Time Series Analysis

Quarterly arrivals prediction from UK to Australia 1981-2011

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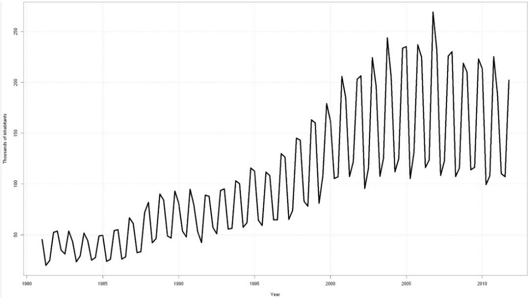
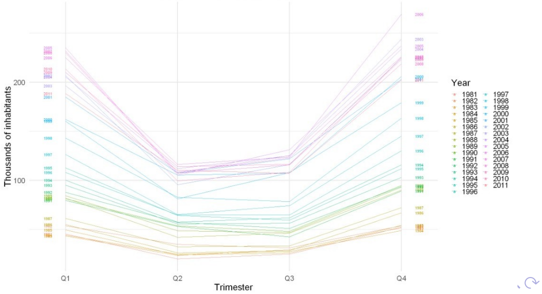
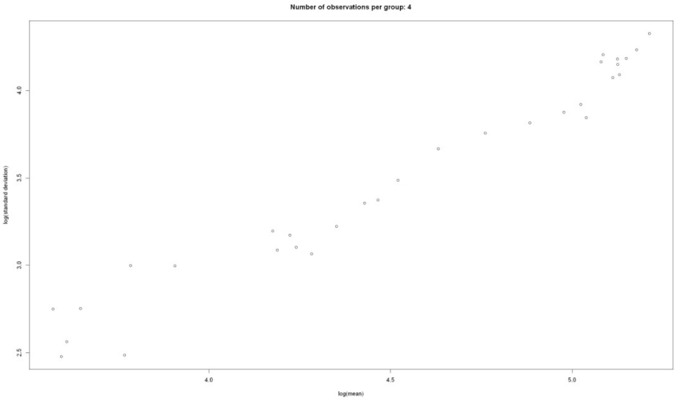
Dataset: 124 quarterly arrivals (in thousands) from the UK to Australia, 1981–2011.

- Let $\{X_t\}$ denote our time series.
- The series shows a trend and increasing variance.
- Due to the data distribution, seasonality with period $s = 4$ is observed.





Variance transformations

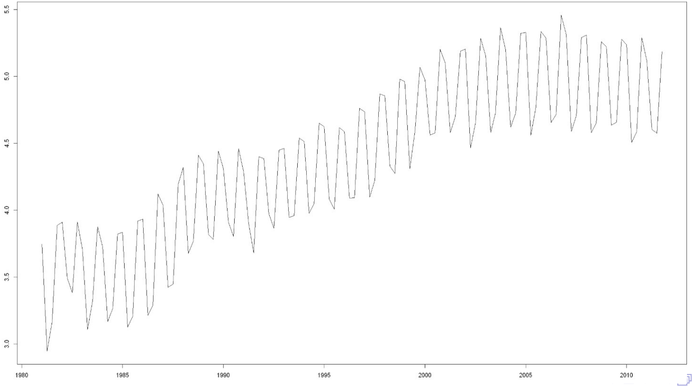


Firstly, we analyze the log-log relationship between mean and standard deviation.

In our case, the points resemble a line with slope 1

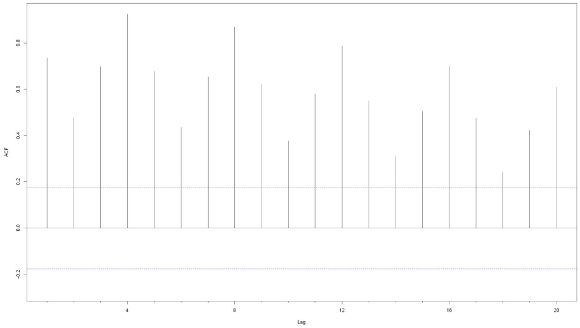
Box-Cox Transformation

The Box-Cox Transformation, with $\widehat{\lambda} = -0,01$. We use logaritmic trasnformation, aiming to remove heterocedasticity. Let $\tilde{X}_t = log(X_t)$.



Mean stabilizing transformations

Now, we would like to remove the trend component.



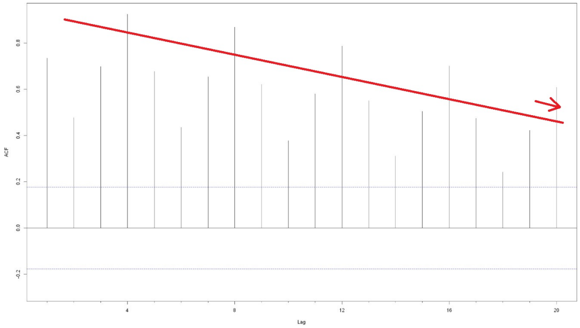
Logarithmic series ACF

Mean stabilizing transformations

Now, we would like to remove the trend component.

In our case, linear decay $\Rightarrow$ apply the **regular difference operator** $\nabla = (1 - B)$

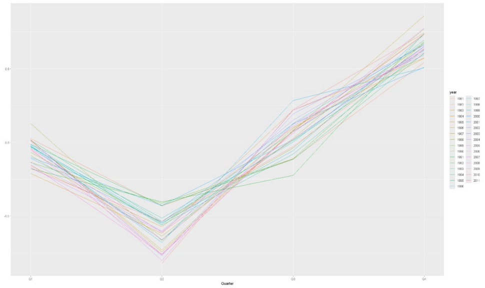
Let $W_t = \nabla \tilde{X}_t = \tilde{X}_t - \tilde{X}_{t-1} =$
$= \log(X_t) - \log(X_{t-1})$



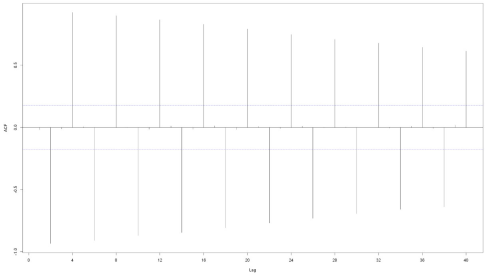
Logarithmic series ACF

Seasonality removal

The seasonal plot and the ACF suggest the presence of a seasonal component of period $s = 4 \Rightarrow$ We apply the **seasonal difference operator** $\nabla_4 = (1 - B^4)$
Let $\tilde{W}_t = \nabla_4 W_t = W_t - W_{t-4}$

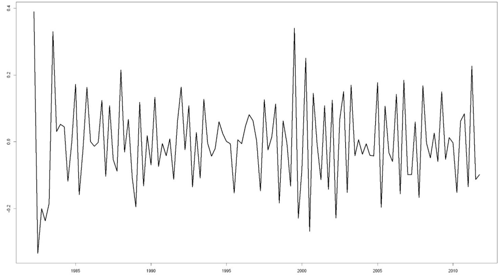


Logaritmic seasonal plot

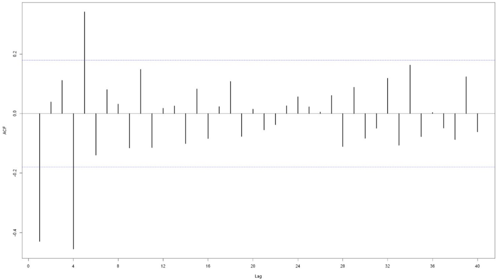


ACF of W_t

Seasonality removal



$\tilde{W}_t$ series plot

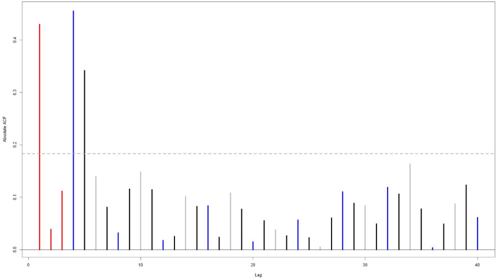


ACF of $\tilde{W}_t$

No deviations from stationarity shown in the Figures. We can asume then that $\{\tilde{W}_t\}$ is stationary, so differencing is not needed anymore

Identification: ACF

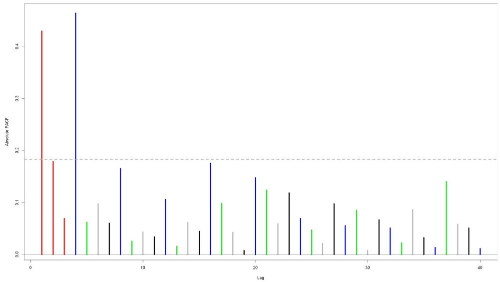
The ACF and PACF help identify significant regular, seasonal, and interaction components in the series.



Absolute value of the ACF of $\{\tilde{W}_t\}$, coloured by lags

- Regular part: one main significant value.
- Seasonal part: significant at lag 1, sinusoidal pattern.
- Only the first interaction is relevant.

Identification: PACF



Absolute value of the PACF of $\{\tilde{W}_t\}$, coloured by lags

- Regular part: one significant PACF coefficient; the second is nearly significant, suggesting exponential decay.
- Seasonal part: one significant coefficient, second nearly significant; also indicates exponential decay.
- Interactions provide no additional information.

Model selection: Suitable models

Based on the properties mentioned above we considered 25 models. But only the following models will be studied:

- M_3: ARIMA$(0,1,1) \times (0,1,1)_4$
- M_{12}: ARIMA$(3,1,0) \times (2,1,0)_4$
- M_{15}: ARIMA$(0,1,1) \times (2,1,0)_4$
- M_{24}: ARIMA$(2,1,2) \times (0,1,1)_4$

Model selection: Metrics

	AIC	AICc	BIC	MSE (90)	MAE (90)	MSE (100)	MAE (100)
Model 3	894.19	894.40	902.53	136.46	9.69	131.92	9.45
Model 12	888.94	889.69	905.61	157.66	9.85	147.94	9.25
Model 15	891.87	892.22	902.99	147.24	9.57	138.70	9.16
Model 24	896.37	897.12	913.05	140.94	9.82	132.77	9.23

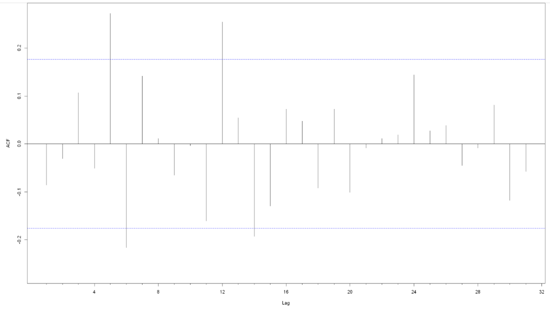
Model selection and prediction metrics for candidate models.

Model selection: Metrics

Aquí, hay que quitar los porcentajes para que se vea la tablita de colores

Model 3

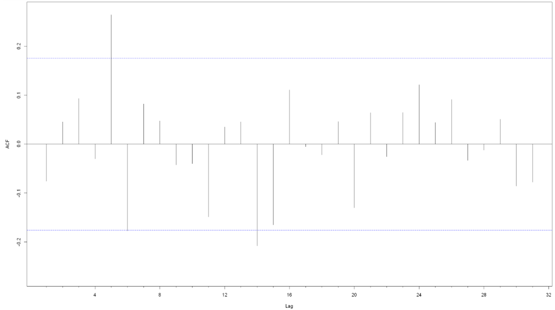
Test	Result	
% Out of Bounds	12.90 %	✗
Turning Point Test	0.25	✓
Difference Sign Test	0.09	✓
Rank Test	0.18	✓
Ljung Test	0.0007	✗
Shapiro Wilks Test	0.0822	✓
Lilliefors Test	0.1401	✓
Pearson Chi-Squared	0.25	✓
$\mu = 0$ Test	0.7877	✓



ACF of Model 3 residuals

Model 12

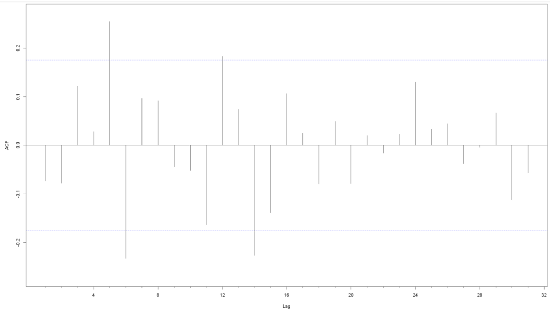
Test	Result	
% Out of Bounds	9.68 %	✗
Turning Point Test	0.89	✓
Difference Sign Test	0.64	✓
Rank Test	0.29	✓
Ljung Test	0.0002	✗
Shapiro Wilks Test	0.1858	✓
Lilliefors Test	0.0684	✓
Pearson Chi-Squared	0.11	✓
$\mu = 0$ Test	0.7758	✓



ACF of Model 12 residuals

Model 15

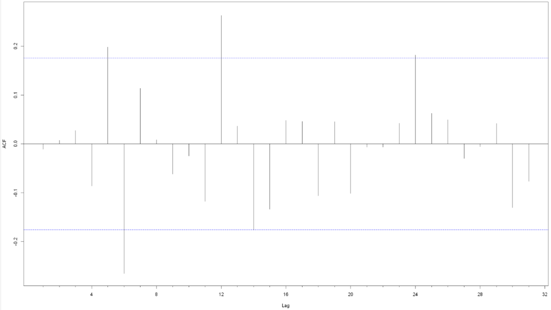
Test	Result	
% Out of Bounds	12.90 %	✗
Turning Point Test	0.47	✓
Difference Sign Test	0.28	✓
Rank Test	0.23	✓
Ljung Test	0.0002	✗
Shapiro Wilks Test	0.1241	✓
Lilliefors Test	0.1305	✓
Pearson Chi-Squared	0.22	✓
$\mu = 0$ Test	0.7800	✓



ACF of Model 15 residuals

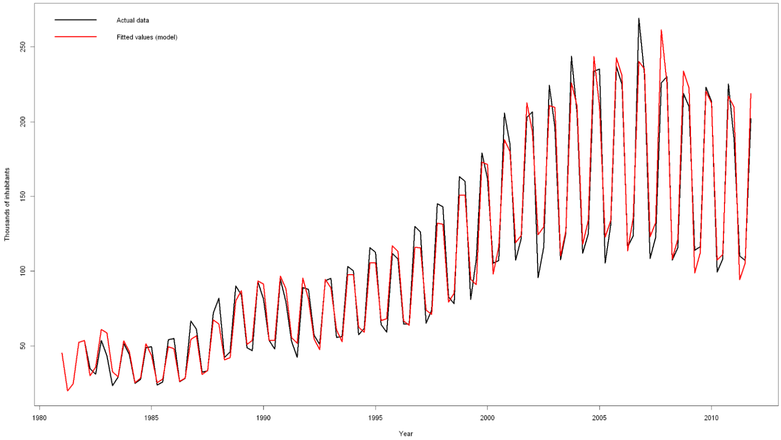
Model 24

Test	Result	
% Out of Bounds	16.13 %	✗
Turning Point Test	0.25	✓
Difference Sign Test	0.88	✓
Rank Test	0.36	✓
Ljung Test	0.0002	✗
Shapiro Wilks Test	0.0138	✗
Lilliefors Test	0.2398	✓
Pearson Chi-Squared	0.18	✓
$\mu = 0$ Test	0.7842	✓



ACF of Model 24 residuals

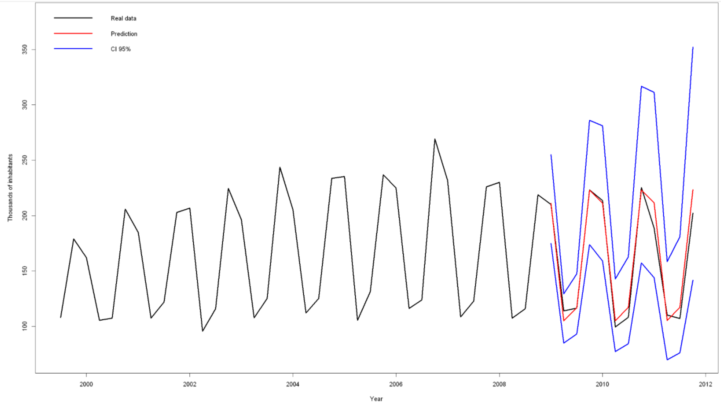
Model Fitting



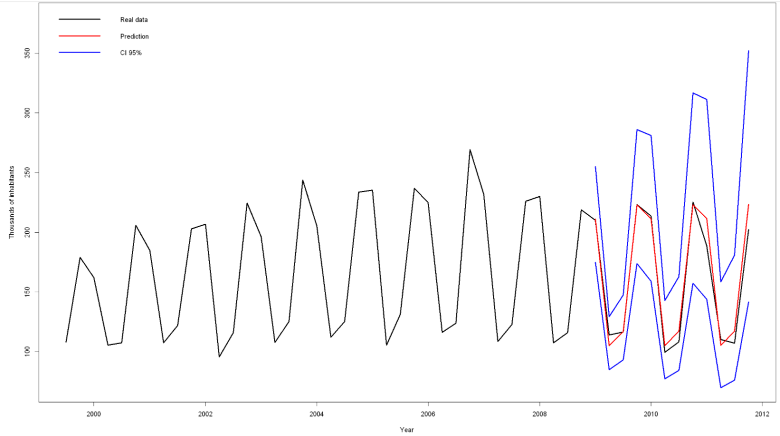
- Fitting done using Model 3
- Predictions good overall, do not fit perfectly but they are consistent throughout the data

Model Prediction

- Training data: first 112 quarters
- Prediction: last 12 quarters
- Predictions in 95 % CI $\Rightarrow$ We assume that the model predicts well



Model Forecasting I



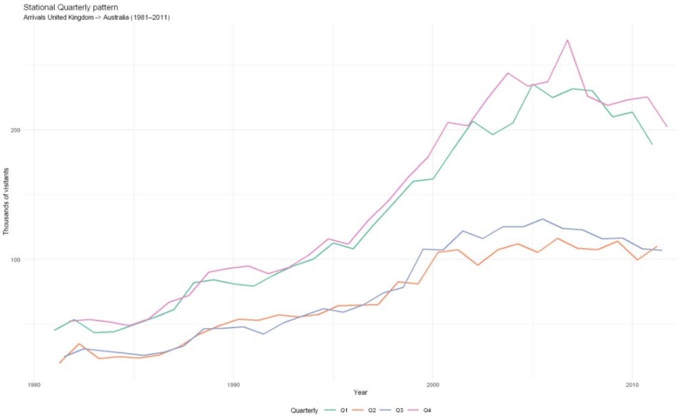
- We would like to forecast the next 10 trimesters
- Also, 95 % and 80 % CI can be observed in the figure

Model Forecasting II

Falta la tabla

Backup I

Clearly, the seasonality is $s = 4$



Backup II: Suitable models (1/2)

- M_1: ARIMA$(1,1,0)\times(0,1,1)_4$
- M_2: ARIMA$(1,1,0)\times(0,1,2)_4$
- M_3: ARIMA$(0,1,1)\times(0,1,1)_4$
- M_4: ARIMA$(0,1,1)\times(1,1,0)_4$
- M_5: ARIMA$(1,1,0)\times(1,1,0)_4$
- M_6: ARIMA$(1,1,1)\times(0,1,1)_4$
- M_7: ARIMA$(1,1,1)\times(1,1,0)_4$
- M_8: ARIMA$(2,1,0)\times(0,1,1)_4$
- M_9: ARIMA$(2,1,0)\times(1,1,0)_4$
- M_{10}: ARIMA$(2,1,0)\times(0,1,2)_4$
- M_{11}: ARIMA$(0,1,2)\times(0,1,2)_4$
- M_{12}: ARIMA$(3,1,0)\times(2,1,0)_4$
- M_{13}: ARIMA$(2,1,1)\times(0,1,1)_4$

Backup II: Suitable models (2/2)

- M_{14}: ARIMA$(1,1,0) \times (1,1,1)_4$
- M_{15}: ARIMA$(0,1,1) \times (2,1,0)_4$
- M_{16}: ARIMA$(1,1,1) \times (1,1,1)_4$
- M_{17}: ARIMA$(2,1,1) \times (0,1,1)_4$
- M_{18}: ARIMA$(0,1,2) \times (1,1,1)_4$
- M_{19}: ARIMA$(1,1,2) \times (0,1,1)_4$
- M_{20}: ARIMA$(1,1,0) \times (1,1,1)_4$
- M_{21}: ARIMA$(2,1,0) \times (1,1,1)_4$
- M_{22}: ARIMA$(0,1,1) \times (1,1,1)_4$
- M_{23}: ARIMA$(1,1,2) \times (1,1,0)_4$
- M_{24}: ARIMA$(2,1,2) \times (0,1,1)_4$
- M_{25}: ARIMA$(0,1,3) \times (0,1,1)_4$