Will be determined automatically at 22.25

(b) Will have a maximum possible value of 22.25

(c) Will have a minimum possible value of 22.25

(d) Will be determined automatically at 22.5

(e) None of these

29. A team of miners planned to mine 1800 tons of ore during a certain number of days. Due to technical difficulties in one-third of the planned number of days, the team was able to achieve an output of 20 tons of ore less than the planned output. To make up for this, the team overachieved for the rest of the days by 20 tons. The end result was that the team completed the task one day ahead of time. How many tons of ore did the team initially plan to ore per day?

(a) 50 tons (b) 100 tons

(c) 150 tons (d) 200 tons

(e) 250 tons

30. According to a plan, a team of woodcutters decided to harvest 216 $m^3$ of wheat in several days. In the first three days, the team fulfilled the daily assignment, and then it harvested 8 $m^3$ of wheat over and above the plan everyday. Therefore, a day before the planned date, they had already harvested 232 $m^3$ of wheat. How many cubic metres of wheat a day did the team have to cut according to the plan?

(a) 12 (b) 13 (c) 24 (d) 25

(e) 26

31. On an average, two liters of milk and one liter of water are needed to be mixed to make 1 kg of sudha shrikhand of type *A*, and 3 liters of milk and 2 liters of water are needed to be mixed to make 1 kg of sudha shrikhand of type *B*. How many kilograms of each type of shrikhand was manufactured if it is known that 130 liters of milk and 80 liters of water were used?

(a) 20 of type *A* and 30 of type *B*

(b) 30 of type *A* and 20 of type *B*

(c) 15 of type *A* and 30 of type *B*

(d) 30 of type *A* and 15 of type *B*

(e) None of these

32. There are 500 seats in Minerva Cinema, Mumbai, placed in similar rows. After the reconstruction of the hall, the total number of seats became 10% less. The number of rows was reduced by 5 but each row contained 5 seats more than before. How many rows and how many seats in a row were there initially in the hall?

(a) 20 rows and 25 seats

(b) 20 rows and 20 seats

(c) 10 rows and 50 seats
(d) 50 rows and 10 seats
(e) 40 rows and 20 seats

33. One fashion house has to make 810 dresses and another one 900 dresses during the same period of time. In the first house, the order was ready 3 days ahead of time and in the second house, 6 days ahead of time. How many dresses did each fashion house make a day if the second house made 21 dresses more a day than the first?
(a) 54 and 75 (b) 24 and 48
(c) 44 and 68 (d) 04 and 25
(e) None of these

34. A shop sold 64 kettles of two different capacities. The smaller kettle cost a rupee less than the larger one. The shop made 100 rupees from the sale of large kettles and 36 rupees from the sale of small ones. How many kettles of either capacity did the shop sell and what was the price of each kettle?
(a) 20 kettles for 2.5 rupees each and 14 kettles for 1.5 rupees each
(b) 40 kettles for 4.5 rupees each and 24 kettles for 2.5 rupees each
(c) 40 kettles for 2.5 rupees each and 24 kettles for 1.5 rupees each
(d) either a or b
(e) None of these

35. An enterprise got a bonus and decided to share it in equal parts between the exemplary workers. It turned out, however, that there were 3 more exemplary workers than it had been assumed. In that case, each of them would have got 4 rupees less. The administration had found the possibility to increase the total sum of the bonus by 90 rupees and as a result each exemplary worker got 25 rupees. How many people got the bonus?
(a) 9 (b) 18 (c) 8 (d) 16
(e) 20

***Directions for Questions 36–39:*** Read the following and answer the questions that follows.

In the island of Hoola Boola Moola, the inhabitants have a strange process of calculating their average incomes and expenditures. According to an old legend prevalent on that island, the average monthly income had to be calculated on the basis of 14 months in a calendar year while the average monthly expenditure was to be calculated on the basis of 9 months per year. This would lead to people having an underestimation of their savings since there would be an underestimation of the income and an overestimation of the expenditure per month.

36. If the minister for economic affairs decided to reverse the process of calculation of average income and average expenditure, what will happen to the estimated savings of a person living on Hoola Boola Moola island?
(a) It will increase
(b) It will decrease
(c) It will remain constant
(d) Will depend on the value

37. If it is known that Mr. Magoo Hoola Boola estimates his savings at 10 Moolahs and if it is further known that his actual expenditure is 288 Moolahs in an year (Moolahs, for those who are not aware, is the official currency of Hoola Boola Moola), then what will happen to his estimated savings if he suddenly calculates on the basis of a 12 month calendar year?
(a) Will increase by 5
(b) Will increase by 15
(c) Will increase by 10
(d) Will triple

38. Mr. Boogie Woogie comes back from the USA to Hoola Boola Moola and convinces his community comprising 546 families to start calculating the average income and average expenditure on the basis of 12 months per calendar year. Now if it is known that the average estimated income on the island is (according to the old system) 87 Moolahs per month, then what will be the change in the average estimated savings for the island of Hoola Boola Moola. (Assume that there is no other change).
(a) 251.60 Moolahs (b) 565.5 Moolahs
(c) 625.5 Moolahs (d) Cannot be determined

39. Mr. Boogle Woogle comes back from the USSR and convinces his community comprising 273 families to start calculating the average income on the basis of 12 months per calendar year. Now if it is known that the average estimated income in his community is (according to the old system) 87 Moolahs per month, then what will be the change in the average estimated savings for the island of Hoola Boola Moola. (Assume that there is no other change).

(a) 251.60 Moolahs (b) 282.75 Moolahs
(c) 312.75 Moolahs (d) Cannot be determined

***Directions for Questions 40–44:*** Read the following and answer the questions that follows.

The Indian cricket team has to score 360 runs on the last day of a test match in 90 overs, to win the test match. This is the target set by the opposing captain Brian Lara after he declared his innings closed at the overnight score of 411 for 7.

The Indian team coach has the following information about the batting rates (in terms of runs per over) of the different batsmen:

Assume that the run rate of a partnership is the weighted average of the individual batting rates of the batsmen involved in the partnership (on the basis of the ratio of the strike each batsman gets, i.e. the run rate of a partnership is defined as the weighted average of the run rates of the two batsmen involved weighted by the ratio of the number of balls faced by each batsman).

Since decimal fractions of runs are not possible for any batsman, assume that the estimated runs scored by a batsman in an inning (on the basis of his run rate and the number of overs faced by him) is rounded off to the next higher integer immediately above the estimated value of the runs scored during the innings.

For example, if a batsman scores at an average of 3 runs per over for 2.1666 overs, then he will be estimated to have scored $2.1666 \times 3 = 6.5$ runs in his innings, but since this is not possible, the actual number of runs scored by the batsman will be taken as 7 (the next higher integer above 6.5).

Also, this rounding off can take place only once for one innings of a batsman.

**Runs scored per over in different batting styles**

| *Name of Batsman* | *Defensive* | *Normal* | *Aggressive* |
|---|---|---|---|
| Das | 3 | 4 | 5 |
| Dasgupta | 2 | 3 | 4 |
| Dravid | 2 | 3 | 4 |
| Tendulkar | 4 | 6 | 8 |
| Laxman | 4 | 5 | 6 |
| Sehwag | 4 | 5 | 6 |
| Ganguly | 3 | 4 | 5 |
| Kumble | 2 | 3 | 4 |
| Harbhajan | 3 | 4 | 5 |
| Srinath | 3 | 4 | 5 |
| Yohannan | 2 | 3 | 4 |

Assume no extras unless otherwise stated.

Assume that the strike is equally shared unless otherwise stated.

40. If the first wicket pair of Das and Dasgupta bats for 22 overs and during this partnership Das has started batting normally and turned aggressive after 15 overs while Dasgupta started off defensively but shifted gears to bat normally after batting for 20 overs, find the expected score after 22 overs.
    (a) 65 (b) 71 (c) 82 (d) 58
41. Of the first-wicket partnership between Das and Dasgupta as per the previous question, the ratio of the number of runs scored by Das to those scored by Dasgupta is:
    (a) 46 : 25 (b) 96 : 46
    (c) 41 : 32 (d) Cannot be determined
42. The latest time by which Tendulkar can come to bat and still win the game, assuming that the run rate at the time of his walking the wicket is into 2.5 runs per over, is (assuming he shares strike equally with his partner and that he gets the maximum possible support at the other end from his batting partner and both play till the last ball).
    (a) After 50 overs (b) After 55 overs
    (c) After 60 overs (d) Cannot be determined
43. For question 42, where Tendulkar batted aggressively and assuming that it is the Tendulkar-Laxman pair that wins the game for India (after Tendulkar walks into bat with the current run rate at 2.5 per over, and at the latest possible time for him to win the game with maximum possible support from the opposite end), what will be Tendulkar's score for the innings (assume equal strike)?
    (a) 105 (b) 120
    (c) 135 (d) None of these
44. For questions 42 and 43, if it was Laxman who batted with Tendulkar for his entire innings, then how many runs would Laxman score in the innings?
    (a) 105 (b) 75
    (c) 90 (d) Cannot be determined

***Directions for Questions 45–49:*** Read the following and answer the questions that follow.

If Sachin Tendulkar walks into bat after the fall of the fifth wicket and has to share partnerships with Ganguly, Kumble, Harbhajan, Srinath and Yohannan, who have batted normally, defensively, defensively, defensively and defensively respectively while Tendulkar has batted normally,

16. Part of the runs scored in the 87th innings will go towards increasing the average of the first 86 innings to the new average and the remaining part of the runs will go towards maintaining the new average for the 87th innings. The only constraint in this problem is that there is an increase in the average by a whole number of runs. This is possible for all three options.
17. Assume $x$ is the average expenditure of 19 people. Then, $19x = 13*79 + 6(x + 4)$.
20. The average weight per ball is asked. Hence, the bag does not have to be counted as the 48th item.
21. $71*48 = 59*46 + x + 11*52 \rightarrow x = 72$.
    Alternately, this can be solved by using the concepts of surpluses and deficits as:
    2*59 (deficit) – 4 * 11(surplus) + 48 (average to be maintained by the 60th number) = 118-44 + 48 = 122.
22. Solve through the same process as the Q. No. 5 of this chapter.
23. (14 * 333 – 2 * 504)/12.
24. $\frac{(84-64)\times 94}{18.8}$.
25. Use alligation to solve, 20———32———68. Thus, 5 corresponds to 12, hence for 36 the answer will be 15.
26. Let the equal distances be '$d$' each. Then $3d/(d/x + d/y + d/z) = 3xyz/(x + y + z)$.

27–30. You have to take between 25th and 30th to mean that both these dates are also included.

27. The maximum average will occur when the maximum possible values are used. Thus:
    *A* should have been born on 30th, *B* on 25th, *C* on 20th, *D* on 10th and *E* on 5th. Further, the months of births in random order will have to be between August to December to maximize the average.
    Hence the total will be 30 + 25 + 20 + 10 + 5 + 12 + 11 + 10 + 9 + 8 = 140. Hence average is 28.
28. The minimum average will be when we have 1 + 5 + 10 + 20 + 25 + 1 + 2 + 3 + 4 + 5 = 76. Hence, average is 15.2.
29. This does not change anything. Hence the answer is the same as Q. 27.
30. The prime dates must be 29th, 23rd, 19th and 5th. Hence, the maximum possible average will reduce by 4/5 = 0.8. Hence, answer will be 27.2.
31. Solve using alligation. Since 15 is the mid-point of 13.25 and 16.75, the ratio is 1:1 and hence there are 20 people who were going for the picnic initially.
32. The number of pass candidates are 2 + 6 + 18 + 40 = 66 out of a total of 100. Hence, 66%.

33&34. Put $x = 10$ in the given equations and find the average of the resultant values.

35. Solve through options.
36. $nz - x + x^1 = nz^1 \rightarrow$ Simplify to get Option (c) correct.
37. By alligation the ratio is 3:8. Hence, only 110 is possible.

Q38-41:

38. You don't know who got out when. Hence, cannot be determined.
39. Since possibilities are asked about, you will have to consider all possibilities. Assume, the sixth and seventh batsmen have scored zero. Only then will the possibility of the first 5 batsmen scoring the highest possible average arise. In this case the maximum possible average for the first 5 batsmen could be 403/5 = 80.6.
40. Again it is possible that only the first batsman has scored runs.
41. We cannot find out the number of runs scored by the 7th batsman. Hence answer is (d).
42. You can take 53 as the base to reduce your calculations. Otherwise the question will become highly calculation intensive.
43. 251.25*4 + 277.52 * 5 + 760 =
44. Solve using options. 20 is the only possible value.
45. Check through options to solve.

## Hints and Solutions III

1. Definitely decrease, since the highest marks in Class *A* is less than the lowest marks in Class *B*.
2. Cannot say since there is no indication of the values of the numbers which are transferred.
3. It will definitely decrease since the highest possible transfer is lower than the lowest value in *C*.
4. The effect on *A* will depend on the profile of the people who are transferred. Hence, anything can happen.
5. Cannot say since there is a possibility that the numbers transferred are such that the average can either increase, decrease or remain constant.
6. If *C* increases, then the average of *C* goes up from 30. For this to happen it is definite that the average of *B* should drop.

7. The maximum possible average for $B$ will occur if all the 5 transferees from $A$ have 22 marks.
8. The average of Group $A$ after the transfer in Q. 7 above is:
(400 – 18*5)/15 = 310/15 = 20.66
9. (400 – 22*5)/15 = 19.33
10. 400 + 23*5 = 515. Average = 515/25 = 20.6
11. 400 + 31*5 = 555. Average = 555/25 = 22.2
12. Will always decrease since the net value transferred from $B$ to $A$ will be higher than the net value transferred from $A$ to $B$.
13. Since the lowest score in Class $B$ is 23 which is more than the highest score of any student in Class $A$. Hence, $A$'s average will always increase.
14. The maximum possible value for $B$ will happen when the $A$ to $B$ transfer has the maximum possible value and the reverse transfer has the minimum possible value.
15. For the minimum possible value of $B$ we will need the $A$ to $B$ transfer to be the lowest possible value while the $B$ to $A$ transfer must have the highest possible value. Thus, $A$ to $B$ transfer → 18* 5 while $B$ to $A$ transfer will be 31*5. Hence answer is 22.4.
16. The maximum value for $A$ will happen in the case of Q. 15. Then the increment for group $A$ is:
31*5 – 18*5 = 5*(31 – 18) = 65.
Thus maximum possible value is 465/20 = 23.25.
17. Minimum possible average will happen for the transfer we saw in Q. 14. Thus the answer will be 405/20 = 20.25.
18. The maximum possible value for $C$ will be achieved when the transfer from $C$ is of five 26's and the transfer back from $B$ is of five 31's. Hence, difference is totals will be +25. Hence, max. average = (900 + 25)/30 = 30.833.
[Note here that 900 has come by 30*30]
19. For the maximum possible value of Class $B$ the following set of operations will have to hold:
Five 33's are transferred from $C$ to $B$, whatever goes from $B$ to $A$ comes back from $A$ to $B$, then five 23's are transferred from $B$ to $C$. This leaves us with:
Increase of 50 marks → average increases by 2 to 27.
20. A will attain maximum value if five 33's come to $A$ from $C$ through $B$ and five 18's leave $A$. In such a case the net result is going to be a change of +75. Thus the average will go up by 75/20 = 3.75 to 23.75.

21-23. Will be solved by the same pattern as the above questions.

24. Only option $A$ will give us the required situation since the transfer of five 31's increases the value of the average of group $A$.

25-28. Will be solved by the same pattern as above questions.

29-35. These are standard questions using the concept of averages. Hence, analyse each and every sentence by itself and link the interpretations. If you are getting stuck, the only reason is that you have not used the information in the questions fully.

36. Monthly estimates of income is reduced as the denominator is increased from 12 to 14 at the same time the monthly estimate of expenditure is increased as the denominator is reduced from 12 to 9. Hence, the savings will be underestimated.

37-39. Use the averages formulae and common sense to answer.

40-49. The questions are commonsensical with a lot of calculations and assumptions involved. You have to solve these using all the information provided.

40. Das's score = 15*2 + 7*2.5 = 47.5 → 48.
Dasgupta's score = 20*1 + 2*1.5 = 23
41. From the above the answer is 48:23 = 96:46.

42-44. By maximum possible support from the other end, you have to assume that he has Laxman or Sehwag batting aggressively for the entire tenure at the crease. Strike has to be shared equally.

42. Through options, After 60 overs, score would be 150. Then Tendulkar can score @ 4 runs per over (sharing the strike and batting aggressively) and get maximum support @ 3 runs per over. Thus in 30 overs left the target will be achieved.
43. Tendulkar's score for the innings will be 30*4 = 120.
44. We do not know when Laxman would have come into bat. Hence this cannot be determined.

45-49. Build in each of the conditions in the problem to form a table like:

| Partnership | Partner | Overs faced | Tendulkar's score | Partner's score |
|---|---|---|---|---|
| 6th wicket | Ganguly | 12 | 6 overs × 6 | 6 overs × 4 |
| 7th wicket | and so on | | | |
| 8th wicket | | | | |
| 9th wicket | | | | |
| 10th wicket | | | | |

**Averages LOD II**

| | | |
|---|---|---|
| 1. b | 33. b | |
| 2. c | 34. a | |
| 3. c | 35. b | |
| 4. c | 36. c | |
| 5. c | 37. c | |
| 6. b | 38. d | |
| 7. a | 39. a | |
| 8. b | 40. d | |
| 9. b | 41. d | |
| 10. c | 42. c | |
| 11. c | 43. b | |
| 12. a | 44. c | |
| 13. a | 45. a | |
| 14. b | | |
| 15. a | | |
| 16. d | | |
| 17. e | | |
| 18. a | | |
| 19. e | | |
| 20. e | | |
| 21. b | | |
| 22. b | | |
| 23. c | | |
| 24. a | | |
| 25. c | | |
| 26. c | | |
| 27. c | | |
| 28. b | | |
| 29. d | | |
| 30. a | | |
| 31. b | | |
| 32. d | | |

**Averages LOD III**

| | |
|---|---|
| 1. b | 33. a |
| 2. d | 34. c |
| 3. b | 35. b |
| 4. d | 36. a |
| 5. d | 37. b |
| 6. b | 38. d |
| 7. b | 39. b |
| 8. a | 40. b |
| 9. c | 41. b |
| 10. d | 42. c |
| 11. b | 43. b |
| 12. b | 44. d |
| 13. b | 45. b |
| 14. c | 46. b |
| 15. a | 47. b |
| 16. c | 48. d |
| 17. d | 49. b |
| 18. a | |
| 19. b | |
| 20. b | |
| 21. b | |
| 22. c | |
| 23. a | |
| 24. c | |
| 25. a | |
| 26. d | |
| 27. c | |
| 28. a | |
| 29. b | |
| 30. c | |
| 31. a | |
| 32. a | |

# 4

# ALLIGATIONS

## INTRODUCTION

The chapter of alligation is nothing but a faster technique of solving problems based on the weighted average situation as applied to the case of two groups being mixed together. I have often seen students having a lot of difficulty in solving questions on alligation. Please remember that all problems on alligation can be solved through the weighted average method. Hence, the student is advised to revert to the weighted average formula in case of any confusion.

The use of the techniques of this chapter for solving weighted average problems will help you in saving valuable time wherever a direct question based on the mixing of two groups is asked. Besides, in the case of questions that use the concept of the weighted average as a part of the problem, you will gain a significant edge if you are able to use the techniques illustrated here.

The relevance of this chapter for the CAT is the use of the alligation technique for solving problems where the concept is used to solve a part of the whole question. Besides, questions of this chapter are directly relevant for the exams like CET (Maharashtra), NMIMS, NIFT, Symbiosis, MAT and all other B level management entrance exams as well as for the bank PO exams and the MCA exams, which are based on the pattern of an aptitude test.

## THEORY

In the chapter on Averages, we had seen the use of the weighted average formula. To recollect, the weighted average is used when a number of smaller groups are mixed together to form one larger group.

If the average of the measured quantity was

| | | | | |
|---|---|---|---|---|
| $A_1$ for group | 1 | containing | $n_1$ | elements |
| $A_2$ for group | 2 | containing | $n_2$ | elements |
| $A_3$ for group | 3 | containing | $n_3$ | elements |
| $A_k$ for group | $k$ | containing | $n_k$ | elements |

We say that the weighted average, $Aw$ is given by:

$$Aw = (n_1A_1 + n_2 A_2 + n_3 A_3 + \ldots\ldots + n_k A_k)/ (n_1 + n_2 + n_3 \ldots + n_k)$$

That is, the weighted average

$$= \frac{\text{Sum total of all groups}}{\text{Total number of elements in all groups together}}$$

In the case of the situation where just two groups are being mixed, we can write this as:

$$Aw = (n_1A_1 + n_2 A_2)/(n_1 + n_2)$$

Rewriting this equation we get: $(n_1 + n_2)\ Aw = n_1A_1 + n_2A_2$

$$n_1(Aw - A_1) = n_2 (A_2 - Aw)$$

or $n_1/n_2 = (A_2 - Aw)/(Aw - A_1) \rightarrow$ The alligation equation.

### The Alligation Situation

Two groups of elements are mixed together to form a third group containing the elements of both the groups.

If the average of the first group is $A_1$ and the number of elements is $n_1$ and the average of the second group is $A_2$ and the number of elements is $n_2$, then to find the average of the new group formed, we can use either the weighted average equation or the alligation equation.

29. Two solutions of 90% and 97% purity are mixed resulting in 21 litres of mixture of 94% purity. How much is the quantity of the first solution in the resulting mixture?
(a) 15 litres (b) 12 litres
(c) 9 litres (d) 6 litres

30. In the Singapore zoo, there are deers and there are ducks. If the heads are counted, there are 180, while the legs are 448. What will be the number of deers in the zoo?
(a) 136 (b) 68 (c) 44 (d) 22

31. A bonus of Rs. 9,85,000 was divided among 300 workers of a factory. Each male worker gets 5000 rupees and each female worker gets 2500 rupees. Find the number of male workers in the factory.
(a) 253 (b) 47 (c) 94 (d) 206

32. What will be the ratio of petrol and kerosene in the final solution formed by mixing petrol and kerosene that are present in three vessels in the ratio 4 : 1, 5 : 2 and 6 : 1 respectively?
(a) 166 : 22 (b) 83 : 22
(c) 83 : 44 (d) None of these

33. A mixture worth Rs. 3.25 a kg is formed by mixing two types of flour, one costing Rs. 3.10 per kg while the other Rs. 3.60 per kg. In what proportion must they have been mixed?
(a) 3 : 7 (b) 7 : 10
(c) 10 : 3 (d) 7 : 3

34. A gain percent of 20 is made by selling the mixture of two types of ghee at Rs. 480 per kg. If the type costing 610 per kg was mixed with 126 kg of the other, how many kilograms of the former was mixed?
(a) 138 kg (b) 34.5 kg
(c) 69 kg (d) Cannot be determined

35. In what proportion must water be mixed with milk so as to gain 20% by selling the mixture at the cost price of the milk? (Assume that water is freely available)
(a) 1 : 4 (b) 1 : 5
(c) 1 : 6 (d) 1 : 12

36. A bartender stole champagne from a bottle that contained 50% of spirit and he replaced what he had stolen with champagne having 20% spirit. The bottle then contained only 25% spirit. How much of the bottle did he steal?
(a) 80% (b) 83.33%
(c) 85.71% (d) 88.88%

37. A bag contains a total of 105 coins of Rs. 1, 50 p and 25 p denominations. Find the total number of coins of Re 1 if there are a total of 50.5 rupees in the bag and it is known that the number of 25 paise coins are 133.33% more than the number of 1 rupee coins.
(a) 56 (b) 25
(c) 24 (d) None of these

38. A man possessing Rs. 6800, lent a part of it at 10% simple interest and the remaining at 7.5% simple interest. His total income after $3\frac{1}{2}$ years was Rs. 1904. Find the sum lent at 10% rates.
(a) Rs. 1260 (b) Rs. 1700
(c) Rs. 1360 (d) None of these

39. If a man decides to travel 80 kilometres in 8 hours partly by foot and partly on a bicycle, his speed on foot being 8 km/h and that on bicycle being 16 km/h, what distance would he travel on foot?
(a) 20 km (b) 30 km
(c) 48 km (d) 60 km

40. Two vessels contain a mixture of spirit and water. In the first vessel the ratio of spirit to water is 8 : 3 and in the second vessel the ratio is 5 : 1. A 35 litre cask is filled from these vessels so as to contain a mixture of spirit and water in the ratio of 4 : 1. How many litres are taken from the first vessel?
(a) 11 litres (b) 22 litres
(c) 16.5 litres (d) 17.5 litres

# ANSWER KEY

**Alligations LOD I**

| | | |
|---|---|---|
| 1. b | 33. d | |
| 2. c | 34. d | |
| 3. c | 35. b | |
| 4. c | 36. b | |
| 5. d | 37. c | |
| 6. a | 38. c | |
| 7. c | 39. c | |
| 8. a | 40. a | |
| 9. c | | |
| 10. c | | |
| 11. b | | |
| 12. c | | |
| 13. a | | |
| 14. d | | |
| 15. c | | |
| 16. c | | |
| 17. a | | |
| 18. c | | |
| 19. d | | |
| 20. c | | |
| 21. a | | |
| 22. b | | |
| 23. d | | |
| 24. a | | |
| 25. c | | |
| 26. b | | |
| 27. c | | |
| 28. d | | |
| 29. c | | |
| 30. c | | |
| 31. c | | |
| 32. b | | |

# BLOCK REVIEW TESTS

## REVIEW TEST ONE

1. Rakshit bought 19 erasers for Rs. 10. He paid 20 paise more for each white eraser than for each brown eraser. What could be the price of a white eraser and how many white erasers could he have bought?
   (a) 60 paise, 8 (b) 60 paise, 12
   (c) 50 paise, 8 (d) 50 paise, 10
2. After paying all your bills, you find that you have Rs. 7.20 in your pocket. You have equal number of 50 paise and 10 paise coins; but no other coins nor any other currency notes. How many coins do you have?
   (a) 8 (b) 24 (c) 27 (d) 30
3. Suresh Kumar went to the market with Rs. 100. If he buys three pens and six pencils he uses up all his money. On the other hand if he buys three pencils and six pens he would fall short by 20%. If he wants to buy equal number of pens & pencils, how many pencils can he buy?
   (a) 4 (b) 5 (c) 6 (d) 7
4. For the above question, what is the amount of money he would save if he were to buy 3 pens and 3 pencils?
   (a) Rs. 50 (b) Rs.25
   (c) Rs.75 (d) Rs.40
5. Abdul goes to the market to buy bananas. If he can bargain and reduce the price per dozen by Rs. 2, he can buy 3 dozen bananas instead of 2 dozen with the money he has. How much money does he have?
   (a) Rs. 6 (b) Rs. 12 (c) Rs. 18 (d) Rs. 24
6. Two oranges, three bananas and four apples cost Rs.15. Three oranges, two bananas and one apple cost Rs.10. I bought 3 oranges, 3 bananas and 3 apples. How much did I pay?
   (a) Rs.10 (b) Rs.8
   (c) Rs.15 (d) cannot be determined
7. John bought five mangoes and ten oranges together for forty rupees. Subsequently, he returned one mango and got two oranges in exchange. The price of an orange would be
   (a) Rs. 1 (b) Rs. 2 (c) Rs. 3 (d) Rs. 4
8. Two towns A and B are 100 km apart. A school is to be built for 100 students of Town B and 30 students of Town A. The Expenditure on transport is Rs.1.20 per km per person. If the total expenditure on transport by all 130 students is to be as small as possible, then the school should be built at
   (a) 33 km from Town A.
   (b) 33 km from Town B
   (c) Town A
   (d) Town B
9. A person who has a certain amount with him goes to the market. He can buy 50 oranges or 40 mangoes. He retains 10% of the amount for taxi fare and buys 20 mangoes and of the balance he purchases oranges. Number of oranges he can purchase is
   (a) 36 (b) 40 (c) 15 (d) 20
10. 72 hens costs Rs._96.7_. Then what does each hen cost, where numbers at "_" are not visible or are written in illegible hand?
   (a) Rs.3.43 (b) Rs.5.31
   (c) Rs.5.51 (d) Rs.6.22

***Directions for Questions 10-12:***
There are 60 students in a class. These students are divided into three groups A, B and C of 15, 20 and 25 students each. The groups A and C are combined to form group D

11. What is the average weight of the students in group D?
   (a) more than the average weight of A.
   (b) more than the average weight of C.
   (c) less than the average weight of C.
   (d) Cannot be determined.
12. If one student from Group A is shifted to group B, which of the following will be true?
   (a) The average weight of both groups increases
   (b) The average weight of both groups decreases
   (c) The average weight of the class remains the same.
   (d) Cannot be determined.
13. If all the students of the class have the same weight then which of the following is false?
   (a) The average weight of all the four groups is the same.
   (b) The total weight of A and C is twice the total weight of B.
   (c) The average weight of D is greater than the average weight of A.
   (d) The average weight of all the groups remains the same even if a number of students are shifted from one group to another.

In such a case go back into the problem and try to identify each statement and see whether you have utilized it or not. If you have already used all the information, you might be interested in knowing whether you have used the information given in the problem in the correct order. If both of these have been done, you might want to explore the next reason for getting stuck.

**Reason 2:** You are stuck because even though you might have used all the information given in the problem, you have not utilized some of the information completely.

In such a case, you need to review each of the parts of the information given in the question and look at whether any additional details can be derived out of the same information. Very often, in Quants, you have situations wherein one sentence might have more than one connotation. If you have used that sentence only in one perspective, then using it in the other perspective will solve the question.

**Reason 3:** You are stuck because the problem does not have a solution.

In such a case, check the question once and if it is correct go back to Reasons 1 and 2. Your solution has to lie there.

My experience in training students tells me that the 1st case is the most common reason for not being able to solve questions correctly. (more than 90% of the times) Hence, if you consider yourself to be weak at Maths, concentrate on the following process in this block of chapters.

## THE LOGIC OF THE STANDARD STATEMENT

What I have been trying to tell you is that most of the times, you will get stuck in a problem only when you are not able to interpret a statement in the problem. Hence, my advise to students (especially those who are weak in these chapters).

Concentrate on developing your ability to decode the mathematical meaning of a sentence in a problem.

To do this, even in problems that you are able to solve (easily or with difficulty) go back into the language of the question and work out the mathematical reaction that you should have with each statement.

It might not be a bad idea to make a list of standard statements along with their mathematical reactions for each chapter in this block of chapters. You will realise that in almost no time, you will come to a situation where you will only rarely encounter new language.

Coming back to the issue of **linear equations**:

Linear equations are expressions about variables that might help us get the value of the variable if we can solve the equation.

Depending upon the number of variables in a problem, a linear equation might have one variable, two variables or even three or more variables. The only thing you should know is that in order to get the value of a variable, the number of equations needed is always equal to the number of variables. In other words, if you have more variables in a system of equations than the number of equations, you cannot solve for the individual values of the variables.

The basic mathematical principle goes like this:

**For a system of equations to be solvable, the number of equations should be equal to the number of variables in the equations.**

Thus for instance, if you have two variables, you need two equations to get the values of the two variables, while if you have three variables you will need three equations.

This situation is best exemplified by the situation where you might have the following equation. $x + y = 7$. If it is known that both $x$ and $y$ are natural numbers, it yields a set of possibilities for the values of x & y as follows: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1). One of these possibilities has to be the answer.

*Contd.*

In fact, it might be a good idea to think of all linear equation situations in this fashion. Hence, before you go ahead to read about the next equation, you should set up this set of possibilities based on the first equation.

Consider the following situation where a question yields a set of possibilities:

> Four enemies $A$, $B$, $C$ and $D$ gather together for a picnic in a park with their wives. $A$'s wife consumes 5 times as many glasses of juice as $A$. $B$'s wife consumes 4 times as many glasses of juice as $B$. $C$'s wife consumes 3 times as many glasses of juice as $C$ and $D$'s wife consumes 2 times as many glasses of juice as $D$. In total, the wives of the four enemies consume a total of 44 glasses of juice. If $A$ consumes at least 5 glasses of juice while each of the other men have at least one glass, find the least number of drinks that could have been consumed by the 4 enemies together.

(1) 9 (2) 12 (3) 11 (4) 10

In the question above, we have 8 variables—$A$, $B$, $C$ & $D$ and $a$, $b$, $c$, $d$ – the number of glasses consumed by the four men and the number of glasses consumed by the four wives.

Also, the question gives us five informations which can be summarized into 5 equations as follows.

a = $5A$

b = $4B$

c = $3C$

d = $2D$

and $a + b + c + d = 44$

Also, $A>5$.

Under this condition, you do not have enough information to get all values and hence you will get a set of possibilities.

Since the minimal value of $A$ is 5, a can take the values 25, 30, 35 and 40 when $A$ takes the values 5, 6, 7 and 8 respectively. Based on these, and on the realization that $b$ has to be a multiple of 4, $c$ a multiple of 3 and $d$ a multiple of 2, the following possibilities emerge:

At $A = 5$

| a | **(multiple of 5)** | 25 | 25 | 25 | 25 | 25 |
|---|---|---|---|---|---|---|
| b | (multiple of 4) | 12 | 8 | 8 | 4 | 4 |
| c | (multiple of 3) | 3 | 9 | 3 | 3 | 9 |
| d | (multiple of 2) | 4 | 2 | 8 | 12 | 6 |
| | *a+b+c+d* | 44 | 44 | 44 | 44 | 44 |

| a | | $A$=6, $a$=30 | $A$=7, $a$= 35 | $A$=8, $a$= 40 |
|---|---|---|---|---|
| b | (multiple of 4) | 4 | 4 | No solution |
| c | (multiple of 3) | 6 | 3 | |
| d | (multiple of 2) | 4 | 2 | |
| | *a+b+c+d* | 44 | 44 | |

In this case the answer will be 10, since in the case of $a$=35, $b$=4, $c$=3 & $d$=2, the values for $A$,$B$,$C$ and $D$ will be respectively 7,1,1 and 1. This solution is the least number of drinks consumed by the 4 enemies together as in all the other possibilities the number of glasses is greater than 10.

**Such utilizations of linear equations are very common in CAT and top level aptitude examinations.**

***The relationship between the decimal value and the percentage value of a ratio:***

Every ratio has a percentage value and a decimal value and the difference between the two is just in the positioning of the decimal point.

Thus 2/4 can be represented as 0.5 in terms of its decimal value and can be represented by 50% in terms of its percentage value.

## PREASSESSMENT TEST

1. Three runners A, Band C run a race, with runner A finishing 24 metres ahead of runner B and 36 metres ahead of runner C, while runner B finishes 16 metres ahead of runner C. Each runner travels the entire distance at a constant speed. .What was the length of the race?
   (a) 72 metres (b) 96 metres
   (c) 120 metres (d) 144 metres
2. A dealer buys dry fruits at Rs.100, Rs. 80 and Rs. 60 per kilogram. He mixes them in the ratio 4:5:6 by weight, and sells at a profit of 50%. At what price per kilogram does he sell the dry fruit?
   (a) Rs.116 (b) Rs.106
   (c) Rs.115 (d) None of these
3. There are two containers: the first contains 500 ml of alcohol, while the second contains 500 ml of water. Five cups of alcohol from the first container is taken out and is mixed well in the second container. Then, five cups of this mixture is taken out from the second container and put back into the first container. Let X and Y denote the proportion of alcohol in the first and the proportion of water in the second container. Then what is the relationship between X & Y? (Assume the size of the cups to be identical)
   (a) X>Y (b) X<Y
   (c) X=Y (d) Cannot be determined
4. Akhilesh took five papers in an examination, where each paper was of 200 marks. His marks in these' papers were in the proportion of 7: 8: 9 :10 : 11. In all papers together, the candidate obtained 60% of the total marks. Then, the number of papers in which he got more than 50% marks is:
   (a) 1 (b) 3
   (c) 4 (d) 5
5. A and B walk up an escalator (moving stairway). The escalator moves at a constant speed, A takes six steps for every four of B's steps. A gets to the top of the escalator after having taken 50 steps, while B (because his slower pace lets the escalator do a little more of the work) takes only 40 steps to reach the top. If the escalator were turned off, how many steps would they have to take to walk up?
   (a) 80 (b) 100
   (c) 120 (d) 160
6. Fifty per cent of the employees of a certain company are men, and 80% of the men earn more than Rs. 2.5 lacs per year. If 60% of the company's employees earn more than Rs. 2.5 lacs per year, then what fraction of the women employed by the company earn more than Rs. 2.5 lacs per year?
   (a) 2/5 (b) 1/4
   (c) 1/3 (d) 3/4
7. A piece of string is 80 centimeters long. It is cut into three pieces. The longest piece is 3 times as long as the middle-sized and the shortest piece is 46 centimeters shorter than the longest piece. Find the length of the shortest piece (in cm).
   (a) 14 (b) 10
   (c) 8 (d) 18
8. Three members of a family A, B, and C, work together to get all household chores done. The time it takes them to do the work together is six hours less than A would have taken working alone, one hour less than B would have taken alone, and half the time C would have taken working alone. How long did it take them to do these chores working together?
   (a) 20 minutes (b) 30 minutes
   (c) 40 minutes (d) 50 minutes
9. Fresh grapes contain 90% water by weight while dried grapes contain 20% water by weight. What is the weight of dry grapes available from 20 kg of fresh grapes?
   (a) 2kg (b) 2.4kg
   (c) 2.5kg (d) None of these
10. At the end of the year 2008, a shepherd bought twelve dozen goats. Henceforth, every year he added p% of the goats at the beginning of the year and sold q% of the goats at the end of the year where p> 0 and q >0. If the shepherd had twelve dozen goats at the end of the year 2012, (after making the sales for that year), which of the following is true?
   (a) p = q (b) p<q
   (c) p>q (d) p = q/2

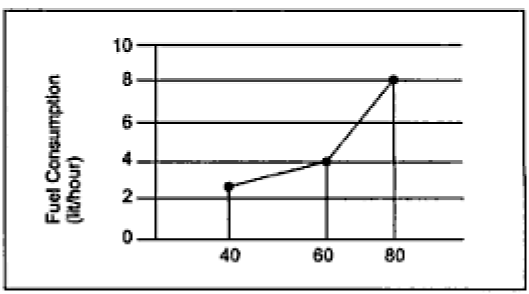
***Directions for Questions 11-12:*** Answer the questions based on the following information.

An Indian company purchases components X and Y from UK and Germany, respectively. X and Y form 40% and 30% of the total production cost. Current gain is 25%. Due to change in the international exchange rate scenario, the cost of the German mark increased by 50% and that of

UK pound increased by 25%. Due to tough competitive market conditions, the selling price cannot be increased beyond 10%.

11. What is the maximum current gain possible?
    (a) 10% (b) 12.5%
    (c) 0% (d) 7.5%
12. If the UK pound becomes cheap by 15% over its original cost and the cost of German mark increased by 20%, what will be the gain if the selling price is not altered.
    (a) 10% (b) 20%
    (c) 25% (d) 7.5%
13. A college has raised 80% of the amount it needs for a new building by receiving an average donation of Rs. 800 from the people already solicited. The people already solicited represent 50% of the people, the college will ask for donations. If the college is to raise exactly the amount needed for the new building, what should be the average donation from the remaining people to be solicited?
    (a) 300 (b) 200
    (c) 400 (d) 500
14. A student gets an aggregate of 60%marks in five subjects in the ratio 10: 9: 8: 7: 6. If the passing marks are 45% of the maximum marks and each subject has the same maximum marks, in how many subjects did he pass the examination?
    (a) 2 (b) 3
    (c) 4 (d) 5
15. After allowing a discount of 12.5 % a trader still makes a gain of 40%. At what per cent above the cost price does he mark on his goods?
    (a) 45% (b) 60%
    (c) 25% (d) None of these
16. The owner of an art shop conducts his business in the following manner. Every once in a while he raises his prices by X%, then a while later he reduces all the new prices by X%. After one such up-down cycle, the price of a painting decreased by Rs. 441. After a second up-down cycle, the painting was sold for Rs. 1944.81. What was the original price of the painting (in Rs.)?
    (a) 2756.25 (b) 2256.25
    (c) 2500 (d) 2000
17. Manas, Mirza, Shorty and Jaipal bought a motorbike for $60,000. Manas paid 50% of the amounts paid by the other three boys, Mirza paid one third of the sum of the amounts paid by the other boys; and Shorty paid one fourth of the sum of the amounts paid by the other boys. How much did Jaipal have to pay?
    (a) $15000 (b) $13000
    (c) $17000 (d) None of these
18. A train X departs from station A at 11.00 a.m. for station B, which is 180 km away. Another train Y departs from station B at 11.00a.m. for station A. Train X travels at an average speed of70 kms/hr and does not stop anywhere until it arrives at station B. Train Y travels at an average speed of 50 km/hr, but has to stop for 10 minutes at station C, which is 60 kms away from station B enroute to station A. Ignoring the lengths of the trains, what is the distance, to the nearest km, from station A to the point where the trains cross each other?
    (a) 110 (b) 112
    (c) 116 (d) None of these
19. In a survey of political preferences, 81% of those asked were in favour of at least one of the three budgetary proposals A, B and C. 50% of those asked favoured proposal A,30% favoured proposal B and 20% favoured proposal C. If 5% of those asked favoured all the three proposals, what percentage of those asked favoured more than one of the three proposals?
    (a) 10% (b) 12%
    (c) 9% (d) 14%

***Directions for Questions 20-21:*** The petrol consumption rate of a new model car 'Palto' depends on its speed and may be described by the graph below:




20. Manasa makes the 240 km trip from Mumbai to Pune at a steady speed of 60 km per hour. What is the amount of petrol consumed for the journey?
    (a) 12.5 litres (b) 16 litres
    (c) 15 litres (d) 19.75 litres

21. Manasa would like to minimize the fuel consumption for the trip by driving at the appropriate speed. How should she change the speed?
    (a) Increase the speed
    (b) Decrease the speed
    (c) Maintain the speed at 60km/hour
    (d) Cannot be determined

***Directions for Questions 22-23:*** Answer the questions based on the following information:

There are five machines -A, B, C, D, and E- situated on a straight line at distances of 10m, 20 m, 30 m, 40 m and 50m respectively from the origin of the line. A robot is stationed at the origin of the line. The robot serves the machines with raw material whenever a machine becomes idle. All the raw materials are located at the origin. The robot is in an idle state at the origin at the beginning of a day. As soon as one or more machines become idle, they send messages to the robot-station and the robot starts and serves all the machines from which it received messages. If a message is received at the station while the robot is away from it, the robot takes notice of the message only when it returns to the station. While moving, it serves the machines in the sequence in which they are encountered, and then returns to the origin. If any messages are pending at the station when it returns, it repeats the process again. Otherwise, it remains idle at the origin till the next message(s) is (are) received.

22. Suppose on a certain day, machines A and D have sent the first two messages to the origin at the beginning of the first second, C has sent a message at the beginning of the 7th second, B at the beginning of the 8th second and E at the beginning of the 10th second. How much distance has the robot traveled since the beginning of the day, when it notices the message of E? Assume that the speed of movement of the robot is 10m/s.
    (a) 140 m (b) 80 m
    (c) 340 m (d) 360 m
23. Suppose there is a second station with raw material for the robot at the other extreme of the line which is 60 m from the origin, i.e., 10m from E. After finishing the services in a trip, the robot returns to the nearest station. If both stations are equidistant, it chooses the origin as the station to return to. Assuming that both stations receive the messages sent by the machines and that all the other data remains the same, what would be the answer to the above question?
    (a) 120 (b) 160
    (c) 140 (d) 170
24. One bacteria splits into eight bacteria of the next generation. But due to environment, only 50% of a generation survive. If the eighth generation number is 8192 million, what is the number in the first generation?
    (a) 1 million (b) 2 million
    (c) 4 million (d) 8 million
25. I bought 10 pens, 14 pencils and 4 erasers. Ravi bought 12 pens, 8 erasers and 28 pencils for an amount which was half more what I had paid. What percent of the total amount paid by me was paid for the pens?
    (a) 37.5% (b) 62.5%
    (c) 50% (d) None of these

**Answers (Block 3 Preassessment Test)**

| | | | | |
|---|---|---|---|---|
| 1. (b) | 2. (a) | 3. (c) | 4. (c) | 5. (b) |
| 6. (a) | 7. (c) | 8. (c) | 9. (c) | 10. (c) |
| 11. (a) | 12. (c) | 13. (b) | 14. (d) | 15. (b) |
| 16. (a) | 17. (b) | 18. (a) | 19. (d) | 20. (b) |
| 21. (b) | 22. (a) | 23. (a) | 24. (b) | 25. (b) |

## SCORE INTERPRETATION ALGORITHM

### Block 3 Preassessment Test

### If You Scored: <7: (In Unlimited Time)

**Step One:** Go through the block three **Back to School** Section carefully. Grasp each of the concepts explained in that part carefully. I would recommend that you go back to your Mathematics school books (ICSE/ CBSE) Class 8,9 & 10 if you feel you need it.

**Step Two:** Move into each of the chapters of the block three by one.

When you do so, concentrate on clearly understanding each of the concepts explained in the chapter theory.

**Step Three:** Then move onto the LOD 1 exercises. These exercises will provide you with the first level of challenge. Try to solve each and every question provided under LOD 1. While doing so do not think about the time requirement. Once you finish solving LOD 1, revise the questions and their solution processes.

don't know the previous year's sales figure (although on the face of it Company $B$ seems to have grown more).

If we had further information saying that company $A$ had a sales turnover of Rs. 1 crore in the previous year and company $B$ had a sales turnover of Rs. 100 crore in the previous year, we can compare growth rates and say that it is company $A$ that has grown by 100%. Hence, company $A$ has a higher growth rate, even though in terms of absolute value increase of sales, company $B$ has grown much more.

## IMPORTANCE OF BASE/ DENOMINATOR FOR PERCENTAGE CALCULATIONS

Mathematically, the percentage value can only be calculated for ratios that, by definition, must have a denominator. Hence, one of the most critical aspects of the percentage is the denominator, which in other words is also called the base value of the percentage. No percentage calculation is possible without knowing the base to which the percentage is to be calculated.

Hence, whenever faced with the question 'What is the percentage ...?' always try first to find out the answer to the question 'Percentage to what base?'

## CONCEPT OF PERCENTAGE CHANGE

Whenever the value of a measured quantity changes, the change can be captured through

(a) Absolute value change or
(b) Percentage change.

Both these measurements have their own advantages and disadvantages.

**Absolute value change:** It is the actual change in the measured quantity. For instance, if sales in year 1 is Rs. 2500 crore and the sales in year 2 is Rs. 2600 crore, then the absolute value of the change is Rs. 100 crore.

**Percentage change:** It is the percentage change got by the formula

$$\text{Percentage change} = \frac{\text{Absolute value change}}{\text{Original quantity}} \times 100$$

$$= \frac{100}{2500} \times 100 = 4\%$$

As seen earlier, this often gives us a better picture of the effect of the change.

*Note:* The base used for the sake of percentage change calculations is always the original quantity unless otherwise stated.

***Example:*** The population of a city grew from 20 lakh to 22 lakh. Find the

(a) percentage change
(b) percentage change based on the final value of population

*Solution:* (a) percentage change = (2/20) × 100 = 10%
(b) percentage change on the final value = (2/22) × 100 = 9.09%

### Difference between the Percentage Point Change and the Percentage Change

The difference between the percentage point change and the percentage change is best illustrated through an example. Consider this:

The savings rate as a percentage of the GDP was 25% in the first year and 30% in the second year. Assume that there is no change in the GDP between the two years. Then:

Percentage point change in savings rate = 30% – 25% = 5 percentage points.

$$\text{Percentage change in savings rate} = \frac{30-25}{25} \times 100$$

= 25%.

## PERCENTAGE RULE FOR CALCULATING PERCENTAGE VALUES THROUGH ADDITIONS

Illustrated below is a powerful method of calculating percentages. In my opinion, the ability to calculate percentage through this method depends on your ability to handle 2 digit additions. Unless you develop the skill to add 2 digit additions in your mind, you are always likely to face problems in calculating percentage through the method illustrated below. In fact, trying this method without being strong at 2-digit additions/subtractions (including 2 digits after decimal point) would prove to be a disadvantage in your attempt at calculating percentages fast.

This process, essentially being a commonsense process, is best illustrated through a few examples:

3. Product constancy application
4. A → B→A application
5. Denominator change to Ratio Change application
6. Use of PCG to calculate Ratio Changes

**Application 1: PCG Applied to Successive Changes**

This is a very common situation in most questions. Suppose you have to solve a question in which a number 30 has two successive percentage increases (20% and 10% respectively).

The situation is handled in the following way using PCG:

$$30 \xrightarrow[+6]{20\%\text{ increase}} 36 \xrightarrow[+3.6]{10\%\text{ increase}} 39.6$$

## Illustration

*A*'s salary increases by 20% and then decreases by 20%. What is the net percentage change in *A*'s salary?

*Solution:*

$$100 \xrightarrow[+20]{20\%\text{ inc.}} 120 \xrightarrow[-24]{20\%\text{ decrease}} 96$$

Hence, *A*'s salary has gone down by 4%

## Illustration

A trader gives successive discounts of 10%, 20% and 10% respectively. The percentage of the original cost price he will recover is:

*Solution:*

$$100 \xrightarrow[-10]{10\%\text{ decrease}} 90 \xrightarrow[-18]{20\%\text{ decrease}} 72 \xrightarrow[-7.2]{10\%\text{ decrease}} 64.8$$

Hence the overall discount is 35.2% and the answer is 64.8%.

## Illustration

A trader marks up the price of his goods by 20%, but to a particularly haggling customer he ends up giving a discount of 10% on the marked price. What is the percentage profit he makes?

*Solution:*

$$100 \xrightarrow[+20]{20\%\text{ increase}} 120 \xrightarrow[-12]{10\%\text{ decrease}} 108$$

Hence, the percentage profit is 8%.

**Application 2: PCG applied to Product Change**

Suppose you have a product of two variables say 10 × 10.

If the first variable changes to 11 and the second variable changes to 12, what will be the percentage change in the product? [Note there is a 10% increase in one part of the product and a 20% increase in the other part.]

The formula given for this situation goes as: $(a + b + ab/100)$

Hence, Required % change = $10 + 20 + \dfrac{10 \times 20}{100}$

(Where 10 and 20 are the respective percentage changes in the two parts of the product) (This is being taught as a shortcut at most institutes across the country currently.)

However, a much easier solution for this case can be visualized as:

$100 \xrightarrow[+20]{20\%\uparrow} 120 \xrightarrow[+12]{10\%\uparrow} 132$. Hence, the final product shows a 32% increase.

Similarly suppose 10 × 10 × 10 becomes 11 × 12 × 13

In such a case the following PCG will be used:

$$100 \xrightarrow[+30]{30\%\uparrow} 130 \xrightarrow[+26]{20\%\uparrow} 156 \xrightarrow[+15.6]{10\%\uparrow} 171.6$$

Hence, the final product sees a 71.6 percent increase (Since, the product changes from 100 to 171.6)

***Note:*** You will get the same result irrespective of the order in which you use the respective percentage changes.

Also note that this process is very similar to the one used for calculating successive percentage change.

**Application for DI:**

Suppose you have two pie charts as follows:


If you are asked to calculate the percentage change in the sales revenue of scooters for the company from year one to year two, what would you do?

The formula for percentage change would give us:

$$\frac{(0.2655 \times 19443.56) - (0.2347 \times 17342.34) \times 100}{(0.2347 \times 17342.34)}$$

i.e. $\dfrac{\text{New Sales Revenue} - \text{Original Sales Revenue}}{\text{Original Sales Revenue}} \times 100$

Obviously this calculation is easier said than done.

However, the Product change application of PCG allows us to execute this calculation with a lot of ease comparatively. Consider the following solution:

Product for year one is: $0.2347 \times 17342.34$

Product for year two is: $0.2655 \times 19443.56$

These can be approximated into:

$234 \times 173$ and $265 \times 194$ respectively (Note that by moving into three digits we do not end up losing any accuracy. We have elaborated this point in the chapter on Ratio and Proportions.)

The overall percentage change depends on two individual percentage changes:

234 increases to 265: A % change of 31/234 = 13.2 % approx. This calculation has to be done using the percentage rule for calculating the percentage value of the ratio

173 increases to 194 – A percentage change of approximately 12%.

Thus PCG will give the answer as follows:

$$100 \xrightarrow[+13.2]{13.2\%\uparrow} 113.2 \xrightarrow[+13.56]{12\%\uparrow} 126.76$$

Hence, 26.76 % increase in the product's value. (Note that the value on the calculator for the full calculation sans any approximations is 26.82 %, and given the fact that we have come extremely close to the answer—the method is good enough to solve the question with a reasonable degree of accuracy.)

**Application 3 of PCG: Product Constancy Application (Inverse proportionality)**

Suppose you have a situation wherein the price of a commodity has gone up by 25%. In case you are required to keep the total expenditure on the commodity constant, you would obviously need to cut down on the consumption. By what percentage? Well, PCG gives you the answer as follows:

$$100 \xrightarrow[\substack{+25\\ \text{Price effect}}]{25\%\uparrow} 125 \xrightarrow[\substack{-25\\ \text{Consumption Effect}}]{} 100$$

Hence, the percentage drop in consumption to offset the price increase is 20%.

I leave it to the student to discover the percentage drop required in the second part of the product if one part increases by 50 percent.

*Note:* Product constancy is just another name for Inverse proportionality.

Table 5.1 gives you some standard values for this kind of a situation.

**Application 4 of PCG: A→B→A.**

Very often we are faced with a situation where we compare two numbers say $A$ and $B$. In such cases, if we are given a relationship from $A$ to $B$, then the reverse relationship can be determined by using PCG in much the same way as the product constancy use shown above.

## Illustration

$B$'s salary is 25% more than $A$'s salary. By what percent is $B$'s salary less than $B$'s salary?

$$100(A) \xrightarrow[+25]{25\%\uparrow} 125(B) \xrightarrow[-25]{} 100(A)$$

A drop of 25 on 125 gives a 20% drop.

Hence $A$'s salary is 20% less than $B$'s.

*Note:* The values which applied for Product Constancy also apply here. Hence Table 4.1 is useful for this situation also.

**Application 5 of PCG → Effect of change in Denominator on the Value of the Ratio**

The denominator has an inverse relationship with the value of a ratio.

Hence the process used for product constancy (and explained above) can be used for calculating percentage change in the denominator.

For instance, suppose you have to evaluate the difference between two ratios:

Ratio 1 : 10/20

Ratio 2 : 10/25

As is evident the denominator is increasing from 20 to 25 by 25%.

If we calculate the value of the two ratios we will get:

Ratio 1 = 0.5, Ratio 2 = 0.4.

% change between the two ratios = $\dfrac{0.1}{0.5} \times 100 = 20\%$ Drop

This value can be got through PCG as:

$100 \longrightarrow 125 \longrightarrow 100$ Hence, 20% drop.

**Note:** This is exactly the same as Product constancy and works here because the numerator is constant.

Hence, $R_1 = N/D_1$ and $R_2 = N/D_2$

i.e. $R_1 \times D_1 = N$ and $R_2 \times D_2 = N$, which is the product constancy situation.

*Direct process for calculation*

To find out the percentage change in the ratio due to a change in the denominator follow the following process:

In order to find the percentage change from 10/20 to 10/25, calculate the percentage change in the denominator in the reverse fashion.

i.e. The required percentage change from $R_1$ to $R_2$ will be given by calculating the percentage change in the denominators from 25 to 20 (i.e. in a reverse fashion) & not from 20 to 25.

**Table 5.1 Product Constancy Table, Inverse Proportionality Table, A → B → A table, Ratio Change to Denominator table**

| *Product XY is Constant* | *X increases (%)* | *Y Decreases (%)* |
|---|---|---|
| *A→B→A* | *A → B % increase* | *B → A% decrease* |
| *X is inversely proportional to Y* | *X increases (%)* | *Y decreases (%)* |
| *Ratio change effect of Denominator change* | *Denominator increases (%)* | *(Ratio decreases(%)* |
| *Denominator change effect of Ratio change* | *Ratio increases (%)* | *As Denominator decreases (%)* |
| Standard Value 1 | 9.09 | 8.33 |
| Standard Value 2 | 10 | 9.09 |
| Standard Value 3 | 11.11 | 10 |
| Standard Value 4 | 12.5 | 11.11 |
| Standard Value 5 | 14.28 | 12.5 |
| Standard Value 6 | 16.66 | 14.28 |
| Standard Value 7 | 20 | 16.66 |
| Standard Value 8 | 25 | 20 |
| Standard Value 9 | 33.33 | 25 |
| Standard Value 10 | 50 | 33.33 |
| Standard Value 11 | 60 | 37.5 |
| Standard Value 12 | 66.66 | 40 |
| Standard Value 13 | 75 | 42.85 |
| Standard Value 14 | 100 | 50 |

**Application 6: Use of PCG to Calculate Ratio Changes:**

Under normal situations, you will be faced with ratios where both numerator and denominator change. The process to handle and calculate such changes is also quite convenient if you go through PCG.

## Illustration

Calculate the percentage change between the Ratios.

Ratio 1 = 10/20 Ratio 2 = 15/25

The answer in this case is 0.5 → 0.6 (20% increase). However, in most cases calculating the values of the ratio will not be easy. The following PCG process can be used to get the answer:

When 10/20 changes to 15/25, the change occurs primarily due to two reasons:

(A) Change in the numerator (Numerator effect)

(B) Change in the denominator (Denominator effect)

By segregating the two effects and calculating the effect due to each separately, we can get the answer easily as follows:

**Numerator Effect** The numerator effect on the value of the ratio is the same as the change in the numerator. Hence, to calculate the numerator effect, just calculate the percentage change in the numerator:

In this case the numerator is clearly changing from 10 to 15 (i.e. a 50% increase.) This signifies that the numerator effect is also 50%.

**Denominator Effect** As we have just seen above, the effect of a percentage change in the denominator on the value of the ratio is seen by calculating the denominator's percentage change in the reverse order.

In this case, the denominator is changing from 20 to 25. Hence the denominator effect will be seen by going reverse from 25 to 20 i.e. 20% drop.

With these two values, the overall percentage change in the Ratio is seen by:

$$100 \xrightarrow[\substack{+50 \\ \text{Numerator} \\ \text{Effect}}]{50\%\uparrow} 150 \xrightarrow[\substack{-30 \\ \text{Denominator} \\ \text{Effect}}]{20\%\downarrow} 120$$

This means that the ratio has increased by 20%.

I leave it to the student to practice such calculations with more complicated values for the ratios.

### Implications for Data Interpretation

Percentage is perhaps one of the most critical links between QA and Data Interpretation.

In the chapter theory mentioned above, the Percentage Rule for Percentage Calculations and the PCG applied to product change and ratio change are the most critical.

As already shown, the use of PCG to calculate the percentage change in a product (as exhibited through the pie chart example above) as well as the use of PCG to calculate ratio changes are two extremely useful applications of the concepts of percentages into DI.

shifting the decimal point by two places to the left. Thus, 83.33% = 0.8333 in decimal value.

- A second learning from this table is in the process of division by any of the numbers such as 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 15, 16, 24 and so on, students normally face problems in calculating the decimal values of these divisions. However, if one gets used to the decimal values that appear in the Table 5.2, calculation of decimals in divisions will become very simple. For instance, when an integer is divided by 7, the decimal values can only be .14, .28, .42, .57, .71, .85 or .00. (There are approximate values)
- This also means that the difference between two ratios like $\frac{x}{6}-\frac{x}{7}$ can be integral if and only if $x$ is divisible by both 6 and 7.

  This principle is very useful as an advanced short cut for option based solution of some questions. I leave it to the student to discover applications of this principle.

### Calculation of Multiplication by Numbers like 1.21, 0.83 and so on

In my opinion, the calculation of multiplication of any number by a number of the form 0.*xy* or of the form 1.*ab* should be viewed as a subtraction/addition situation and not as a multiplication situation. This can be explained as follows.

***Example:*** Calculate 1.23 × 473.

*Solution:* If we try to calculate this by multiplying, we will end up going through a very time taking process, which will yield the final value at the end but nothing before that (i.e. you will have no clue about the answer's range till you reach the end of the calculation).

Instead, one should view this multiplication as an addition of 23% to the original number. This means, the answer can be got by adding 23% of the number to itself.

Thus 473 × 1.23 = 473 + 23% of 473 = 473 + 94.6 + 3% of 473 = 567.6 + 14.19 = 581.79

(The percentage rule can be used to calculate the addition and get the answer.)

The similar process can be utilised for the calculation of multiplication by a number such as 0.87

(Answer can be got by subtracting 13% of the number from itself and this calculation can again be done by percentage rule.)

Hence, the student is advised to become thorough with the percentage rules. Percentage calculation & additions of 2 & 3 digit numbers.

## WORKED-OUT PROBLEMS

**Problem 5.1** A sells his goods 30% cheaper than *B* and 30% dearer than *C*. By what percentage is the cost of *C*'s goods cheaper than *B*'s goods.

**Solution** There are two alternative processes for solving this question:

**1. Assume the price of *C*'s goods as *p*.:** Then *A*'s goods are at 1.3 p and *B*'s goods are such that *A*'s goods are 30% cheaper than *B*'s goods. i.e. *A*'s goods are priced at 70% of *B*'s goods.

$$\text{Hence, } 1.3\,p \rightarrow 70$$
$$B\text{'s price} \rightarrow 100$$
$$B\text{'s price} = 130\,p/70 = 1.8571\,p$$

Then, the percentage by which *C*'s price is cheaper than *B*'s price =

$$(1.8571\,p - p) \times 100/(1.8571\,p) = 600/13 = \mathbf{46.15\%}$$

**Learning task for student** Could you answer the question: Why did we assume *C*'s price as a variable *p* and then work out the problem on its basis. What would happen if we assumed *B*'s price as *p* or if we assumed *A*'s price as *p*?

**2. Instead of assuming the price of one of the three as p, assume the price as 100.**

Let *B* = 100. Then *A* = 70, which is 30% more than *C*. Hence *C* = 23.07% less than *A* (from Table 4.1) = approx. 53.84. Hence answer is 46.15% approximately.

(This calculation can be done mentally if you are able to work through the calculations by the use of percentage rule. The students are advised to try to assume the value of 100 for each of the variables *A*, *B* and *C* and see what happens to the calculations involved in the problem. Since the value of 100 is assumed for a variable to minimise the requirements of calculations to solve the problems, we should ensure that the variable assumed as 100 should have the maximum calculations associated with it.)

5. Mr. Abhimanyu Banerjee is worried about the balance of his monthly budget. The price of petrol has increased by 40%. By what percent should he reduce the consumption of petrol so that he is able to balance his budget?
   (a) 33.33 (b) 28.56
   (c) 25 (d) 14.28
   (e) None of these
6. In question 5, if Mr. Banerjee wanted to limit the increase in his expenditure to 5% on his basic expenditure on petrol then what should be the corresponding decrease in consumption so that expenditure exceeds only by 5%?
   (a) 33.33 (b) 28.56
   (c) 25 (d) 20
   (e) None of these
7. Ram sells his goods 25% cheaper than Shyam and 25% dearer than Bram. How much percentage is Bram's goods cheaper than Shyam's?
   (a) 33.33% (b) 50%
   (c) 66.66% (d) 40%
   (e) None of these
8. In an election between 2 candidates, Bhiku gets 65% of the total valid votes. If the total votes were 6000, what is the number of valid votes that the other candidate Mhatre gets if 25% of the total votes were declared invalid?
   (a) 1625 (b) 1575 (c) 1675 (d) 1525
   (e) None of these
9. In a medical certificate, by mistake a candidate gave his height as 25% more than normal. In the interview panel, he clarified that his height was 5 feet 5 inches. Find the percentage correction made by the candidate from his stated height to his actual height.
   (a) 20 (b) 28.56
   (c) 25 (d) 16.66
   (e) None of these
10. Arjit Sharma generally wears his father's coat. Unfortunately, his cousin Shaurya poked him one day that he was wearing a coat of length more than his height by 15%. If the length of Arjit's father's coat is 120 cm then find the actual length of his coat.
    (a) 105 (b) 108 (c) 104.34 (d) 102.72
    (e) 101.11
11. A number is mistakenly divided by 5 instead of being multiplied by 5. Find the percentage change in the result due to this mistake.
    (a) 96% (b) 95%
    (c) 2400% (d) 200%
    (e) None of these
12. Hursh wanted to subtract 5 from a number. Unfortunately, he added 5 instead of subtracting. Find the percentage change in the result.
    (a) 300% (b) 66.66%
    (c) 50% (d) 33.33%
    (e) Cannot be determined
13. If 65% of $x$ = 13% of $y$, then find the value of $x$ if $y$ = 2000.
    (a) 200 (b) 300 (c) 400 (d) 500
    (e) None of these
14. In a mixture of 80 litres of milk and water, 25% of the mixture is milk. How much water should be added to the mixture so that milk becomes 20% of the mixture?
    (a) 20 litres (b) 15 litres
    (c) 25 litres (d) 24 litres
    (e) None of these
15. 50% of $a$% of $b$ is 75% of $b$% of $c$. Which of the following is $c$?
    (a) $1.5a$ (b) $0.667a$ (c) $0.5a$ (d) $1.25a$
    (e) $1.66a$
16. A landowner increased the length and the breadth of a rectangular plot by 10% and 20% respectively. Find the percentage change in the cost of the plot assuming land prices are uniform throughout his plot.
    (a) 33% (b) 35%
    (c) 22.22% (d) 28.56%
    (e) None of these
17. The height of a triangle is increased by 40%. What can be the maximum percentage increase in length of the base so that the increase in area is restricted to a maximum of 60%?
    (a) 50% (b) 20% (c) 14.28% (d) 25%
    (e) 33.33%
18. The length, breadth and height of a room in the shape of a cuboid are increased by 10%, 20% and 50% respectively. Find the percentage change in the volume of the cuboid.
    (a) 77% (b) 75% (c) 88% (d) 98%
    (e) 99%
19. The salary of Amit is 30% more than that of Varun. Find by what percentage is the salary of Varun less than that of Amit?
    (a) 26.12% (b) 23.07%

(c) 21.23% (d) 27.27%
(e) None of these

20. The price of sugar is reduced by 25% but inspite of the decrease, Aayush ends up increasing his expenditure on sugar by 20%. What is the percentage change in his monthly consumption of sugar ?
(a) +60% (b) –10% (c) +33.33% (d) 50%
(e) None of these

21. The price of rice falls by 20%. How much rice can be bought now with the money that was sufficient to buy 20 kg of rice previously?
(a) 5 kg (b) 15 kg (c) 25 kg (d) 30 kg
(e) 20 kg

22. 30% of a number when subtracted from 91, gives the number itself. Find the number.
(a) 60 (b) 65
(c) 70 (d) 75
(e) None of these

23. When 60% of a number $A$ is added to another number $B$, $B$ becomes 175% of its previous value. Then which of the following is true regarding the values of $A$ and $B$?
(a) $A > B$
(b) $B > A$
(c) $B \geq A$
(d) Either (a) or (b) can be true depending upon the values of $A$ and $B$
(e) Nothing can be said

24. At an election, the candidate who got 56% of the votes cast won by 144 votes. Find the total number of voters on the voting list if 80% people cast their vote and there were no invalid votes.
(a) 360 (b) 720 (c) 1800 (d) 1500
(e) 1600

25. The population of a village is 1,00,000. The rate of increase is 10% per annum. Find the population at the start of the third year?
(a) 1,33,100 (b) 1,21,000
(c) 1,18,800 (d) 1,20,000
(e) None of these

26. The population of the village of Gavas is 10,000 at this moment. It increases by 10% in the first year. However, in the second year, due to immigration, the population drops by 5%. Find the population at the end of the third year if in the third year the population increases by 20%.
(a) 12,340 (b) 12,540
(c) 1,27,540 (d) 12,340
(e) 13,240

27. A man invests Rs. 10,000 in some shares in the ratio 2 : 3 : 5 which pay dividends of 10%, 25% and 20% (on his investment) for that year respectively. Find his dividend income.
(a) 1900 (b) 2000
(c) 2050 (d) 1950
(e) 1850

28. In an examination, Mohit obtained 20% more than Sushant but 10% less than Rajesh. If the marks obtained by Sushant is 1080, find the percentage marks obtained by Rajesh if the full marks is 2000.
(a) 86.66% (b) 72%
(c) 78.33% (d) 77.77%
(e) None of these

29. In a class, 25% of the students were absent for an exam. 30% failed by 20 marks and 10% just passed because of grace marks of 5. Find the average score of the class if the remaining students scored an average of 60 marks and the pass marks are 33 (counting the final scores of the candidates).
(a) 37.266 (b) 37.6
(c) 37.8 (d) 36.93
(e) 37.5

30. Ram spends 20% of his monthly income on his household expenditure, 15% of the rest on books, 30% of the rest on clothes and saves the rest. On counting, he comes to know that he has finally saved Rs. 9520. Find his monthly income.
(a) 10000 (b) 15000
(c) 20000 (d) 12000
(e) None of these

31. Hans and Bhaskar have salaries that jointly amount to Rs. 10,000 per month. They spend the same amount monthly and then it is found that the ratio of their savings is 6 : 1. Which of the following can be Hans's salary?
(a) Rs. 6000 (b) Rs. 5000
(c) Rs. 4000 (d) Rs. 3000

32. The population of a village is 5500. If the number of males increases by 11% and the number of females increases by 20%, then the population becomes 6330. Find the population of females in the town.
(a) 2500 (b) 3000
(c) 2000 (d) 3500

(b) 100 (for) and 200 (against)
(c) 150 (for), 300 (against)
(d) 200 (for) and 300 (against)

40. Of the adult population in Nagpur, 45% of men and 25% of women are married. What percentage of the total population of adults is married (assume that no man marries more than one woman and vice versa)?
(a) 33.33% (b) 32.14%
(c) 31.1% (d) None of these

41. The weight of an iron bucket increases by 33.33% when filled with water to 50% of its capacity. Which of these may be 50% of the weight of the bucket when it is filled with water (assume the weight of bucket and its capacity in kg to be integers)?
(a) 7 kg (b) 6 kg (c) 5 kg (d) 8 kg

42. Australia scored a total of $x$ runs in 50 overs. India tied the scores in 20% less overs. If India's average run rate had been 33.33% higher the scores would have been tied 10 overs earlier. Find how many runs were scored by Australia?
(a) 250 (b) 240
(c) 200 (d) Cannot be determined

43. Due to a 25% hike in the price of rice per kilogram, a person is able to purchase 20 kg less for Rs. 400. Find the increased price of rice per kilogram?
(a) Rs. 5 (b) Rs. 6 (c) Rs. 10 (d) Rs. 4

44. A salesman is appointed on the basic salary of Rs. 1200 per month and the condition that for every sales of Rs. 10,000 above Rs. 10,000, he will get 50% of basic salary and 10% of the sales as a reward. This incentive scheme does not operate for the first Rs. 10000 of sales. What should be the value of sales if he wants to earn Rs. 7600 in a particular month?
(a) Rs. 60,000 (b) Rs. 50,000
(c) Rs. 40,000 (d) None of these

45. In question 44, which of the following income cannot be achieved in a month?
(a) Rs. 6000
(b) Rs. 9000
(c) Both a and b
(d) Any income can be achieved

46. In question 44 despite a 5 percentage point increment on the commission from 20%, the total commission remained unaltered. Find the change in the volume of the transaction?
(a) –10% (b) –16%
(c) –25% (d) –20%

47. In an assembly election at Surat, the total turnout was 80% out of which 16% of the total voters on the voting list were declared invalid. Find which of the following can be the percentage votes got by the winner of the election if the candidate who came second got 20% of the total voters on the voting list. (There were only three contestants, only one winner and the total number of voters on the voters' list was 20000.)
(a) 44.8% (b) 46.6%
(c) 48% (d) None of these

48. A watch gains by 2% per hour when the temperature is in the range of 40°C–50°C and it loses at the same rate when the temperature is in the range of 20°C–30°C. However, the watch owner is fortunate since it runs on time in all other temperature ranges. On a sunny day, the temperature started soaring up from 8 a.m. in the morning at the uniform rate of 2°C per hour and sometime during the afternoon it started coming down at the same rate. Find what time will it be by the watch at 7 p.m. if at 8 a.m. the temperature was 32°C and at 4 p.m., it was 40°C.
(a) 6 : 55 p.m. (b) 6 : 55 : 12 p.m.
(c) 6 : 55 : 24 p.m. (d) None of these

***Questions 49–50:*** Study the following table and answer the questions that follow.

| *Beverages* | *% of Vitamin* | *% of Minerals* | *% of Micronutrients* | *Cost per 250 gram(In Rs.)* |
|---|---|---|---|---|
| Pepsi | 12 | 18 | 30 | 8 |
| Coke | 15 | 20 | 10 | 10 |
| Sprite | 20 | 10 | 40 | 7 |

49. Which of the following beverages contains the maximum amount of vitamins?
(a) Pepsi worth Rs. 16
(b) Coke worth Rs. 15
(c) Sprite worth Rs. 8
(d) All the three worth Rs. 12.5 (125 grams of each)

50. Which of these is the cheapest?
(a) 200 grams of Pepsi + 200 grams of Coke
(b) 300 grams of Coke +100 grams of Pepsi
(c) 100 grams of Coke + 100 grams of Pepsi + 100 grams of Sprite
(d) 300 grams of Coke +100 grams of Sprite

## Level of Difficulty (LOD) III

1. The price of raw materials has gone up by 15%, labour cost has also increased from 25% of the cost of raw material to 30% of the cost of raw material. By how much percentage should there be a reduction in the usage of raw materials so as to keep the cost same?
   (a) 17% (b) 24%
   (c) 28% (d) 25%
   (e) Cannot be determined
2. Mr. *A* is a computer programmer. He is assigned three jobs for which time allotted is in the ratio of 5 : 4 : 2 (jobs are needed to be done individually). But due to some technical snag, 10% of the time allotted for each job gets wasted. Thereafter, owing to the lack of interest, he invests only 40%, 30%, 20% of the hours of what was actually allotted to do the three jobs individually. Find how much percentage of the total time allotted is the time invested by *A*?
   (a) 38.33% (b) 39.4545%
   (c) 32.72% (d) 36.66%
   (e) Cannot be determined
3. In the MOCK CAT paper at AMS, questions were asked in five sections. Out of the total students, 5% candidates cleared the cut-off in all the sections and 5% cleared none. Of the rest, 25% cleared only one section and 20% cleared four sections. If 24.5% of the entire candidates cleared two sections and 300 candidates cleared three sections, find out how many candidates appeared at the MOCK CAT at AMS?
   (a) 1000 (b) 1200 (c) 1500 (d) 2000
   (e) 1800
4. There are three galleries in a coal mine. On the first day, two galleries are operative and after some time, the third gallery is made operative. With this, the output of the mine became half as large again. What is the capacity of the second gallery as a percentage of the first, if it is given that a four-month output of the first and the third galleries was the same as the annual output of the second gallery?
   (a) 70% (b) 64%
   (c) 60% (d) 65%
   (e) None of these
5. 10% of salty sea water contained in a flask was poured out into a beaker. After this, a part of the water contained in the beaker was vapourised by heating and due to this, the percentage of salt in the beaker increased $M$ times. If it is known that after the content of the beaker was poured into the flask, the percentage of salt in the flask increased by $x$%, find the original quantity of sea water in the flask.
   (a) $\frac{9M+1\%}{M-1}$ (b) $\frac{(9M+1)x\%}{M-1}$
   (c) $\frac{9M-1x\%}{M+1}$ (d) $\frac{9M+x\%}{M+1}$
   (e) None of these
6. In an election of 3 candidates *A*, *B* and *C*, *A* gets 50% more votes than *B*. *A* also beats *C* by 1,80,00 votes. If it is known that *B* gets 5 percentage point more votes than *C*, find the number of voters on the voting list (given 90% of the voters on the voting list voted and no votes were illegal)
   (a) 72,000 (b) 81,000
   (c) 90,000 (d) 1,00,000
   (e) 1,10,000
7. A clock is set right at 12 noon on Monday. It loses 1/2% on the correct time in the first week but gains 1/4% on the true time during the second week. The time shown on Monday after two weeks will be
   (a) 12 : 25 : 12 (b) 11 : 34 : 48
   (c) 12 : 50 : 24 (d) 12 : 24 : 16
   (e) None of these
8. The petrol prices shot up by 7% as a result of the hike in the price of crudes. The price of petrol before the hike was Rs. 28 per litre. Vawal travels 2400 kilometres every month and his car gives a mileage of 18 kilometres to a litre. Find the increase in the expenditure that Vawal has to incur due to the increase in the price of petrol (to the nearest rupee)?
   (a) Rs. 270 (b) Rs. 262
   (c) Rs. 276 (d) Rs. 272
   (e) Rs. 280
9. For question 8, by how many kilometres should Vawal reduce his travel if he wants to maintain his expenditure at the previous level (prior to the price increase)?
   (a) 157 km (b) 137 km
   (c) 168 km (d) 180 km
   (e) None of these

10. In question 8, if Vawal wants to limit the increase in expenditure to Rs. 200, what strategy should he adopt with respect to his travel?
    (a) Reduce travel to 2350 kilometres
    (b) Reduce travel to 2340 kilometres
    (c) Reduce travel to 2360 kilometres
    (d) Reduce travel to 2370 kilometres
    (e) None of these
11. A shopkeeper announces a discount scheme as follows: for every purchase of Rs. 3000 to Rs. 6000, the customer gets a 15% discount or a ticket that entitles him to get a 7% discount on a further purchase of goods costing more than Rs. 6000. The customer, however, would have the option of reselling his right to the shopkeeper at 4% of his initial purchase value (as per the right refers to the 7% discount ticket). In an enthusiastic response to the scheme, 10 people purchase goods worth Rs. 4000 each. Find the maximum. Possible revenue for the shopkeeper.
    (a) Rs. 38,400 (b) Rs. 38,000
    (c) Rs. 39,400 (d) Rs. 39,000
    (e) Rs. 40,000
12. For question 11, find the maximum possible discount that the shopkeeper would have to offer to the customer.
    (a) Rs. 1600 (b) Rs. 2000
    (c) Rs. 6000 (d) Rs. 4000
    (e) None of these

***Directions for Questions 13–16:*** Read the following and answer the questions that follow.

Two friends Shayam and Kailash own two versions of a car. Shayam owns the diesel version of the car, while Kailash owns the petrol version.

Kailash's car gives an average that is 20% higher than Shayam's (in terms of litres per kilometre). It is known that petrol costs 60% of its price higher than diesel.

13. The ratio of the cost per kilometre of Kailash's car to Shayam's car is
    (a) 3 : 1 (b) 1 : 3
    (c) 1.92 : 1 (d) 2 : 1
    (e) Cannot be determined
14. If Shayam's car gives an average of 20 km per litre, then the difference in the cost of travel per kilometre between the two cars is
    (a) Rs. 4.3 (b) Rs. 3.5
    (c) Rs. 2.5 (d) Rs. 3
    (e) Cannot be determined
15. For question 14, the ratio of the cost per kilometre of Shayam's travel to Kailash's travel is.
    (a) 3 : 1 (b) 1 : 3
    (c) 1 : 1.92 (d) 2 : 1
    (e) Cannot be determined
16. If diesel costs Rs. 12.5 per litre, then the difference in the cost of travel per kilometre between Kailash's and Shayam's is (assume an average of 20 km per litre for Shayam's car and also assume that petrol is 50% of its own price higher than diesel)
    (a) Rs. 1.75 (b) Rs. 0.875
    (c) Rs. 1.25 (d) Rs. 1.125
    (e) None of these

***Directions for Questions 17–23:*** Read the following and answer the questions that follow.

In the island of Hoola Boola Moola, the inhabitants have a strange process of calculating their average incomes and expenditures. According to an old legend prevalent on that island, the average monthly income had to be calculated on the basis of 14 months in a calendar year while the average monthly expenditure was to be calculated on the basis of 9 months per year. This would lead to people having an underestimation of their savings since there would be an underestimation of the income and an overestimation of the expenditure per month.

17. Mr. Boogle Woogle comes back from the USSR and convinces his community comprising 273 families to start calculating the average income and the average expenditure on the basis of 12 months per calendars year. Now if it is known that the average estimated income in his community is (according to the old system) 87 moolahs per month, then what will be the percentage change in the savings of the community of Mr. Boogle Woogle (assume that there is no other change)?
    (a) 12.33% (b) 22.22%
    (c) 31.31% (d) 33.33%
    (e) Cannot be determined
18. For question 17, if it is known that the average estimated monthly expenditure is 19 moolahs per month for the island of Hoola Boola Moola, then what will be the percentage change in the estimated savings of the community?
    (a) 32.42% (b) 38.05%
    (c) 25.23% (d) 26.66%
    (e) Cannot be determined

19. For question 18, if it is known that the average estimated monthly expenditure was 22 moolahs per month for the community of Boogle Woogle (having 273 families), then what will be the percentage change in the estimated savings of the community?

(a) 30.77% (b) 28.18%
(c) 25.23% (d) 25.73%
(e) Cannot be determined

20. For question 19, what will be the percentage change in the estimated average income of the community (calculated on the basis of the new estimated average)?

(a) 14.28% increase (b) 14.28% decrease
(c) 16.66% increase (d) 16.66% decrease
(e) None of these

21. If the finance minister of the island Mr. Bhola Ram declares that henceforth the average monthly income has to be estimated on the basis of 12 months per year while the average monthly expenditure is to be estimated on the basis of 11 months to the year, what will happen to the savings in the economy of Hoola Boola Moola?

(a) Increase (b) Decrease
(c) Remain constant (d) Either (b) or (c)
(e) Cannot be determined

22. For question 21, what will be the percentage change in savings?

(a) 3.1% (b) 1.52%
(c) 2.5% (d) 3.2%
(e) Cannot be determined

23. For question 22, what will be the percentage change in the estimated monthly expenditure?

(a) 22.22% decrease (b) 22.22% increase
(c) 18.18% decrease (d) 18.18% increase
(e) None of these

24. Abhimanyu Banerjee has 72% vision in his left eye and 68% vision in his right eye. On corrective therapy, he starts wearing contact lenses, which augment his vision by 15% in the left eye and 11% in the right eye. Find out the percentage of normal vision that he possesses after corrective therapy. (Assume that a person's eyesight is a multiplicative construct of the eyesight's of his left and right eyes)

(a) 52.5% (b) 62.5% (c) 72.5% (d) 68.6%
(e) None of these

25. A shopkeeper gives 3 consecutive discounts of 10%, 15% and 15% after which he sells his goods at a percentage profit of 30.05% on the C.P. Find the value of the percentage profit that the shopkeeper would have earned if he had given discounts of 10% and 15% only.

(a) 53% (b) 62.5% (c) 72.5% (d) 68.6%
(e) 69.2%

26. If the third discount in question 25 was Rs. 2,29,50, then find the original marked price of the item.

(a) Rs. 1,00,000 (b) Rs. 1,25,000
(c) Rs. 2,00,000 (d) Rs. 2,50,000
(e) Rs. 2,25,000

27. Krishna Iyer, a motorist uses 24% of his fuel in covering the first 20% of his total journey (in city driving conditions). If he knows that he has to cover another 25% of his total journey in city driving conditions, what should be the minimum percentage increase in the fuel efficiency for non-city driving over the city driving fuel efficiency, so that he is just able to cover his entire journey without having to refuel? (Approximately)

(a) 39.2% (b) 43.5%
(c) 45.6% (d) 41.2%

***Directions for Questions 28–30:*** Read the following and answer the questions that follow the BSNL announced a cut in the STD rates on 27 December 2001. The new rates and slabs are given in the table below and are to be implemented from the 14 January 2002.

**Slab Details**

| *Distance* | *Rates (Rs./min)* | | | |
|---|---|---|---|---|
| | *Peak Rates* | | *Off Peak* | |
| | *Old* | *New* | *Old* | *New* |
| 50–200 | 4.8 | 2.4 | 1.2 | 1.2 |
| 200–500 | 11.6 | 4.8 | 3 | 2.4 |
| 500–1000 | 17.56 | 9.00 | 4.5 | 4.5 |
| 1000+ | 17.56 | 9.00 | 6 | 4.5 |

28. The maximum percentage reduction in costs will be experienced for calls over which of the following distances?

(a) 50–200 (b) 500–1000
(c) 1000+ (d) 200–500

29. The percentage difference in the cost of a set of telephone calls made on the 13th and 14th January having durations of 4 minutes over a distance of 350 km, 3 minutes for a distance of 700 km and 3 minutes for a

distance of 1050 km is (if all the three calls are made in peak times)

(a) 51.2% (b) 51.76%
(c) 59.8 % (d) Cannot be determined

30. If one of the three calls in question 29 were made in an off peak time on both days, then the percentage reduction in the total cost of the calls between 13th and 14th January will
(a) Definitely reduce
(b) Definitely increase
(c) Will depend on which particular call was made in an off peak time
(d) Cannot be determined

***Directions for Questions 31–35:*** Read the following caselet and answer the questions that follow.

The circulation of the *Deccan Emerald* newspaper is 3,73,000 copies, while its closest competitors are *The Times of Hindustan* and *India's Times*, which sell 2,47,000 and 20% more than that respectively (rounded off to the higher thousand). All the newspapers cost Rs. 2 each. The hawker's commissions offered by the three papers are 20%, 25% and 30% respectively (these commissions are calculated on the sale price of the newspaper). Also, it is known that newspapers earn primarily through sales and advertising.

31. Taking the base as the net revenue of *Deccan Emerald*, the percentage difference of the net revenue (revenues – commission disbursed to hawkers) between *Deccan Emerald* and *India's Times* is
(a) 24.62% (b) 30.32%
(c) 26.28% (d) None of these

32. The ratio of the percentage difference in the total net revenue between Deccan Emerald and India's Times to the percentage difference in the total revenue between Deccan Emerald and India's Times is
(a) 1.488 (b) 0.3727
(c) 0.6720 (d) Cannot be determined

33. If the cost of printing the newspaper is Rs. 8, 7.5 and 7 respectively per day for *Deccan Emerald*, *Times of Hindustan* and *India's Times* respectively and on any day the available advertising space in the *Deccan Emerald* newspaper is 800 cc (column centimetres) and the advertising rate for *Deccan Emerald* is Rs. 3000 per cc then the percentage of the advertising space that must be utilised to ensure the full recovery of the day's cost for *Deccan Emerald* is
(a) 95.83% (b) 99.46%
(c) 97.28% (d) Cannot be determined

34. Based on the data in the previous question and the additional information that the space availability in *India's Times* is 1000 cc and that in the *Times of Hindustan* is 1100 cc, find the percentage point difference in the percentage of advertising space to be utilised in *India's Times* and that which must be utilised in *Times of Hindustan* so that both newspapers just break even.
(a) 4.5 (b) 5.2
(c) 10 (d) Cannot be determined

35. For the data in questions 33 and 34 if it is known that the advertising rate in *Times of Hindustan* is Rs. 1800 per cc and that in the *India's Times* is Rs. 2100 per cc, then what is the percentage point difference in the percentage of advertising space to be utilised by *Times of Hindustan* and *India's Times* so that both of them are just able to break even?
(a) 4.18 (b) 5.6
(c) 4.09 (d) Cannot be determined

36. On a train journey, there are 5 kinds of tickets AC I, AC II, AC III, 3-tier, and general. The relationship between the rates of the tickets for the Eurail is:
AC II is 20% higher than AC III and AC I is 70% of AC III's value higher than the AC II ticket's value. The 3-tier ticket is 25% of the AC I's ticket cost and the general ticket is 1/3 the price of the AC II ticket. The AC II ticket costs 780 euros between London and Paris. The difference in the rates of 3 tier and general ticket is
(a) 41.25 euros (b) 55.8 euros
(c) 48.75 euros (d) 52.75 euros

37. For the above question, the total cost of one ticket of each class will be
(a) 3233.75 (b) 3533.75
(c) 4233.75 (d) 3733.75

***Directions for Question 38–40:*** Read the following and answer the questions that follow.

A Eurailexpress train has 2 AC I bogeys having 24 berths each, 3 AC II bogeys having 45 berths each, 2 AC III bogeys having 64 berths each and 12 3-tier bogeys having 64 berths each. There are no general bogeys in the train. If 200 euros is the cost of an AC 3-tier berth from London to Glasgow, answer the following questions: