

EXERCISE 16.1

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In each of the following Exercises 1 to 7, describe the sample space for the indicated experiment.

1. A coin is tossed three times.

Solution:-

Since either coin can turn up Head (H) or Tail (T), the possible outcomes may be

When 1 coin is tossed once the sample space = 2

Then,

The coin is tossed 3 times the sample space = $2^3 = 8$

Thus, the sample space is $S = \{HHH, THH, HTH, HHT, TTT, HTT, THT, TTH\}$

2. A die is thrown two times.

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

Then, the total number of the sample space = (6×6)

= 36

Thus, the sample space is

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

3. A coin is tossed four times.

Solution:-

Since either coin can turn up Head (H) or Tail (T), the possible outcomes are

When 1 coin is tossed once the sample space = 2

Then,

The coin is tossed 3 times the sample space = $2^4 = 16$

Thus, the sample space is $S = \{HHHH, THHH, HTHH, HHHT, TTTT, HTTT, THTT, TTHT, TTTH, TTHH, HHTT, THTH, HTHT, THHT, HTTH\}$

4. A coin is tossed, and a die is thrown.

Solution:-

A coin can turn up either Head (H) or Tail (T), and this is the possible outcome.

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible numbers that come when the die is thrown.

Then, total number of space = $(2 \times 6) = 12$

Thus, the sample space is,

$$S = \{(H,1),(H,2),(H,3),(H,4),(H,5),(H,6),(T,1),(T,2),(T,3),(T,4),(T,5),(T,6)\}$$

5. A coin is tossed, and then a die is rolled only in case a head is shown on the coin.

Solution:-

Since either coin can turn up Head (H) or Tail (T), these are the possible outcomes.

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible numbers that come when the die is thrown.

When the head is encountered,

Then, number of space = $(1 \times 6) = 6$

Sample Space $S_H = \{H1, H2, H3, H4, H5, H6\}$

Now, the tail is encountered, Sample space $S_T = \{T\}$

Therefore the total sample space $S = \{H1, H2, H3, H4, H5, H6, T\}$

6. 2 boys and 2 girls are in Room X, and 1 boy and 3 girls are in Room Y. Specify the sample space for the experiment in which a room is selected and then a person.

Solution:-

From the question, it is given that

2 boys and 2 girls are in Room X

1 boy and 3 girls in Room Y

Let us assume b_1, b_2 and g_1, g_2 be 2 boys and 2 girls in Room X.

And also, assume b_3 and g_3, g_4, g_5 be 1 boy and 3 girls in Room Y.

The problem is solved by dividing it into two cases.

Case 1: Room X is selected

Sample Space $S_x = \{(X,b_1),(X,b_2),(X,g_1),(X,g_2)\}$

Case 2: Room Y is selected

Sample Space $S_y = \{(Y,b_3),(Y,g_3),(Y,g_4),(Y,g_5)\}$

The overall sample space

$$S = \{(X, b1), (X, b2), (X, g1), (X, g2), (Y, b3), (Y, g3), (Y, g4), (Y, g5)\}$$

7. One die of red colour, one of white colour and one of blue colour are placed in a bag. One die is selected at random and rolled, its colour and the number on its uppermost face is noted. Describe the sample space.

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible numbers that come when the die is thrown.

And also, assume die of red colour be 'R', die of white colour be 'W', die of blue colour be 'B'.

So, the total number of sample space = $(6 \times 3) = 18$

The sample space of the event is

$$S = \{(R, 1), (R, 2), (R, 3), (R, 4), (R, 5), (R, 6), (W, 1), (W, 2), (W, 3), (W, 4), (W, 5), (W, 6), (B, 1), (B, 2), (B, 3), (B, 4), (B, 5), (B, 6)\}$$

8. An experiment consists of recording boy–girl composition of families with 2 children.

(i) What is the sample space if we are interested in knowing whether it is a boy or girl in the order of their births?

(ii) What is the sample space if we are interested in the number of girls in the family?

Solution:-

Let us assume the boy be 'B' and the girl be 'G'.

(i) The sample space if we are interested in knowing whether it is a boy or girl in the order of their births, $S = \{GG, BB, GB, BG\}$

(ii) The sample space if we are interested in the number of girls in the family when there are two children in the family then

$$\text{Sample space } S = \{2, 1, 0\}$$

9. A box contains 1 red and 3 identical white balls. Two balls are drawn at random in succession without replacement. Write the sample space for this experiment.

Solution:-

From the question, it is given that a box contains 1 red and 3 identical white balls.

Let us assume 'R' be the event of the red ball being drawn, and 'W' be the event of the white ball being drawn.

Given in the question that white balls are identical; therefore, the event of drawing any one of the three white balls is the same.

Then, total number of sample space = $(2^2 - 1) = 3$

$$\therefore \text{Sample space } S = \{WW, WR, RW\}$$

10. An experiment consists of tossing a coin and then throwing it the second time if a head occurs. If a tail occurs on the first toss, then a die is rolled once. Find the sample space.

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

Since either coin can turn up Head (H) or Tail (T), these are the possible outcomes.

Let us take,

Case 1: Head is encountered

Sample space $S_1 = \{HT, HH\}$

Case 2: Tail is encountered

Sample Space $S_2 = \{(T,1), (T,2), (T,3), (T,4), (T,5), (T,6)\}$

Then,

The Overall Sample space

$S = \{(HT), (HH), (T1), (T2), (T3), (T4), (T5), (T6)\}$

11. Suppose 3 bulbs are selected at random from a lot. Each bulb is tested and classified as defective (D) or non-defective (N). Write the sample space of this experiment.

Solution:-

From the question,

‘D’ denotes the event that the bulb is defective, and ‘N’ denotes the event of non-defective bulbs.

Then,

Total number of Sample space $= 2 \times 2 \times 2 = 8$

Thus, Sample space $S = \{DDD, DDN, DND, NDD, DNN, NDN, NND, NNN\}$

12. A coin is tossed. If the outcome is a head, a die is thrown. If the die shows up an even number, the die is thrown again. What is the sample space for the experiment?

Solution:-

Since either coin can turn up Head (H) or Tail (T), these are the possible outcomes.

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

The problem can be solved by dividing it into 3 cases.

Case 1: The outcome is Head, and the corresponding number on the die shows an Odd number.

Total number of sample space = $(1 \times 3) = 3$

Sample space $S_{H_0} = \{(H,1), (H,3), (H,5)\}$

Case 2: The outcome is Head, and the corresponding number on the die shows an Even number.

Total number of sample space = $(1 \times 3 \times 6) = 18$

$S_{H_E} = \{(H,2,1), (H,2,2), (H,2,3), (H,2,4), (H,2,5), (H,2,6), (H,4,1), (H,4,2), (H,4,3), (H,2,4), (H,4,5), (H,4,6), (H,6,1), (H,6,2), (H,6,3), (H,6,4), (H,6,5), (H,6,6)\}$

Case 3: The outcome is Tail

Total number of sample space = 1

Sample space $S_T = \{(T)\}$

The overall sample spaces

$S = \{(H,1), (H,3), (H,5), (H,2,1), (H,2,2), (H,2,3), (H,2,4), (H,2,5), (H,2,6), (H,4,1), (H,4,2), (H,4,3), (H,2,4), (H,4,5), (H,4,6), (H,6,1), (H,6,2), (H,6,3), (H,6,4), (H,6,5), (H,6,6), (T)\}$

13. The numbers 1, 2, 3 and 4 are written separately on four slips of paper. The slips are put in a box and mixed thoroughly. A person draws two slips from the box, one after the other, without replacement. Describe the sample space for the experiment.

Solution:-

From the question, it is given that 1, 2, 3, and 4 are the numbers written on the four slips.

When two slips are drawn without replacement, the first event has 4 possible outcomes, and the second event has 3 possible outcomes because 1 slip is already picked.

Therefore, the total number of possible outcomes = $(4 \times 3) = 12$

Thus, the sample space is,

$S = \{(1,2), (1,3), (1,4), (2,1), (2,3), (2,4), (3,1), (3,2), (3,4), (4,1), (4,2), (4,3)\}$

14. An experiment consists of rolling a die and then tossing a coin once if the number on the die is even. If the number on the die is odd, the coin is tossed twice. Write the sample space for this experiment.

Solution:-

Since either coin can turn up Head (H) or Tail (T), these are the possible outcomes.

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

The following problem can be divided into two cases.

(i) The number on the die is even.

The sample space $S_E = \{(2,H), (4,H), (6,H), (2,T), (4,T), (6,T)\}$

(ii) The number on the die is odd, and the coin is tossed twice.

The sample space

$S_o = \{(1,H,H), (3,H,H), (5,H,H), (1,H,T), (3,H,T), (5,H,T), (1,T,H), (3,T,H), (5,T,H), (1,T,T), (3,T,T), (5,T,T)\}$

Hence, the overall sample space for the problem = $S_E + S_o$

$S = \{(2,H), (4,H), (6,H), (2,T), (4,T), (6,T), (1,H,H), (3,H,H), (5,H,H), (1,H,T), (3,H,T), (5,H,T), (1,T,H), (3,T,H), (5,T,H), (1,T,T), (3,T,T), (5,T,T)\}$

15. A coin is tossed. If it shows a tail, we draw a ball from a box which contains 2 red and 3 black balls. If it shows head, we throw a die. Find the sample space for this experiment.

Solution:-

Since either coin can turn up Head (H) or Tail (T), these are the possible outcomes.

Let us assume R_1, R_2 denote the event the red balls are drawn, and B_1, B_2, B_3 denote the events black balls are drawn.

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

(i) Coin shows the Tail.

So, the sample space $S_T = \{(TR_1), (TR_2), (TB_1), (TB_2), (TB_3)\}$

(ii) Coin shows the head.

So, the sample space $S_H = \{(H,1), (H,2), (H,3), (H,4), (H,5), (H,6)\}$

Hence, the overall sample space for the problem = $S_T + S_H$

$S = \{(T, R_1), (T, R_2), (T, B_1), (T, B_2), (T, B_3), (H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6)\}$

16. A die is thrown repeatedly until a six comes up. What is the sample space for this experiment?

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

As per the condition given in the question, a die is thrown repeatedly until a six comes up.

If six may come up for the first throw or six may come up on the second throw, this process will go continuously until the six comes.

The sample space when 6 comes on the very first throw $S_1 = \{6\}$

The sample space when 6 comes on the second throw $S_2 = \{(1,6), (2,6), (3,6), (4,6), (5,6)\}$

This event can go on for infinite times.

So, the sample space is infinitely defined

$$S = \{(6), (1,6), (2,6), (3,6), (4,6), (5,6), (1,1,6), (1,2,6), \dots\}$$

EXERCISE 16.2

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1. A die is rolled. Let E be the event “die shows 4” and F be the event “die shows even number”. Are E and F mutually exclusive?

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

So, $S = (1, 2, 3, 4, 5, 6)$

As per the conditions given in the question,

E be the event “die shows 4”

$E = (4)$

F be the event “die shows even number”.

$F = (2, 4, 6)$

$E \cap F = (4) \cap (2, 4, 6)$

$= 4$

$4 \neq \phi \dots$ [because there is a common element in E and F]

Therefore, E and F are not mutually exclusive events.

2. A die is thrown. Describe the following events:

(i) A: a number less than 7 (ii) B: a number greater than 7

(iii) C: a multiple of 3 (iv) D: a number less than 4

(v) E: an even number greater than 4 (vi) F: a number not less than 3

Also, find $A \cup B$, $A \cap B$, $B \cup C$, $E \cap F$, $D \cap E$, $A - C$, $D - E$, $E \cap F^c$, F^c

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

So, $S = (1, 2, 3, 4, 5, 6)$

As per the conditions given in the question,

(i) A: a number less than 7

All the numbers in the die are less than 7,

$A = (1, 2, 3, 4, 5, 6)$

(ii) B: a number greater than 7

There is no number greater than 7 on the die.

Then,

$$B = (\varnothing)$$

(iii) C: a multiple of 3

There are only two numbers which are multiple of 3.

Then,

$$C = (3, 6)$$

(iv) D: a number less than 4

$$D = (1, 2, 3)$$

(v) E: an even number greater than 4

$$E = (6)$$

(vi) F: a number not less than 3

$$F = (3, 4, 5, 6)$$

Also, we have to find, $A \cup B$, $A \cap B$, $B \cup C$, $E \cap F$, $D \cap E$, $D - E$, $A - C$, $E \cap F'$, F'

So,

$$A \cap B = (1, 2, 3, 4, 5, 6) \cap (\varnothing)$$

$$= (\varnothing)$$

$$B \cup C = (\varnothing) \cup (3, 6)$$

$$= (3, 6)$$

$$E \cap F = (6) \cap (3, 4, 5, 6)$$

$$= (6)$$

$$D \cap E = (1, 2, 3) \cap (6)$$

$$= (\varnothing)$$

$$D - E = (1, 2, 3) - (6)$$

$$= (1, 2, 3)$$

$$A - C = (1, 2, 3, 4, 5, 6) - (3, 6)$$

$$= (1, 2, 4, 5)$$

$$F' = S - F$$

$$= (1, 2, 3, 4, 5, 6) - (3, 4, 5, 6)$$

$$= (1, 2)$$

$$E \cap F' = (6) \cap (1, 2)$$

$$= (\varphi)$$

3. An experiment involves rolling a pair of dice and recording the numbers that come up. Describe the following events: A: the sum is greater than 8, B: 2 occurs on either die C: the sum is at least 7 and a multiple of 3. Which pairs of these events are mutually exclusive?

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

In the question, it is given that pair of die is thrown, so the sample space will be

$$S = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{array} \right\}$$

A: the sum is greater than 8

$$\therefore A = \left\{ \begin{array}{l} (3,6), (4,5), (5,4), (6,3), (4,6), \\ (5,5), (6,4), (5,6), (6,5), (6,6) \end{array} \right\}$$

Possible sum greater than 8 are 9, 10, 11 & 12

B: 2 occurs on either die

$$B = \left\{ \begin{array}{l} (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (1,2), (3,2), (4,2), (5,2), (6,2) \end{array} \right\}$$

In this conditions possibilities are there that the number 2 will come on either first die or second die or both the die simultaneously

C: The sum is at least 7 and multiple of 3

$$C = \{(3,6), (4,5), (5,4), (6,3), (6,6)\}$$

So the sum can be only 9 or 12

Now, we shall find whether pairs of these events are mutually exclusive or not.

(i) $A \cap B = \varnothing$

There is no common element in A and B.

Therefore, A & B are mutually exclusive.

(ii) $B \cap C = \varnothing$

There is no common element between.

Therefore, B and C are mutually exclusive.

(iii) $A \cap C = \{(3,6), (4,5), (5,4), (6,3), (6,6)\}$

$\Rightarrow \{(3,6), (4,5), (5,4), (6,3), (6,6)\} \neq \varnothing$

A and C have common elements.

Therefore, A and C are mutually exclusive.

4. Three coins are tossed once. Let A denotes the event ‘three heads show’, B denotes the event “two heads and one tail show”, C denotes the event” three tails show and D denote the event ‘a head shows on the first coin’. Which events are

(i) Mutually exclusive? (ii) Simple? (iii) Compound?

Solution:-

Since either coin can turn up Head (H) or Tail (T), these are the possible outcomes.

But, now three coins are tossed once, so the possible sample space contains

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

Now,

A: ‘three heads’

$A = \{HHH\}$

B: “two heads and one tail”

$B = \{HHT, THH, HTH\}$

C: ‘three tails’

$C = \{TTT\}$

D: a head shows on the first coin

$D = \{HHH, HHT, HTH, HTT\}$

(i) Mutually exclusive

$$A \cap B = (HHH) \cap (HHT, THH, HTH)$$

$$= \varnothing$$

Therefore, A and C are mutually exclusive.

$$A \cap C = (HHH) \cap (TTT)$$

$$= \varnothing$$

There, A and C are mutually exclusive.

$$A \cap D = (HHH) \cap (HHH, HHT, HTH, HTT)$$

$$= (HHH)$$

$$A \cap D \neq \varnothing$$

So they are not mutually exclusive

$$B \cap C = (HHT, HTH, THH) \cap (TTT)$$

$$= \varnothing$$

Since there is no common element in B & C, so they are mutually exclusive.

$$B \cap D = (HHT, THH, HTH) \cap (HHH, HHT, HTH, HTT)$$

$$= (HHT, HTH)$$

$$B \cap D \neq \varnothing$$

There are common elements in B & D.

So, they are not mutually exclusive.

$$C \cap D = (TTT) \cap (HHH, HHT, HTH, HTT)$$

$$= \varnothing$$

There is no common element in C & D.

So they are not mutually exclusive.

(ii) Simple event

If an event has only one sample point of a sample space, it is called a simple (or elementary) event.

$$A = (HHH)$$

$$C = (TTT)$$

Both A & C have only one element.

So they are simple events.

(iii) Compound events

If an event has more than one sample point, it is called a Compound event.

$B = (HHT, HTH, THH)$

$D = (HHH, HHT, HTH, HTT)$

Both B & D have more than one element.

So, they are compound events.

5. Three coins are tossed. Describe

(i) Two events which are mutually exclusive.

(ii) Three events which are mutually exclusive and exhaustive.

(iii) Two events which are not mutually exclusive.

(iv) Two events which are mutually exclusive but not exhaustive.

(v) Three events which are mutually exclusive but not exhaustive.

Solution:-

Since either coin can turn up Head (H) or Tail (T), these are the possible outcomes.

But, now three coins are tossed once, so the possible sample space contains

$S = (HHH, HHT, HTH, HTT, THH, THT, TTH, TTT)$

(i) Two events which are mutually exclusive.

Let us assume A be the event of getting only head,

$A = (HHH)$

And also, let us assume B be the event of getting only Tail,

$B = (TTT)$

So, $A \cap B = \varnothing$

Since there is no common element in A & B, these two are mutually exclusive.

(ii) Three events which are mutually exclusive and exhaustive.

Now,

Let us assume P be the event of getting exactly two tails,

$$P = (\text{HTT}, \text{TTH}, \text{THT})$$

Let us assume Q be the event of getting at least two heads,

$$Q = (\text{HHT}, \text{HTH}, \text{THH}, \text{HHH})$$

Let us assume R be the event of getting only one tail,

$$C = (\text{TTT})$$

$$P \cap Q = (\text{HTT}, \text{TTH}, \text{THT}) \cap (\text{HHT}, \text{HTH}, \text{THH}, \text{HHH})$$

$$= \varnothing$$

There is no common element in P and Q.

Therefore, they are mutually exclusive.

$$Q \cap R = (\text{HHT}, \text{HTH}, \text{THH}, \text{HHH}) \cap (\text{TTT})$$

$$= \varnothing$$

There is no common element in Q and R.

Hence, they are mutually exclusive.

$$P \cap R = (\text{HTT}, \text{TTH}, \text{THT}) \cap (\text{TTT})$$

$$= \varnothing$$

There is no common element in P and R.

So they are mutually exclusive.

Now, P and Q, Q and R, and P and R are mutually exclusive

\therefore P, Q, and R are mutually exclusive.

And also,

$$P \cup Q \cup R = (\text{HTT}, \text{TTH}, \text{THT}, \text{HHT}, \text{HTH}, \text{THH}, \text{HHH}, \text{TTT}) = S$$

Hence P, Q and R are exhaustive events.

(iii) Two events which are not mutually exclusive.

Let us assume 'A' be the event of getting at least two heads,

$$A = (\text{HHH}, \text{HHT}, \text{THH}, \text{HTH})$$

Let us assume 'B' be the event of getting only head,

$$B = (HHH)$$

$$\text{Now, } A \cap B = (HHH, HHT, THH, HTH) \cap (HHH)$$

$$= (HHH)$$

$$A \cap B \neq \varnothing$$

There is a common element in A and B.

So they are not mutually exclusive.

(iv) Two events which are mutually exclusive but not exhaustive.

Let us assume 'P' be the event of getting only Head,

$$P = (HHH)$$

Let us assume 'Q' be the event of getting only tail,

$$Q = (TTT)$$

$$P \cap Q = (HHH) \cap (TTT)$$

$$= \varnothing$$

Since there is no common element in P and Q,

These are mutually exclusive events.

But,

$$P \cup Q = (HHH) \cup (TTT)$$

$$= \{HHH, TTT\}$$

$$P \cup Q \neq S$$

Since $P \cup Q \neq S$, these are not exhaustive events.

(v) Three events which are mutually exclusive but not exhaustive.

Let us assume 'X' be the event of getting only head,

$$X = (HHH)$$

Let us assume 'Y' be the event of getting only tail,

$$Y = (TTT)$$

Let us assume 'Z' be the event of getting exactly two heads,

$$Z = (HHT, THH, HTH)$$

Now,

$$X \cap Y = (HHH) \cap (TTT)$$

$$= \varnothing$$

$$X \cap Z = (HHH) \cap (HHT, THH, HTH)$$

$$= \varnothing$$

$$Y \cap Z = (TTT) \cap (HHT, THH, HTH)$$

$$= \varnothing$$

Therefore, they are mutually exclusive.

Also,

$$X \cup Y \cup Z = (HHH, TTT, HHT, THH, HTH)$$

$$X \cup Y \cup Z \neq S$$

So, X, Y and Z are not exhaustive.

Hence, it is proved that X, Y and X are mutually exclusive but not exhaustive.

6. Two dice are thrown. The events A, B and C are as follows:

A: getting an even number on the first die.

B: getting an odd number on the first die.

C: getting the sum of the numbers on the dice ≤ 5 .

Describe the events

(i) A^c (ii) not B (iii) A or B

(iv) A and B (v) A but not C (vi) B or C

(vii) B and C (viii) $A \cap B^c \cap C^c$

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

In the question, it is given that pair of die is thrown, so the sample space will be

$$S = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{array} \right\}$$

As per the condition given the question,

A: getting an even number on the first die.

$$A = \left\{ \begin{array}{l} (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{array} \right\}$$

B: getting an odd number on the first die.

$$B = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \end{array} \right\}$$

C: getting the sum of the numbers on the dice ≤ 5

$$C = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (3,1), (3,2), (2,3), (4,1)\}$$

Then,

$$(i) A' = \left\{ \begin{array}{l} \left\{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \right\} \\ \left\{ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \right\} \\ \left\{ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \right\} \end{array} \right\} = B$$

$$(ii) B' = \left\{ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \right\} = A$$

$$(iii) A \cup B \text{ (A or B)} = \left\{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \right\} = S$$

$$(iv) A \text{ and } B \text{ (} A \cap B \text{)} = \phi$$

$$(v) A \text{ but not } C = A - C = \left\{ (2,4), (2,5), (2,6), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \right\}$$

$$(vi) B \text{ or } C = B \cup C = \left\{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \right\}$$

$$(vii) B \text{ and } C = B \cap C = \{(1,1), (1,2), (1,3), (1,4), (3,1), (3,2)\}$$

(viii)

$$C' = \left\{ (1,5), (1,6), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (3,6), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \right\}$$

$$\therefore A \cap B' \cap C' = A \cap A \cap C' = A \cap C'$$

$$= \left\{ (2,4), (2,5), (2,6), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \right\}$$

7. Refer to question 6 above and state true or false. (Give reasons for your answer)

(i) A and B are mutually exclusive

(ii) A and B are mutually exclusive and exhaustive

(iii) $A = B'$

(iv) A and C are mutually exclusive

(v) A and B' are mutually exclusive.

(vi) A', B', C are mutually exclusive and exhaustive.

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

In the question, it is given that pair of die is thrown, so the sample space will be,

By referring the question 6 above,

$$S = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{array} \right\}$$

As per the condition given the question,

A: getting an even number on the first die.

$$A = \left\{ \begin{array}{l} (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{array} \right\}$$

B: getting an odd number on the first die.

$$B = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \end{array} \right\}$$

C: getting the sum of the numbers on the dice ≤ 5

$$C = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (3,1), (3,2), (2,3), (4,1)\}$$

(i) A and B are mutually exclusive.

So, $(A \cap B) = \varnothing$

So, A & B are mutually exclusive.

Hence, the given statement is true.

(ii) A and B are mutually exclusive and exhaustive.

$$A \cup B = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{array} \right\} = S$$

$$\Rightarrow A \cup B = S$$

From statement (I), we have A and B are mutually exclusive.

\therefore A and B are mutually exclusive and exhaustive.

Hence, the statement is true.

(iii) $A = B$

$$B' = \left\{ \begin{array}{l} (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \end{array} \right\} = A$$

Therefore, the statement is true.

(iv) A and C are mutually exclusive.

We have,

$$A \cap C = \{(2, 1), (2, 2), (2, 3), (4, 1)\}$$

$$A \cap C \neq \varnothing$$

A and C are not mutually exclusive.

Hence, the given statement is false.

(v) A and B' are mutually exclusive.

We have,

$$A \cap B' = A \cap A = A$$

$$\therefore A \cap B' \neq \varnothing$$

So, A and B' are not mutually exclusive.

Therefore, the given statement is false.

(vi) A^I, B^I, C are mutually exclusive and exhaustive.

$$\text{Here } A^I = \left\{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \right. \\ \left. (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \right\}$$

$$B = \left\{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \right. \\ \left. (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \right\} \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$$

$$\text{And } C = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (3,1), (3,2), (2,3), (4,1)\}$$

$$A^I \cap B^I = \phi$$

Hence there is no common element in A^I and B^I

So they are mutually exclusive.

$$B^I \cap C = \{(2,1), (2,2), (2,3), (4,1)\}$$

$$B^I \cap C \neq \phi$$

$$B = \left\{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \right. \\ \left. (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \right\} \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)$$

$$\text{And } C = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (3,1), (3,2), (2,3), (4,1)\}$$

$$A^I \cap B^I = \phi$$

Hence there is no common element in A^I and B^I

So they are mutually exclusive.

$$B^I \cap C = \{(2,1), (2,2), (2,3), (4,1)\}$$

$$B^I \cap C \neq \phi$$

They are not mutually exclusive.

B^I and C are not mutually exclusive.

Therefore, A^I, B^I and C are not mutually exclusive and exhaustive.

So, the given statement is false.

EXERCISE 16.3

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1. Which of the following cannot be a valid assignment of probabilities for outcomes of sample Space $S = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7\}$

Assignment

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(a)	0.1	0.01	0.05	0.03	0.01	0.2	0.6
(b)	1/7	1/7	1/7	1/7	1/7	1/7	1/7
(c)	0.1	0.2	0.3	0.4	0.5	0.6	0.7
(d)	-0.1	0.2	0.3	0.4	-0.2	0.1	0.3
(e)	1/14	2/14	3/14	4/14	5/14	6/14	15/14

Solution:-

(a) Condition (i): Each of the numbers $p(\omega_i)$ is positive and less than zero. Condition (ii): Sum of probabilities

$$0.01 + 0.05 + 0.03 + 0.01 + 0.2 + 0.6 = 1$$

Therefore, the given assignment is valid.

b) Condition (i): Each of the numbers $p(\omega_i)$ is positive and less than zero. Condition (ii): Sum of probabilities

$$= (1/7) + (1/7) + (1/7) + (1/7) + (1/7) + (1/7) + (1/7)$$

$$= 7/7$$

$$= 1$$

Therefore, the given assignment is valid.

c) Condition (i): Each of the numbers $p(\omega_i)$ is positive and less than zero. Condition (ii): Sum of probabilities

$$= 0.1 + 0.2 + 0.3 + 0.4 + 0.5 + 0.6 + 0.7$$

$$= 2.8 > 1$$

Therefore, the 2nd condition is not satisfied

Which states that $p(w_i) \leq 1$

So, the given assignment is not valid.

d) The conditions of the axiomatic approach don't hold true in the given assignment, that is

1) Each of the numbers $p(w_i)$ is less than zero but also negative.

To be true, each of the numbers $p(w_i)$ should be less than zero and positive.

So, the assignment is not valid.

e) Condition (i): Each of the numbers $p(w_i)$ is positive and less than zero. Condition (ii): Sum of probabilities

$$= (1/14) + (2/14) + (3/14) + (4/14) + (5/14) + (6/14) + (7/14)$$

$$= (28/14) \geq 1$$

The second condition doesn't hold true, so the assignment is not valid.

2. A coin is tossed twice, what is the probability that at least one tail occurs?

Solution:-

Since either coin can turn up Head (H) or Tail (T), these are the possible outcomes.

Here coin is tossed twice, then the sample space is $S = (TT, HH, TH, HT)$

\therefore Number of possible outcomes $n(S) = 4$

Let A be the event of getting at least one tail.

$$\therefore n(A) = 3$$

$P(\text{Event}) = \text{Number of outcomes favourable to the event} / \text{Total number of possible outcomes}$

$$P(A) = n(A)/n(S)$$

$$= \frac{3}{4}$$

3. A die is thrown, find the probability of the following events.

(i) A prime number will appear.

(ii) A number greater than or equal to 3 will appear.

(iii) A number less than or equal to one will appear.

(iv) A number more than 6 will appear.

(v) A number less than 6 will appear.

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

Here, $S = \{1, 2, 3, 4, 5, 6\}$

$$\therefore n(S) = 6$$

(i) A prime number will appear.

Let us assume 'A' be the event of getting a prime number,

$$A = \{2, 3, 5\}$$

$$\text{Then, } n(A) = 3$$

$P(\text{Event}) = \text{Number of outcomes favourable to the event} / \text{Total number of possible outcomes}$

$$\therefore P(A) = n(A)/n(S)$$

$$= 3/6$$

$$= 1/2$$

(ii) A number greater than or equal to 3 will appear.

Let us assume 'B' be the event of getting a number greater than or equal to 3,

$$B = \{3, 4, 5, 6\}$$

$$\text{Then, } n(B) = 4$$

$P(\text{Event}) = \text{Number of outcomes favourable to the event} / \text{Total number of possible outcomes}$

$$\therefore P(B) = n(B)/n(S)$$

$$= 4/6$$

$$= 2/3$$

(iii) A number less than or equal to one will appear.

Let us assume 'C' be the event of getting a number less than or equal to 1,

$$C = \{1\}$$

$$\text{Then, } n(C) = 1$$

$P(\text{Event}) = \text{Number of outcomes favourable to the event} / \text{Total number of possible outcomes}$

$$\therefore P(C) = n(C)/n(S)$$

$$= 1/6$$

(iv) A number more than 6 will appear.

Let us assume 'D' be the event of getting a number more than 6, then

$$D = \{0\}$$

$$\text{Then, } n(D) = 0$$

$$P(\text{Event}) = \text{Number of outcomes favourable to the event} / \text{Total number of possible outcomes}$$

$$\therefore P(D) = n(D)/n(S)$$

$$= 0/6$$

$$= 0$$

(v) A number less than 6 will appear.

Let us assume 'E' be the event of getting a number less than 6, then

$$E = \{1, 2, 3, 4, 5\}$$

$$\text{Then, } n(E) = 5$$

$$P(\text{Event}) = \text{Number of outcomes favourable to the event} / \text{Total number of possible outcomes}$$

$$\therefore P(E) = n(E)/n(S)$$

$$= 5/6$$

4. A card is selected from a pack of 52 cards.

(a) How many points are there in the sample space?

(b) Calculate the probability that the card is an ace of spades.

(c) Calculate the probability that the card is (i) an ace (ii) black card

Solution:-

From the question, it is given that there are 52 cards in the deck.

(a) Number of points in the sample space = 52 (given)

$$\therefore n(S) = 52$$

(b) Let us assume 'A' be the event of drawing an ace of spades.

$$A = 1$$

$$\text{Then, } n(A) = 1$$

$$P(\text{Event}) = \text{Number of outcomes favourable to the event} / \text{Total number of possible outcomes}$$

$$\therefore P(A) = n(A)/n(S)$$

$$= 1/52$$

(c) Let us assume 'B' be the event of drawing an ace. There are four aces.

Then, $n(B) = 4$

$P(\text{Event}) = \text{Number of outcomes favourable to the event} / \text{Total number of possible outcomes}$

$$\therefore P(B) = n(B)/n(S)$$

$$= 4/52$$

$$= 1/13$$

(d) Let us assume 'C' be the event of drawing a black card. There are 26 black cards.

Then, $n(C) = 26$

$P(\text{Event}) = \text{Number of outcomes favourable to the event} / \text{Total number of possible outcomes}$

$$\therefore P(C) = n(C)/n(S)$$

$$= 26/52$$

$$= \frac{1}{2}$$

5. A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. Find the probability that the sum of numbers that turn up is (i) 3 (ii) 12

Solution:-

Let us assume that 1, 2, 3, 4, 5 and 6 are the possible outcomes when the die is thrown.

So, the sample space $S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$

Then, $n(S) = 12$

(i) Let us assume 'P' be the event having the sum of numbers as 3.

$$P = \{(1, 2)\},$$

Then, $n(P) = 1$

$P(\text{Event}) = \text{Number of outcomes favourable to the event} / \text{Total number of possible outcomes}$

$$\therefore P(P) = n(P)/n(S)$$

$$= 1/12$$

(ii) Let us assume 'Q' be the event having the sum of the number as 12.

Then $Q = \{(6, 6)\}$, $n(Q) = 1$

$P(\text{Event}) = \text{Number of outcomes favourable to the event} / \text{Total number of possible outcomes}$

$$\therefore P(Q) = n(Q)/n(S)$$

$$= 1/12$$

6. There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?

Solution:-

From the question, it is given that there are four men and six women on the city council.

Here total members in the council $= 4 + 6 = 10$,

Hence, the sample space has 10 points.

$$\therefore n(S) = 10$$

The number of women is 6 ... [given]

Let us assume 'A' be the event of selecting a woman.

$$\text{Then } n(A) = 6$$

$P(\text{Event}) = \text{Number of outcomes favourable to the event} / \text{Total number of possible outcomes}$

$$\therefore P(A) = n(A)/n(S)$$

$$= 6/10 \dots [\text{divide both numerator and denominators by 2}]$$

$$= 3/5$$

7. A fair coin is tossed four times, and a person wins Rs 1 for each head and loses Rs 1.50 for each tail that turns up.

From the sample space, calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

Solution:-

Since either coin can turn up Head (H) or Tail (T), these are the possible outcomes.

But, now coin is tossed four times, so the possible sample space contains,

$S = (\text{HHHH}, \text{HHHT}, \text{HHTH}, \text{HTHH}, \text{THHH}, \text{HHTT}, \text{HTHT}, \text{THHT}, \text{HTTH}, \text{THTH}, \text{TTHH}, \text{TTTH}, \text{TTHT}, \text{THTT}, \text{HTTT}, \text{TTTT})$

As per the condition given the question, a person will win or lose money depending upon the face of the coin, so

(i) For 4 heads = $1 + 1 + 1 + 1 = ₹ 4$

So, he wins ₹ 4.

(ii) For 3 heads and 1 tail = $1 + 1 + 1 - 1.50$

$$= 3 - 1.50$$

$$= ₹ 1.50$$

So, he will be winning ₹ 1.50.

(iii) For 2 heads and 2 tails = $1 + 1 - 1.50 - 1.50$

$$= 2 - 3$$

$$= - ₹ 1$$

So, he will be losing ₹ 1.

(iv) For 1 head and 3 tails = $1 - 1.50 - 1.50 - 1.50$

$$= 1 - 4.50$$

$$= - ₹ 3.50$$

So, he will be losing Rs. 3.50.

(v) For 4 tails = $- 1.50 - 1.50 - 1.50 - 1.50$

$$= - ₹ 6$$

So, he will be losing Rs. 6.

Now, the sample space of amounts is

$$S = \{4, 1.50, 1.50, 1.50, 1.50, -1, -1, -1, -1, -1, -1, -1, -3.50, -3.50, -3.50, -3.50, -6\}$$

Then, $n(S) = 16$

$$P(\text{winning ₹ 4}) = 1/16$$

$$P(\text{winning ₹ 1.50}) = 4/16 \dots [\text{divide both numerator and denominator by 4}]$$

$$= 1/4$$

$$P(\text{winning ₹ 1}) = 6/16 \dots [\text{divide both numerator and denominator by 2}]$$

$$= 3/8$$

$$P(\text{winning ₹ 3.50}) = 4/16 \dots [\text{divide both numerator and denominator by 4}]$$

$$= 1/4$$

$$P(\text{winning ₹ 6}) = 1/16$$

$$= 3/8$$

8. Three coins are tossed once. Find the probability of getting

(i) 3 heads (ii) 2 heads (iii) at least 2 heads

(iv) at most 2 heads (v) no head (vi) 3 tails

(vii) Exactly two tails (viii) no tail (ix) at most two tails

Solution:-

Since either coin can turn up Head (H) or Tail (T), these are the possible outcomes.

But, now three coins are tossed, so the possible sample space contains

$$S = \{HHH, HHT, HTH, THH, TTH, HTT, TTT, THT\}$$

Where s is sample space and here $n(S) = 8$

(i) 3 heads

Let us assume 'A' be the event of getting 3 heads.

$$n(A) = 1$$

$$\therefore P(A) = n(A)/n(S)$$

$$= 1/8$$

(ii) 2 heads

Let us assume 'B' be the event of getting 2 heads.

$$n(A) = 3$$

$$\therefore P(B) = n(B)/n(S)$$

$$= 3/8$$

(iii) at least 2 heads

Let us assume 'C' be the event of getting at least 2 heads.

$$n(C) = 4$$

$$\therefore P(C) = n(C)/n(S)$$

$$= 4/8$$

$$= 1/2$$

(iv) at most 2 heads

Let us assume 'D' be the event of getting at most 2 heads.

$$n(D) = 7$$

$$\therefore P(D) = n(D)/n(S)$$

$$= 7/8$$

(v) no head

Let us assume 'E' be the event of getting no heads.

$$n(E) = 1$$

$$\therefore P(E) = n(E)/n(S)$$

$$= 1/8$$

(vi) 3 tails

Let us assume 'F' be the event of getting 3 tails.

$$n(F) = 1$$

$$\therefore P(F) = n(F)/n(S)$$

$$= 1/8$$

(vii) Exactly two tails

Let us assume 'G' be the event of getting exactly 2 tails.

$$n(G) = 3$$

$$\therefore P(G) = n(G)/n(S)$$

$$= 3/8$$

(viii) no tail

Let us assume 'H' be the event of getting no tails.

$$n(H) = 1$$

$$\therefore P(H) = n(H)/n(S)$$

$$= 1/8$$

(ix) at most two tails

Let us assume 'I' be the event of getting at most 2 tails.

$$n(I) = 7$$

$$\therefore P(I) = n(I)/n(S)$$

$$= 7/8$$

9. If $2/11$ is the probability of an event, what is the probability of the event 'not A'.

Solution:-

From the question, it is given that $2/11$ is the probability of an event A.

$$\text{i.e. } P(A) = 2/11$$

Then,

$$P(\text{not } A) = 1 - P(A)$$

$$= 1 - (2/11)$$

$$= (11 - 2)/11$$

$$= 9/11$$

10. A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that the letter is (i) a vowel (ii) a consonant

Solution:-

The word given in the question is 'ASSASSINATION'.

Total letters in the given word = 13

Number of vowels in the given word = 6

Number of consonants in the given word = 7

Then, the sample space $n(S) = 13$

(i) a vowel

Let us assume 'A' be the event of selecting a vowel.

$$n(A) = 6$$

$$\therefore P(A) = n(A)/n(S)$$

$$= 6/13$$

(ii) Let us assume 'B' be the event of selecting the consonant.

$$n(B) = 7$$

$$\therefore P(B) = n(B)/n(S)$$

$$= 7/13$$

11. In a lottery, a person chooses six different natural numbers at random, from 1 to 20, and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game? [Hint order of the numbers is not important.]

Solution:-

From the question, it is given that

Total numbers of numbers in the draw = 20

Numbers to be selected = 6

$$\therefore n(S) = {}^{20}C_6$$

Let us assume 'A' be the event that six numbers match with the six numbers already fixed by the lottery committee.

$$\therefore n(A) = {}^6C_6 = 1$$

Probability of winning the prize

$$\begin{aligned} P(A) &= \frac{n(A)}{n(S)} = \frac{{}^6C_6}{{}^{20}C_6} = \frac{6!14!}{20!} \\ &= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 14!}{20 \times 19 \times 18 \times 17 \times 16 \times 15 \times 14!} \\ &= \frac{1}{38760} \end{aligned}$$

12. Check whether the following probabilities $P(A)$ and $P(B)$ are consistently defined

(i) $P(A) = 0.5$, $P(B) = 0.7$, $P(A \cap B) = 0.6$

(ii) $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cup B) = 0.8$

Solution:-

(i) $P(A) = 0.5$, $P(B) = 0.7$, $P(A \cap B) = 0.6$

$$P(A \cap B) > P(A)$$

Therefore, the given probabilities are not consistently defined.

(ii) $P(A) = 0.5$, $P(B) = 0.4$, $P(A \cup B) = 0.8$

Then,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.8 = 0.5 + 0.4 - P(A \cap B)$$

Transposing $-P(A \cap B)$ to LHS and it becomes $P(A \cap B)$ and 0.8 to RHS, and it becomes -0.8 .

$$P(A \cap B) = 0.9 - 0.8$$

$$= 0.1$$

Therefore, $P(A \cap B) < P(A)$ and $P(A \cap B) < P(B)$

So, the given probabilities are consistently defined.

13. Fill in the blanks in the following table.

	P(A)	P(B)	P(A ∩ B)	P(A ∪ B)
(i)	1/3	1/5	1/15
(ii)	0.35	0.25	0.6
(iii)	0.5	0.35	0.7

Solution:-

From the given table,

(i) $P(A) = 1/3$, $P(B) = 1/5$, $P(A \cap B) = 1/15$, $P(A \cup B) = ?$

We know that,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= (1/3) + (1/5) - (1/15)$$

$$= ((5 + 3)/15) - (1/15)$$

$$= (8/15) - (1/15)$$

$$= (8 - 1)/15$$

$$= 7/15$$

$$(ii) P(A) = 0.35, P(B) = ?, P(A \cap B) = 0.25, P(A \cup B) = 0.6$$

Then,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.6 = 0.35 + P(B) - 0.25$$

Transposing $-0.25, 0.35$ to LHS, and it becomes 0.25 and -0.35 .

$$P(B) = 0.6 + 0.25 - 0.35$$

$$= 0.5$$

$$(iii) P(A) = 0.5, P(B) = 0.35, P(A \cup B) = 0.7, P(A \cap B) = ?$$

Then,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.7 = 0.5 + 0.35 - P(A \cap B)$$

Transposing $-P(A \cap B)$ to LHS and it becomes $P(A \cap B)$ and 0.7 to RHS, and it becomes -0.7 .

$$P(A \cap B) = 0.85 - 0.7$$

$$= 0.15$$

14. Given $P(A) = 5/3$ and $P(B) = 1/5$. Find $P(A \text{ or } B)$, if A and B are mutually exclusive events.

Solution:-

From the question, it is given that

$$P(A) = 5/3 \text{ and } P(B) = 1/5$$

Then, $P(A \text{ or } B)$, if A and B are mutually exclusive

$$P(A \cup B) \text{ or } P(A \text{ or } B) = P(A) + P(B)$$

$$= (3/5) + (1/5)$$

$$= 4/5$$

15. If E and F are events such that $P(E) = 1/4$, $P(F) = 1/2$ and $P(E \text{ and } F) = 1/8$, find

(i) $P(E \text{ or } F)$, (ii) $P(\text{not } E \text{ and not } F)$

Solution:-

From the question, we have $P(E) = 1/4$, $P(F) = 1/2$ and $P(E \cap F) = 1/8$

$$(i) P(E \text{ or } F) \text{ i.e., } P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

$$= \frac{1}{4} + \frac{1}{2} - \left(\frac{1}{8}\right)$$

$$= \frac{5}{8}$$

$$(ii) P(\text{not } E \text{ and not } F) = P(\bar{E} \cap \bar{F}) = P(\overline{E \cup F}) = 1 - P(E \cup F)$$

$$= 1 - \left(\frac{5}{8}\right)$$

$$= \frac{8 - 5}{8}$$

$$= \frac{3}{8}$$

16. Events E and F are such that $P(\text{not } E \text{ or not } F) = 0.25$, State whether E and F are mutually exclusive.

Solution:-

From the question it is given that, $P(\text{not } E \text{ and not } F) = 0.25$

$$\text{So, } P(\bar{E} \cap \bar{F}) = 0.25$$

Then we have,

$$\Rightarrow P(\overline{E \cap F}) = 0.25$$

$$\Rightarrow 1 - P(E \cap F) = 0.25$$

$$\Rightarrow P(E \cap F) = 1 - 0.25$$

$$= 0.75$$

$$P(E \cap F) \neq 0$$

Hence, E and F are not mutually exclusive events.

17. A and B are events such that $P(A) = 0.42$, $P(B) = 0.48$ and $P(A \text{ and } B) = 0.16$. Determine (i) $P(\text{not } A)$, (ii) $P(\text{not } B)$ and (iii) $P(A \text{ or } B)$

Solution:-

From the question, it is given that $P(A) = 0.42$, $P(B) = 0.48$ and $P(A \text{ and } B) = 0.16$.

$$(i) P(\text{not } A) = 1 - P(A)$$

$$= 1 - 0.42$$

$$= 0.58$$

$$(ii) P(\text{not } B) = 1 - P(B)$$

$$= 1 - 0.48$$

$$= 0.52$$

$$(iii) P(A \text{ not } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.42 + 0.48 - 0.16$$

$$= 0.74$$

18. In Class XI of a school, 40% of the students study Mathematics, and 30% study Biology. 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology.

Solution:-

Let us assume 'A' be the event that the student is studying mathematics and 'B' be the event that the student is studying biology.

$$\text{So, } P(A) = 40/100$$

$$= 2/5$$

$$\text{And, } P(B) = 30/100$$

$$= 3/10$$

$$\text{Then, } P(A \cap B) = (10/100)$$

$$= 1/10, P(A \cap B) \text{ is probability of studying both mathematics and biology.}$$

Here, Probability of studying mathematics or biology will be given by $P(A \cup B)$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= (2/5) + (3/10) - (1/10)$$

$$= 6/10$$

$$= 3/5$$

Hence, $(3/5)$ is the probability that the student will studying mathematics or biology

19. In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8, and the probability of passing the second examination is 0.7. The probability of passing at least one of them is 0.95. What is the probability of passing both?

Solution:-

Let us assume the probability of a randomly chosen student passing the first examination is 0.8 be $P(A)$.

And also, assume the probability of passing the second examination is 0.7 be $P(B)$.

Then,

$P(A \cup B)$ is the probability of passing at least one of the examinations.

Now,

$$P(A \cup B) = 0.95, P(A) = 0.8, P(B) = 0.7$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.95 = 0.8 + 0.7 - P(A \cap B)$$

Transposing $-P(A \cap B)$ to LHS and it becomes $P(A \cap B)$ and 0.95 to RHS, and it becomes

$$-0.95$$

$$P(A \cap B) = 1.5 - 0.95$$

$$= 0.55$$

Hence, 0.55 is the probability that the student will pass both examinations.

20. The probability that a student will pass the final examination in both English and Hindi is 0.5, and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination?

Solution:-

Let us assume the probability of passing the English examination is 0.75 be $P(A)$.

And also, assume the probability of passing the Hindi examination is $P(B)$.

Here given, $P(A) = 0.75$, $P(A \cap B) = 0.5$, $P(A' \cap B') = 0.1$

We know that, $P(A' \cap B') = 1 - P(A \cup B)$

$$\text{Then, } P(A \cup B) = 1 - P(A' \cap B')$$

$$= 1 - 0.1$$

$$= 0.9$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.9 = 0.75 + P(B) - 0.5$$

Transposing $0.75, -0.5$ to LHS, and it becomes $-0.75, 0.5$.

$$P(B) = 0.9 + 0.5 - 0.75$$

$$= 0.65$$

21. In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that

(i) The student opted for NCC or NSS.

(ii) The student has opted for neither NCC nor NSS.

(iii) The student has opted for NSS but not NCC.

Solution:-

From the question, it is given that

The total number of students in the class = 60

Thus, the sample space consists of $n(S) = 60$

Let us assume that the students opted for NCC to be 'A'.

And also, assume that the students opted for NSS to be 'B'.

$$\text{So, } n(A) = 30, n(B) = 32, n(A \cap B) = 24$$

$$\text{We know that, } P(A) = n(A)/n(S)$$

$$= 30/60$$

$$= \frac{1}{2}$$

$$P(B) = n(B)/n(S)$$

$$= 32/60$$

$$= 8/15$$

$$P(A \cap B) = n(A \cap B)/n(S)$$

$$= 24/60$$

$$= \frac{2}{5}$$

$$\text{Therefore, } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(i) The student opted for NCC or NSS.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \left(\frac{8}{15}\right) - \left(\frac{2}{5}\right)$$

$$= \frac{19}{30}$$

(ii) $P(\text{student opted neither NCC nor NSS})$

$$P(\text{not } A \text{ and not } B) = P(A' \cap B')$$

$$\text{We know that, } P(A' \cap B') = 1 - P(A \cup B)$$

$$= 1 - \left(\frac{19}{30}\right)$$

$$= \frac{11}{30}$$

(iii) $P(\text{student opted NSS but not NCC})$

$$n(B - A) = n(B) - n(A \cap B)$$

$$\Rightarrow 32 - 24 = 8$$

The probability that the selected student has opted for NSS and not NCC is

$$= \left(\frac{8}{60}\right) = \frac{2}{15}$$

MISCELLANEOUS EXERCISE

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1. A box contains 10 red marbles, 20 blue marbles and 30 green marbles. 5 marbles are drawn from the box, what is the probability that

(i) all will be blue? (ii) at least one will be green?

Solution:-

From the question, it is given that

Number of red marbles in the box = 10

Number of blue marbles in the box = 20

Number of green marbles in the box = 30

So, total number of marbles in the box = $10 + 20 + 30 = 60$

Number of ways of drawing 5 marbles from 60 marbles = ${}^{60}C_5$

(i) All the drawn marbles will be blue if we draw 5 marbles out of 20 blue marbles.

We have,

Number of ways of drawing 5 blue marbles from 20 blue marbles = ${}^{20}C_5$

Then,

The probability that all marbles will be blue = ${}^{20}C_5 / {}^{60}C_5$

(ii) Number of ways in which the drawn marble is not green = ${}^{(20+10)}C_5$

We have,

The probability that no marble is green = ${}^{30}C_5 / {}^{60}C_5$

Then,

The probability that at least one marble is green = $1 - {}^{30}C_5 / {}^{60}C_5$

2. 4 cards are drawn from a well-shuffled deck of 52 cards. What is the probability of obtaining 3 diamonds and one spade?

Solution:-

From the question, it is given that

4 cards are drawn from a well-shuffled deck of 52 cards.

Number of ways of drawing 4 cards from 52 cards = ${}^{52}C_4$

In a deck of 52 cards, there are 13 diamonds and 13 spades.

Number of ways of drawing 3 diamonds and one spade = ${}^{13}C_3 \times {}^{13}C_1$

Therefore, the probability of obtaining 3 diamonds and one spade

$$= ({}^{13}C_3 \times {}^{13}C_1) / {}^{52}C_4$$

3. A die has two faces, each with the number '1', three faces, each with the number '2' and one face with the number '3'. If a die is rolled once, determine

(i) P(2) (ii) P(1 or 3) (iii) P(not 3)

Solution:-

From the question, it is given that

Die has two faces, each with the number '1'.

Three faces, each with the number '2'.

And one face with the number '3'.

Then, the Total number of faces of the die = 6

(i) P(2)

Number faces with number '2' = 3 ... [given]

$$\text{So, } P(2) = 3/6$$

$$= 1/2$$

(ii) P(1 or 3)

We know that, $P(1 \text{ or } 3) = P(\text{not } 2) = 1 - P(2)$

$$\text{So, } P(1 \text{ or } 3) = 1 - 1/2$$

$$= (2 - 1)/2$$

$$= 1/2$$

(iii) P(not 3)

Number of faces with number '3' = 1

$$P(3) = 1/6$$

$$P(\text{not } 3) = 1 - P(3)$$

$$= 1 - 1/6$$

$$= (6 - 1)/6$$

$$= 5/6$$

4. In a certain lottery, 10,000 tickets are sold, and ten equal prizes are awarded. What is the probability of not getting a prize if you buy (a) one ticket (b) two tickets (c) 10 tickets?

Solution:-

From the question, it is given that

Number of lottery tickets sold = 10,000

Number prizes awarded = 10

(a) The probability of not getting a prize if we buy one ticket,

$$P(\text{getting a prize}) = 10/10000$$

$$= 1/1000$$

Then,

$$P(\text{not getting a prize}) = 1 - (1/1000)$$

$$= (1000 - 1)/1000$$

$$= 999/1000$$

(b) The probability of not getting a prize if we buy two tickets,

Then,

$$\text{Number of tickets not awarded} = 10,000 - 10 = 9990$$

$$\text{Therefore, } P(\text{not getting a prize}) = {}^{9990}C_2 / {}^{10000}C_2$$

(iii) The probability of not getting a prize if we buy 10 tickets, then

$$\text{Number of tickets not awarded} = 10,000 - 10 = 9990$$

$$\text{Therefore, } P(\text{not getting a prize}) = {}^{9990}C_{10} / {}^{10000}C_{10}$$

5. Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100 students, what is the probability that

(a) you both enter the same section?

(b) you both enter the different sections?

Solution:-

From the question,

Total number of students = 100

I and my friend are among the 100 students.

Then, two sections of 40 and 60 are formed.

Total number of ways of selecting 2 students out of 100 students =

(a) Let S = the two of us will enter the same section if both of us are among 40 students or among 60 students.

Number of ways in which both of us enter the same section = P(S)

$$P(S) = ({}^{40}C_2 + {}^{60}C_2) / {}^{100}C_2$$

$$P(S) = \frac{\frac{40!}{2! \times 38!} + \frac{60!}{2! \times 58!}}{\frac{100!}{2! \times 98!}} = \frac{39 \times 40 + 59 \times 60}{99 \times 100} = \frac{5100}{9900} = \frac{17}{33}$$

(b) P(we enter different sections)

$$= 1 - P(\text{we enter the same section})$$

$$P(\text{we enter different sections}) = 1 - (17/33)$$

$$= (33 - 17)/33$$

$$= 16/33$$

6. Three letters are dictated to three persons, and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

Solution:-

Let us assume L1, L2, L3 be three letters and E1, E2, and E3 be their corresponding envelopes, respectively.

Then, the sample space is

L₁E₁, L₂E₃, L₃E₂,

L₂E₂, L₁E₃, L₃E₁,

L₃E₃, L₁E₂, L₂E₁,

L₁E₁, L₂E₂, L₃E₃,

L₁E₂, L₂E₃, L₃E₁,

L₁E₃, L₂E₁, L₃E₂,

Hence, there are 6 ways of inserting 3 letters in 3 envelopes.

And there are 4 ways in which at least one letter is inserted in the proper envelope. (first 4 rows of sample space)

The probability that at least one letter is inserted in the proper envelope,

$$= 4/6$$

$$= 2/3$$

7. A and B are two events such that $P(A) = 0.54$, $P(B) = 0.69$ and $P(A \cap B) = 0.35$. Find

(i) $P(A \cup B)$ (ii) $P(A^c \cap B^c)$ (iii) $P(A \cap B^c)$ (iv) $P(B \cap A^c)$

Solution:-

From the question, it is given that A and B are two events such that,

$$P(A) = 0.54, P(B) = 0.69, P(A \cap B) = 0.35$$

(i) $P(A \cup B)$

$$\text{We know that, } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A \cup B) = 0.54 + 0.69 - 0.35 = 0.88$$

$$\therefore P(A \cup B) = 0.88$$

(ii) $P(A^c \cap B^c)$

We know that $A^c \cap B^c = (A \cup B)^c$ [by De Morgan's law]

$$\text{So, } P(A^c \cap B^c) = P(A \cup B)^c$$

$$= 1 - P(A \cup B)$$

$$= 1 - 0.88 = 0.12$$

$$\therefore P(A^c \cap B^c) = 0.12$$

(iii) $P(A \cap B^c)$

We have,

$$P(A \cap B^c) = P(A) - P(A \cap B)$$

$$= 0.54 - 0.35$$

$$= 0.19$$

Therefore, $P(A \cap B^c) = 0.19$

(iv) $P(B \cap A^c)$

We know that: $P(B \cap A^c) = P(B) - P(A \cap B)$

$$\Rightarrow P(B \cap A^c) = 0.69 - 0.35$$

Hence, $P(B \cap A^c) = 0.34$

8. From the employees of a company, 5 persons are selected to represent them in the managing committee of the company. Particulars of five persons are as follows:

S.No.	Name	Sex	Age in years
1	Harish	M	30
2	Rohan	M	33
3	Sheetal	F	46
4	Alis	F	28
5	Salim	M	41

A person is selected at random from this group to act as a spokesperson. What is the probability that the spokesperson will be either male or over 35 years?

Solution:-

From the given table,

The number of people = 5

Out of 5 persons, 3 are Male.

Out of 5 persons, 2 are 35 years of age.

Let us assume 'A' be the event in which the spokesperson will be a male and B be the event in which the spokesperson will be over 35 years of age.

Accordingly, $P(A) = 3/5$ and $P(B) = 2/5$

Since there is only one male who is over 35 years of age,

$$P(A \cap B) = 1/5$$

We know that: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= (3/5) + (2/5) - (1/5)$$

$$= 4/5$$

Hence, the probability that the spokesperson will either be a male or over 35 years of age is $4/5$.

9. If 4-digit numbers greater than 5,000 are randomly formed from the digits 0, 1, 3, 5, and 7, what is the probability of forming a number divisible by 5 when, (i) the digits are repeated? (ii) the repetition of digits is not allowed?

Solution:-

(i) When the digits are repeated

Since four-digit numbers greater than 5000 are formed,

The thousand's place digit is either 7 or 5.

$$\text{Total number of 4-digit numbers greater than 5000} = 2 \times 5 \times 5 \times 5 - 1$$

$$= 250 - 1$$

$$= 249$$

A number is divisible by 5 if the digit at its unit's place is either 0 or 5.

\therefore Total number of 4-digit numbers greater than 5000 that are divisible by 5

$$= 2 \times 5 \times 5 \times 2 - 1$$

$$= 100 - 1 = 99$$

Hence, the probability of forming a number divisible by 5 when the digits are repeated is $= P(\text{a number divisible by 5 when digits are repeated})$

$$P(\text{number divisible by 5 when digits repeated}) = 99/249$$

$$= 33/83$$

(ii) When repetition of digits is not allowed

The thousands place can be filled with either of the two digits 5 or 7, i.e., by 2 ways.

The remaining 3 places can be filled with any of the remaining 4 digits.

$$\text{Total number of 4-digit numbers greater than 5000} = 2 \times 4 \times 3 \times 2 = 48$$

Here, the number of 4-digit numbers starting with 5 and divisible by 5

$$= 1 \times 3 \times 2 \times 1 = 6$$

Here, the number of 4-digit numbers starting with 7 and divisible by 5

$$= 1 \times 2 \times 3 \times 2 = 12$$

Total number of 4-digit numbers greater than 5000 that are divisible by 5

$$= 6 + 12 = 18$$

Thus, the probability of forming a number divisible by 5 when the repetition of digits is not allowed

$$= P(\text{a number divisible by 5 when digits are not repeated})$$

$$P(\text{a number divisible by 5 when digits are not repeated}) = 18/48$$

$$= 3/8$$

10. The number lock of a suitcase has 4 wheels, each labelled with ten digits, i.e., from 0 to 9. The lock opens with a sequence of four digits with no repeats. What is the probability of a person getting the right sequence to open the suitcase?

Solution:-

From the question, it is given that

The number lock has 4 wheels, each labelled with ten digits, i.e., from 0 to 9.

Then,

The number of ways of selecting 4 different digits out of the 10 digits = ${}^{10}C_4$

Now, each combination of 4 different digits can be arranged in 4! Ways.

We have,

$$\text{Number of four digits with no repetitions} = 4! \times {}^{10}C_4 = 5040$$

There is only one number that can open the suitcase.

Hence, the required probability is $1/5040$